Magnetic moments of mirror nuclei

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(Received 6 November 2000; published 31 January 2001)

The linear correlations between *ground* state gyromagnetic ratios and superallowed β-decay transition strengths of mirror nuclei are reexamined in the light of more extensive experimental data. Predictions are made for the (as yet unmeasured) ground state magnetic moments of 11 nuclei in the mass range $45 \leqslant A \leqslant 59$, using shell model calculations to resolve sign ambiguities. The linear correlations are tentatively extended to superallowed decays between *excited* states of mirror nuclei.

DOI: 10.1103/PhysRevC.63.037301 PACS number(s): 23.40.Hc, 21.10.Ky, 23.40.Bw, 27.40.+z

Linear relations have previously been reported between the ground state gyromagnetic ratios and superallowed β-decay transition strengths of mirror nuclei in the mass range $3 \leqslant A \leqslant 43$ [1]. Based on data from 14 mirror pairs [2], the following linear relations with correlation coefficient magnitudes in excess of 0.997 were deduced:

$$\gamma_p = -(1.145 \pm 0.012)\gamma_n + (1.056 \pm 0.021),$$
$$\gamma_p = (4.38 \pm 0.10)\gamma_\beta + (0.939 \pm 0.046),$$
$$\gamma_n = -(3.82 \pm 0.10)\gamma_\beta + (0.101 \pm 0.043), \tag{1}$$

where γ_p and γ_n are the ground state gyromagnetic ratios (i.e., magnetic dipole moment divided by angular momentum, μ/J) of the odd-proton and odd-neutron members of a mirror pair, respectively, and γ_β is related to the *ft* value for the β transition by

$$|\gamma_\beta| = \frac{1}{2}\sqrt{\left(\frac{6170}{ft} - 1\right)\frac{1}{J(J+1)}}. \tag{2}$$

The sign of γ_β is obtained from systematics.

When the ground state magnetic moment of one member of a mirror pair has been measured, these relations allow the prediction of the other. Such predictions have previously been reported [3] for ^{23}Mg, ^{33}Cl, ^{37}Ar, ^{43}Ti, ^{51}Fe, and ^{55}Ni. Since then, measurements in good agreement with these expectations have been made on the first four of these nuclei [4–7]. These experimental results mean that magnetic moments are now known for all mirror pairs in the mass range $11 \leqslant A \leqslant 43$. With the inclusion of the $A=3$ values, this allows the linear correlations to be reexamined with a larger database of 18 mirror pairs, rather than the 14 used previously.

The ground state magnetic moment of one member of the pair and/or the *ft* value of the superallowed β decay have been measured for all mirror nuclei in the mass range $45 \leqslant A \leqslant 59$ [8,9]. Thus, predictions may be made for the 11 unmeasured ground state magnetic moments of the associated mirror pairs in this mass range. This is done first. Table I lists all the experimentally known values of the ground state spins, magnetic moments, and the β-decay log *ft* values for mirror nuclei in the mass range $3 \leqslant A \leqslant 59$. To ease the application of the linear correlations under discussion, Table II lists gyromagnetic ratios and values of γ_β for these same nuclei. We then speculate on the possible extension of the linear correlations to superallowed β decays between the excited states of mirror nuclei. We conclude with a summary of our results.

On repeating the analysis of Ref. [1] with the data listed in Table II for the 18 mirror nuclei with mass numbers $A=3$ and $11 \leqslant A \leqslant 43$ we deduce the linear relations

$$\gamma_p = -(1.1485 \pm 0.0104)\gamma_n + (1.0515 \pm 0.0158),$$
$$\gamma_p = (4.3736 \pm 0.0903)\gamma_\beta + (0.9140 \pm 0.0370),$$
$$\gamma_n = -(3.8044 \pm 0.0830)\gamma_\beta + (0.1192 \pm 0.0340), \tag{3}$$

