Nuclear multifragmentation as a branching cascade

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(Received 4 October 2000; published 14 February 2001)

Analytic branching cascade is considered as a model of nuclear multifragmentation and corresponding stretched lognormal distribution of fragments size is compared with experimental data obtained in nuclear emulsion. Branching dimension of the cascade corresponding to these data is found to be equal to 3. It is also shown that the same branching dimension can be extracted from the data on the branching cascades in turbulence, while for multiparticle production at high energy heavy ion collisions the branching dimension turned out to be equal to 2.

DOI: 10.1103/PhysRevC.63.037602 PACS number(s): 25.70.Pq, 24.10.−i

In the nuclear collisions at lower bombarding energies (~1 GeV/nucleon) particle production is strongly suppressed and fragmentation processes dominate. An analogy with critical phenomena was the first and very productive approach to theoretical description of nuclear multifragmentation. In Ref. [1] the best data on nuclear multifragmentation (the breakup of 1.04 GeV gold nuclei incident on a carbon tagest) was examined from this point of view, and it was shown that corresponding values of the critical exponents are close to those for liquid-gas systems and clearly different than those for three dimensional (3D) percolation. This description implies utilization of the *lower* order moments, while another approach widely used at investigations of the intermittency phenomenon in multiparticle production at high energy heavy ion collisions, is related to *higher* order moments (see [2] for a recent review). Since the authors of Ref. [1] did not use their excellent data to explore the intermittency phenomenon in higher order moments we should compare our theoretical approach with more earlier data represented in Ref. [3], where the intermittency phenomenon at nuclear multifragmentation was studied just for the *higher* order moments.

Cascades are natural models for the fragmentation processes and it is shown in [4] that there is a deep connection between cascades and critical processes. The cascade processes, however, imply lognormal distribution of the fragments size, which is generally not observed in the real nuclear multifragmentation processes. It is shown in Ref. [5] that the Central Limit Theorem (which results in the lognormal distribution for the ordinary cascade models) applied to fractals leads to a generalization of the normal distribution: *stretched* normal distribution and, consequently, to the stretched lognormal distribution for the branching cascades on fractals (at least for the analytic case of the integer branching dimensions considered in the present report). Namely, the stretched normal distribution for integer values of the branching dimension has following form:

$$P(x) \sim \exp-\left(\frac{(x-x_c)^{2d}}{2\sigma^2}\right), \qquad (1)$$

where σ and x_c are some parameters and d is the so-called branching dimension [5]. Corresponding stretched lognormal distribution is

$$P(m) \sim \frac{1}{m}\exp-\left(\frac{(\ln m-\ln m_c)^{2d}}{2\sigma^2}\right). \qquad (2)$$

Since we are interested in the analytic case here (i.e., in the integer values of the branching dimensions $d=1, 2, 3,...$) we should not use absolute value in the exponent.

To make our approach more reliable we have compared the results obtained at lower bombarding energies (nuclear multifragmentation) with those obtained at higher bombarding energies (multiparticle production) and have found an interesting analogy: both of them can be described by the stretched lognormal distribution, although with different values of the branching dimension d ($d=3$ for nuclear multifragmentation at lower bombarding energies and $d=2$ for multiparticle production at heavy ion collisions at higher energies).

Two methods are used to compare theoretical predictions for probability distribution with experimental data. They are the direct comparison and the moments method. Therefore, let us calculate moments corresponding to the stretched lognormal distribution Eq. (2).

The moments corresponding to the stretched lognormal distribution can be estimated as

$$\langle m^p\rangle \sim \int_0^\infty m^p \exp-\frac{(\ln m-\ln m_c)^{2d}}{2\sigma^2}\left(\frac{1}{m}dm\right). \qquad (3)$$

Let us introduce a new variable

$$x=\ln m-\ln m_c. \qquad (4)$$

Then

$$\langle m^p\rangle \sim e^{ap}\int_0^\infty \exp\left(px-\frac{x^{2d}}{2\sigma^2}\right)dx. \qquad (5)$$

Let us denote

$$f(x)=px-\frac{x^{2d}}{2\sigma^2} \qquad (6)$$

and let us find the value of x, where this function has its maximum using the following equation:

$$f'(x)=p-\frac{d}{\sigma^2}x^{2d-1}=0. \tag{7}$$

Equation (7) has a solution

$$x_0=\left(\frac{\sigma^2 p}{d}\right)^{\alpha}, \tag{8}$$

where

$$\alpha=\frac{1}{2d-1} \tag{9}$$

(let us recall that we consider only positive integer values of d here).

