# $T=0$ and $T=1$ pairing and the formation of four-particle correlated structures in the ground states of $Z=N$ nuclei

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The $T=0$ and $T=1$ pairing correlations are discussed within the framework of two simple models: the SO(8) algebraic model and the single-$j$ model with a surface-$\delta$ interaction. The possibility of an approximate description of the ground-state wave functions in terms of correlated four-particle $T=0$, $S=0$ ($\alpha$-like) structures is investigated. The overlap of the approximate and the exact wave functions is shown to be larger than 0.93 for any relation between the $T=1$ and $T=0$ pairing strengths. The influence of the neutron-proton pairing interaction terms on the quadrupole sum rule and on the ground-state magnetic moments of the odd nuclei with $Z=N\pm 1$ is analyzed.

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## I. INTRODUCTION

The treatment of the $T=1$ pair correlations among nucleons of the same kind in heavy nuclei is significantly simplified because a simple approximate expression for the ground-state wave function of the even-even nucleus—the BCS wave function [1,2]—can be here employed. A specific feature of the BCS wave function is that it is not an eigenfunction of the particle number operator. However, it is possible to restore the corresponding symmetry by a projection. The projected wave function is represented by an expression in which a two-particle-creation operator, whose structure is determined by the pairing interaction matrix elements and by the mean-field single-particle energies, is repeatedly applied to the inert core wave function to get the corresponding number of particles. Such a description forms a basis for the generalized seniority scheme [3] or the alternative broken pair approximation approach [4,5]. Of course, it would be useful to find an analogous simple approximate expression for the ground-state wave function of an even-even nucleus also in the situation when the neutron-proton ($np$) pair correlations (either in the single $T=1$ channel or in both the $T=1$ and $T=0$ channels) are important.

A generalization of the $u$-$v$ Bogoliubov transformation to the case of the $np$ pairing has been discussed many times (see, e.g., [6–9]). Since the particle number and the isospin are not conserved, this procedure should be treated with care [10], particularly when both the $T=1$ and $T=0$ pair correlations are present.

A useful insight into the role of $np$ pairing can be obtained within the framework of exactly solvable algebraic models. The model with both $T=1$ and $T=0$ pairing channels has SO(8) symmetry [10–13]. A simple expression for the SO(8) ground-state wave function has been constructed in such a way that the maximum possible number of fermions form correlated four-particle $T=0$, $S=0$ structures [10]. These correlated four-particle structures are characterized by the same quantum numbers as $\alpha$ particles and this result is in a correspondence with an $\alpha$-cluster model applied to light and medium mass nuclei. However, it is necessary to note that the four-particle correlated structures that emerge are not real $\alpha$ particles. It is more appropriate to call them $\alpha$-like structures.

In the present paper, we investigate the structure of the wave functions of the ground and excited states of the system with $np$ pairing in more detail. A simple one-term expression written with the help of the creation operators of the four-particle correlated structures is discussed. The case with interplay of both $T=0$ and $T=1$ pairing interaction channels is discussed.

## II. SO(8) ALGEBRAIC MODEL

The Hamiltonian of the SO(8) algebraic model which includes both $T=1$ and $T=0$ pairing terms has the form [13]

$$\hat{H}=-(1+x)\sum_{\mu}(P^{\dagger}_{\mu})_f(P_{\mu})_f-(1-x)\sum_{\mu}(D^{\dagger}_{\mu})_f(D_{\mu})_f, \tag{1}$$

where

$$(P^{\dagger}_{\mu})_f=\sqrt{l+\tfrac{1}{2}}\sum_{m,\sigma,\tau,\tau'}C^{00}_{lml-m}C^{00}_{(1/2)\sigma(1/2)-\sigma}C^{1\mu}_{(1/2)\tau(1/2)\tau'}\times a^{\dagger}_{lm,(1/2)\sigma,(1/2)\tau}a^{\dagger}_{l-m,(1/2)-\sigma,(1/2)\tau'} \tag{2}$$

$$(D^{\dagger}_{\mu})_f=\sqrt{l+\tfrac{1}{2}}\sum_{m,\sigma,\sigma',\tau}C^{00}_{lml-m}C^{1\mu}_{(1/2)\sigma(1/2)\sigma'}C^{00}_{(1/2)\tau(1/2)-\tau}\times a^{\dagger}_{lm,(1/2)\sigma,(1/2)\tau}a^{\dagger}_{l-m,(1/2)\sigma',(1/2)-\tau}. \tag{3}$$

Above, $a^{\dagger}_{lm,(1/2)\sigma,(1/2)\tau}$ is the fermion creation operator describing a nucleon with orbital momentum $l$, spin projection $\sigma$, and isospin projection $\tau$. The parameter $x$, which varies from $-1$ (pure isoscalar pairing) to 1 (pure isovector pairing), governs the relative importance of isoscalar and isovector pairing in the Hamiltonian (1).

