Broadening of transverse momentum of partons propagating through a medium

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Broadening of the transverse momentum of a parton propagating through a medium is treated using the color dipole formalism, which has the advantage of being a well developed phenomenology in deep-inelastic scattering and soft processes. Within this approach, nuclear broadening should be treated as color filtering, i.e., absorption of large-size dipoles leading to diminishing (enlarged) transverse separation (momentum). We also present a more intuitive derivation based on the classic scattering theory of Molière. This derivation helps one to understand the origin of the dipole cross section, part of which comes from attenuation of the quark, while another part is due to multiple interactions of the quark. It also demonstrates that the lowest-order rescattering term provides an A dependence very different from the generally accepted $A^{1/3}$ behavior. The effect of broadening increases with energy, and we evaluate it using different phenomenological models for the unintegrated gluon density. Although the process is dominated by soft interactions, the phenomenology we use is tested using hadronic cross section data.

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I. INTRODUCTION

A high-energy parton propagating through a medium experiences multiple interactions that increase its transverse momentum. This broadening of the transverse momentum can be measured in different reactions, a few examples of which follow.

In the Drell-Yan process [1] the lepton pair produced carries undisturbed information about the transverse momentum of the projectile quark which undergoes initial state interactions. The important condition is the shortness of the coherence time of the Drell-Yan process [2], which allows one to factorize the initial state interaction from the cross section for the Drell-Yan reaction. In the opposite regime of very long coherence time, the effect of transverse momentum broadening also exists, although the lepton pair is produced as a fluctuation long in advance the nucleus (in the nucleus rest frame, see Ref. [2]) and does not ''communicate'' with the quark any more. Nevertheless, the nucleus supplies the fluctuation with a larger mean value of momentum transfer than a nucleon target, therefore it is able to liberate harder fluctuations, i.e., those which have larger intrinsic transverse momenta k_T. As a result, the value of $\langle k_T^2\rangle$ of the lepton pair increases. The nuclear modification of the transverse momentum distribution of lepton pairs in the limit of long coherence time is calculated in Refs. [3,4].

In a similar way, the production of heavy quarkonia off nuclei [1] can measure the k_T broadening of a projectile gluon. Final state elastic rescattering of the produced quarkonium can be neglected since the cross section is very small.

One can also study the k_T broadening of a quark originating from DIS on a nuclear target, which also includes the Fermi motion of the participating nucleon. Although the available data from the EMC experiment [5] do not show any significant effect, this is related to a suppression by a factor z^2, where z is the fraction of the quark momentum carried by the produced hadron. In this experiment $\langle z\rangle \sim 0.25$. Nevertheless, new precise data are expected soon from the HERMES experiment.

In the production of two high-p_T back-to-back hadrons (or jets) off nuclei, one of the hadrons defines the scattering plane, while the acoplanarity of the other one serves as a measure for the nuclear broadening of the transverse momenta by initial and final state interactions. Available data [6,7] demonstrate an unusually strong effect (see the interpretation in Ref. [8]).

The propagation of a high-energy parton through a medium can be treated intuitively as a random walk in transverse momentum space leading to a linear increase of $\langle k_T^2\rangle$ with the thickness of the matter covered [9,10]. Such an interpretation faces, however, certain difficulties. The cross section for the interaction of a colored quark diverges at small momentum transfer, and one has to introduce an infrared cutoff. On the other hand, the mean value of the momentum transfer squared diverges at the ultraviolet limit.

It is demonstrated in Sec. II that the broadening of the transverse momentum of a parton propagating through a nucleus can be treated within the light-cone dipole approach introduced in Ref. [11] as a color filtering effect [12] for a color $\bar{q}q$ dipole traveling through nuclear matter. Color filtering is a size-dependent attenuation of $\bar{q}q$ color dipoles propagating through nuclear matter.

Since the formal transition from a single quark to a $\bar{q}q$ dipole propagating in a medium might be difficult to understand, a more intuitive derivation based on the classic multiple scattering theory of Molière is presented in Sec. III. It goes along with the treatment of multiple interactions in the Abelian case by Levin and Ryskin [13]. Although it does not have explicit QCD input, the final result has the form of a color-screened dipole cross section, and one can see where different parts of this cross section come from. One can also trace the origin of the $A^{1/3}$ dependence of $\langle k_T^2\rangle$ broadening to multiple interactions and see that the lowest-order contribution actually has a quite different behavior.

The energy-dependent dipole cross section is evaluated in Sec. IV via the unintegrated gluon density, which is estimated within models adjusted to describe data for the proton structure function over a wide range of x and Q^2, as well as for the hadronic total cross sections. The predicted broadening is expected to rise steeply with energy.

Since k_T broadening of a high energy parton results from multiple gluonic exchanges with nucleons, nuclear shadowing of gluons diminishes the effect of broadening. This can also be interpreted as the Landau-Pomeranchuk effect, or coherence-suppressing gluon radiation, which gives an important contribution to the broadening. In Sec. V we evaluate the effect of shadowing relying on the light-cone Green function formalism, which includes the strong nonperturbative interaction of gluons that dramatically diminishes the effect of shadowing. The reduction of broadening due to gluon shadowing turns out to be rather small, less than 10%.

Nuclear broadening of k_T can be affected also by the finiteness of the experimental aperture. In Sec. VI we estimate the corresponding correction factor, which is rather close to unity if the aperture covers a range of k_T exceeding a few GeV.

The main observations are summarized and discussed in the last section.

II. TRANSVERSE MOMENTUM BROADENING AS COLOR FILTERING

Nuclear broadening of the transverse momentum of a quark propagating through a medium was treated in Ref. [14] in terms of eikonalized multiple interactions of a colorless $\bar{q}q$ pair via gluon exchanges. It was found that the mean transverse momentum squared grows as

$$\delta\langle k_T^2\rangle = 2C\rho_A L. \tag{1}$$

Here L is the length of the path covered by the quark in nuclear matter up to the point that the lepton pair is produced, and ρ_A is the nuclear density. The factor C originates from the expression for the total cross section for the interaction between a nucleon and a colorless $\bar{q}q$ dipole having transverse separation r_T and c.m. energy squared s [11],

$$\sigma_{\bar{q}q}(r_T,s) = C(r,s)r_T^2. \tag{2}$$

The r_T^2 behavior at small $r_T \to 0$ is dictated by gauge invariance and the non-Abelian nature of QCD. The energy-independent part of C was estimated in the Born approximation to be $C \approx 3$ [11,15]. However higher order perturbative corrections lead to a rising energy dependence of the $C(r_T,s)$. At small r_T the factor $C(r_T,s)$ is related to the gluon density in the proton [16,17],

$$C(r_T,s) = \frac{\pi^2}{3}G(x,Q^2), \tag{3}$$

where $G(x,Q^2) = xg(x,Q^2)$ is the gluon distribution function which depends on $Q^2 \sim 1/r_T^2$ and $x = Q^2/s$. The broadening corresponding to Eq. (1) takes the form

$$\delta\langle k_T^2\rangle = \frac{2\pi^2}{3}G(x,Q^2)\rho_A L. \tag{4}$$

This expression is also derived in Ref. [18]. Here, in a target-rest-frame formulation using considerations similar to [19,20,3,34], we derive a general expression (16) for nuclear modification of the transverse momentum distribution which is free of these approximations.

