Baryon spectrum in the chiral constituent quark model

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The low-lying baryon spectrum predicted by a chiral constituent quark model is examined within an exact Faddeev approach. The reliability of the solutions provided by a truncated hyperspherical harmonic calculation is discussed. We demonstrate that the obtained spectrum is quite reasonable and compatible with the description of the *NN* phenomenology in the presence of a standard one-gluon exchange force.

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The complexity of quantum chromodynamics (QCD), the quantum field theory of the strong interaction, has so far prevented a rigorous deduction of its predictions even for the simplest hadronic systems. Therefore, one has to rely on QCD-inspired models to get some insight into many of the phenomena of the hadronic world. One of the central issues that such models should be able to address is a quantitative description of baryons, still one of the major challenges in hadronic physics.

The very success of QCD-inspired quark models supports the picture that has emerged from more fundamental studies: namely, that below a certain distance-scale QCD is a weakly coupled theory with asymptotically free quark and gluon degrees of freedom; but above this scale a strong coupling regime emerges in which color is confined and chiral symmetry is broken. These two aspects, confinement and chiral symmetry breaking, are now recognized as basic ingredients in any QCD-inspired model for the low-energy (and therefore nonperturbative) sector. Along this line, the simplest approach is undoubtedly the successful constituent quark model, where multigluon degrees of freedom are eliminated in favor of confined constituent quarks with effective masses coming from chiral symmetry breaking and quark-quark effective interactions [1]. Concerning confinement, although little is known about the mechanism which confines the quarks inside hadrons, the study of heavy meson spectra and lattice calculation results suggested a linear form for the short-range part of the confining interaction [2]. Finally, apart from the previous interactions, a lot of evidence has been accumulated about the important role played by a color-spin force with respect to the low-energy hadronic phenomenology [3]. This interaction is obtained from the one-gluon exchange derived long ago by de Rújula *et al.* [4].

With these basic ingredients several quark models have been proposed in the literature [5]. Among them, the quark model of Ref. [6] has tried to undertake a simultaneous description of the baryon-baryon interaction [6–8] and the baryon spectrum [9,10]. Recently, the performance of quark models, including the one-gluon exchange to describe the baryon spectrum, has been questioned [11], adducing the results to the failure of the mathematical method used. It is the purpose of this paper to clarify the situation by performing an exact Faddeev calculation of the baryon spectrum of Ref. [9], analyzing in detail the results obtained to demonstrate if models including the usual one-gluon exchange force are able to produce reasonable predictions for the baryon spectrum.

The model of Ref. [9] includes a linear-rising confining potential, a one-gluon exchange (OGE) term simulating the perturbative effects of QCD and a pseudoscalar and scalar Goldstone-boson exchange at the level of quarks. The form of the OGE interaction is taken from Ref. [4]. To explicitly derive the form of the chiral potential, one proceeds to a nonrelativistic reduction of an effective Hamiltonian that mimics QCD in the regime between the scale of chiral symmetry breaking and the confinement scale. The emerging image is that of constituent quarks interacting through the one-gluon exchange and the Goldstone modes associated to the spontaneous breaking of the chiral symmetry [1]. Explicitly, the form of the potential is given by [6]

$$V_{qq}(\vec{r}_{ij})=V_{\rm CON}(\vec{r}_{ij})+V_{\rm OGE}(\vec{r}_{ij})+V_{\rm OPE}(\vec{r}_{ij})+V_{\rm OSE}(\vec{r}_{ij}), \tag{1}$$

where $\vec{r}_{ij}$ is the interquark distance ($\vec{r}_{ij}=\vec{r}_i-\vec{r}_j$). $V_{\rm CON}$ is the confining potential taken to be linear,

$$V_{\rm CON}(\vec{r}_{ij})=-a_c\vec{\lambda}_i\cdot\vec{\lambda}_j\, r_{ij}\,. \tag{2}$$

