Unfactorized versus factorized calculations for ^{2}H$(e,e'p)$ reactions at GeV energies

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(Received 27 September 2000; published 14 February 2001)

In the literature, one often finds calculations of $(e,e'p)$ reactions at GeV energies using the factorization approach. Factorization implies that the differential cross section can be written as the product of an off-shell electron-proton cross section and a distorted missing momentum distribution. While this factorization appears in the nonrelativistic plane wave impulse approximation, it is broken in a more realistic approach. The main source of factorization breaking is final state interactions. In this paper, sources of factorization breaking are identified and their numerical relevance is examined in the reaction ^{2}H$(e,e'p)$ for various kinematic settings in the GeV regime. The results imply that factorization should not be used for precision calculations, especially as unfactorized calculations are available.

DOI: 10.1103/PhysRevC.63.034609 PACS number(s): 24.10.−i, 25.30.Fj, 25.10.+s

I. INTRODUCTION

The study of electron scattering from nuclei has brought us many insights over the past decades, starting with Hofstaedter's classic inclusive electron scattering experiments which determined charge radii, and continuing to the modern day coincidence experiments which yield detailed information on the nuclear responses which allow us to study the short range structure of nuclei and the properties of nucleons in the nuclear medium.

In the past years, with the advent of high duty cycle machines with several GeV of beam energy, coincidence experiments with GeV energy and three-momentum transfers have become feasible and are carried out mainly at Jefferson Lab, and with some limitations in beam energy also at MAMI and Bates. These high energy and momentum transfers permit us to study the transition from hadronic degrees of freedom to quark-gluon or quark and flux tube degrees of freedom in the nucleus. Naturally, the interpretation of the data and the extraction of the desired information is feasible only with a detailed knowledge of the whole reaction. The general philosophy is that if we cannot describe a data set with the best ''conventional nuclear physics'' calculation, which would involve just hadronic degrees of freedom—one-body currents and meson exchange currents, isobars, initial and final state correlations—we would see evidence for genuine quark effects in the nucleus. The main practical problem for the time being is that for the realm of several GeV, where the chance to see quark effects is expected to be highest, the ''conventional nuclear physics'' calculations have not yet been fully developed.

The main problems are a consistent or at least realistic description of the final and initial hadronic states, proper inclusion of relativistic effects [1], especially the development of relativistic meson exchange currents [2], and isobar states. While all this has been achieved and worked out in great detail over the past 20 years for the regime of lower energy and three-momentum transfers of the order of a few hundred MeV, see, e.g., Refs. [3–6], a lot of work still needs to be done in the GeV regime. In this regime, one needs new techniques: the nature of the NN interaction changes and takes on a diffractive character, a description in terms of partial waves becomes impractical, particle production is possible and indeed the most frequent process, and relativity plays an important role.

Currently, even in the best available theoretical calculations, approximations are necessary. However, in many cases, even more approximations than necessary are used, and one of them, the approximation of *factorization*, is the topic of this paper. Numerical results for the validity of this approximation in $(e,e'p)$ reactions at GeV energies presented in this paper are for deuteron targets and have been obtained using the Argonne V18 wave function [7]. Note that the factorization approximation is in general not used in calculations at lower energies, see, e.g., Refs. [3–6].

This paper is organized as follows: after giving a brief overview over the general formalism and notation in the following, I will discuss the factorization approximation in Sec. II, and illustrate the mechanism of factorization breaking by final state interaction with the simple example of a strictly nonrelativistic one-body current. In Sec. III, I present numerical examples for factorization breaking with a relativistic current operator, and then summarize my results in the last section.

Brief overview over the formalism and notation. In order to compare the full calculation with the factorized approach, I start by introducing some notation and giving a brief summary of the basic formalism of $(e,e'p)$ reactions. More details can be found in Refs. [3,8].