Similarly to [10], we employ the boson mapping procedure [14] to obtain the solution of the fermion problem with

 

Hamiltonian (1). Using the generalized Dyson boson representation of the fermion operators [14–16]

$$a_s^\dagger a_{s'}^\dagger \to b_{ss'}^\dagger - [\hat{C}, b_{ss'}^\dagger],$$

$$a_{t'} a_t \to b_{tt'},$$

$$a_s^\dagger a_{s'} \to \sum_p b_{sp}^\dagger b_{s'p}, \tag{4}$$

where

$$\hat{C} = \tfrac{1}{4} \sum b_{ss'}^\dagger b_{tt'}^\dagger b_{st} b_{s't'},$$

$$b_{ss'}^\dagger = -b_{s's}^\dagger,$$

$$[b_{ss'}, b_{tt'}^\dagger] = \delta_{st}\delta_{s't'} - \delta_{st'}\delta_{s't},$$

$$[b_{ss'}, b_{tt'}] = 0, \tag{5}$$

we can get a boson image of the Hamiltonian (1). In our case $s = lm, \frac{1}{2}\sigma, \frac{1}{2}\tau$. Introducing the boson operators

$$P_\nu^\dagger = \frac{1}{\sqrt{2}} \sum_{m,\sigma,\tau,\tau'} C_{lml-m}^{00} C_{(1/2)\sigma(1/2)-\sigma}^{00} C_{(1/2)\tau(1/2)\tau'}^{1\nu} \times b_{lm,(1/2)\sigma,(1/2)\tau,l-m,(1/2)-\sigma,(1/2)\tau'}^\dagger,$$

$$D_\mu^\dagger = \frac{1}{\sqrt{2}} \sum_{m,\sigma,\sigma',\tau} C_{lml-m}^{00} C_{(1/2)\sigma(1/2)\sigma'}^{1\mu} C_{(1/2)\tau(1/2)-\tau}^{00} \times b_{lm,(1/2)\sigma,(1/2)\tau,l-m,(1/2)\sigma',(1/2)-\tau}^\dagger, \tag{6}$$

we obtain the boson representations of the fermion operators $(P_\mu^\dagger)_f$, $(D_\mu^\dagger)_f$, $(P_\mu)_f$, and $(D_\mu)_f$:

$$(P_\mu^\dagger)_f \to \sqrt{2l+1}(P_\mu^\dagger - [\hat{C}, P_\mu^\dagger]),$$

$$(P_\mu)_f \to \sqrt{2l+1} P_\mu,$$

$$(D_\mu^\dagger)_f \to \sqrt{2l+1}(D_\mu^\dagger - [\hat{C}, D_\mu^\dagger]),$$

$$(D_\mu)_f \to \sqrt{2l+1} D_\mu, \tag{7}$$

where for $\hat{C}$ we get, from Eqs. (5),

$$\hat{C} = \frac{1}{2l+1}[\tfrac{1}{2}(\hat{n}_p + \hat{n}_d)^2 - \tfrac{1}{2}(\hat{n}_p + \hat{n}_d) + \tfrac{1}{4}[(P^\dagger \cdot P^\dagger)(D \cdot D) + (D^\dagger \cdot D^\dagger)(P \cdot P) - (P^\dagger \cdot P^\dagger)(P \cdot P) - (D^\dagger \cdot D^\dagger)(D \cdot D)]]. \tag{8}$$

Above,

$$\hat{n}_p \equiv \sum_\mu P_\mu^\dagger P_\mu,$$

$$\hat{n}_d \equiv \sum_\mu D_\mu^\dagger D_\mu,$$

$$P^\dagger \cdot P^\dagger = \sum_\mu (-1)^\mu P_\mu^\dagger P_{-\mu}^\dagger,$$

$$D^\dagger \cdot D^\dagger = \sum_\mu (-1)^\mu D_\mu^\dagger D_{-\mu}^\dagger.$$

Using Eqs. (7) and (8) we obtain the boson representation of the Hamiltonian (1):

$$\begin{aligned}\hat{H} = &-(1+x)(2l+1)\hat{n}_p - (1-x)(2l+1)\hat{n}_d \\ &+ (1+x)\hat{n}_p(\hat{n}_p + \hat{n}_d - 1) + (1-x)\hat{n}_d(\hat{n}_p + \hat{n}_d - 1) \\ &- \tfrac{1}{2}(1+x)(P^\dagger \cdot P^\dagger)(P \cdot P) - \tfrac{1}{2}(1-x)(D^\dagger \cdot D^\dagger)(D \cdot D) \\ &+ \tfrac{1}{2}(1+x)(D^\dagger \cdot D^\dagger)(P \cdot P) + \tfrac{1}{2}(1-x)(P^\dagger \cdot P^\dagger)(D \cdot D).\end{aligned} \tag{9}$$

The above boson Hamiltonian is equivalent to the boson Hamiltonian from [10] where a slightly different form has been used.