The nth derivative of $f(x)$ at point x_0 is

$$f^{(n)}(x_0)=-\frac{d(2d-1)\dots(2d-n+1)}{\sigma^2}x_0^{2d-n}. \tag{10}$$

Then Taylor series for function $f(x)$ at point x_0 can be written as follows:

$$f(x)=f(x_0)-\frac{(\sigma^2 p)^{2d\alpha}}{\sigma^2}g\left(\frac{x-x_0}{x_0}\right), \tag{11}$$

where

$$g\left(\frac{x-x_0}{x_0}\right)=\sum_{n=2}^{\infty}\frac{d^{-\alpha}}{n!}(2d-1)\dots(2d-n+1)\left(\frac{x-x_0}{x_0}\right)^n. \tag{12}$$

The moments can now be estimated as

$$\langle m^p\rangle \sim e^{ap+f(x_0)}\int_{-\infty}^{\infty}\exp[-hg((x-x_0)/x_0)]dx, \tag{13}$$

where

$$h=\frac{1}{\sigma^2}(\sigma^2 p)^{2d\alpha}. \tag{14}$$

Let us introduce a new variable

$$y=\frac{x-x_0}{x_0}. \tag{15}$$

Then we can rewrite the representation (13) as

$$\langle m^p\rangle \sim x_0 e^{ap+f(x_0)}\int_{-\infty}^{\infty}\exp[-hg(y)]dy. \tag{16}$$

Now the function

$$g(y)=\sum_{n=2}^{\infty}\frac{d^{-\alpha}}{n!}(2d-1)\dots(2d-n+1)y^n \tag{17}$$

is independent of σ and on p, i.e., this function is independent of parameter h. For large h, i.e., for

$$h \gg 1, \tag{18}$$

the integral on the right-hand side of Eq. (16) is dominated by $\min_y\{g(y)\}$, i.e.,

$$\langle m^p\rangle \sim x_0 e^{ap+f(x_0)}e^{-h\,\min_y\{g(y)\}}. \tag{19}$$

The saddle-point method used above has exponential convergence and, for example, for $\sigma^2=3$, $p=2$, and $d=3$ the errors of the calculations can be estimated to be about 3% (also see below). Since for $2d-1>0$ the real function $f(x)$ has generally one maximum at point x_0 only, this is the absolute maximum of this function and, therefore, $g(y)\geqslant 0$. Hence, $\min_y\{g(y)\}=0$ and we obtain from Eq. (19)

$$\langle m^p\rangle \sim x_0 e^{ap+f(x_0)}. \tag{20}$$

It is easy to show using Eqs. (14) and (18) that for

$$\sigma^2>\left(\frac{2d}{(2d-1)^2 e}\right)^{2d} \tag{21}$$

we can neglect by the multiplier x_0 in comparison with the exponent on the right-hand side of representation (20) and we obtain from Eq. (20)

$$\langle m^p\rangle \sim \exp[ap+p(1-1/2d)(\sigma^2 p/d)^{\alpha}]. \tag{22}$$

For $d=3$, for instance, condition (21) gives restriction $\sigma^2>4.7\times 10^{-7}$, i.e., the restriction imposed by the condition (21) is very weak.

Using Eq. (22) we then obtain a generalized scaling relationship based on the stretched lognormal distribution

$$\frac{\langle m^q\rangle}{\langle m^z\rangle^{q/z}} \sim \left(\frac{\langle m^p\rangle}{\langle m^z\rangle^{p/z}}\right)^{q(q^{\alpha}-z^{\alpha})/p(p^{\alpha}-z^{\alpha})}. \tag{23}$$

Since analytic branching cascades the branching dimension $d=1,2,3,\dots$, then the corresponding values of the exponent $\alpha=1,1/3,1/5,\dots$.

If s is the charge of the nuclear fragments, then for a particular partition of the region of interest Δs in M bins of the size $\delta s=\Delta s/M$ the multiplicity moments can exhibit self-similar properties (scaling) [3]

$$\langle m^q\rangle \sim \left(\frac{\Delta s}{\delta s}\right)^{\zeta_q}. \tag{24}$$

Substituting Eq. (24) into Eq. (23) we obtain a functional equation for ζ_q, which for a particular case $z=1$ has the following form:

$$\frac{\zeta_q/q-\zeta_1}{\zeta_q/p-\zeta_1}=\frac{q^{\alpha}-1}{p^{\alpha}-1}. \tag{25}$$

The general solution of this equation is

$$\frac{\zeta_p}{p}=(\zeta_1-a)+ap^{\alpha}, \tag{26}$$

where a is some constant.