The differential cross section in the lab frame is

$$\left(\frac{d\sigma^5}{d\epsilon'\, d\Omega_e\, d\Omega_N}\right)^h_{fi} = \frac{m_N\, m_f\, p_N}{8\pi^3\, m_i}\sigma_{\mathrm{Mott}}\, f_{\mathrm{rec}}^{-1} \times [v_L R^L_{fi} + v_T R^T_{fi} + v_{TT} R^{TT}_{fi} + v_{TL} R^{TL}_{fi} + h(v_{T'} R^{T'}_{fi} + v_{TL'} R^{TL'}_{fi})], \tag{1}$$

where m_i, m_N, and m_f are the masses of the target nucleus, the ejectile nucleon, and the residual system, p_N and Ω_N are the momentum and solid angle of the ejectile, ϵ' is the energy of the detected electron, and Ω_e is its solid angle. The helicity of the electron is denoted by h. The Mott cross section is

$$\sigma_{\rm Mott}=\left(\frac{\alpha\cos(\theta_e/2)}{2\varepsilon\sin^2(\theta_e/2)}\right)^2, \tag{2}$$

and the recoil factor is given by

$$f_{\rm rec}=\left|1+\frac{\omega p_x-E_x q\cos\theta_x}{m_i\, p_x}\right|. \tag{3}$$

The coefficients v_K are the leptonic coefficients, and the R_K are the response functions which are defined by

$$\begin{aligned}
R^L_{fi}&\equiv|\rho(\vec{q})_{fi}|^2,\\
R^T_{fi}&\equiv|J_+(\vec{q})_{fi}|^2+|J_-(\vec{q})_{fi}|^2,\\
R^{TT}_{fi}&\equiv 2\,\Re[J^*_+(\vec{q})_{fi}\,J_-(\vec{q})_{fi}],\\
R^{TL}_{fi}&\equiv -2\,\Re[\rho^*(\vec{q})_{fi}\,(J_+(\vec{q})_{fi}-J_-(\vec{q})_{fi})],\\
R^{T'}_{fi}&\equiv|J_+(\vec{q})_{fi}|^2-|J_-(\vec{q})_{fi}|^2,\\
R^{TL'}_{fi}&\equiv -2\,\Re[\rho^*(\vec{q})_{fi}\,(J_+(\vec{q})_{fi}+J_-(\vec{q})_{fi})],
\end{aligned} \tag{4}$$

where the $J_\pm$ are the spherical components of the electromagnetic current. For my calculations, I have chosen the following kinematic conditions: the z axis is parallel to $\vec{q}$, the missing momentum is defined as $\vec{p}_m\equiv\vec{q}-\vec{p}_N$, so that in plane wave impulse approximation (PWIA), the missing momentum is equal to the negative initial momentum of the struck nucleon in the nucleus, $\vec{p}_m=-\vec{p}$. I denote the angle between $\vec{p}_m$ and $\vec{q}$ by θ_m, and the term ''parallel kinematics'' indicates $\theta_m=0°$, ''perpendicular kinematics'' indicates $\theta_m=90°$, and ''antiparallel kinematics'' indicates $\theta_m=180°$. Note that both this definition of the missing momentum and the definition with the other sign are used in the literature. In this paper, I assume that the experimental conditions are such that either the kinetic energy of the outgoing nucleon and the angles of the missing momentum, θ_m, and the azimuthal angle ϕ_m, are fixed, or that the transferred energy ω, the transferred momentum $\vec{q}$, and the azimuthal angle ϕ_m, are fixed. In the former case, the transferred energy and momentum change for changing missing momentum, in the latter situation, the kinetic energy and polar angle of the outgoing proton change for changing missing momentum.

II. WHAT IS FACTORIZATION?

Factorization appears naturally in the nonrelativistic plane wave impulse approximation (PWIA) (see, e.g., Ref. [9]). There, one can describe the differential cross section for the full process as proportional to the product of the electron-proton cross section and the spectral function. The spectral function $S(E,\vec{p})$ describes the probability to find a proton with a certain energy E and momentum $\vec{p}$ inside the nucleus.

$$\frac{d^6\sigma}{d\epsilon'\,d\Omega_e\,d\Omega_N\,dE_N}=\frac{m_N\,m_f\,p_N}{E_f}\,\sigma_{eN}\;S(E,\vec{p}). \tag{5}$$