The Hermicity of Hamiltonian (9) can be restored by the following transformation which conserves commutation relations:

$$P_\mu^\dagger \to (1+x)^{1/4} P_\mu^\dagger, \quad P_\mu \to (1+x)^{-1/4} P_\mu,$$

$$D_\mu^\dagger \to (1-x)^{1/4} D_\mu^\dagger, \quad D_\mu \to (1-x)^{-1/4} D_\mu. \tag{10}$$

Applying the transformation (10) to the Hamiltonian (9), we get

$$\begin{aligned}\hat{H} = &-(1+x)(2l+1)\hat{n}_p - (1-x)(2l+1)\hat{n}_d \\ &+ (1+x)\hat{n}_p(\hat{n}_p + \hat{n}_d - 1) + (1-x)\hat{n}_d(\hat{n}_p + \hat{n}_d - 1) \\ &+ \tfrac{1}{2}(1+x)(P^\dagger \cdot P^\dagger)(P \cdot P) + \tfrac{1}{2}(1-x)(D^\dagger \cdot D^\dagger)(D \cdot D) \\ &+ \tfrac{1}{2}\sqrt{1-x^2}[(D^\dagger \cdot D^\dagger)(P \cdot P) + (P^\dagger \cdot P^\dagger)(D \cdot D)].\end{aligned} \tag{11}$$

The Hamiltonian (11) has been diagonalized and the eigenvectors have been constructed using the basis

$$|NSTk\rangle = (P^\dagger \cdot P^\dagger)^k (D^\dagger \cdot D^\dagger)^{(N-S-T)/2-k} (P_1^\dagger)^T (D_1^\dagger)^S |0\rangle. \tag{12}$$

Here $N$ is the total number of bosons, $S$ is the spin ($L=0$), and $T$ is the isospin. For simplicity we have considered as a basis the state vectors with the maximum values of the spin and isospin projections.

Having obtained exact eigenstates, we investigate the possibility to represent them within terms of $\alpha$-like two-boson $T=0$, $S=0$ structures. Those are obtained as the linear combinations of the operators $(P^\dagger \cdot P^\dagger)$ and $(D^\dagger \cdot D^\dagger)$:

$$A^\dagger = (P^\dagger \cdot P^\dagger)\cos\theta - (D^\dagger \cdot D^\dagger)\sin\theta \tag{13}$$

and the orthogonal one

$$A'^{\dagger}=(P^{\dagger}\cdot P^{\dagger})\sin\theta+(D^{\dagger}\cdot D^{\dagger})\cos\theta. \tag{14}$$

First, let us consider eigenstates with $S=0$ and $T=0$. The lowest state is approximated by

$$A^{\dagger N/2}|0\rangle. \tag{15}$$

The parameter $\theta$ in Eq. (13) is determined so as to get a maximum overlap of the state vector (15) with the lowest $S=0$, $T=0$ exact eigenstate. The next excited eigenstate with $S=0$, $T=0$ is approximated by the form

$$A^{\dagger N/2-1}A'^{\dagger}|0\rangle, \tag{16}$$

from which a projection on the vector (15) is subtracted. The higher eigenstates are constructed similarly by increasing the degree of the $A'^{\dagger}$ operator in the expression for the state vector and orthogonalizing this approximate state to the previously obtained lower-lying $\alpha$-correlated expressions. For every eigenstate we determine the value of $\theta$, which gives a maximum overlap of the corresponding approximate state vector with the exact one. Thus, the values of $\theta$ are state dependent.

The calculations have been done for a total number of nucleons equal to 16 (number of bosons $N=8$), orbital angular momentum $l=3$, and the eigenstates with isospin $T=0$ and spin $S=0$.

The results of the calculations are shown in Fig. 1. All overlaps of the exact and the corresponding approximate state vectors are larger than 0.985 for all values of $x$ between $-1$ and 1.[1] The values of $\theta$ do not depend on the state in the dynamical symmetry limits at $x=0$ [SU(4)], $x=1$ [$\mathrm{SO}^T(5)$], and $x=-1$ [$\mathrm{SO}^S(5)$]. Between the dynamical symmetry limits, the values of $\theta$ are state dependent but this dependence is not strong. The calculations show that if we take for all eigenstates the same (average) value of $\theta$, we get for the overlaps values equal to or larger than 0.97 for all values of $x$.

These results mean that the $\alpha$-like bi-bosons, which are the boson analogs of the four-fermion $T=0$, $S=0$ correlated structures, are very important structure units. A representation of the eigenstates is simplified significantly with the help of them. It stresses the important role of the four-nucleon $\alpha$-like correlations in the regime of $np$ pairing.

The lowest eigenvectors with a nonzero $T$ and $S$ can be described approximately by the expression

$$(A^{\dagger})^{(N-S-T)/2}(P_1^{\dagger})^T(D_1^{\dagger})^S|0\rangle. \tag{17}$$

Again, the calculations confirm the almost perfect overlap of this construction with the exact solution.

