Nuclear pairing in the $T=0$ channel reexamined

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Recent published data on the isoscalar gap in symmetric nuclear matter using the Paris force and the corresponding BHF single particle dispersion are corrected, leading to an extremely high proton-neutron gap of $\Delta\sim8$ MeV at $\rho\sim0.5\rho_0$. Arguments of whether this value can be reduced due to screening effects are discussed. A density dependent delta interaction with cutoff is adjusted so as to approximately reproduce the nuclear matter values with the Paris force.

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In a recent publication [1], the possibility to reproduce the gap in nuclear matter, as obtained, e.g., from the Paris *NN* force, by an effective density dependent zero range force was investigated. Supplied with an energy cutoff, such effective forces indeed turned out to be able to reproduce very reasonably the gap values in the isospin $T=0$ and $T=1$ channels over the whole relevant range of densities. The adjustments were performed on previously published solutions of the gap equation using Brückner-Hartree-Fock results for the single particle spectra [2]. Such effective forces may possess some analogies, with similar ones frequently used in recent structure calculation of superfluid nuclei [3]. Unfortunately, due to the subtleties connected with the numerical solution of the gap equation, the published results in the $T=0$ channel were not accurate enough so that the corresponding gap is underestimated in Refs. [1,2] by about 20%. It is the purpose of this note to give the corrected results for the gap in the $T=0$ channel and also to readjust the corresponding density-dependent δ-force. We also discuss again the issue of whether screening affects the $T=1$ and $T=0$ channels differently.

In Fig. 1 we show the correct result for the isoscalar gap as obtained with the Paris force [4] using two independent numerical codes. We also checked that the Argonne V14 force [5] gives practically the same result. What is striking is the giant gap value of ~8 MeV at maximum, which is of the same order as the Fermi energy at the corresponding density. Even around saturation, Δ is still of the order of several MeV. This is clearly a strong coupling situation, as expected from the fact that at low density the n-p Cooper pair turns into the deuteron wave function [2]. The above values are actually much more compatible with earlier calculations of the critical temperature in Ref. [6] than the previous results [2]. Indeed, considering the usual relation $\Delta=1.76\ T_c$ [7], quantitative agreement between the results of Ref. [6] and the ones in Fig. 1 is obtained. In order to obtain an estimate of the typical magnitude of the isoscalar gap in a finite nucleus, we apply the local density approximation and average the local gap over the density at the Fermi energy. This procedure has given reliable estimates of the average energy dependent gap in the isovector channel [8]. We therefore calculate

$$\Delta=\frac{\int drr^2\Delta(k_F(r))k_F(r)}{\int drr^2k_F(r)}, \tag{1}$$

where the local Fermi momentum is defined as

$$k_F(r)=\sqrt{(\mu-V(r))2m/\hbar^2}, \tag{2}$$

with μ the chemical potential. We take the same single particle potential $V(r)$ as in Ref. [8] and the result for, e.g., $N=Z=35$ is that Δ is of the order of 3 MeV. Compared to the neutron-neutron and proton-proton channels this is a very high value.

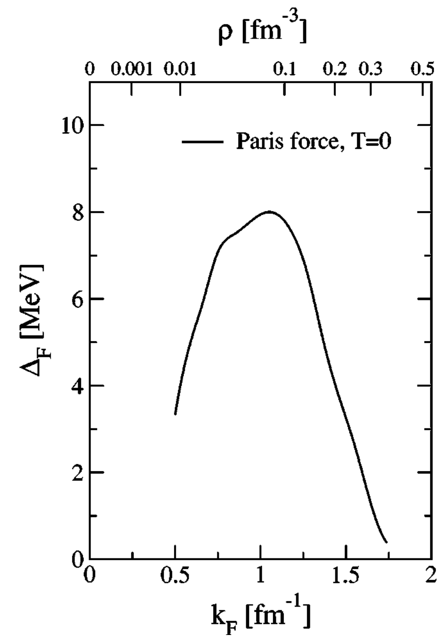

FIG. 1. Pairing gap vs Fermi momentum for symmetric nuclear matter in the $T=0$ channel from the Paris potential.

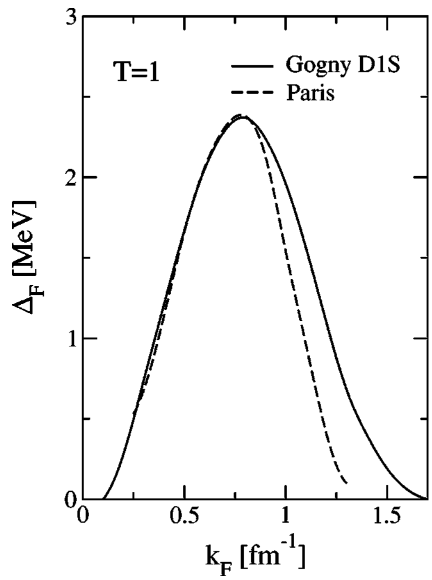

FIG. 2. Pairing gap Δ_F in the $^1S_0$ channel in symmetric nuclear matter calculated with the Gogny force D1S compared with results from the Paris force.