After integrating over the ejected nucleon's energy one finds

$$\frac{d^5\sigma}{d\epsilon'\,d\Omega_e\,d\Omega_N}=\frac{m_N\,m_f\,p_N}{m_i}\,\sigma_{eN}\,f^{-1}_{\rm rec}\,n(\vec{p}), \tag{6}$$

where $n(\vec{p})$ is the momentum distribution. The eN cross section is given by

$$\sigma_{eN}=\sigma_{\rm Mott}\sum_K v_K R_K^{\text{single nucleon}}, \tag{7}$$

and the single nucleon responses are related to the nuclear responses by

$$R_K^{\text{nucleus}}=(2\pi)^3\,R_K^{\text{single nucleon}}\,n(\vec{p}) \tag{8}$$

so that one has in total

$$\frac{d^5\sigma}{d\epsilon'\,d\Omega_e\,d\Omega_N}=\frac{m_N\,m_f\,p_N}{m_i}\,f^{-1}_{\rm rec}\,\sigma_{\rm Mott}\,n(\vec{p})\times\sum_K v_K R_K^{\text{single nucleon}}. \tag{9}$$

These simple and intuitive results are valid only under the special conditions of the nonrelativistic PWIA: (1) There is no final state interaction between the ejected nucleon and the residual nucleus. (2) The negative energy states present in a relativistic treatment are neglected. (3) The nucleon struck by the virtual photon is the one which is detected in coincidence with the electron. The last condition is commonly referred to as impulse approximation (IA).

The main culprit for breaking factorization is the final state interaction, which is always present in the general case. The factorization breaking introduced by relaxing the other two conditions are a bit more subtle. The size of the factorization breaking introduced by negative energy states and terms beyond the impulse approximation depends on the observable and kinematic region one considers. For the unpolarized cross section in the GeV region, these effects and the associated factorization breaking are small. The negative energy states which are present in the relativistic treatment lead to a breaking of factorization, as was pointed out in Refs. [10–12]. An illustrative example for the case of a deuteron target is shown in Ref. [1]. There, it was also shown that a relativistic, positive-energy current operator reproduces the fully relativistic, manifestly covariant result for missing momenta up to 400 MeV/c, and that deviations for higher missing momenta stem from off-shell effects and not from the negative energy states.

The assumption of the impulse approximation is quite good for high energy and momentum transfers. The additional graph present in the Born approximation (BA) describes the situation that the nucleus breaks up and a nucleon that did not interact with the virtual photon is detected. When high energies and momenta are transferred, it is very unlikely that the initially struck nucleon transfers all of its momentum to another nucleon in the final state interaction, or that another nucleon could have such high momentum already in the ground state. Therefore, in the region of GeV energy and momentum transfers relevant to this paper, a full Born approximation calculation differs from the impulse approximation calculation at most by a few percent.

As stated above, the final state interactions are the main source of factorization breaking in the kinematics considered in this paper. Nevertheless, one finds many calculations in the GeV regime assuming factorization, even in the presence of final state interactions [13–16]:

$$\frac{d^5\sigma^{\text{factorized}}}{d\epsilon'\, d\Omega_e\, d\Omega_N} = \frac{m_N\, m_f\, p_N}{m_i}\, \sigma_{eN}\, f_{\text{rec}}^{-1}\, n^{\text{distorted}}(\vec{p},\vec{p}_m), \tag{10}$$

where the distorted missing momentum distribution is given by

$$n^{\text{distorted}}(\vec{p},\vec{p}_m) = \frac{1}{(2\pi)^3} \overline{\sum_f}\, |\mathcal{M}_f|^2 \tag{11}$$

with

$$\begin{aligned}\mathcal{M}_f &= \langle f|\hat{S}_{\text{FSI}}|i\rangle \\ &= \int d\vec{R}_1, \ldots, d\vec{R}_{A-1}\, \Psi_f^*(\vec{R}_1, \ldots, \vec{R}_{A-2}) \\ &\quad \times \hat{S}_{\text{FSI}}(\vec{r}_1, \ldots, \vec{r}_A)\, \exp(i\vec{p}_m \vec{R}_{A-1})\, \Psi_i(\vec{R}_1, \ldots, \vec{R}_{A-1}).\end{aligned} \tag{12}$$