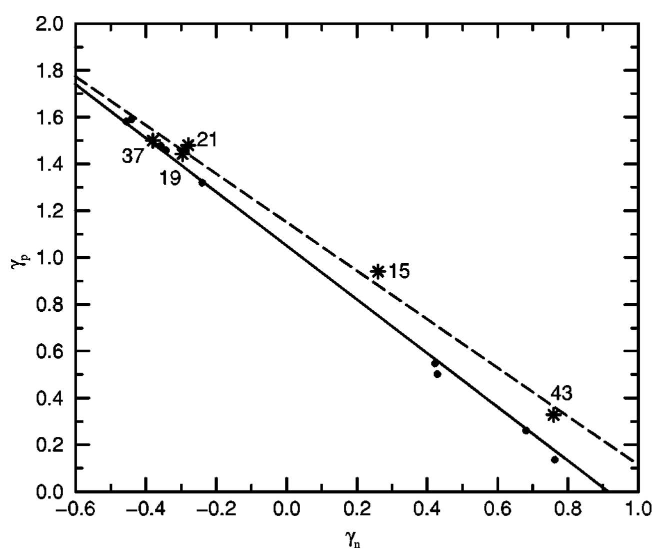

FIG. 1. Plot of gyromagnetic ratios γ_p vs γ_n for ground states (solid circles, data from Table II) and excited states (asterisks, data from Table IV, labeled by mass number) of mirror nuclei. The solid line represents the linear correlation of Eq. (3) for the ground states, and the dashed line that of Eq. (4) for the excited states.

TABLE I. Experimental ground state magnetic dipole moments, spins, and log ft values for mirror nuclei in the mass range $3 \leqslant A \leqslant 59$.

Odd p	μ_p	Odd n	μ_n	J^π	log ft
p	$+2.79284739(7)$	n	$-1.91304275(45)$	$1/2^+$	
^{3}H	$+2.97896248(7)$	^{3}He	$-2.12762485(7)$	$1/2^+$	3.058(1)
^{5}Li	—	^{5}He	—	$3/2^-$	—
^{7}Li	$+3.2564268(17)$	^{7}Be	—	$3/2^-$	3.32
^{9}B	—	^{9}Be	$-1.1778(9)$	$3/2^-$	—
^{11}B	$+2.6886489(10)$	^{11}C	$-0.964(1)$	$3/2^-$	3.599(2)
^{13}N	$-0.3222(4)$	^{13}C	$+0.7024118(14)$	$1/2^-$	3.667(1)
^{15}N	$-0.28318884(5)$	^{15}O	$+0.71951(12)$	$1/2^-$	3.637
^{17}F	$+4.72130(20)$	^{17}O	$-1.89379(9)$	$5/2^+$	3.358(2)
^{19}F	$+2.628868(8)$	^{19}Ne	$-1.88542(8)$	$1/2^+$	3.237(2)
^{21}Na	$+2.38630(10)$	^{21}Ne	$-0.661797(5)$	$3/2^+$	3.61(1)
^{23}Na	$+2.217520(2)$	^{23}Mg	$-0.5364(3)$	$3/2^+$	3.67(2)
^{25}Al	$+3.6455(12)$	^{25}Mg	$-0.85545(8)$	$5/2^+$	3.57(1)
^{27}Al	$+3.6415069(7)$	^{27}Si	$-0.8554(4)$	$5/2^+$	3.61(2)
^{29}P	$+1.2349(3)$	^{29}Si	$-0.55529(2)$	$1/2^+$	3.686(8)
^{31}P	$+1.13160(3)$	^{31}S	$-0.48793(8)$	$1/2^+$	3.682(7)
^{33}Cl	$+0.7523(16)$	^{33}S	$+0.6438212(14)$	$3/2^+$	3.755(7)
^{35}Cl	$+0.8218743(2)$	^{35}Ar	$+0.633(2)$	$3/2^+$	3.761(6)
^{37}K	$+0.20321(6)$	^{37}Ar	$+1.145(2)$	$3/2^+$	3.66(1)
^{39}K	$+0.3914662(3)$	^{39}Ca	$+1.02168(12)$	$3/2^+$	3.632(3)
^{41}Sc	$+5.535(4)$	^{41}Ca	$-1.594781(9)$	$7/2^-$	3.461(7)
^{43}Sc	$+4.62(4)$	^{43}Ti	$-0.85(2)$	$7/2^-$	3.561(6)
^{45}V	—	^{45}Ti [a]	0.095(2)	$7/2^-$	3.636(17)
^{47}V	—	^{47}Cr	—	$3/2^-$	3.705(11)
^{49}Mn	—	^{49}Cr [a]	0.476(3)	$5/2^-$	3.671(24)
^{51}Mn	$+3.5683(13)$	^{51}Fe	—	$5/2^-$	3.655(10)
^{53}Co	—	^{53}Fe	—	$7/2^-$	3.634(42)
^{55}Co	$+4.822(3)$	^{55}Ni	—	$7/2^-$	3.634(11)
^{57}Cu	—	^{57}Ni	$-0.7975(14)$	$3/2^-$	—
^{59}Cu	—	^{59}Zn	—	$3/2^-$	3.69(2)