We already discussed in Ref. [1] and show again in Fig. 2 that the use of the Paris force in conjunction with the *k*-mass, m^*/m, yields gap values as a function of density that are globally very similar to the ones of the Gogny force for $T=1$, and therefore, the use of a bare force seems not unreasonable in the $T=1$ channel. The fact that Δ for $T=1$ drops off quite a bit faster close to saturation for the Paris force than for the Gogny D1S force may be attenuated in a finite nucleus to quite some extent, since a certain averaging over all densities $\rho<\rho_0$ takes place. Therefore, the needed medium renormalization of the bare force seems to be of minor importance in the $T=1$ channel.[1] However, the situation may not be the same for $T=0$ pairing. The extremely strong $T=0$ pairing stems essentially from the fact that with respect to the $T=1$ channel the tensor force is acting additionally. Without the tensor force np $(T=0)$ and nn $(T=1)$ pairing would be of comparable magnitude. The screening of the tensor force in the medium is, however, still a controversial subject [9]. On the other hand, even for very low densities where screening should not be so important, $T=0$ pairing remains strong. Therefore, there may be a good chance that the new heavier exotic nuclei with $N=Z$ experience quite pronounced np superfluidity. This may well be the cause for the so-called Wigner energy of the nuclear mass formula, since it can be shown [10] that away from symmetric nuclei, $T=0$ pairing very quickly loses its strength.

Let us now proceed to the readjustment of the effective $T=0$ delta force. We use the standard ansatz [1,11]

[1]Of course, it cannot be excluded that the medium completely reshuffles the distribution of gap values, still reproducing experimental pairing phenomena in finite nuclei.

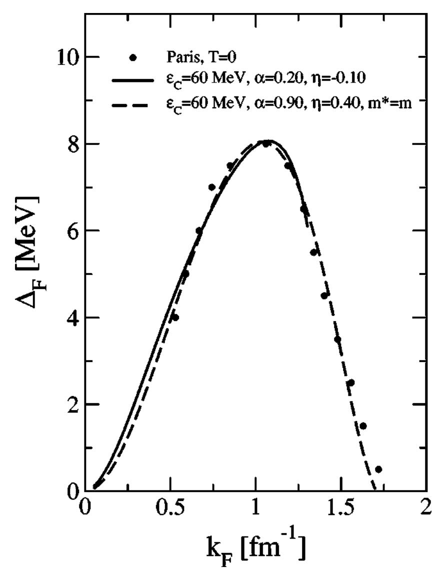

FIG. 3. $T=0$ pairing gap in nuclear matter. The dots are the results obtained for the Paris potential. The curves are fits with Eq. (3) using an energy cutoff $\epsilon_C=60$ MeV, $v_0=-480$ MeV fm^3, and different parameters for the fit with effective mass m^* (solid line, $\eta=-0.10$, $\alpha=0.20$) and for the fit with the bare mass (dashed line, $\eta=0.40$, $\alpha=0.90$).

$$v(\vec{r}_1,\vec{r}_2)=v_0\left\{1-\eta\left[\rho\left(\frac{r_1+r_2}{2}\right)\Big/\rho_0\right]^{\alpha}\right\}\delta(\vec{r}_1-\vec{r}_2)(1+P_\sigma)/2. \tag{3}$$

With the above density-dependent zero range force, the gap equation reads

$$1=-\frac{v_0}{\pi^2}[1-\eta(\rho/\rho_0)^{\alpha}]\times\left(\frac{m^*(\rho)}{2\hbar^2}\right)^{3/2}\int_0^{\epsilon_C}d\epsilon\sqrt{\frac{\epsilon}{(\epsilon-\epsilon_F)^2+\Delta^2}}. \tag{4}$$

In Fig. 3 we present two fits for the above ansatz, one of the fits is obtained from the following parameters: $\alpha=0.2$, $\eta=-0.10$ and a cutoff energy $\epsilon_C=60$ MeV (see Ref. [1]), using the effective mass m^*/m as obtained from the Gogny force. The other fit is obtained by using a bare mass and parameters $\alpha=0.90$, $\eta=0.40$, and $\epsilon_C=60$ MeV.