A similar discussion of the SO(8) wave functions in terms of $\alpha$-like structures has been performed in [10]. There, the ground-state wave functions have been analyzed. The variational principle has been employed and the genuine fermionic wave functions (both the exact and approximate) have been considered in [10].

[1]For $x=0$ [SU(4) symmetry limit], the overlap is 100%.

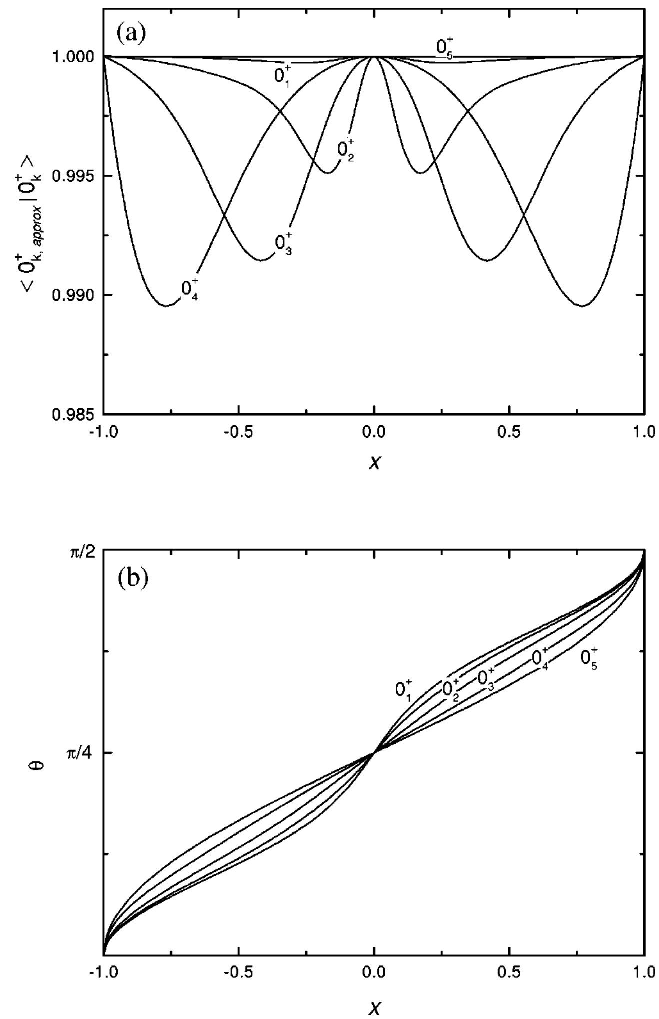


FIG. 1. (a) Dependence of the overlap of the exact and approximate state vectors of the $0^+$ states in the SO(8) algebraic model on the parameter $x$. (b) Dependence of the angle $\theta$ introduced in Eqs. (13) and (14) on $x$. Calculations are done for the total number of nucleons equal to 12.

In the present study, we search for the approximate wave function by the principle of maximal overlap. In view of the almost perfect agreement of the approximate and exact solutions, we have found a little difference between the maximal overlap and variational procedures.

The other aspect which makes a difference between present calculations and those of Ref. [10] is our employment of the bosonized and Hermitized SO(8) Hamiltonian and comparison of the exact solutions of this Hamiltonian with the bosonic analogs of the $\alpha$-like correlated wave functions. This approach does not agree completely with the genuine fermionic procedure but again the differences are not large and are of order up to $1/(l+\frac{1}{2})$.

## III. SINGLE-$j$-LEVEL WITH SURFACE $\delta$ INTERACTION

The SO(8) model comprising only the fermion pairs with the values of the angular momentum $J=0$ and $J=1$ is, of course, an idealization of the real situation. In fact, we should take into account also the fermion pairs with other values of the angular momentum $J$. As a rule, fermion pairs with $T=0$ and angular momentum $J$ equal to the maximum angular momentum allowed by a corresponding shell model configuration have low energy and play an important role. One should clarify whether the conclusions of the preceding section about the possibility of approximating the exact wave functions by four-particle $T=0$, $J=0$ correlated structures are also valid in the more general case.

We consider a model Hamiltonian with nucleons of both kinds interacting by the surface-$\delta$ interaction (SDI) $[1+y(\vec{\tau}_1\cdot\vec{\tau}_2)]\delta(|\vec{r}_1-\vec{r}_2|)\delta(r_1-R_0)$ and occupying isolated single-particle level with angular momentum $j$:

$$\hat{H}=-(1-x)\sum_J G_{0,J}\sum_M A^\dagger_{JM,00}A_{JM,00} -(1+x)\sum_J G_{1,J}\sum_{M,M_T} A^\dagger_{JM,1M_T}A_{JM,1M_T}. \quad (18)$$