[a]Signs for $\mu_n(^{45}\mathrm{Ti})$ and $\mu_n(^{49}\mathrm{Cr})$ are experimentally undetermined, but shell model calculations suggest positive and negative, respectively.

with correlation coefficients -0.9994, 0.9968, and -0.9965, respectively. These linear relations are clearly compatible with the previous results, but have somewhat smaller uncertainties in the values of slopes and intercepts (as is to be expected for an increased database).

Knowledge of any one of γ_n, γ_p, or γ_β allows a prediction from Eqs. (3) of the other two quantities (with uncertainties combined in the standard way). However, if two of the γ's are known, these equations yield two separate values for the third. The best estimate of this third γ is obtained by a weighted mean of the two separate values deduced from Eqs. (3). Furthermore, as pointed out in Ref. [3], if γ_β is known, together with either γ_n or γ_p, a smaller error is generated by considering the equation for the isoscalar quantity $\gamma_n+\gamma_p$ as a function of γ_β than by using the equations for γ_n and γ_p as functions of γ_β individually.

Table III shows predictions based on this reanalysis for 11 ground state magnetic dipole moments of mirror nuclei in the mass range $45 \leqslant A \leqslant 59$. For five of these nuclei a value for the magnetic moment of its mirror partner has been measured [8]. However, the other six predictions, for masses $A = 47$, 53, and 59, are based on the experimental ft value alone [10,11], which yields only the magnitude of γ_β [see Eq. (2)]. For these latter six cases, and also for the predictions of ^{45}V and ^{49}Mn where only the magnitudes of the mirror partner dipole moments are known, there are ambiguities of sign arising from our use of the linear correlations. For the most part, these sign ambiguities lead to such widely differing predictions that even a small basis shell model calculation can decide between them [12]. For example, the predicted magnetic moments for ^{47}V and ^{47}Cr are $(2.16 \pm 0.08)\mu_N$ and $(-0.51 \pm 0.07)\mu_N$, respectively for positive γ_β, but $(0.58 \pm 0.08)\mu_N$ and $(0.87 \pm 0.07)\mu_N$, respectively, for negative γ_β. Shell model calculations suggest $2.14\mu_N$ and $-0.47\mu_N$, respectively [13], clearly favoring the positive γ_β predictions. In fact the shell model basis used in that calculation was rather large, but in general even a few active particles in a small configuration space are sufficient to resolve our ambiguities. This is true for all the other cases listed in Table III except for ^{45}V. In that example the mea-

TABLE II. Experimental ground state gyromagnetic ratios and values of γ_β for mirror nuclei in the mass range $3 \leqslant A \leqslant 59$.