Although the result (1) is correct it was poorly motivated in Ref. [14]. Better derivations can be found in Refs. [18,21]. They, however, assume Gaussian shape for the k_T distribution and employ the approximation of constant $C(r_T)$ which cannot be justified for soft multiple interactions. Here we derive a general expression (16) for nuclear modification of the transverse momentum distribution which is free of these approximations.

The effect of the mean-square transverse momentum of the quark after propagation through a medium depends on the reaction, nevertheless, the broadening $\delta\langle k_T^2\rangle$ is universal. This is demonstrated below for the example of a hadron-nucleus interaction where we are interested in the final transverse momentum distribution of one of the projectile quarks. This can be expressed in terms of the density matrix of the final quark, $\Omega_f^q(\vec{b},\vec{b}\,')$, where $\vec{b}$ is an impact parameter,

$$\frac{dN_q}{d^2k_T} = \int d^2b\, d^2b'\, \exp[i\vec{k}_T(\vec{b}-\vec{b}\,')]\Omega_f^q(\vec{b},\vec{b}\,'). \tag{5}$$

The distribution dN_q/d^2k_T is normalized to one. The final density matrix is related to the initial one as

$$\Omega_f^q(\vec{b},\vec{b}\,') = \mathrm{Tr}\,\hat{S}^\dagger(\vec{b}\,'+\vec{B})\hat{\Omega}_{in}^q(\vec{b},\vec{b}\,')\hat{S}(\vec{b}+\vec{B}). \tag{6}$$

Here $\hat{S}(\vec{b}+\vec{B})$ is the S matrix for a quark-nucleus collision with impact parameter $\vec{b}+\vec{B}$ where $\vec{B}$ and $\vec{b}$ are the impact parameters between the center of gravity of the projectile hadron and the center of the nucleus or the quark, respectively. We take the trace over the color indices of the quark. The initial density matrix reads

$$\hat{\Omega}_{in}^q(\vec{b}_1,\vec{b}_1') = \sum_{n,\{in\}} |C_n|^2 \int d^2b_2\, d^2b_3 \cdots d^2b_n\, \Psi_n^\dagger \times(\vec{b}_1,\vec{b}_2,\ldots,\vec{b}_n)\Psi_n(\vec{b}_1',\vec{b}_2,\ldots,\vec{b}_n). \tag{7}$$

Here we sum over different Fock components of the hadron containing different numbers n of (anti)quarks with weight factors $|C_n|^2$. We also sum over the initial state polarizations and colors of all quarks except for the first one. This makes the matrix $\Omega_{in}^q(\vec{b}_1,\vec{b}_1')$ diagonal in color space. We do not show explicitly the dependence of the hadron light-cone wave function on the longitudinal momenta of the quarks, assuming integration over all longitudinal momenta except for that of the first quark. In the high energy approximation we neglect the energy loss of the quark propagating over a finite path in the nuclear medium. This effect, if it becomes important, should be treated separately.

Let us consider the S matrix of a quark-nucleus collision in the approximation where all the coordinates $\vec{r}_i$ of the bound nucleons, as well as the intrinsic quark coordinates $\vec{\rho}_j$ in the nucleons, are ''frozen'' during the interaction time. In this case the S matrix acquires the eikonal form

$$\hat{S}(\vec{b}+\vec{B},\vec{r}_i;\{\vec{\rho},\mu\}_i)=\sum_P \Theta(z_2-z_1)\cdots\Theta(z_A-z_{A-1}) \times\hat{s}_1(\vec{b}+\vec{B}-\vec{r}_{T_1};\{\vec{\rho},\mu\}_1)\cdots\hat{s}_A \times(\vec{b}+\vec{B}-\vec{r}_{T_A};\{\vec{\rho},\mu\}_A). \tag{8}$$

Here we sum over permutations of the nucleons. $\{\vec{\rho},\mu\}_i$ denotes the set of intrinsic quark coordinates and color indices in the ith nucleon. The single quark-nucleon S matrix reads

$$\hat{s}_i(\vec{b}+\vec{B}-\vec{r}_{T_i};\{\vec{\rho},\mu\}_i)=\exp\left[\frac{i}{4}\sum_{j=1}^{3}\hat{\lambda}_a\hat{\lambda}_a(j)\chi(\vec{b}+\vec{B}-\vec{r}_{T_j})\right], \tag{9}$$

where $\hat{\lambda}_a$ are the Gell-Mann matrices, and index j refers to one of the quarks in the target nucleon

$$\chi(\vec{b})=\int_{\Lambda^2}^{\infty}\frac{dq^2}{q^2}\alpha_s(q^2)\,J_0(b\cdot q). \tag{10}$$

Here J_0 is a Bessel function, $\vec{q}$ is the transverse momentum of the gluon exchanged in the t channel, and Λ^2 is an infrared cutoff.