Here, $\hat{S}_{\text{FSI}}$ is the final state interaction operator. Jacobi coordinates are denoted by $\vec{R}$, the laboratory system coordinates are denoted by $\vec{r}$. The factorization approximation reduces the numerical effort as only one integral needs to be evaluated. In the unfactorized approach, every part of the electromagnetic current operator is evaluated separately, and the cross section is built up from the different response functions based on the matrix elements $\langle f|\hat{S}_{\text{FSI}}\, J_{em}|i\rangle$, as written out in Eq. (4). Of course, when assuming factorization, any difference in the behavior of the different response functions is neglected. There are some cases when this obviously cannot work, e.g., for the fifth response, $R_{TL'}$, which is measurable only with a polarized electron beam. In the absence of final state interaction, the fifth response is identically zero. While it is quite clear from this example that factorization does not work for polarization observables, the quality of the factorization approximation for the unpolarized cross section and response functions is not clear *a priori*, and is investigated in this paper. It largely depends on which components of the current operator are involved in calculating a specific observable.

A simple example of factorization breaking. In order to illustrate this point, I will consider the strictly nonrelativistic reduction of the electromagnetic one-body current operator. In Sec. III, I also include the full relativistic, positive-energy form as discussed in Ref. [17], but for the moment, the familiar nonrelativistic form is completely sufficient to illustrate why and where factorization fails. The nonrelativistic current operator consists of a charge part and of a magnetization current and a convection current:

$$J^o_{\text{nonrel}} = G_E,$$

$$J^\perp_{\text{nonrel}} = -\frac{i}{2\, m_N} G_M\, (\vec{q}\times\vec{\sigma}) + \frac{1}{m_N} G_E \left(\vec{p} - \frac{\vec{q}\cdot\vec{p}}{q^2}\vec{q} \right). \tag{13}$$

It is clear from the structure of the current operator that matrix elements which contain the charge operator, G_E, or the magnetization current, $-(i/2\, m_N)\, G_M\, (\vec{q}\times\vec{\sigma})$, differ only in the spin structure, but not in their structure in coordinate or momentum space. So, as long as the final state interaction operator is purely central, factorization is valid for the matrix elements of the charge operator and the magnetization current. However, the convection current contains a gradient operator in coordinate space, coming from the $\vec{p}_\perp$ in momentum space, and therefore, the matrix element of the convection current differs from the other matrix elements in coordinate or momentum space—it does not factorize. Now, the validity of factorization depends on the importance of the convection current contribution to the observable in question. As the key observable is the cross section, I will discuss the responses that contribute to it. The convection current obviously does not contribute to R_L, and the magnetization current is dominant in R_T, so one might expect a factorization breaking of only a few percent in R_T. So far, factorization would be acceptable, but there are also the interference responses R_{TL} and R_{TT} which contribute to the cross section. While the interference responses are at least an order of magnitude smaller for low missing momenta, they become comparable to R_L and R_T for higher missing momenta, and are therefore quite important for the cross section. From the spin structure, it is clear that R_{TL} in the nonrelativistic approach is proportional to the product of the charge operator and convection current matrix elements. Due to the presence of the convection current, it is not going to factorize. The same holds for R_{TT}, as this response contains only the convection current matrix elements. So in the general case, even for the simple nonrelativistic current operator, factorization will not hold for higher missing momenta. Factorization will be approximately valid in parallel and antiparallel kinematics, as the interference responses do not contribute there. In the next section, I will show results for a relativistic current operator. There, it will be obvious that factorization works even less well in the relativistic case. The relativistic operator contains additional, nonfactorizing operator structures and new coefficients for the old operator structures, which may contain kinematic factors like $\vec{p}^{\,2}$, which break factorization, too. For now, I will just show the results for the validity of factorization in the nonrelativistic approach for several different kinematic settings. In Fig. 1, I show the ratio of the cross section calculated in the factorization approximation to the unfactorized cross section in parallel and perpendicular kine-