$V_{\rm OGE}$ is the one-gluon exchange potential with a smeared (of r_0 range) δ function term in order to avoid an unbound spectrum [12],

$$V_{\rm OGE}(\vec{r}_{ij})=\frac{1}{4}\,\alpha_s\vec{\lambda}_i\cdot\vec{\lambda}_j\left\{\frac{1}{r_{ij}}-\frac{1}{4m_q^2}\left[1+\frac{2}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j\right]\frac{e^{-r_{ij}/r_0}}{r_0^2\,r_{ij}}-\frac{1}{4m_q^2r_{ij}^3}S_{ij}\right\}, \tag{3}$$

where α_s is the effective quark-quark-gluon coupling constant and the $\lambda' s$ are the SU(3) color matrices. The $\sigma' s$ stand for the spin Pauli matrices and S_{ij} is the quark tensor operator $S_{ij}=3(\vec{\sigma}_i\,.\hat{r}_{ij})(\vec{\sigma}_j.\hat{r}_{ij})-\vec{\sigma}_i\,.\vec{\sigma}_j$.

$V_{\rm OPE}$ and $V_{\rm OSE}$ are the pseudoscalar and scalar Goldstone-boson exchange potentials, given, respectively, by

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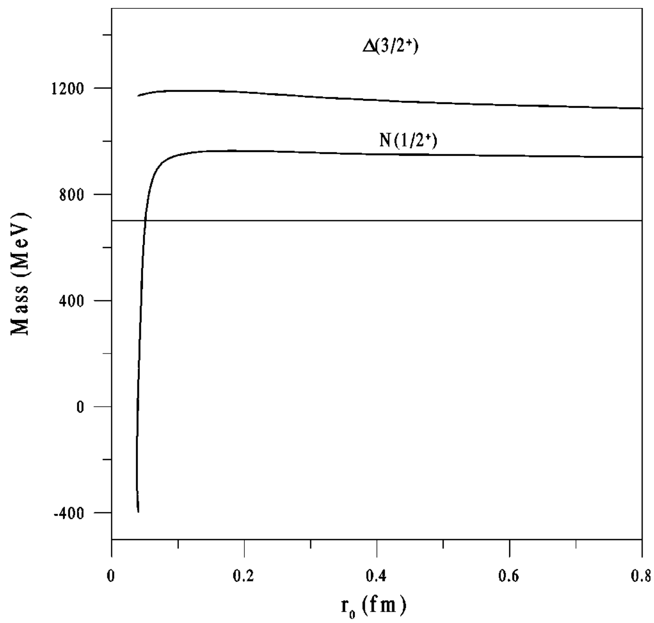

FIG. 1. $N(1/2^+)$ and $\Delta(3/2^+)$ ground state mass as a function of the reguralization parameter r_0.

$$V_{\mathrm{OPE}}(\vec{r}_{ij})=\frac{1}{3}\alpha_{ch}\frac{\Lambda_\pi^2}{\Lambda_\pi^2-m_\pi^2}m_\pi \times\left\{\left[Y(m_\pi r_{ij})-\frac{\Lambda_\pi^3}{m_\pi^3}Y(\Lambda_\pi r_{ij})\right]\vec{\sigma}_i\cdot\vec{\sigma}_j +\left[H(m_\pi r_{ij})-\frac{\Lambda_\pi^3}{m_\pi^3}H(\Lambda_\pi r_{ij})\right]S_{ij}\right\}\vec{\tau}_i\cdot\vec{\tau}_j, \tag{4}$$

$$V_{\mathrm{OSE}}(\vec{r}_{ij})=-\alpha_{ch}\frac{4m_q^2}{m_\pi^2}\frac{\Lambda_\sigma^2}{\Lambda_\sigma^2-m_\sigma^2}m_\sigma \times\left[Y(m_\sigma r_{ij})-\frac{\Lambda_\sigma}{m_\sigma}Y(\Lambda_\sigma r_{ij})\right], \tag{5}$$

TABLE I. Quark model parameters for the calculation of Fig. 3.