As soon as the initial density matrix $\hat{\Omega}^q_{in}(\vec{b},\vec{b}\,')$ is diagonal in the color indices of the quark (see above), we can average the product $\hat{S}^\dagger\hat{S}$ over the colors and coordinates of the quarks in the target nucleons in inverse sequence starting from the last one. Then we average over the positions of the nucleons $\{\vec{r}\}$ (compare with Ref. [22]),

$$\begin{aligned}&\langle\langle\hat{S}^\dagger(\vec{b}\,'+\vec{B},\vec{r}_i;\{\vec{\rho},\mu\}_i)\hat{S}(\vec{b}+\vec{B},\vec{r}_i;\{\vec{\rho},\mu\}_i)\rangle_{\{\vec{\rho},\mu\}}\rangle_{\{\vec{r}\}}\\&=\Big\langle\sum_P\Theta(z_2-z_1)\cdots\Theta(z_A-z_{A-1})\\&\times\langle\hat{s}_1^\dagger(\vec{b}\,'+\vec{B}-\vec{r}_{T_1})\\&\times\langle\hat{s}_2^\dagger(\vec{b}\,'+\vec{B}-\vec{r}_{T_2})\cdots\langle\hat{s}_A^\dagger(\vec{b}\,'+\vec{B}-\vec{r}_{T_A})\cdot\hat{s}_A\\&\times(\vec{b}+\vec{B}-\vec{r}_{T_A})\rangle_{\{\vec{\rho},\mu\}_A}\cdots\\&\times\hat{s}_2(\vec{b}+\vec{B}-\vec{r}_{T_2})\rangle_{\{\vec{\rho},\mu\}_2}\hat{s}_1(\vec{b}+\vec{B}-\vec{r}_{T_1})\rangle_{\{\vec{\rho},\mu\}_1}\Big\rangle_{\{\vec{r}\}}.\end{aligned} \tag{11}$$

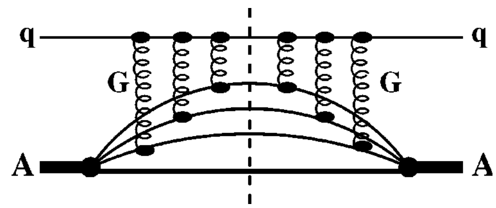

FIG. 1. The probability of multiple interactions via one gluon exchange for the quark in the nucleus. The dashed line shows the unitarity cut.

This expression is illustrated in Fig. 1, where we use the two-gluon model for the Pomeron for the sake of simplicity. One can see that the Pomerons are enclosed within each other.

After averaging over the target nucleon wave function including the target quark color indices, the functions

$$U_k(\vec{b}\,'+\vec{B}-\vec{r}_{T_k},\vec{b}+\vec{B}-\vec{r}_{T_k}) =\langle\hat{s}_k^\dagger(\vec{b}\,'+\vec{B}-\vec{r}_{T_k})\cdot\hat{s}_k(\vec{b}+\vec{B}-\vec{r}_{T_k})\rangle_{\{\vec{\rho},\mu\}_k} \tag{12}$$

commute, and the sum over permutations of the nucleon coordinates of the product of Θ functions equals one. This expression becomes independent of index k after integration over $\vec{r}_k$. The integral can be represented as

$$\begin{aligned}&\frac{1}{A}\int d^3r\,\rho_A(\vec{r}_T,z)U(\vec{b}\,'+\vec{B}-\vec{r}_T,\vec{b}+\vec{B}-\vec{r}_T)\\&=\frac{1}{A}\int d^3r\,\rho_A(\vec{r}_T,z)-\frac{1}{A}\int d^3r\,\rho_A(\vec{r}_T,z)\\&\times[1-U(\vec{b}\,'+\vec{B}-\vec{r}_T,\vec{b}+\vec{B}-\vec{r}_T)]\\&\approx1-\frac{1}{2A}T_A\left(\frac{\vec{b}+\vec{b}\,'}{2}+\vec{B}\right)\sigma_{\bar{q}q}(\vec{b}-\vec{b}\,').\end{aligned} \tag{13}$$

We use the standard approximation of uncorrelated nuclear wave functions, and $\rho_A(\vec{r})$ is the one-body nuclear density normalized to A. $T_A(\vec{b})$ is the nuclear thickness function,

$$T_A(b)=\int_{-\infty}^{\infty}dz\,\rho_A(b,z) \tag{14}$$

and

$$\sigma_{\bar{q}q}(\vec{\rho})=\frac{2}{A}\int d^2r_T[1-U(\vec{b}\,'+\vec{B}-\vec{r}_T,\vec{b}+\vec{B}-\vec{r}_T)] \tag{15}$$

is the total cross section for the interaction of a $\bar{q}q$ dipole of transverse separation $\vec{\rho}=\vec{b}-\vec{b}\,'$ [11] with a nucleon.

Eventually we arrive at the expression for the transverse momentum distribution of a quark that has propagated through the nucleus,

$$\frac{dN_q}{d^2k_T}=\int d^2b\, d^2b'\, \exp[i\vec{k}_T(\vec{b}-\vec{b}\,')]\Omega^q_{in}(\vec{b},\vec{b}\,')\times\exp\left[-\frac{1}{2}\sigma_{q\bar{q}}(\vec{b}-\vec{b}\,')T_A\left(\frac{\vec{b}+\vec{b}\,'}{2}+\vec{B}\right)\right]. \quad (16)$$

This equation describes the full k_T distribution of the final quarks, as well as the mean transverse momentum squared of an ejectile quark in a hadron-nucleus collision,

$$\langle k_T^2\rangle_{hA}=\frac{1}{A}\int d^2B\int_{-\infty}^{\infty}dz\,\rho_A(B,z)\int d^2k_T\,k_T^2\times\int d^2b_1\,d^2b_2\,\Omega^q_{in}(\vec{b}_1,\vec{b}_2)\exp[i\vec{k}(\vec{b}_1-\vec{b}_2)]\times\exp\left[-\frac{1}{2}\sigma_{q\bar{q}}(\vec{b}_1-\vec{b}_2)T_A\left(\frac{\vec{b}_1+\vec{b}_2}{2}+\vec{B},z\right)\right]. \quad (17)$$

The integration in Eq. (17) is easy to perform by replacing $k_T^2\exp[i\vec{k}(\vec{b}_1-\vec{b}_2)]$ by derivatives $\vec{\nabla}(b_1)\cdot\vec{\nabla}(b_2)\exp[i\vec{k}(\vec{b}_1-\vec{b}_2)]$. Integrating by parts we arrive at the expression

$$\langle k_T^2\rangle_{hA}=\frac{1}{A}\int d^2B\int_{-\infty}^{\infty}dz\,\rho_A(B,z)\int d^2b_1\times\{\vec{\nabla}(b_1)\cdot\vec{\nabla}(b_2)\Omega^q_{in}(\vec{b}_1,\vec{b}_2)+\vec{\nabla}(b_1)\cdot\vec{\nabla}(b_2)\times\exp[-\tfrac{1}{2}\sigma_{q\bar{q}}(\vec{b}_1-\vec{b}_2)T_A(B,z)]\}_{\vec{b}_2=\vec{b}_1}, \quad (18)$$

where we assume that the nuclear radius is much larger than that of the projectile hadron.