As one can see in Fig. 3, the fit obtained using the bare mass is able to reproduce the microscopic calculation up to the highest values of k_F ($k_F\sim1.7$ fm^{-1}), while the fit obtained using the effective mass breaks down at lower densities corresponding to $k_F\sim1.35$ fm^{-1}. The reason for this different behavior can be traced back to the dependence on the effective mass inside the integral of the gap equation. It turns out that in order to get a solution of the gap equation (4), the energy cutoff ϵ_C should be larger than the Fermi energy ϵ_F. Otherwise, no value of Δ satisfies the equation. In the case of the energy cutoff used in Figs. 3 and 4 ($\epsilon_C=60$ MeV), the largest k_F reachable is $k_F\sim1.7$ fm^{-1} when

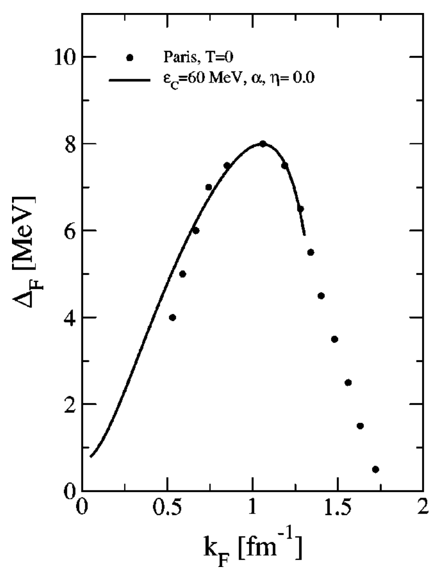

FIG. 4. Same as in Fig. 3 but suppressing the density dependence ($\eta=0$) and using $v_0=-530$ MeV fm^3.

bare masses are used, but only $k_F\sim 1.35$ when effective masses are used instead. Therefore, we plot in Figs. 3 and 4 the fits obtained only up to those values of k_F, when $m^*/m\neq 1$. Nevertheless, the fits cover all the physically relevant range of densities form zero to saturation ($\rho_0=0.16$ fm^{-3}).

In principle, in the $T=0$ channel, v_0 should be chosen such that the deuteron binding energy is reproduced in free space. However, we have found that with this condition the fit obtained is very poor. Therefore, for a given energy cut ϵ_C, we vary the parameter v_0 from the value that produces a bound state at zero energy $v_0=-(\hbar^2/m)(2\pi^2/\sqrt{2m\epsilon_C})$, up to the value that produces the bound state at the deuteron energy [11], and choose the best fit. The fits in Fig. 3 have been obtained with $v_0=-480$ MeV fm^3 as it corresponds to a bound state at zero energy. This reduces the value of the gap at low densities but improves significantly the fit at higher energies. On the other hand, as we shall see in Fig. 4, the value of v_0 is chosen between the two extreme values considered: bound state at zero energy and at the deuteron energy. In any case, the values used for v_0 are quoted in each case.

In Fig. 4 we present a similar fit for the case with $m^*/m\neq 1$, however, suppressing the density dependence completely, which is $\eta=0$. Since this parameter was already small for the case in Fig. 3, the fit is still acceptable and only a slight deterioration at the low density end is visible. Let us mention also that the use of the bare mass $m^*=m$ allows an excellent fit of the microscopically calculated gap values at all densities (see Fig. 3). However, realistic calculations of finite nuclei are rarely performed with the bare nucleon mass.

As a first guess we may try to use the effective pairing force obtained with the present fit also for finite nucleus calculations. This will give a rough account of whether the use of a bare force in a finite nucleus is at all reasonable in the $T=0$ channel. We would, however, like to point out that the expression of Eq. (3) for finite nuclei may not give precise reproduction of the results one would obtain with direct use of the Paris force in the gap equation. Indeed, in the meantime, we compared in the $T=1$ channel the results of the genuine Gogny force and its density dependent δ-force substitute elaborated in Ref. [1] in a half-infinite matter calculation [12]. Preliminary results show that the detailed surface dependence of the gap and the anomalous density seem to be quite different in both cases. However, integrated quantities, like the correlation energy, may still be rather similar.

Of course, it should be interesting for the future to derive also an effective finite range force in the $T=0$ channel that is as efficient as the Gogny force for $T=1$ pairing. In fact, the Gogny force has never been used for np pairing. However, since in this channel the density dependent zero range force enters, one has to introduce an additional cutoff that is an unknown adjustable parameter.

In summary, we give corrected values of the np ($T=0$) gap in nuclear matter using the Paris force together with Brückner-Hartree-Fock single-particle energies. An extremely high value of $\Delta\sim 8$ MeV at $\rho\sim 0.5\rho_0$ is obtained, leading to a gap value in finite nuclei of ~ 3 MeV. Arguments are advanced that the pairing force in the $T=0$ channel may be more strongly screened than in the $T=1$ channel. We then adjust a density-dependent δ-force to the nuclear matter gap values. The fit is reasonably successful for densities below saturation.

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