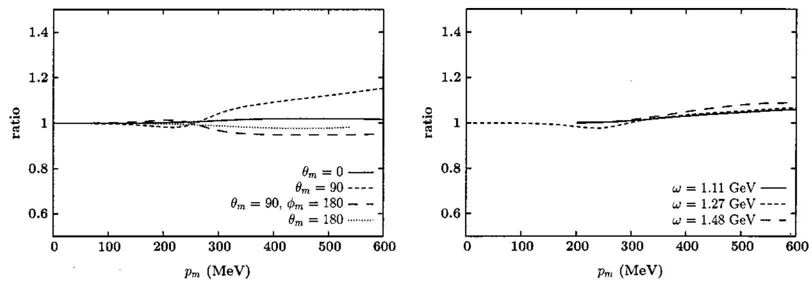

FIG. 1. The ratio of the cross section calculated in the factorization approximation to the unfactorized cross section for different kinematic settings. The nonrelativistic form of the current operator was employed. The left panel shows the ratio for fixed kinetic energy of 1 GeV of the outgoing proton and various fixed angles of the missing momentum, the right panel shows the ratio for fixed three-momentum transfer $q=2.0$ GeV/c and different values of the fixed energy transfers ω: 1.11 GeV, 1.27 GeV, and 1.48 GeV.

matics for fixed kinetic energy of 1 GeV for the outgoing proton, and for different values of the energy transfer ω for fixed three-momentum transfer $|\vec{q}|=2.0$ GeV/c.

From the left panel in Fig. 1, it is clear that the violation of factorization is smallest in (anti)parallel kinematics. There, the interference responses vanish and the breaking of factorization stems solely from the small convection current contribution to the transverse response. The resulting deviation from 1 of the ratio is of the order of a few percent only. In perpendicular kinematics, the deviations from 1 are much larger. The two curves shown for $\theta_m=90°$ differ by the azimuthal angle ϕ_m of the neutron. The ejected nucleon's azimuthal angle is $\phi=\phi_m+\pi$. The response R_{TL} implicitly contains a cos(ϕ) dependence. So, the only difference in the two cross sections is that in one case, the transverse-longitudinal interference response is added, and in the other case, it is subtracted from the sum of the other responses. The breaking of factorization increases with the missing momentum. This can be understood as FSI is mainly responsible for the factorization breaking. At the energies considered here, FSI is mainly diffractive and short ranged, so that it leads to large contributions at large missing momenta (see, e.g., Refs. [16,18]). Also, the interference responses become comparable to the other responses at larger p_m. In perpendicular kinematics, the deviations from 1 are considerable for $p_m>300$ MeV/c and range from 5% to 15%. A comparison of perpendicular kinematics and (anti)parallel kinematics clearly shows that the contribution of the interference responses leads to strong factorization breaking.

In the right panel of Fig. 1, the situation is depicted for fixed energy and three-momentum transfer. In such a setting, the angle of the missing momentum changes with p_m. Therefore, the interference responses are present in the cross section, and the factorization breaking is noticeable for $p_m>400$ MeV/c. For larger missing momenta, deviations range roughly from 6% to 9% and seem to grow with increasing transferred energy. This comes about as R_{TL} increases with the energy transfer.

After identifying the source of factorization breaking due to FSI and discussing the mechanism for the (too) simple case of a strictly nonrelativistic current operator, I proceed to give a realistic estimate of the validity of the factorization approximation in the next section.

III. REALISTIC NUMERICAL EXAMPLES

In this section, I use the relativistic, on-shell form, positive energy (OSPE) current operator discussed in Refs. [17,1]. Using this form of the current operator, I choose a specific off-shell prescription. Currently, there exists no microscopic description of the off-shell behavior that can be applied for a wide range of kinematic conditions—there are only *ad hoc* prescriptions [19,20,9]. Here, I use the popular ansatz of applying the electromagnetic current operator in its on-shell form. In principle, one can perform the same analysis of the validity of factorization using a current operator with a more general off-shell behavior. However, there is no reason to assume that factorization would work any better with a more general—and therefore more complicated—off-shell behavior. The OSPE current has the following form:

$$J^{\mu}(P\Lambda;P'\Lambda')\equiv\chi^{\dagger}_{\Lambda'}\overline{J^{\mu}}(P;P')\ \chi_{\Lambda} \tag{14}$$

with

$$\overline{J^0}=\rho=f_0(\xi_0+i\ \xi_0'(\vec{q}\times\vec{p})\cdot\vec{\sigma}),$$

$$\overline{J^3}=\frac{\omega}{q}\overline{J^0},$$

$$\overline{J^{\perp}}=f_0\Bigg(\xi_1\left[\vec{p}-\left(\frac{\vec{q}\cdot\vec{p}}{q^2}\right)\vec{q}\right]-i\Bigg\{\xi_1'(\vec{q}\times\vec{\sigma})+\xi_2'(\vec{q}\cdot\vec{\sigma})(\vec{q}\times\vec{p})+\xi_3'[(\vec{q}\times\vec{p})\cdot\vec{\sigma}]\left[\vec{p}-\left(\frac{\vec{q}\cdot\vec{p}}{q^2}\right)\vec{q}\right]\Bigg\}\Bigg). \tag{15}$$

Here, f_0,ξ_i,ξ_i' are all functions of ω,q,p^2; their explicit forms are

$$f_0\equiv\frac{1}{\mu_1\sqrt{1+\frac{\tau}{4(1+\tau)}\mu_2^2\delta^2}}, \tag{16}$$

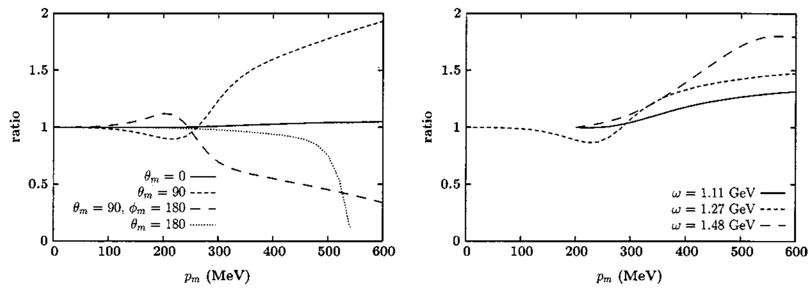

FIG. 2. The ratio of the cross section calculated in the factorization approximation to the unfactorized cross section for different kinematic settings. The full relativistic form of the current operator was employed. The left panel shows the ratio for fixed kinetic energy of 1 GeV of the outgoing proton and various fixed angles of the missing momentum, the right panel shows the ratio for fixed three-momentum transfer $q=2.0$ GeV/c and different values of the fixed energy transfers ω: 1.11 GeV, 1.27 GeV, and 1.48 GeV. Note that the scales in this figure are different from the ones in Fig. 1.

$$
\begin{aligned}
\xi_0 &= \frac{\kappa}{\sqrt{\tau}}\left[G_E+\frac{\mu_1\mu_2}{2(1+\tau)}\,\delta^2\tau G_M\right],\\
\xi_0' &= \frac{1}{\sqrt{1+\tau}}\left[\mu_1 G_M-\frac{1}{2}\,\mu_2 G_E\right],\\
\xi_1 &= \frac{1}{\sqrt{1+\tau}}\left[\mu_1 G_E+\frac{1}{2}\,\mu_2\tau G_M\right],\\
\xi_1' &= \frac{\sqrt{\tau}}{\kappa}\left(1-\frac{\mu_1\mu_2}{2(1+\tau)}\,\delta^2\right)G_M\,,\\
\xi_2' &= \frac{\lambda\sqrt{\tau}}{2\kappa^3}\,\mu_1\mu_2 G_M\,,\\
\xi_3' &= \frac{\sqrt{\tau}}{2\kappa(1+\tau)}\,\mu_1\mu_2[G_E-G_M].
\end{aligned}
\tag{17}
$$