Here,

$$A^\dagger_{JM,TM_T}=\sum_{m,m'} C^{JM}_{jm,jm'}\sum_{\tau,\tau'} C^{TM_T}_{(1/2)\tau,(1/2)\tau'}a^\dagger_{jm,(1/2)\tau}a^\dagger_{jm',(1/2)\tau'}, \quad (19)$$

where $a^\dagger_{jm,(1/2)\tau}$ is a creation operator of a nucleon with angular momentum $j$, its projection $m$, and isospin projection $\tau$. Another notation is

$$x=-\frac{1}{3}+\frac{2}{3}y,$$

$$G_{0,J}=[1-(-1)^J](f^2_{j,J}+g^2_{j,J}),$$

$$G_{1,J}=[1+(-1)^J]\frac{1}{3}f^2_{j,J} \quad (20)$$

and

$$f_{j,J}=\frac{2j+1}{\sqrt{2J+1}}C^{J0}_{j(1/2),j-(1/2)},$$

$$g_{j,J}=\frac{2j+1}{\sqrt{2J+1}}C^{J1}_{j(1/2),j(1/2)}. \quad (21)$$

The parameter $x$ regulates the relative role of the $T=1$ and $T=0$ pairing in the Hamiltonian (18).

The calculations have been done for the single-particle angular momentum $j=\frac{7}{2}$, total number of nucleons equal to 8 (midshell), and eigenstates with isospin $T=0$ and angular momentum $J=0$. The following fermionic basis has been used for exact diagonalization of the Hamiltonian (18):

$$|I,\alpha,\beta\rangle=[\nu^\alpha_I\times\pi^\beta_I]_0|0\rangle, \quad (22)$$

where

$$\nu^\alpha_{I,M}=(a^\dagger_n a^\dagger_n a^\dagger_n a^\dagger_n)^\alpha_{I,M},$$

$$\pi^\alpha_{I,M}=(a^\dagger_p a^\dagger_p a^\dagger_p a^\dagger_p)^\alpha_{I,M} \quad (23)$$

are orthonormal, fully antisymmetric basis vectors of neutron ($n$) and proton ($p$) subsystems, respectively. These vectors are constructed using coefficients of fractional percentage (CFP) with definite angular momentum $I$ and have an additional quantum number $\alpha$ to distinguish orthonormal states with the same $I$.

We investigate the possibility to approximate exact ground states of the Hamiltonian (18) by using $\alpha$-like correlated structures. The creation operator of the four-particle $T=0$, $J=0$ correlated structure is introduced as

$$A^\dagger=\tfrac{1}{2}\sum_\lambda c_\lambda[[a^\dagger_n\times a^\dagger_n]_\lambda\times[a^\dagger_p\times a^\dagger_p]_\lambda]_0. \quad (24)$$

In Eq. (24) the coefficients $c_\lambda$ should satisfy the relation

$$4c_0=\sum_{\lambda=\mathrm{even}}\sqrt{2\lambda+1}c_\lambda \quad (25)$$

in order to get a $T=0$ operator. Moreover, the coefficients $c_\lambda$ in Eq. (24) are normalized so that

$$\langle 0|AA^\dagger|0\rangle=1.$$

Some of the $c_\lambda$ coefficients are negative.

The lowest state with $J=0$ and $T=0$ is approximated by

$$|J=0,T=0\rangle_{\mathrm{app}}=\mathcal{N}^{-1/2}(A^\dagger)^2|0\rangle, \quad (26)$$

and the coefficients $c_\lambda$ are determined so as to get a maximum overlap of Eq. (26) with the corresponding exact eigenvector. In our case of $j=\frac{7}{2}$, there are only two free parameters in the expression (26).

As is shown in Fig. 2, the overlap of the exact and approximate ground-state vectors is larger than 0.93 for all values of the Hamiltonian parameter $x$. The dependence of the coefficients $c_\lambda$ on the parameter $x$ is displayed in Fig. 2(c). For the $x=1$ case with a pure $T=1$ pair interaction, the coefficient $c_\lambda$ with $\lambda=0$ is much larger than the other ones. This finding reflects the prevailing role of the $T=1$, $J=0$ pair and the usefulness of the seniority classification in that case.

Thus for the model with SDI's, we obtain a similar picture as in Sec. II where the SO(8) algebraic model has been considered. However, the model with SDI's is more realistic than the SO(8) model. What is especially important is that it includes fermion pairs with angular momentum $J\neq 0,1$.