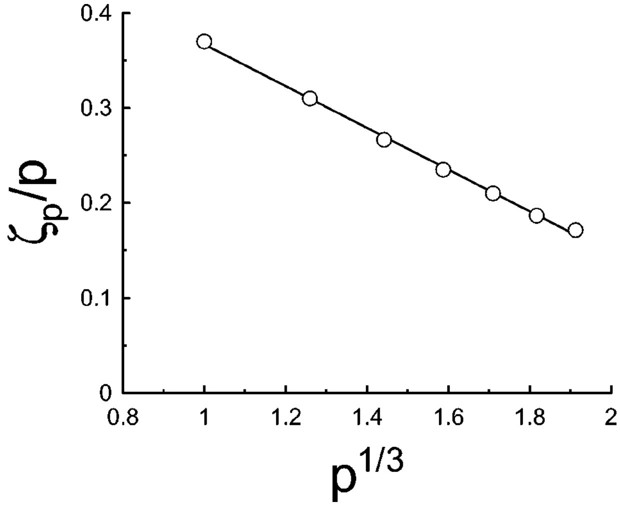

FIG. 1. ζ_p/p vs $p^{1/5}$ for the turbulent mixing data obtained in the atmosphere [7]. The straight line (the best fit) indicates agreement between the data and Eq. (26) with $\alpha=1/5$ (that corresponds to the analytic branching cascade with branching dimension $d=3$).

It should be noted that the branching cascades also take place at some other physical processes. Cascade decay of the admixture spots in turbulent fluid [5,6] is the most well known. In this case m is the absolute value of the admixture concentration increment

$$m=|c(x+r)-c(x)|, \tag{27}$$

where r is the distance between two space points under consideration. The scaling for the concentration increment is

$$\langle m^p\rangle \sim r^{\zeta_p}. \tag{28}$$

If the scaling is formed as a result of a branching cascade then one can use representation (26). Figure 1 shows corresponding data obtained in turbulent atmosphere [7]. The axes in this figure are chosen for comparison with the representation (26) with $\alpha=1/5$ [which correspond to the analytic branching cascade with $d=3$; see Eq. (9)]. The straight line (the best fit) is drawn to indicate agreement between the data and this representation.

In Ref. [3] the scaled factorial moments were used to analyze experimental data on the fragment mass distributions produced by interactions in nuclear emulsion, in which ${}^{197}_{79}\mathrm{Au}_{118}$ nuclei of energy $E=1$ GeV/nucleon break up into fragments. Definition of the scaled factorial moments is

$$\langle F_q\rangle = M^{q-1}\left\langle \sum_{p=1}^{M} \frac{n_p(n_p-1)\cdots(n_p-q+1)}{\langle N\rangle^q}\right\rangle, \tag{29}$$

where $\langle N\rangle$ is the mean fragments multiplicity in the interval Δs (s is the charge of the fragments), with a particular partition of the region of interest Δs in M bins of size $\delta s = \Delta s/M$, n_p is the number of fragments in the pth bin, and the brackets $\langle ...\rangle$ denote the average over many events (the authors of [3] also used some smoothing operation). It is shown in [3] that these (smoothed) factorial moments exhibit scaling

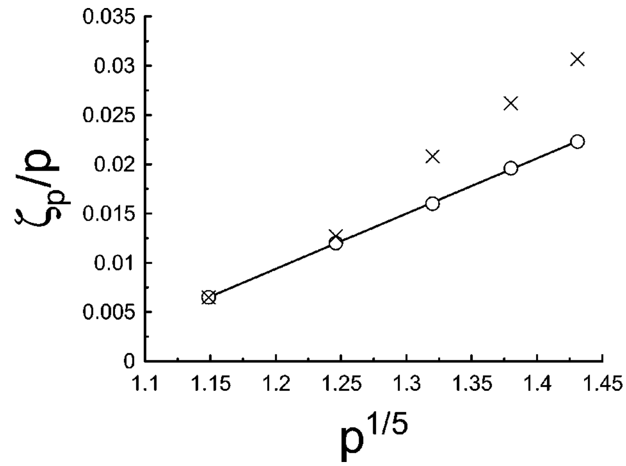

FIG. 2. ζ_p/p vs $p^{1/5}$ for the data [3] (circles) obtained in nuclear emulsion in which ${}^{197}_{79}\mathrm{Au}_{18}$ nuclei of energy $E=1$ GeV/nucleon break up into fragments. The straight line (the best fit) indicates agreement between the data and Eq. (31) with $\alpha=1/5$ (which corresponds to the analytic branching cascade with branching dimension $d=3$). Crosses correspond to analogous calculations performed for a bond percolation model [3].

$$\langle F_p\rangle \sim (\Delta s/\delta s)^{\zeta_p}, \tag{30}$$

where ζ_p is some function on p. Analogously to the previous sections it can be shown that in the case of the stretched lognormal distribution

$$\zeta_p \sim p(p^{\alpha}-1), \tag{31}$$

where α is given by the same equation (9).

