Random phase approximation for odd nuclei and its application to the description of the electric dipole modes in ^{17}O

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(Received 22 September 2000; published 7 February 2001)

A consistent generalization of the random phase approximation (RPA) for odd nuclei is suggested. The derivation is based on the Green function method using the equation for the three-particle Green function. The model developed combines properties of both the standard RPA, and the particle-vibration coupling model. This gives a possibility to describe both the single-particle and collective parts of the excitation spectrum including giant resonances in the continuum and splitting of discrete collective states [particle (hole) $\otimes$ phonon multiplets] on a common basis. In the framework of this model, where the single-particle continuum is taken into account exactly, the $E1$ photoabsorption cross section and the isoscalar $E1$ resonance in ^{17}O are calculated. The comparative RPA calculations of the same $E1$ modes in ^{16}O nucleus are presented. The results obtained are compared with experiment for the $E1$ resonance in ^{16}O and ^{17}O. For the isoscalar $E1$ strength in ^{17}O we obtained an additional and noticeable low-lying contribution below 12.5 MeV caused by the odd neutron only.

DOI: 10.1103/PhysRevC.63.034304 PACS number(s): 21.60.Jz, 24.30.Cz, 27.20.+n

I. INTRODUCTION

One of the first realistic approaches, developed for the description of odd nuclei excitations, was the particle-vibration coupling model of Bohr and Mottelson [1]. The following problem is solved in this model and its further modifications and variants [2,3]: how is the excitation energy and the transition density of the isolated state of even-even core (i.e., of the phonon) changed under the influence of the odd particle or hole. This is the so-called problem of the particle-vibration multiplets splitting in odd nuclei. Another approach to the same question was worked out later (see Ref. [4]) within a self-consistent variant of the theory of finite Fermi systems (TFFS) [5].

However, this statement of the problem has at least two shortcomings. First, the state of the even-even core is supposed to be discrete, i.e., to have the normalizable wave function. This assumption limits application of the theory in the region of giant resonances and other states lying in the continuum. Second, if we take from the very beginning the single-particle states and the phonons as basis elements of the configurational space formed by the odd nucleus states, it is difficult to control the realization of the basis completeness and the Pauli principle fulfillment (see Ref. [6], and references therein where this problem is solved in the framework of the quasiparticle-phonon model).

Formally these shortcomings are absent in the variant of the TFFS [5] which treats the degenerate ground state of an odd nucleus on the average. Essentially this model is equivalent to the random phase approximation (RPA) with the changed occupation number of the valence level. This makes it possible to calculate the strength function for the transitions from the odd nucleus ground state into continuum (see, e.g., Ref. [7]), but it does not produce any multiplet splitting in view of averaged treatment of the ground state.

This makes one consider attempting to construct a model which would combine merits of both standard RPA and the particle-vibration coupling model. Here we suggest such a model which is termed the RPA for odd nuclei or, for brevity, odd RPA (ORPA). Our derivation is based on the Green function (GF) method. The exact formula for the response function of the odd nucleus is derived. The model is constructed on the base of this exact formula allowing control of the realization of Pauli principle fulfillment and related issues. The ORPA gives a possibility to describe both the single-particle and collective parts of the excitation spectrum, including giant resonances in the continuum, on a common basis. In addition, the splitting of particle (hole) $\otimes$ phonon multiplets appears in the model both for discrete collective states and for the collective states lying in the continuum, which is especially important for exotic nuclei. The $E1$ photoabsorption cross section and the isoscalar $E1$ resonance in ^{17}O have been calculated in the ORPA framework. The results obtained are compared with the experimental data available and with the RPA calculations of the same modes in ^{16}O nucleus.

II. THEORY

A. Description of the odd nucleus excitations in the Green function method: general relations

1. Basic formula for the three-particle Green function

Our consideration is based on the general form of the nuclear many-body Hamiltonian

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$$H=\sum_{12} h^0_{12}\, a^\dagger_1 a_2+\frac{1}{4}\sum_{1234} w_{12,34}\, a^\dagger_1 a^\dagger_2 a_4 a_3+W^{\rm MN}, \tag{2.1}$$

where $a^\dagger_1$ and a_1 are creation and annihilation operators of nucleons, $h^0=\mathbf{p}^2/2m$ is free single-particle Hamiltonian, $w_{12,34}$ is an antisymmetrized matrix element of the two-nucleon interaction, and the term $W^{\rm MN}$ contains the contributions of the many-nucleon interactions (three-nucleon one and others). The figures in subscripts hereinafter include quantum numbers of the single-particle wave functions of some complete basis set or single-particle variables of the coordinate representation (in the latter case, $1=\{\mathbf{r}_1,\sigma_1,\tau_1\}$, where σ_1 and τ_1 are the spin and isospin variables, respectively, and summation over 1 includes integration over $\mathbf{r}_1$).

The one-, two-, and three-particle Green functions we shall need are defined by

$$G_{12}=-i\langle 0|\, T\psi_1\psi^\dagger_2\,|0\rangle, \tag{2.2}$$

$$G^{(2)}_{12,34}=-i^2\langle 0|\, T\psi_1\psi_2\psi^\dagger_3\psi^\dagger_4\,|0\rangle, \tag{2.3}$$

$$G^{(3)}_{123,456}=-i^3\langle 0|\, T\psi_1\psi_2\psi_3\psi^\dagger_4\psi^\dagger_5\psi^\dagger_6\,|0\rangle, \tag{2.4}$$

where $\psi_1=\exp(iHt_1)\,a_1\exp(-iHt_1)$ is the time-dependent Heisenberg operator corresponding to a_1 (for brevity we shall not write out the time arguments of the Green functions and related quantities explicitly except when necessary; it will not lead to confusion because the time arguments are numbered by the same figures as the single-particle variables).