Odd p	γ_p	Odd n	γ_n	J^π	γ_β
p	+5.5857(-)	n	−3.8261(−)	$1/2^+$	
^{3}H	+5.9579(-)	^{3}He	−4.2552(−)	$1/2^+$	+1.211(2)
^{5}Li	—	^{5}He	—	$3/2^-$	—
^{7}Li	+2.1710(-)	^{7}Be	—	$3/2^-$	+0.361(6)
^{9}B	—	^{9}Be	−0.7582(6)	$3/2^-$	—
^{11}B	+1.7924(-)	^{11}C	−0.6427(7)	$3/2^-$	+0.192(1)
^{13}N	−0.6444(8)	^{13}C	+1.4048(-)	$1/2^-$	−0.331(1)
^{15}N	−0.5664(−)	^{15}O	+1.4390(2)	$1/2^-$	−0.376(2)
^{17}F	+1.8885(1)	^{17}O	−0.7575(−)	$5/2^+$	+0.221(1)
^{19}F	+5.2577(-)	^{19}Ne	−3.7708(2)	$1/2^+$	+0.926(3)
^{21}Na	+1.5909(1)	^{21}Ne	−0.4412(−)	$3/2^+$	+0.185(6)
^{23}Na	+1.4783(-)	^{23}Mg	−0.3576(2)	$3/2^+$	+0.146(14)
^{25}Al	+1.4582(5)	^{25}Mg	−0.3422(−)	$5/2^+$	+0.137(4)
^{27}Al	+1.4566(-)	^{27}Si	−0.3422(2)	$5/2^+$	+0.121(8)
^{29}P	+2.4698(4)	^{29}Si	−1.1106(−)	$1/2^+$	+0.301(13)
^{31}P	+2.2632(6)	^{31}S	−0.9759(2)	$1/2^+$	+0.307(11)
^{33}Cl	+0.5015(10)	^{33}S	+0.4292(-)	$3/2^+$	−0.075(8)
^{35}Cl	+0.5479(-)	^{35}Ar	+0.4220(13)	$3/2^+$	−0.068(7)
^{37}K	+0.1355(-)	^{37}Ar	+0.7633(13)	$3/2^+$	−0.153(7)
^{39}K	+0.2610(-)	^{39}Ca	+0.6811(1)	$3/2^+$	−0.171(2)
^{41}Sc	+1.5814(11)	^{41}Ca	−0.4557(−)	$7/2^-$	+0.134(2)
^{43}Sc	+1.3200(114)	^{43}Ti	−0.24(1)	$7/2^-$	+0.105(2)
^{45}V	—	^{45}Ti	0.027(-)	$7/2^-$	+0.082(5)
^{47}V	—	^{47}Cr	—	$3/2^-$	+0.120(8)
^{49}Mn	—	^{49}Cr	0.1904(12)	$5/2^-$	+0.095(11)
^{51}Mn	+1.4273(5)	^{51}Fe	—	$5/2^-$	+0.102(4)
^{53}Co	—	^{53}Fe	—	$7/2^-$	+0.083(14)
^{55}Co	+1.3777(9)	^{55}Ni	—	$7/2^-$	+0.083(4)
^{57}Cu	—	^{57}Ni	−0.532(−)	$3/2^-$	—
^{59}Cu	—	^{59}Zn	—	$3/2^-$	+0.132(15)

sured value of γ_n for the mirror partner ^{45}Ti is 0.027, with undetermined sign. Although shell model calculations suggest a positive sign, favoring $\mu(^{45}\mathrm{V}) = (3.83 \pm 0.04)\mu_N$, the value of γ_n is so close to zero that we present results for both sign possibilities.

TABLE III. Predicted ground state magnetic dipole moments.

Nucleus	μ (nm)
^{45}V	3.63±0.04 or 3.83±0.04
^{47}V	2.16±0.08
^{47}Cr	−0.51±0.07
^{49}Mn	3.18±0.03
^{51}Fe	−0.83±0.03
^{53}Co	4.47±0.24
^{53}Fe	−0.69±0.21
^{55}Ni	−1.02±0.04
^{57}Cu	2.49±0.03
^{59}Cu	2.24±0.11
^{59}Zn	−0.57±0.10

The justification for the linear correlations between magnetic moments given in Ref. [1] requires that the contributions from the even number particles of the nucleus be very small (ideally zero). While this is expected to hold for ground states of mirror nuclei, there is no obvious reason for it to be valid for excited states. Nevertheless, we have examined the available data for gyromagnetic ratios (magnetic moments) of pairs of excited mirror states [8], and found the values listed in Table IV. Figure 1 shows that all five of them lie very close to the $\gamma_p - \gamma_n$ straight line deduced from the ground state data. In fact, an explicit fit to the five pairs of moments leads to the linear correlation

$$\gamma_p = -1.037\gamma_n + 1.1516. \tag{4}$$

with a somewhat different slope and intercept from the ground state relation. Unfortunately, this line is not determined accurately enough to allow reasonable predictions of other excited states (in the same, or other, nuclei).