## IV. INFLUENCE OF THE *np*-PAIR CORRELATIONS ON SOME PHYSICAL QUANTITIES

It was discussed above that the ground-state wave functions of the even-even $Z=N$ nuclei can be approximated quite well by expressions in which the maximum possible number of fermions forms the correlated four-particle $T=0$, $J=0$ structures. All information about the ground state

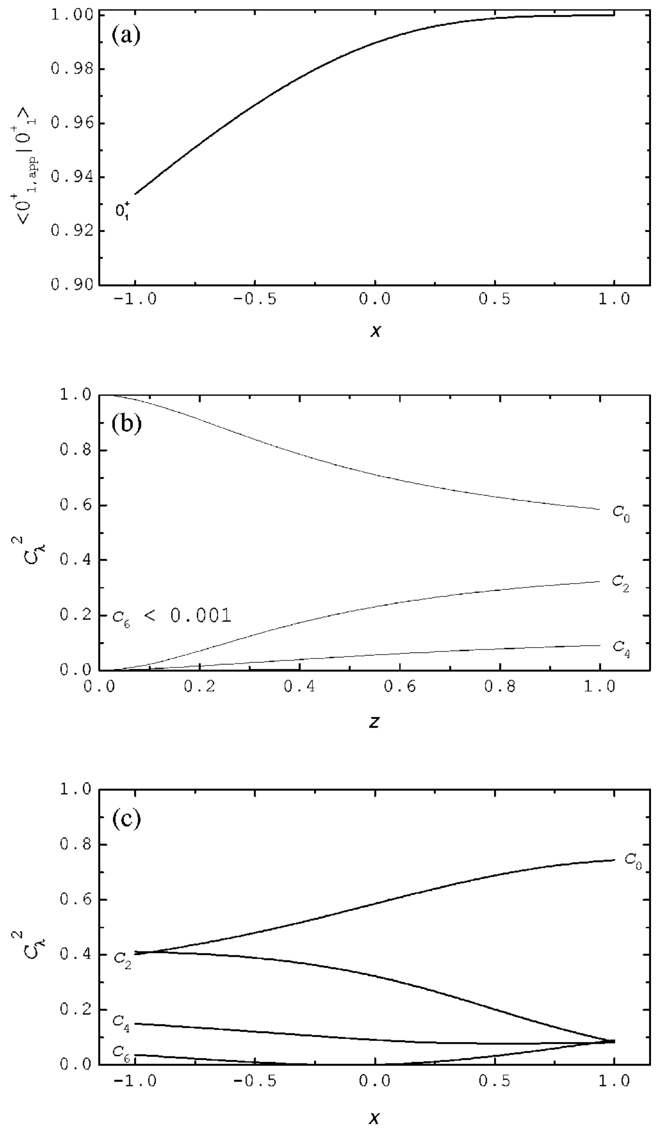


FIG. 2. (a) Dependence of the overlap of the exact and approximate ground-state vectors on the parameter $x$ in the single-$j$ model with SDI's. (b) Dependence of the squares of the coefficients of the ground-state vector multipole expansion on the parameter $z$ regulating a strength of the $np$ interaction. The results are shown for $x=0$. (c) The same as in (b) but for the dependence on the parameter $x$ $(z=1)$.

is thus contained in the structure of the four-particle creation operators (13) and (24). The important question arises as to whether relevant experimental quantities exist in which such an $\alpha$-like structure would be revealed and which would confirm the importance of the $np$ pairing degrees of freedom.

In the present section, we discuss the quadrupole sum rule calculated for the ground state of the even-even $Z=N$ nucleus and the ground-state magnetic moment of the odd nucleus with $Z=N\pm1$. The former quantity mainly characterizes the $E2$-transition probability from the ground to the first $2^+$ state. Both the $E2$ sum rule and magnetic moment depend on the angular momenta of the neutron and proton subsystems. As in the preceding section, we consider eight nucleons in the $j=\frac{7}{2}$ ($f_{7/2}$) shell. An additional odd particle is taken to be a neutron.

To see the effect of the different types of pairing correlations on the quantities mentioned above it would be convenient to have the possibility to switch off or to scale these correlations in the wave functions. However, using the wave functions given as expansions in the basis states (22) it is difficult to realize this idea. It is easier to use the corresponding scaling factor in the Hamiltonian. One of them is already presented in $H$. It is the parameter $x$ which regulates the relative role of the $T=1$ and $T=0$ pairing correlations. To investigate the influence of the $np$ pairing forces, we multiply $np$-interaction terms both in the $T=1$ and $T=0$ parts of the Hamiltonian (18) by a factor $z$ which varies from zero (absence of the $np$-pairing force) to one (full presence of the $np$-pairing force). Of course, the isospin invariance of the Hamiltonian is broken for $z\neq1$. Using this artificial procedure we get some insight into the effect of different types of pairing correlations. Although isospin is not significantly broken in nuclei, usually, we are dealing with the effective nuclear Hamiltonians derived for the restricted configurational spaces. In these cases different numbers of the proton and neutron configurations can be left outside a restricted configurational space, especially for $N\neq Z$ nuclei. For this reason the relation between proton-proton, neutron-neutron, and proton-neutron interaction constants in the effective Hamiltonian can differ from that in the total Hamiltonian.