m_q(MeV)	313
α_s	0.72
a_c(MeV$\cdot$fm^{-1})	72.518
α_{ch}	0.0269
r_0(fm)	0.2
m_σ(fm^{-1})	3.42
m_π(fm^{-1})	0.7
Λ(fm^{-1})	4.2
Λ_σ(fm^{-1})	4.2

where m_π (m_σ) is the pion (sigma) mass; α_{ch} is the chiral coupling constant related to the πNN coupling constant through $\alpha_{ch}=(\frac{3}{5})^2(g_{\pi NN}^2/4\pi)(m_\pi^2/4m_N^2)$; Λ_π and Λ_σ are cutoff parameters; and $Y(x),H(x)$ are the Yukawa functions defined as

$$Y(x)=\frac{e^{-x}}{x},\quad H(x)=\left(1+\frac{3}{x}+\frac{3}{x^2}\right)Y(x). \tag{6}$$

As mentioned above, this model has been applied to two-baryon systems, giving reasonable results for the deuteron observables, the NN phase shifts, and the main features of the $N-\Delta$ and $\Delta-\Delta$ interactions [6–8].

As already mentioned, its application to the baryon spectra should be done with care, because the contact interaction (Dirac delta) of the OGE potential has to be regularized. When regularization of the delta interaction is done by means of a spreading Yukawa function, reasonable baryon spectra have been reported using a hyperspherical harmonic approach [9]. It is important to note that the regularization depends on the model space in which the calculation is done and the parameter of this regularization (r_0) should not be understood as a true parameter of the model Hamiltonian. The regularization scheme chosen in Ref. [9] consisted of two steps: (i) the model space was truncated leaving only up to $k=2$ excitations in the hyperspherical harmonic approach, and (ii) the δ was regularized by demanding that the matrix

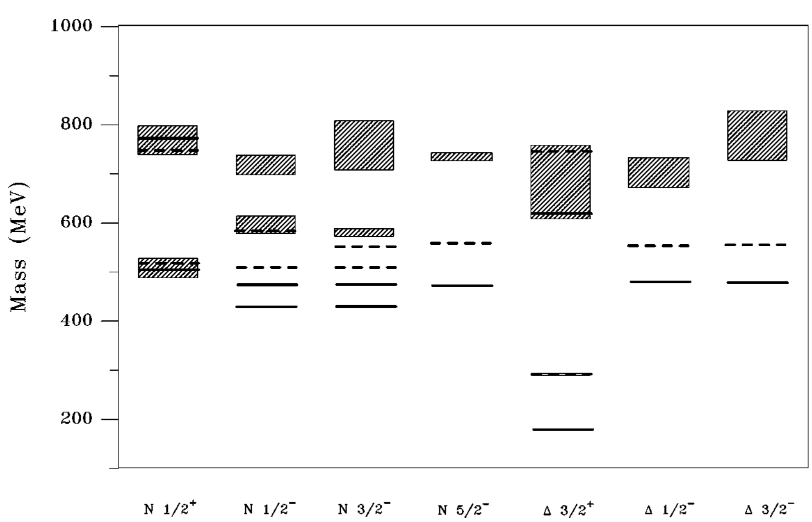

FIG. 2. Relative energy nucleon and delta spectrum up to 1.0 GeV excitation energy. The dashed line corresponds to the predictions of the original model in Ref. [9]. The solid line corresponds to the results of the present calculation within a Faddeev scheme.

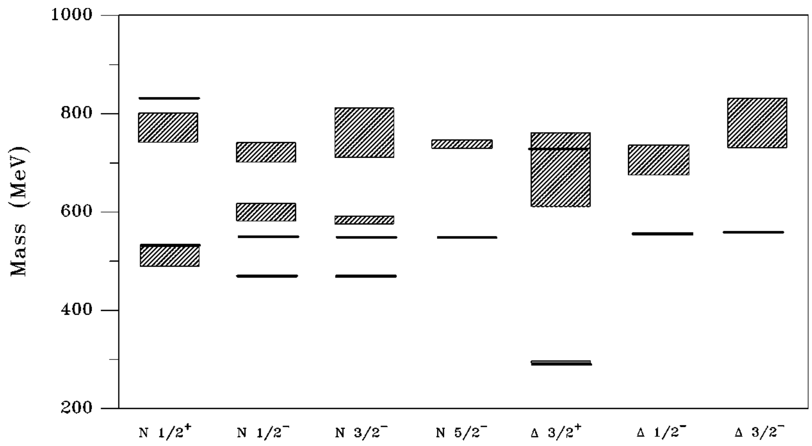

FIG. 3. Same as solid line of Fig. 2 for the set of parameters of Table I.