Using a Gaussian for the hadronic wave function, the density matrix of the initial quark in the case of a proton beam reads

$$\Omega^q_{\rm in}(\vec{b}_1,\vec{b}_2)=\frac{2}{3\pi\langle r^2_{\rm ch}\rangle}\exp\left[-\frac{b_1^2+b_2^2}{3\langle r^2_{\rm ch}\rangle}\right], \quad (19)$$

where $\langle r^2_{\rm ch}\rangle$ is the mean square of the proton charge radius. Note that we do not introduce any unreasonably large primordial transverse momentum for the projectile quark, which is usually assumed for hard reactions in leading order in the parton model. In the light cone approach, the higher order perturbative corrections are already included in the phenomenological cross section (2), and they generate the energy dependence of $C(r_T,s)$.

In the second term on the right-hand side (rhs) of (18) one should apply both derivatives to $\sigma_{q\bar{q}}(\vec{b}_1-\vec{b}_2)$ in the exponential. This is because $\sigma_{q\bar{q}}(\vec{b}_1-\vec{b}_2)\propto(\vec{b}_1-\vec{b}_2)^2$ as $\vec{b}_1\to\vec{b}_2$, otherwise the result is zero. Using Eq. (2) for the dipole cross section, one gets from Eq. (17) the mean-square transverse momentum of a valence quark,

$$\langle k_T^2\rangle_{pA}=\frac{2}{3\langle r^2_{\rm ch}\rangle}+2\,C(0,s)\langle T_A\rangle, \quad (20)$$

where

$$\langle T_A\rangle=\frac{1}{A}\int d^2b\,T^2(b) \quad (21)$$

is the mean nuclear thickness. Thus, with a better derivation we confirm the result of Ref. [14], Eq. (1).

III. MOLIÈRE THEORY: A MORE INTUITIVE DERIVATION

The result we obtained in Eqs. (18) and (20) might look puzzling. Indeed, in the expansion of the exponential in Eq. (18) only the lowest order term $\propto\sigma_{q\bar{q}}T_A\propto A^{1/3}$ survives differentiation and contributes to $\langle k_T^2\rangle$. This observation seems to lead to the conclusion that only single rescattering contributes to $\langle k_T^2\rangle$, while higher multifold scatterings do not affect the broadening of the transverse momentum (compare with Ref. [23]). Such an interpretation contradicts the intuitive expectation that broadening is caused by multiple interactions in the medium resulting in random walk of the particle in the transverse momentum plane. Here we present a different, more intuitive derivation for the same result (20) based on Molière's theory [24] of multiple interactions in a medium (see also Ref. [13]). This result shows that broadening is indeed caused by multiple interactions rather than by the single rescattering term, which has an A dependence very different from $A^{1/3}$.

We concentrate on nuclear broadening of $\langle k_T^2\rangle$ and for the sake of clarity neglect the primordial transverse momentum distribution of the projectile quark which is responsible for the first term on the rhs of Eq. (20). Evolution of the transverse momentum distribution $D(k_T,z)$ as a function of longitudinal coordinate z is described by the kinetic equation

$$\frac{dD(k_T,z)}{dz}=-\sigma_{\rm tot}\rho_A(z)\,D(k_T,z)+\int d^2k_T'\,\rho_A(z)\,\frac{d\,\sigma(\vec{k}_T-\vec{k}_T')}{d^2k_T'}D(k_T',z). \quad (22)$$

Here $d\sigma/d^2k_T$ is the differential cross section of quark scattering on a nucleon summed over final states and quark colors. The first term on the rhs of Eq. (22) and the total cross section,

$$\sigma_{\rm tot}=\int d^2k_T\frac{d\sigma}{d^2k_T}, \quad (23)$$

describe the leakage of quarks from the element of phase space d^2k_T. At the same time, this region gains new quarks from other parts of phase space via scattering as is described by the second term in Eq. (22).

Equation (22) is easy to solve switching to coordinate space,

$$D(k_T,z)=\frac{1}{2\pi}\int d^2r_T\, e^{i\vec{r}_T\vec{k}_T}D(r_T,z). \tag{24}$$

The solution is (up to the initial condition)

$$D(k_T,z)\propto\frac{1}{2\pi}\int d^2r_T\, e^{i\vec{r}_T\vec{k}_T}\exp[-(\sigma_{\rm tot}-\gamma(r_T))T_A(z)], \tag{25}$$

where

$$\gamma(r_T)=\int d^2k_T\, e^{-i\vec{k}_T\vec{r}_T}\frac{d\sigma}{dk_T^2}, \tag{26}$$

$$\gamma(0)=\sigma_{\rm tot}, \tag{27}$$

$$T_A(z)=\int_{-\infty}^{z}dz'\,\rho_A(z'). \tag{28}$$

We are now in a position to discuss the meaning of expansion of the exponential in Eq. (25). First of all, the expansion,

$$\exp(-\sigma_{\rm tot}T_A)=\sum_{n=0}\frac{(-1)^n}{n!}(\sigma_{\rm tot}T_A)^n, \tag{29}$$

cannot be treated as a multiple scattering series. This is just an attenuation exponential, and the particle (quark) cannot be absorbed (knocked out of a phase space cell) twice. This simple exponential should not be confused with the Glauber eikonal multiple scattering series, which has a nontrivial interpretation in terms of unitarity relation and AGK cutting rules [25].

At the same time, the expansion of the second term, $\gamma(r_T)T_A$, in the exponent of Eq. (25) does have the interpretation of a multiple scattering series. Indeed,

$$\int d^2r_T\exp(i\vec{k}_T\cdot\vec{r}_T)\exp[\gamma(r_T)T_A]$$
$$=\delta(\vec{k}_T)+\sum_{n=1}\frac{T_A^n}{n!}\int\prod_{j=1}^{n}d^2k_j\frac{d\sigma(\vec{k}_j)}{d^2k_j}\delta\left(\vec{k}_T-\sum_{i=1}^{n}\vec{k}_i\right). \tag{30}$$

The nth term on the sum in the rhs of this equation clearly corresponds to the n-fold scattering of the quark. Thus, all multifold rescatterings contribute to the shape of the k_T distribution of a quark propagating through a medium.

Amazingly, the exponent in Eq. (25) can be represented as a color dipole cross section, making it similar to the results of the preceding section,

$$\sigma_{\rm tot}-\gamma(r_T)=\frac{1}{2\pi}\int d^2k_T(1-e^{i\vec{k}_T\cdot\vec{r}_T})\frac{d\sigma}{d^2k_T}=\frac{1}{2}\sigma_{q\bar{q}}(r_T). \tag{31}$$

We have made use of the fact that $d\sigma/d^2k_T\propto 1/k_T^4$. This expression is also derived in more detail at the end of this section.