As is seen from Fig. 2(b), the values of the coefficients $c_\lambda$ depend strongly on the parameter $z$. Of course, when $z$ is less than 1, the isospin invariance is broken and the relation (25) does not hold. For $z=0$, i.e., for only $nn$- and $pp$-pairing forces, the coefficient $c_0=1$ and the neutron and proton subsystems have zero angular momenta separately. This finding can be explained by the separation of the neutron and proton degrees of freedom and by the seniority conservation for the SDI in a single-$j$ shell. With increasing $z$, $c_0^2$ decreases and $c_\lambda^2$'s with $\lambda\neq0$ increase. Therefore with increasing $z$, the neutron and proton parts of the four-particle correlated structures possess nonzero angular momenta and can influence the ground-state magnetic moment and quadrupole sum rule.

In Fig. 2(c), similar correlations are observed between the values of $c_\lambda$ and the parameter $x$. However, as follows from Eq. (25), $c_0^2<1$ when $z=1$ even for $x=+1$, i.e., for only $T=1$ pairing force being present in the Hamiltonian. The reason for this is the presence of the $T=1$, $J=0$ neutron-proton pair correlations which create some angular momentum in the neutron and proton subsystems separately.

The square of the quadrupole proton operator $Q_{2\mu}$,

$$Q_{2\mu}^p=\sum_{m,m'} C_{jm',2\mu}^{jm} a_{jm,p}^\dagger a_{jm',p},$$

averaged over the ground state

$$\langle 0_1^+|(Q_2^p\cdot Q_2^p)|0_1^+\rangle, \tag{27}$$

gives us a value of the quadrupole sum rule. The dependence of the quantity (27) on $z$ and $x$ is illustrated in Figs. 3(a) and 3(b). No significant change of the sum rule with $x$ is observed. Therefore, the quadrupole sum rule cannot be used to get information on the $T=1$ and $T=0$ pairing competition.

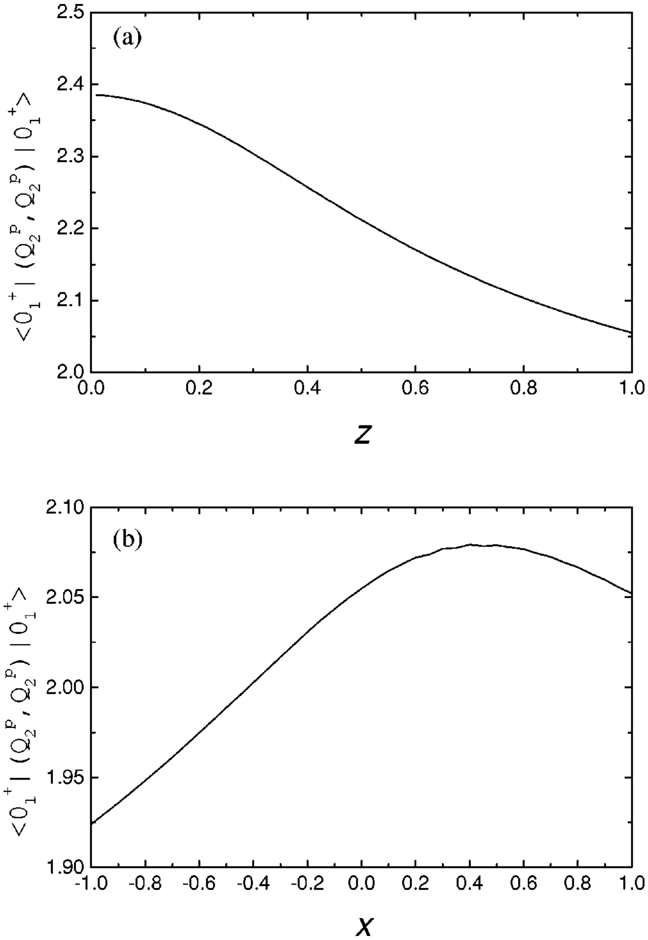


FIG. 3. Dependence of the ground-state quadrupole sum rule of the $N=Z$ nucleus on the parameters $z$ $(x=0)$ (a) and $x$ $(z=1)$ (b) in the single-$j$ model calculations.