That the excited state data should lie at all close to the ground state line, or indeed fall on a straight line of their own, is frankly a surprise. It would be interesting to know if

TABLE IV. Experimental gyromagnetic ratios for excited states of mirror nuclei.

Odd p (E^*[MeV])	γ_p	Odd n (E^*[MeV])	γ_n	J^π
^{15}N (5.270)	+0.94(7)	^{15}O (5.241)	+0.26(3)	$5/2^+$
^{19}F (0.197)	+1.4428(32)	^{19}Ne (0.238)	−0.296(3)	$5/2^+$
^{21}Na (0.332)	+1.48(10)	^{21}Ne (0.351)	−0.28(3)	$5/2^+$
^{37}K (1.379)	+1.5(1)	^{37}Ar (1.611)	−0.38(1)	$7/2^-$
^{43}Sc (3.123)	+0.3286(7)	^{43}Ti (3.066)	+0.760(1)	$19/2^-$

linear relations between excited state gyromagnetic ratios hold more widely and we strongly urge experimentalists to make measurements to shed more light on the matter.

We have reexamined the data on superallowed beta decays between the ground states of mirror nuclei. There are now measurements on all such nuclei in the mass range $11 \leqslant A \leqslant 43$, which together with $A=3$ means that the analysis of Ref. [1] may be repeated with a database enlarged from 14 pairs to 18. Compatible linear relations between γ_p, γ_n, and γ_β are obtained, with reduced uncertainties in slopes and intercepts. We use these results, aided by shell model considerations to eliminate sign ambiguities, to predict the ground state magnetic moments of 11 mirror nuclei (Table III). We also note that these linear relations are quite closely obeyed by five pairs of excited states in mirror nuclei (Table IV and Fig. 1). We speculate that this may be a general feature of low-lying excitations in mirror nuclei.

We would like to thank Prof. Alfredo Poves for very helpful correspondence concerning his shell model calculations. In addition, A.C.M. would like to thank the U.K. Engineering and Physical Science Research Council (EPSRC) for financial support. S.M.P. would like to thank the S.A. Foundation for Research and the University of Cape Town for financial support.

[1] B. Buck and S. M. Perez, Phys. Rev. Lett. **50**, 1975 (1983).
[2] S. Raman, C. A. Houser, T. A. Walkiewicz, and I. S. Towner, At. Data Nucl. Data Tables **21**, 567 (1978).
[3] S. M. Perez, Phys. Rev. C **36**, 1202 (1987).
[4] M. Fukuda, T. Izumikawa, T. Ohtsubo, M. Tanigaki, S. Fukuda, Y. Nakayama, K. Matsuta, Y. Nojiri, and T. Minamisono, Phys. Lett. B **307**, 278 (1993).
[5] W. F. Rogers, D. L. Clark, S. B. Dutta, and A. G. Martin, Phys. Lett. B **177**, 293 (1986).
[6] J. Vanhaverbeke, N. Severijns, W. Vanderpoorten, D. Vandeplassche, J. Wouters, L. Vanneste, and K. Schlösser, Hyperfine Interact. **43**, 373 (1988).
[7] K. Matsuta, A. Ozawa, Y. Nojiri, T. Minamisono, M. Fukuda, T. Ohtsubo, S. Fukuda, K. Sugimoto, I. Tanihata, K. Yoshida, K. Omata, J. R. Alonso, G. F. Krebs, and T. J. M. Symons, Hyperfine Interact. **78**, 123 (1993).
[8] P. Raghavan, At. Data Nucl. Data Tables **42**, 189 (1989).
[9] N. J. Stone (private communication).
[10] T. W. Burrows, J. W. Olness, and D. E. Alburger, Phys. Rev. C **31**, 1490 (1985).
[11] C. M. Baglin, Nucl. Data Sheets **69**, 733 (1993).
[12] A. Poves (private communication).
[13] G. Martínez-Pinedo, A. P. Zuker, A. Poves, and E. Caurier, Phys. Rev. C **55**, 187 (1997).