The appearance of the dipole cross section, as in Eq. (17), is the result of an artificial construction. Clearly, the object participating in the scattering is not the colored $\bar{q}q$ dipole but rather a single colored quark. Now it turns out that the different parts of this dipole cross section have quite different origins. Namely, the first term in parentheses in Eq. (31) (the ''1'') corresponds to simple attenuation of the projectile quark detected in a given phase space cell. However, the second term $[\exp(i\vec{k}_T\cdot\vec{k}_T)]$ originates from multiple scattering of the quark.

Thus, if one needs to establish a relation between the expansion of the exponential $\exp[-\frac{1}{2}\sigma_{q\bar{q}}(r_T)T_A]$ in Eq. (17) and the multiple quark interaction, it would be incorrect to think that the nth order term of this expansion corresponds to the probability W_n to have n-fold quark multiple scattering,

$$W_n(r_T)\neq\frac{(-1)^n}{n!}[\sigma_{q\bar{q}}(r_T)T_A]^n, \tag{32}$$

but, instead,

$$W_n(r_T)=e^{-\sigma_{\rm tot}T_A}\frac{\gamma^n(r_T)T_A^n}{n!}. \tag{33}$$

In contrast to Eq. (32), all the terms in Eq. (33) are positive as they should be. This standard result of the multiple scattering theory has also been revived recently in Ref. [26].

Now we are in a position to figure out where the mean k_T^2 comes from. We find

$$\langle k_T^2\rangle=\frac{1}{\sigma_{\rm tot}}\int d^2k_T\,k_T^2\int d^2r_T\,e^{i\vec{k}_T\cdot\vec{r}_T}\sum_{n=0}W_n(r_T)=k_0^2\langle n\rangle, \tag{34}$$

where

$$k_0^2=\frac{1}{\sigma_{\rm tot}}\int d^2k_T\,k_T^2\frac{d\sigma}{dk_T^2} \tag{35}$$

is the mean transverse momentum squared gained by the quark in a single scattering, and

$$\langle n\rangle=\sigma_{\rm tot}T_A \tag{36}$$

is the mean number of interactions of the quark propagating though nuclear thickness T_A.

Equation (34) explicitly demonstrates that nuclear broadening of the quark mean transverse momentum is the result of multiple interactions leading to a random walk in the transverse momentum plane. Therefore, it would be incorrect to interpret the result of the preceding section, Eq. (17), as the contribution of a single scattering, which must contain an extra factor $\exp(-\sigma_{\rm tot}T_A)$ and has quite a different A dependence compared to $T_A\propto A^{1/3}$ of Eq. (17).

Concluding this section we present a derivation of Eqs. (17)–(20) within the model of potential scattering. The amplitude for elastic scattering of a particle from a potential reads

$$f(k_T)=\frac{i}{2\pi}\int d^2b\, e^{i\vec{b}\cdot\vec{k}_T}\omega(b), \tag{37}$$

where

$$\omega(b)=1-e^{i\chi(b)}, \tag{38}$$

$$\chi(b)=-\frac{1}{v}\int_{-\infty}^{\infty}dz\, V(\vec{b},z). \tag{39}$$

Correspondingly,

$$\sigma_{\rm tot}=4\pi\,{\rm Im}\,f(0)=2\,{\rm Re}\int d^2b\;\omega(b), \tag{40}$$

$$\frac{d\sigma}{d^2k_T}=|f(k_T)|^2, \tag{41}$$

$$\sigma_{\rm el}=\int d^2k_T\frac{d\sigma}{d^2k_T}=\int d^2b(2-2\cos\chi(b))=\sigma_{\rm tot}, \tag{42}$$

as one should have expected for potential scattering.

Substituting Eq. (41) into Eq. (26) we get

$$\gamma(r_T)=\int d^2b\;\omega^*(\vec{b}+\alpha\vec{r}_T)\omega(\vec{b}-(1-\alpha)\vec{r}_T)$$
$$=\sigma_{\rm tot}-\tfrac{1}{2}\sigma_{q\bar{q}}(r_T), \tag{43}$$

where $0<\alpha<1$ is an arbitrary number, and

$$\sigma_{q\bar{q}}(r_T)=2\,{\rm Re}\int d^2b\{1-\exp[i\chi(\vec{b}+\alpha\vec{r}_T)$$
$$-i\chi(\vec{b}-(1-\alpha)\vec{r}_T)]\} \tag{44}$$

is the total cross section for a quark-antiquark pair with transverse separation r_T, called dipole cross section. In this case α can be interpreted as a share of the total light-cone momentum carried by a quark or antiquark. Thus, we confirm Eq. (31).

IV. CALCULATION OF THE $C(0,s)$

As mentioned above, all total cross sections rise with energy, so the factor $C(r_T,s)$ does also. The energy dependence is steeper towards small r_T [27] and may lead to dramatic changes in $C(r_T,s)$ compared to the oversimplified estimate $C\approx 3$ [11,15].

Intuitively, it is clear that a fast quark scatters off the gluon clouds of bound nucleons. The gluon density at small Bjorken x relevant to such a high-energy interaction increases with $1/x$, i.e., with quark energy [see Eq. (4)]. Said differently, multiple interactions of the quark are accompanied by gluon bremsstrahlung. As the energy is increased, more phase space becomes available for the radiated gluons. Since gluon radiation enhances transverse motion of the quark, $\langle k_T^2\rangle$ should grow with energy [via $C(0,s)$].

Thus, the energy dependence of $C(r_T,s)$ can be expressed in terms of the unintegrated gluon density [17],

$$C(0,s)=\frac{\pi}{3}\int d^2k\frac{\alpha_s(k^2)k^2}{k^4}\mathcal{F}(x,k^2), \tag{45}$$

where $\mathcal{F}(x,k^2)=\partial G(x,k^2)/\partial(\ln k^2)$ and $G(x,Q^2)=xg(x,Q^2)$. Here $\vec{k}$ is the transverse momentum that a quark acquires scattering off the gluon cloud of a nucleon. $\alpha_s(k^2)$ is the QCD running constant, which we calculate in the one-loop approximation.