The dimensionless variables are defined as follows:

$$
\begin{aligned}
\kappa &= \frac{|\vec{q}|}{2m_N},\\
\delta &= \frac{p_\perp}{m_N},\\
\tau &= \kappa^2-\lambda^2,\\
\lambda &= \frac{\omega}{2m_N}
\end{aligned}
\tag{18}
$$

and μ_1,μ_2 are shorthand for

$$
\mu_1 \equiv \frac{\kappa\sqrt{1+\tau}}{\sqrt{\tau}(\varepsilon+\lambda)} = \frac{1}{\sqrt{1+\dfrac{\delta^2}{1+\tau}}},
\tag{19}
$$

$$
\mu_2 \equiv \frac{2\kappa\sqrt{1+\tau}}{\sqrt{\tau}(1+\tau+\varepsilon+\lambda)} = \frac{2\mu_1}{1+\dfrac{\sqrt{\tau(1+\tau)}}{\kappa}\,\mu_1}.
\tag{20}
$$

For the reasons explained in Ref. [17], I refer to the operator associated with ξ_0 as zeroth-order charge operator, I call the term containing the ξ_0' first-order spin-orbit operator, the term containing ξ_1 first-order convection current, the term containing ξ_1' zeroth-order magnetization current, the term containing ξ_2' first-order convective spin-orbit term, and the term containing ξ_3' second-order convective spin-orbit term. In this paper, the current is used in this unexpanded, full form, which is possible as the evaluation of the FSI integrals takes place in momentum space. Some technical problems pertaining to the coordinate space treatment can be avoided this way.

The final state interaction is calculated using Glauber theory, see, e.g., Ref. [18]. For the purpose of this paper, considering the central, dominating part of the FSI is sufficient, as the breaking of factorization is strong already in this case. Spin-dependent FSI will break factorization even for the nonrelativistic forms of the charge operator and the magnetization current. However, it is quite small compared to the central FSI, and makes its major contribution to the smallest response, R_{TT}, and to the fifth response, which does not enter the unpolarized quantities I consider here. In other words, the case against factorization is obvious already from using only central FSI, and any spin-dependent FSI will only increase the problem. In this paper, I use only the central FSI for simplicity, although calculations including the spin-orbit FSI are available in the literature, see, e.g., Refs. [21,22,18]. Glauber theory is the main tool used for calculating FSI at GeV energies. The details of the employed FSI operators and the parameters used for it are not important for the current purpose, as the breaking of factorization depends only on the presence of final state interaction. The detailed form of the current operator is much more important, as can be seen from the comparison of Figs. 1 and 2.

In Fig. 2, I show the ratio of the factorized to the unfactorized cross section for the same kinematic conditions as in Fig. 1, but with the full relativistic OSPE current of Eq. (15).

Note that the scales are different in the two figures in order to accommodate the larger deviations from unity in the relativistic case. Comparing the two figures, it is obvious that the more complicated structure of the relativistic current operator leads to a much larger breaking of the factorization assumption in all considered kinematics.

The ratio in parallel and antiparallel kinematics (left panel) is still rather close to 1, deviations at higher missing momentum are of the order of 5%. The larger deviations in antiparallel kinematics occur close to the kinematic threshold (values larger than a certain $p_{m,\mathrm{max}}$ cannot be reached for a fixed proton momentum, which is implied by a fixed proton kinetic energy), and are not of great practical relevance. The only responses contributing in these kinematics are R_L and R_T, and the deviations from 1 now stem not only from the convection current, but also from the first- and second-order convective spin-orbit contributions to the transverse part of the current operator and from the spin-orbit operator in the charge operator. In addition, the factors ξ_0 and ξ_1' which multiply the zeroth-order charge operator and the magnetization current depend on $\delta^2=(p_\perp^2/m^2)$, and therefore not even the matrix elements $\langle f|\hat{S}_{\mathrm{FSI}}J_{em}|i\rangle$ of the zeroth-order charge operator and the magnetization current are proportional, and factorization does not hold at all in this relativistic setting. This is reflected by the larger amount of factorization breaking in the relativistic case, Fig. 2, compared to the nonrelativistic case, Fig. 1. Factorization breaking on the order of 5%, as observed in parallel kinematics, is not a terribly large effect. However, one needs to take into account that in an actual experiment, exactly parallel kinematics are not necessarily achieved, and that sometimes data corresponding to a larger range in acceptance may be combined in a single bin. For example, for $\theta_m=10°$, the deviations are rising to 6% at missing momenta around 400 MeV/c and to 11% at missing momenta around 600 MeV/c. For $\theta_m=15°$, the deviations are rising to 8% at missing momenta around 400 MeV/c and to 14% at missing momenta around 600 MeV/c. While this is still good enough for count rate estimates, one certainly does not want to incur this error in a precise theoretical prediction by making an entirely unnecessary approximation like factorization.