Figure 2 shows the data [3] (circles) obtained in nuclear emulsion in which ${}^{197}_{79}\mathrm{Au}_{18}$ nuclei of energy E

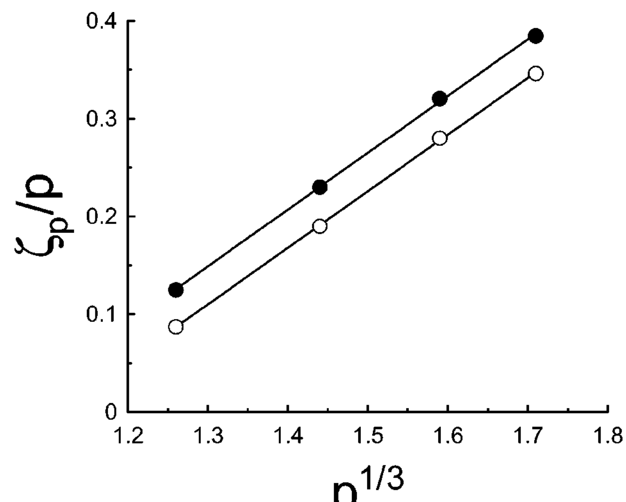

FIG. 3. ζ_p/p vs $p^{1/3}$ for the data [10] obtained at ultrarelativistic heavy ion collisions (for secondary particles produced in ^{16}O-AgBr interactions at 60 GeV/nucleon). Filled circles correspond to the pseudorapidity and open circles correspond to the azimuthal phase spaces. The straight lines (the best fit) indicate agreement between the data and Eq. (26) with $\alpha=1/3$ (which corresponds to the analytic branching cascade with branching dimension $d=2$).

$=1$ GeV/nucleon break up into fragments. The axes in this figure are chosen for comparison with the representation (31) with $\alpha=1/5$ (that corresponds to the analytic branching cascade with $d=3$, as for the turbulent branching cascade considered previously). The straight line (the best fit) is drawn to indicate agreement between the data and the model with the branching dimension $d=3$. The universality of the branching dimension $d=3$ is significant in light of the discussion on the finite size effects in the nuclear reactions (see, for instance [8,9] and references therein).

In Ref. [3] the intermittency exponent ζ_p was also calculated for a bond percolation model, and in Fig. 2 we show these data (crosses) calculated with randomly distributed bond parameter. One can see that ζ_p for the percolation model is close to ζ_p obtained for the nuclear multifragmentation for small values of p only (see also [1]).

It is also interesting to compare the results obtained for nuclear multifragmentation at low bombarding energies with the intermittency data on multiparticle production obtained at ultrarelativistic heavy ion collisions. In the last case nuclear breakup into fragments is suppressed and the multiparticle production dominates. In a recent paper [10] the intermittency phenomenon for the distribution of secondary particles produced in ^{16}O-AgBr interactions at 60 GeV/nucleon was studied using the scaled factorial moments. In Fig. 3 we show the data represented in Ref. [10] for the pseudorapidity (filled circles) and for the azimuthal (open circles) phase spaces in the case of largest bin regions. As in the previous figures the axes are chosen for comparison with the stretched lognormal distribution (26) (straight lines, the best fit, are drawn to indicate this correspondence). However, now the horizontal axis is scaled as $p^{1/3}$, which corresponds to the parameter $\alpha=1/3$ (or to the ranching dimension $d=2$).

The author is grateful to T. Nakano for discussions, to the Machanaim Center (Jerusalem), and to the Graduate School of Science and Engineering of the Chuo University (Tokyo) for support.

[1] V. L. Gilkes *et al.*, Phys. Rev. Lett. **73**, 1590 (1994).
[2] E. A. De Wolf, I. M. Dremin, and W. Kittel, Phys. Rep. **270**, 1 (1996).
[3] M. Ploszajczak and A. Tucholski, Phys. Rev. Lett. **65**, 1539 (1990).
[4] A. Bershadskii, J. Phys. A **27**, L191 (1994).
[5] A. Bershadskii, Europhys. Lett. **39**, 587 (1997).
[6] A. C. Monin and A. M. Yaglom, *Statistical Fluid Mechanics 2* (MIT Press, Cambridge, 1975).
[7] F. Schmitt, D. Schertzer, S. Lovejoy, and Y. Brunet, Europhys. Lett. **34**, 195 (1996).
[8] F. Gulminelli and Ph. Chomaz, in *XXXVI International Winter Meeting on Nuclear Physics*, Bormio, 1998, edited by I. Iori [Ric. Sci. **112**, 349 (1998)].
[9] B. Elattaari, J. Richert, and P. Wager, J. Phys. G **24**, 601 (1998).
[10] D. Ghosh *et al.*, Europhys. Lett. **41**, 371 (1998).