The value of x for gluons in the rhs of Eq. (45) is related to the quark-nucleon c.m. energy squared s. The minimal value of x corresponding to a collinear quark-gluon collision reads

$$x_{\rm min}=\frac{4k^2}{s}. \tag{46}$$

The density of the gluon cloud around the proton should vanish at large distances because of the color neutrality of the proton. Therefore, the unintegrated gluon distribution $\mathcal{F}(x,k^2)\propto k^2$ as $k^2\to 0$. This provides infrared stability of the integral in Eq. (45).

There are quite a few models for the unintegrated gluon distribution $\mathcal{F}(x,k^2)$ at high k^2 (e.g., see Ref. [28]); however, very little information is available in the region of small k^2 where perturbative QCD cannot be used. Since the integral in Eq. (45) is dominated by soft gluons, one should develop a phenomenology for $\mathcal{F}(x,k^2)$ and use other observables to restrict possible uncertainties. One of the sensitive probes for $\mathcal{F}(x,k^2)$ [even more sensitive than $C(0,s)$] is the total hadronic cross section. The simplest case is the pion-proton cross section, which is given by the same approach as Eq. (45),

$$\sigma^{\pi p}(s)=\frac{4\pi}{3}\int d^2k\frac{\alpha_s(k^2)}{k^4}[1-F^{\pi}_{qq}(k)]\mathcal{F}(x,k^2). \tag{47}$$

Here $F^{\pi}_{qq}(k)=\langle\pi|\exp[i\vec{k}(\vec{r}_2-\vec{r}_1)]|\pi\rangle\approx\exp(-k^2\langle r^2_{\pi}\rangle/3)$ is the two-quark form factor of the pion, and $\langle r^2_{\rm ch}\rangle_{\pi}=0.44\pm 0.01\,{\rm fm}^2$ [29] is the mean-square pion charge radius.

In the Born approximation [30,31,11] the gluon density takes a simple form

$$\mathcal{F}(x,k^2)\Rightarrow\frac{4}{\pi}\alpha_s(k^2)[1-F^N_{qq}(k)], \tag{48}$$

where $F^p_{qq}(k)=\langle N|\exp[i\vec{k}(\vec{r}_2-\vec{r}_1)]|N\rangle\approx\exp(-k^2\langle r^2_{\rm ch}\rangle_p/2)$ is the two-quark form factor of the nucleon, and $\langle r^2_{\rm ch}\rangle_p=0.79\pm 0.03\,{\rm fm}^2$ [32] is the mean-square charge radius of the proton. In fact, because Coulomb gluons have no partonic inter-

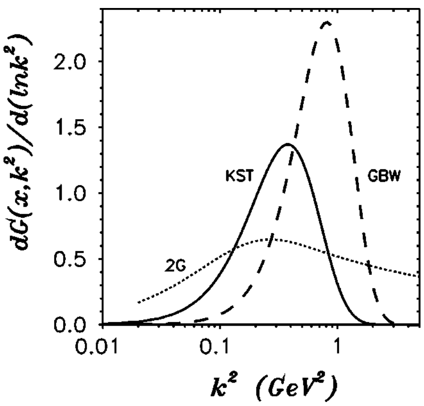

FIG. 2. The nonintegrated gluon density $\mathcal{F}(x,k^2)$ calculated at the quark-nucleon energy $s=500\,\mathrm{GeV}^2$ as a function of k^2. The dashed and solid curves correspond to the models GBW and KST. The dotted curve shows the two-gluon contribution (48).

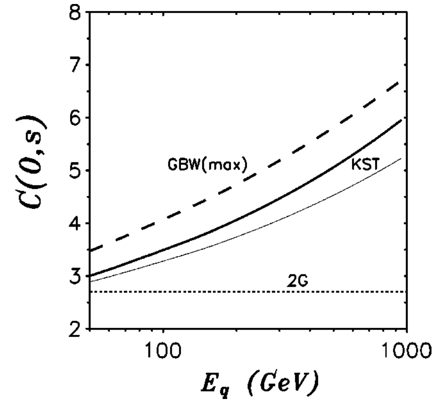

FIG. 3. Factor $C(0,s)$ as a function of the quark energy calculated with Eq. (45). The curve assignments are the same as in Fig. 2, and the thin solid curve is the KST curve corrected for gluon shadowing in lead.

pretation, quark elastic scattering through the exchange of two such gluons cannot be expressed in terms of the gluon density as given in Eq. (4). Therefore, Eq. (48) cannot be treated as a model for the gluon distribution function $\mathcal{F}(x,k^2)$ by any means; nevertheless we plot it vs k^2 in Fig. 2 (as the dotted curve) to compare with other models. This contribution is independent of x. As was mentioned, x dependence originates from gluon bremsstrahlung.

Comparing the factor $(1-F^{\pi}_{qq}(k))$ in Eq. (47) to k^2 in Eq. (45), it is clear that the total hadronic cross section is much more sensitive to the behavior of $\mathcal{F}(x,k^2)$ than $C(0,s)$ is in the region of interest, namely at small k^2. Therefore, reproduction of the total hadronic cross section is an important test for any model of the gluon density. At large values k^2 perturbative QCD is supposed to be valid, and different models should therefore converge there.

Note that one should interpret Eq. (47) as the inelastic rather than the total cross section because it is linear in gluon density, i.e., corresponds to a color octet-octet unitarity cut, which does not contain elastic or diffractive states. Therefore, one should add to Eq. (47) the elastic and single diffraction cross sections to get the total cross section. However, such an amplitude linear in the gluon density is subject to unitarization. The first unitarity correction, which is quadratic in the gluon density, is just the same elastic and diffractive cross sections, but with a negative sign. Hence, this cancels the contributions of the elastic and diffractive cross sections, and one can treat Eq. (47) as the total cross section neglecting the higher order unitarity corrections $O(G^3)$, which are known to be quite small at the present energies.

We will try a few models for the gluon density to calculate the $C(0,s)$, hoping that the spread in the results may serve as a measure for the theoretical uncertainty.

A. GBW model

Golec-Biernat and Wüsthoff [33] suggested a model for the dipole cross section (2) which saturates at large $\bar{q}q$ separations and reproduces well the DIS cross sections:

$$\sigma_{\bar{q}q}(r_T,x)=\sigma_0\left[1-\exp\left(-\frac{r_T^2}{R_0^2(x)}\right)\right], \tag{49}$$

where $R_0(x)=0.4\,\mathrm{fm}\times(x/x_0)^{\lambda/2}$ and $\sigma_0=23.03\,\mathrm{mb}$; $\lambda=0.288$; $x_0=3.04\times10^{-4}$. This cross section obviously satisfies the behavior suggested by data for $F_2(x,Q^2)$: the smaller the separation r_T, the steeper the growth with $1/x$. However, the value of x is not well defined for a dipole of a given energy (there is no problem with the definition of x in our next model).