In perpendicular kinematics (dashed curves, left panel), the deviations from 1 are now large for missing momenta $p_m>$ 300 MeV/c, and they are non-negligible for missing momenta from 100 MeV/c to 300 MeV/c. In addition to the factorization breaking in R_L and R_T, the interference responses contribute strongly to the factorization breaking. The reason for the huge increase in factorization breaking going from the nonrelativistic to the relativistic treatment is that in the relativistic treatment, the interference responses pick up large contributions [17], and are now much more important in the cross section, specifically for large missing momenta.

When fixing transferred energy and three-momentum (right panel), the factorization breaking is present for missing momenta from 200 to 300 MeV/c, and very large for missing momenta beyond that. In these kinematics, the missing momentum angle varies, so that at any value of p_m, one can expect the interference responses to contribute. Therefore, the factorization breaking is large, even though not quite as large as in perpendicular kinematics, where the contribution of the interference responses is maximized. Again, one sees a large increase in factorization breaking going from the nonrelativistic case to the relativistic case, due to the interference responses.

IV. SUMMARY AND CONCLUSIONS

I have pointed out the sources of factorization breaking in $(e,e'p)$ reactions at GeV energies, and given numerical examples for the reaction $^2\mathrm{H}(e,e'p)$. Both in the (oversimplifying) nonrelativistic treatment and in the relativistic case, the factorization breaking is significant. The strength of the factorization breaking depends considerably on the chosen kinematics. Only at very low missing momenta, $p_m<100$ MeV/c, factorization works. In strictly parallel kinematics, the deviations are about 5% or smaller. However, one needs to keep in mind that in an experiment, a range of angles around 0° may contribute, and the deviations will be correspondingly larger, around 10% for large missing momenta. In perpendicular kinematics or for fixed transferred energy and three-momentum, the factorization assumption clearly fails for $p_m>$ 300 MeV/c.

While it is well known that factorization is insufficient when $(e,e'p)$ reactions at lower energies are calculated, this fact does not seem to be widely appreciated when it comes to GeV energy and momentum scales. This paper serves to draw attention to the fact that this approximation is lacking and that correct treatments of at least this problem are available, both in the GeV regime and in the transition region from lower to higher energies, see, e.g., Refs. [23,21,22,18,24,10]. It is especially important to be accurate as far as factorization is concerned, as there are other aspects of the problem, e.g., relativistic two-body currents, which are not yet worked out to a satisfactory degree, and which are going to cause uncertainties in the theoretical calculations. Note that many color transparency calculations, see, e.g., Refs. [14,13], assume factorization. The color transparency effects predicted for experiments at Jefferson Lab are relatively small, and the additional uncertainty introduced by assuming factorization may very well be of the same order of magnitude as the predicted effects and rather misleading in the interpretation of the data.

In this paper, I have considered the effects of factorization on the unpolarized cross section only. It is clear that, e.g., polarization observables or single responses are more sensitive to this type of approximation. One hardly needs to point out that wherever a dip is predicted for an observable in a factorized calculation, it will most likely be filled in when the correct, unfactorized calculation is performed. Furthermore, interesting new information will be obtained from separating the responses R_L+R_{TT}, R_T, and R_{TL}, which cannot be interpreted in a factorized approach.

ACKNOWLEDGMENTS

I would like to thank T. W. Donnelly, N. N. Nikolaev, and J. W. Van Orden for many stimulating discussions on the subject of electron scattering. I am grateful to Shalev Gilad for clarifying some points concerning the experimental setup which I had misunderstood. I would like to thank R. Schiavilla for providing a parametrization of the Argonne V18 deuteron wave functions. This work was supported by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement No. DE-AC05-84ER40150.

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