elements of the regularized form between harmonic oscillator wave functions are the same as the matrix elements of the exact δ. This prescription was adopted as a criterion to avoid the freedom associated to the existence of a new parameter: r_0. It cannot be fixed on the NN sector, where the δ is used directly, because the calculation is done perturbatively using harmonic oscillator wave functions. The results of Ref. [9] were reanalyzed in Ref. [11], using the same potential and the same regularization but a different model space in a Faddeev-Noyes approach. The results obtained were strikingly different, not only for the model of Ref. [9] but also for other models including the one-gluon exchange force [13]. From this analysis, the authors of Ref. [11] concluded that interactions including an OGE force, like the one in Ref. [9], are not appropriate.

Although there has already been some discussion in Refs. [14,15] with respect to the different working schemes, we would like to definitively clarify the performance of the model of Ref. [9] in order to describe the baryon spectrum. For this purpose, we have proceeded to recalculate the spectrum making use of an exact Faddeev calculation. The method has been tested by reproducing existing results in the literature [16], and therefore we do not consider it deserves a wider discussion. Thus let us directly present and analyze the results we have obtained. First, we show in Fig. 1 the dependence of the $N(1/2^+)$ and the $\Delta(3/2^+)$ ground state energy on the regularization parameter r_0. As can be seen, for values of r_0 below 0.1 fm, the $N(1/2^+)$ ground state energy decreases very quickly while the $\Delta(3/2^+)$ remains almost stable. As a consequence, the excitation energy of the whole spectrum becomes too high, of the order of 1 GeV. This result can be easily understood, taking into account that the short-ranged spin-spin force is attractive for the nucleon but repulsive for the Δ, and therefore it lowers the $L=0$ nucleon states. In the case of the truncated hyperspherical harmonic calculation, the increase on the strength of the spin-spin force through a smaller value of r_0 translates into a stronger mixing with higher-order components ($k=4,6$). In Ref. [9] the expansion was cut in $k=2$ (therefore, this mixing did not occur), and the results were stable for the Hilbert space used. However, at the light of Fig. 1, it is obvious that the Faddeev calculation shows that a larger basis would have been necessary in order to fully account for the effect of the regularization of the δ function present in the one-gluon exchange with the small parameter used, $r_0=0.0367$ fm.

It is also worth noticing another aspect of Fig. 1. The nucleon ground state energy almost does not change for values of r_0 greater than 0.1 fm. One should bear in mind that in the calculation of Ref. [9], the parameter r_0 could have been taken as free, and therefore any value would have been acceptable. In particular, without modifying any of the parameters of Ref. [9] but taking a bigger value for r_0 (e.g., $r_0=0.8$ fm), the spectrum obtained is presented in Fig. 2. Al-

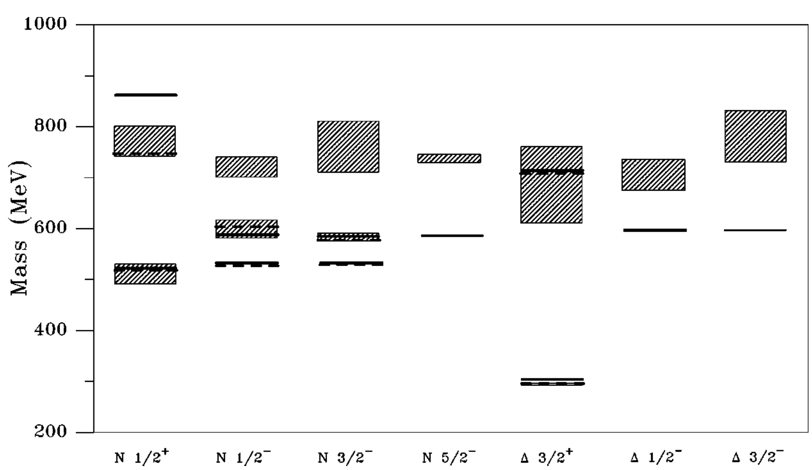

FIG. 4. Same as Fig. 2 for the set of parameters of Table II.