The gluon density corresponding to this dipole cross section reads [33]

$$\mathcal{F}(x,k^2)=\frac{3\sigma_0}{16\pi^2\alpha_s(k^2)}k^4\,R_0^2(x)\exp\left[-\frac{1}{4}R_0^2(x)k^2\right]. \tag{50}$$

This function is depicted in Fig. 2 by the dashed curve. Note that according to Eq. (50) the right wing of this curve at large k^2 is a steeply rising function of energy.

This model faces obvious problems with hadronic cross sections. Indeed, the dipole cross section (49) grows steeply with r_T and saturates for $r_T>R_0(x)$ at $\sigma_{\bar{q}q}(r_T,x)\leqslant\sigma_0$. Averaging Eq. (49) weighted with the pion wave function squared, one never can reach a pion-proton total cross section larger than $\sigma_0=23.03\,\mathrm{mb}$.

In order to calculate the energy dependence of $C(0,s)$ we need to know the value of x, which is poorly defined in this model. If we use the minimal value (46) permitted by kinematics, the result shown by the dashed curve in Fig. 3 should be an upper bound for $C(0,s)$ in this model. We see that $C(0,s)$ is rather steep as a function of energy.

B. KST model

The advantage of the GBW parametrization (49) is simplicity and convenience for analytical calculations. One may keep this form, but modify it as suggested in Ref. [34] to address the problems mentioned above. An explicit energy dependence is introduced in the parameter σ_0 in Eq. (49) in a way that guarantees the reproduction of the correct hadronic cross sections,

$$\sigma_0(s)=\sigma_{\rm tot}^{\pi p}(s)\left(1+\frac{3R_0^2(s)}{8\langle r_{\rm ch}^2\rangle_\pi}\right), \tag{51}$$

where $\sigma_{\rm tot}^{\pi p}(s)=23.6\times(s/s_0)^{0.08}$ is the Pomeron part of the πp total cross section [35], and $R_0(s)=0.88\,{\rm fm}\times(s/s_0)^{-\lambda/2}$ with $\lambda=0.28$ and $s_0=1000\,{\rm GeV}^2$ is the energy-dependent radius. With these parameters the proton structure function $F_2(x,Q^2)$ was calculated [34] using the nonperturbative distribution functions for the $\bar{q}q$ component of the photon [34] and the dipole cross section (49) and (51). The results agree well with the data up to $Q^2\sim 10\,{\rm GeV}^2$, which is sufficient for our interval of k^2.

The corresponding gluon density (50) is shown as a function of k^2 by the dashed curve in Fig. 2 with the redefined functions $\sigma_0(s)$ and $R_0(s)$. The factor $C(0,s)$ calculated with this gluon density is shown by the thick solid curve as a function of energy in Fig. 3. $C(0,s)$ rises with energy similar to the prediction of the GBW model.

Summarizing, the two models under consideration provide quite different values and k^2 dependences for the nonintegrated gluon density, as one can see from Fig. 2. Nevertheless, in the energy range of the E772/E866 experiment at Fermilab the value $C(0,s)\approx 3-4$ is pretty certain and is about twice as big as the simple two-gluon approximation predicts.

V. NUCLEAR SHADOWING OF GLUONS

An important source of broadening in the transverse momentum of a quark is the gluon radiation that accompanies the multiple interactions of the quark in the nucleus. Indeed, one can see from Fig. 3 that the factor $C(0,s)$ is enhanced compared to the contribution of two-gluon exchange, which corresponds to a quark scattering on Coulomb gluons without gluon radiation.

It is known that radiation in multiple scattering is subject to Landau-Pomeranchuk suppression [36], which is a coherence phenomenon in radiation. Namely, if the gluons radiated due to scattering of a quark off different nucleons are in phase, one should add up the amplitudes rather than the probabilities. Interferences substantially suppress the radiation compared to the classical expectation (Bethe-Heitler approximation).

The same phenomenon may be treated quite differently as gluon fusion in the parton model in the infinite momentum frame of the nucleus, where it is known as nuclear shadowing for the gluon density at small x [37–41]. Indeed, only the fast part of parton clouds of bound nucleons are squeezed by Lorentz contraction, but the low-x partons may be spread in the longitudinal direction more than the longitudinal size of the nucleus in its infinite momentum frame. Therefore, partons originating from different nucleons at the same impact parameter overlap and may fuse. This effect diminishes the number of partons at small x. Thus, we replace the gluon density in Eq. (45) by a shadowed one, suppressed by a factor $S_A(x,k^2)$ compared to the unintegrated gluon density in a free proton,

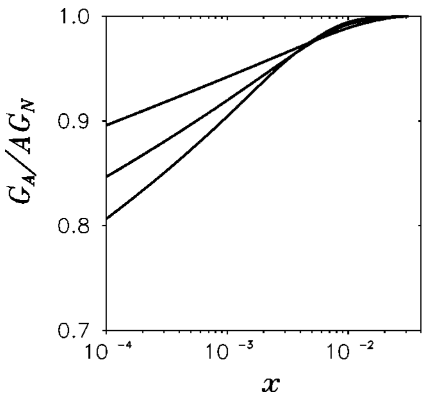

FIG. 4. The nuclear shadowing factor $S_A(x,k^2)$ for soft gluons for lead, copper, and carbon (from bottom to top) calculated with the light-cone Green function approach [34].

$$C(0,s)=\frac{\pi}{3}\int d^2k\frac{\alpha_s(k^2)}{k^2}\mathcal{F}(x,k^2)S_A(x,k^2), \tag{52}$$

where $S_A(x,k^2)$ is calculated at $x=4k^2/s$.

The nuclear suppression factor $S_A(x,k^2)$ is calculated in Ref. [34] using the light-cone Green function approach. At small k^2, shadowing is controlled by the strong nonperturbative interactions of gluons, ensuring that shadowing is nearly independent of k^2 for $k^2<4\ \ {\rm GeV}^2$. This covers the region of k^2 we are interested in, therefore we can safely disregard the k^2 dependence of $S_A(x,k^2)$, except for that which comes from the x dependence. The factor $S_A(x,k^2)$ is depicted in Fig. 4 at small k^2 for a few nuclei as calculated in Ref. [34]. We see that the onset of gluon shadowing takes place at $x<10^{-2}$, which are smaller values of x than one needs for the shadowing of quarks ($F_2^A(x,Q^2)$). Nevertheless, one can see from Fig. 2 that the values of k^2 where the unintegrated gluon density is large are quite small, corresponding to very small x where one may expect strong shadowing effects.