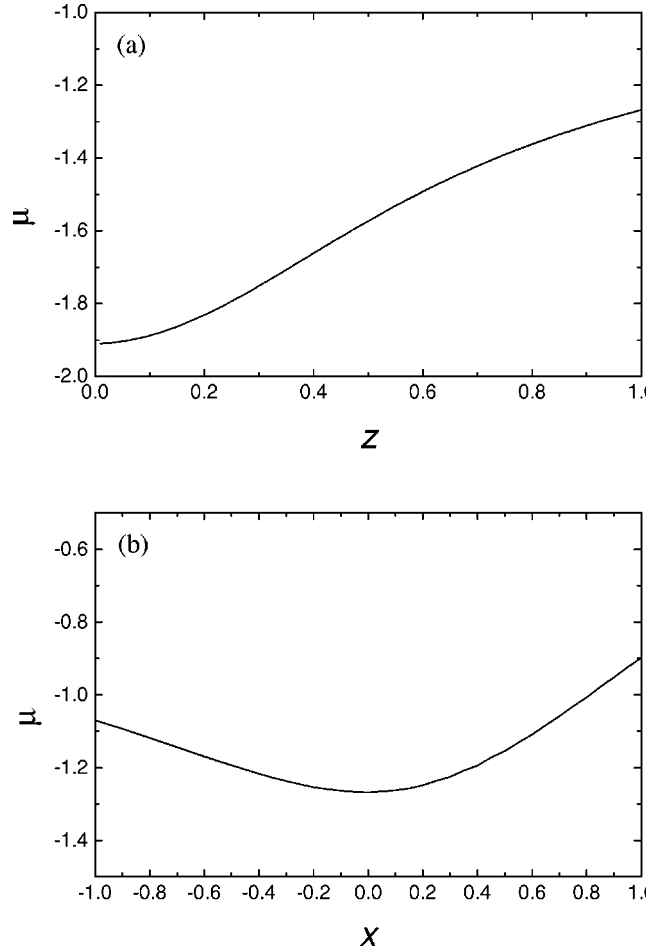


FIG. 4. The same as in Fig. 3 but for the ground-state magnetic moment of the $N=Z+1$ odd nucleus.

The sensitivity of this quantity on $z$ is more pronounced. The sum rule value is larger when the $np$ correlations are absent.

For the ground-state wave function of the odd nucleus adjacent to the $T=0$ line, we use the $\alpha$-like correlated form

$$|jm\rangle = \mathcal{N}_{\rm odd}^{-1/2} a_{jm}^{\dagger}(A^{\dagger})^2|0\rangle. \tag{28}$$

The magnetic moment operator is written as

$$\hat{\mu} = g_{j,n}\hat{j}_{n,z} + g_{j,p}\hat{j}_{p,z}, \tag{29}$$

with $\hat{j}_{n(p),z}$ being the $z$ component of the neutron (proton) angular momentum. For the $f_{7/2}$ shell, $g_{j,n}=-0.55$ and $g_{j,p}=1.66$.

Using the state vector (28) and its expansion in the basis (22), we obtain, for the magnetic moment by a direct calculation,

$$\mu \equiv \langle jj|\hat{\mu}|jj\rangle = g_{j,n}j - (g_{j,n}-g_{j,p})\frac{\mathcal{N}_{\rm even}}{\mathcal{N}_{\rm odd}}\frac{1}{(j+1)(2j+1)}\langle \hat{I}^2\rangle_p, \tag{30}$$

where $\langle \hat{I}^2\rangle_p$ is the average value of the squared angular momentum of the proton (the same for the neutron) subsystem of the even–even core described by the state vector $\mathcal{N}_{\rm even}^{-1/2}(A^{\dagger})^2|0\rangle$.

The first term in Eq. (30) is the single-particle magnetic moment of an odd neutron. The second term is the contribution of the protons and neutrons forming the four-particle $T=0$, $J=0$ correlated structures. Of course, this $Z=N$ core contribution appears due to the nonzero angular momenta of the proton and neutron subsystems of the core. It is proportional to the core average of the squared proton (neutron) angular momentum operator. If the proton and neutron subsystems of the even-even $Z=N$ nucleus have zero angular momenta, the core term is equal to zero and the magnetic moment approaches its single-particle value. This is illustrated in Fig. 4(a) where the dependence of the magnetic moment $\mu$ on the Hamiltonian parameter $z$ is displayed. For $z$ equal to 1, i.e., for the $np$ correlations fully included, the contribution of the core nucleons becomes essential.

The sensitivity of the magnetic moment to the parameter $x$, shown in Fig. 4(b), is weak and insufficient to study the $T=1$ and $T=0$ pairing competition. This finding is connected with the fact that both the isovector and isoscalar $np$-pair correlations introduce nonzero angular momentum into the neutron and proton subsystems and cause a deviation of the magnetic moment from the single-particle value.

## V. CONCLUSIONS

We have discussed the $T=0$ and $T=1$ pairing correlations within the framework of two simple models: the SO(8) algebraic model and the single-$j$ model with the surface-$\delta$ interaction. We investigate the possibility to represent the wave vector of the ground state by a simple one-term expression obtained by using the creation operator of the four-particle $T=0$, $J=0$ correlated structures. In the cases studied, an accuracy of better than 93% of the overlap of the exact and approximate wave functions has been obtained. Thus, the possibility opens to formulate an approximate approach, similar to the broken pair approximation for nuclei with like-nucleon pairing, to describe nuclei in which both like-nucleon and $np$-pair correlations are important.

Employing this approximation, we have investigated the influence of the different kinds of pair correlations on the ground-state magnetic moments of an odd nucleus and on the electric quadrupole sum rule. The magnetic moment appears to be quite sensitive to the presence of the $np$ correlations. However, both the magnetic moment and quadrupole sum rule are not sensitive enough to the competition between the $T=1$ and $T=0$ pair correlations.

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