though the obtained spectrum shows all the states with a small excitation energy, the ordering of the states is not very different from the one appearing in Fig. 1 of Ref. [9]. The different behavior observed in the second excited state of the $N(1/2^+)$, which is mainly an $L=2$ state, is due to the fact that in the present calculation we have neglected the OGE and OPE tensor forces considered in Ref. [9]. The excitation energy can be easily increased just by changing the confinement constant and a slight modification of the strong coupling constant. In this way, using a value of r_0 in the region of stability, we have recalculated the spectrum with the set of parameters of Table I. We present the results in Fig. 3. As can be seen, apart from the energy difference between the positive and negative excitations of the nucleon, they are almost the same as those reported in Ref. [9]. Therefore, the validity of a model, where the OGE is combined with Goldstone-boson exchanges, is without any doubt after this calculation. Even more, one should say that it presents many advantages with respect to models based only on Goldstone-boson exchanges, as has been recently emphasized in several works [3,17].

Let us make a final comment with respect to the so-called *level ordering problem*. As in many other spectroscopy quark models, with the standard value of the parameters, the position of the Roper resonances (first radial excited states) and the corresponding first negative parity states are predicted inverted with respect to data. This is so, in spite of the fact that the $(\vec{\sigma}_i\cdot\vec{\sigma}_j)(\vec{\tau}_i\cdot\vec{\tau}_j)$ structure of the pseudoscalar potential favors their correct location, since it gives attraction (repulsion) for symmetric (antisymmetric) spin-isospin pairs. The model can be forced to generate the inversion of the Roper and the negative parity excitations of the nucleon, slightly increasing the strength of the pseudoscalar potential, $\Lambda_\pi=5.4\ \mathrm{fm}^{-1}$. To avoid any doubt with the Dirac delta in the OGE one can use a smoother regularization than in Ref. [9], taking $r_0=0.8$ fm, just by increasing the strong coupling constant, $\alpha_s=0.65$. Using a confinement constant $a_c=60.12\ \mathrm{MeV\,fm}^{-1}$, we present the results in Fig. 4. This set of parameters, quoted in Table II, has been used in Ref. [10]. One can see how in this case the results obtained with the truncated hyperspherical harmonic approach up to $k=2$ are almost exact, as can be understood from the previous discussion. It can be also checked how the correct level ordering between the positive and negative parity can be achieved at the expense of having a stronger pseudoscalar exchange. However, by artificially enhancing the pseudoscalar potential, the agreement for the two-baryon sector, especially the binding energy of the deuteron, is destroyed [17]. Therefore, the complete inversion of these two levels cannot be generated from the pseudoscalar potential if consistency with the two-baryon system is required.

TABLE II. Quark model parameters for the calculation of Fig. 4.

m_q(MeV)	313
α_s	0.65
a_c(MeV$\cdot$fm^{-1})	60.12
α_{ch}	0.0269
r_0(fm)	0.8
m_σ(fm^{-1})	3.42
m_π(fm^{-1})	0.7
Λ_π(fm^{-1})	5.4
Λ_σ(fm^{-1})	4.2

In summary, we would like to emphasize that the chiral constituent quark model of Ref. [9] is able to generate a quite reasonable description of the baryon spectrum with a set of parameters that allows to understand the NN phenomenology. The correct level ordering can be easily obtained, as first explained in Ref. [9], by means of a strong pionic interaction at the expense of loosing the correct description of the NN system. The simultaneous description of the one and two-baryon systems still remains an open problem.

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