The results of calculation of $C(0,s)$ for lead with the KST model for the unintegrated gluon density are shown in Fig. 3 by the thin solid curve. It turns out that the effect of gluon shadowing is not dramatic, i.e., does not exceed 10%.

VI. THE EFFECT OF FINAL APERTURE

The factor $C(0,s)$ for the nuclear broadening of the transverse quark momentum in Eq. (20) appears as a result of integration over k_T up to infinity. In reality, the angular acceptance of the experimental apparatus restricts the accessible values of k_T to be less than some k_m. Introducing this upper cutoff for k_T in Eq. (17) we arrive at a modified value $\tilde{C}(s)$ that has to replace $C(0,s)$,

$$\tilde{C}(s)=\frac{2\pi k_m}{A\langle T_A\rangle}\int_0^\infty dB\,B\int_0^\infty dr\,J_1(k_m r)F(B,r), \tag{53}$$

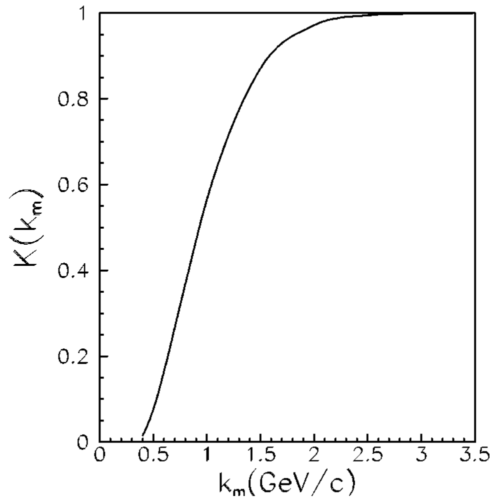

FIG. 5. The suppression factor $K(k_m)=\widetilde{C}(s)/C(0,s)$ caused by a finite aperture calculated with Eq. (53) as a function of k_m.

where

$$F(B,r)=\frac{8}{\sigma^2_{\bar{q}q}(r,s)}\left[\frac{\partial\sigma_{\bar{q}q}(r,s)}{\partial r^2}\left(1-\frac{r^2}{3\langle r^2_{\rm ch}\rangle_p}\right)+\frac{\partial^2\sigma_{\bar{q}q}(r,s)}{\partial^2 r^2}r^2\right]f_1(B,r)-\frac{16}{\sigma^3_{\bar{q}q}(r,s)}r^2\left[\frac{\partial\sigma_{\bar{q}q}(r,s)}{\partial r^2}\right]^2 f_2(B,r) \qquad (54)$$

and

$$f_1(B,r)=1-(1+a)e^{-a}, \qquad (55)$$

$$f_2(B,r)=1-\left(1+a+\frac{a^2}{2}\right)e^{-a}, \qquad (56)$$

$$a=\tfrac{1}{2}\sigma_{\bar{q}q}(r,s)T_A(b). \qquad (57)$$

Integration over B and r in Eq. (53) needs to be done numerically. The suppression factor $K(k_m)=\widetilde{C}(s)/C(0,s)$ is plotted in Fig. 5 as function of k_m. This substantially deviates from unity for a cutoff $k_m<2$ GeV.

VII. DISCUSSION OF THE RESULTS

We performed calculations for broadening of the transverse momentum of a quark propagating through a medium based on the light-cone color dipole representation, which treats the broadening of transverse momenta as a color filtering of transverse sizes of $\overline{q}q$ dipoles propagating through nuclear matter. It is natural that the broadening is proportional to the length of the path times the density of the medium, since multiple interactions cause the quark to undergo a random walk in the transverse momentum plane. The main task is to calculate the coefficient C which turns out to be the same as in the dipole cross section $\sigma_{\bar{q}q}(r_T)=C\,r_T^2$ [see Eq. (20)]. This is not surprising since, in our approach, the k_T distribution of the final quarks is proven to have the form of an eikonalized dipole cross section. In addition to the formal derivation, we present another one based on the Molière scattering theory. This simple and intuitive approach helps to understand how the propagation of a single quark can be described in terms of the color dipole cross section. In particular, it turns out that a part of the dipole cross section originates from the simple attenuation of the quark traveling through the medium, while the rest of the cross section is due to multiple interactions of the quark. It is demonstrated that the wide-spread belief that the lowest order rescattering leads to a broadening $\delta\langle k_T^2\rangle\propto A^{1/3}$ is incorrect.

We have also performed a parameter-free evaluation of the effect of broadening. We calculated the factor $C(0,s)$ using a phenomenological description of the dipole cross section that is adjusted to data for the proton structure function and total hadronic cross sections. We found that the effect of k_T broadening steeply rises with energy.

Several corrections diminishing the effect of broadening, such as nuclear shadowing of gluons and the finiteness of the experimental aperture, are evaluated as well.

We make a few remarks in conclusion.

For gluons propagating through a medium, our results for k_T broadening must be enlarged by the Casimir factor 9/4. An example of a process relevant to gluon propagation is heavy quarkonium production.

Numerically, our results are in a reasonable agreement with k_T broadening observed in production of J/Ψ and Υ off nuclei, but are somewhat higher than what is observed in the Drell-Yan processes [1]. A detailed comparison with available data for Drell-Yan processes, heavy quarkonium production, dijet and dihadron production off nuclei, etc., will be presented elsewhere.

Although we concentrate in this paper on the calculation of the nuclear broadening of the mean-square transverse momentum $\delta\langle k_T^2\rangle$, we are in a position to calculate the full momentum transfer distribution of partons after they escape from the nuclear medium. This distribution is given by Eq. (16).

Nuclear broadening of the transverse momentum of partons originating from relativistic heavy ion collisions is enhanced, since the parton experiences multiple interactions propagating through both nuclei. On top of that, if the parton is produced at mid-rapidity with a high transverse momentum it should substantially increase broadening relative to the original direction if it propagates through a dense matter (i.e., quark-gluon plasma) since it probes the density of the medium. This needs measurements with two back-to-back hadrons or jets. Thus, the effect of broadening of $\langle k_T^2\rangle$ can serve as a sensitive probe for creation of dense matter in heavy ion collisions.

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