The above definitions are only formal ones. Practically the quantities G and $G^{(2)}$ are defined in the framework of the GF method by the well-known equations of motion, namely, by the Dyson and the Bethe-Salpeter equations (see Refs. [5,8] for more details). As a starting point for the practical definition of the three-particle Green function $G^{(3)}$ let us take the following ansatz (see Ref. [8] and also Ref. [9] where the GF formalism for the fermion systems with many-particle interactions was developed)

$$G^{(3)}_{123,456}=G_{36}G^{(2)}_{12,54}+\int d\,7\,d\,8\,R_{63,78}\frac{\delta G^{(2)}_{12,54}}{\delta G_{78}}. \tag{2.5}$$

Here and further the integrals mean

$$\int d\,1=\sum_1\int_{-\infty}^{\infty} d\,t_1, \tag{2.6}$$

quantity R is the particle-hole (p-h) response function

$$R_{12,34}=G^{(2)}_{23,14}-G_{21}G_{34}. \tag{2.7}$$

It satisfies the Bethe-Salpeter equation

$$R_{12,34}=-G_{31}G_{24}-i\int d\,5\,d\,6\,d\,7\,d\,8\,G_{51}G_{26}\,\mathcal{U}_{56,78}R_{78,34}, \tag{2.8}$$

where $\mathcal{U}$ is the irreducible kernel of this equation. $\mathcal{U}$ has a sense of an irreducible amplitude of the effective nucleon-nucleon interaction in the p-h channel and is defined by the ansatz

$$\mathcal{U}_{12,34}=i\frac{\delta\Sigma_{34}}{\delta G_{12}}=i\frac{\delta\Sigma_{21}}{\delta G_{43}}, \tag{2.9}$$

where Σ is the exact single-particle mass operator.

Using Eqs. (2.7)–(2.9) we get from Eq. (2.5) the following formula for the three-particle Green function which is one of basic formulas of our approach:

$$G^{(3)}_{123,456}=G^{(3)\ \rm disc}_{123,456}+G^{(3)\ \rm core}_{123,456}+G^{(3)\ \rm SP}_{123,456}+G^{(3)\ \rm rest}_{123,456}, \tag{2.10}$$

where

$$G^{(3)\ \rm disc}_{123,456}=-G_{36}G_{14}G_{25}-G_{14}R_{63,25}-G_{25}R_{41,36}, \tag{2.11}$$

$$G^{(3)\ \rm core}_{123,456}=-G_{36}R_{41,25}, \tag{2.12}$$

$$G^{(3)\ \rm SP}_{123,456}=-\int d\,1'\,d\,5'\,R_{41,31'}G^{-1}_{1'5'}R_{65',25}-\int d\,2'\,d\,4'\,R_{41,4'6}G^{-1}_{2'4'}R_{2'3,25}, \tag{2.13}$$

$$G^{(3)\ \rm rest}_{123,456}=i\int d\,1'\,d\,2'\,d\,3'\,d\,4'\,d\,5'\,d\,6'\,R_{41,4'1'}\,R_{2'5',25}\times(G^{-1}_{1'5'}\,\Gamma_{4'2',3'6'}+G^{-1}_{2'4'}\,\Gamma_{5'1',3'6'}+B^{(3)}_{4'1',2'5';3'6'})\,G_{33'}\,G_{6'6}. \tag{2.14}$$

Quantity $G^{(3)\ \rm disc}$ is the disconnected part of $G^{(3)}$, quantity $G^{(3)\ \rm core}$ contains the contributions of the even-even core, quantity $G^{(3)\ \rm SP}$ is the single-particle part, quantity $G^{(3)\ \rm rest}$ contains the rest (correction) terms. The quantity Γ in Eq. (2.14) is the exact two-particle scattering amplitude (in terms of TFFS, Ref. [5]) which is defined by the relation

$$R_{12,34}=-G_{31}\,G_{24}+i\int d\,5\,d\,6\,d\,7\,d\,8\,G_{51}G_{26}\Gamma_{56,78}G_{37}G_{84}. \tag{2.15}$$

Quantity $B^{(3)}$ is the effective three-particle interaction amplitude defined by the ansatz

$$B^{(3)}_{12,34;56}=\frac{\delta\mathcal{U}_{12,34}}{\delta G_{65}}-i\int d\,3'\,d\,4'\,d\,5'\,d\,6'\frac{\delta\mathcal{U}_{12,34}}{\delta G_{6'5'}}\times G_{3'5'}\,G_{6'4'}\,\Gamma_{3'4',56}. \tag{2.16}$$

The block of the diagrams corresponding to the amplitude $B^{(3)}_{12,34;56}$ is reducible in the channel ''1234 → 56.'' The diagrams corresponding to the terms from Eqs. (2.11)–(2.14) are presented in Figs. 1–3. The numbering of indices has been changed in conformity with Eq. (2.28) of the next sub-

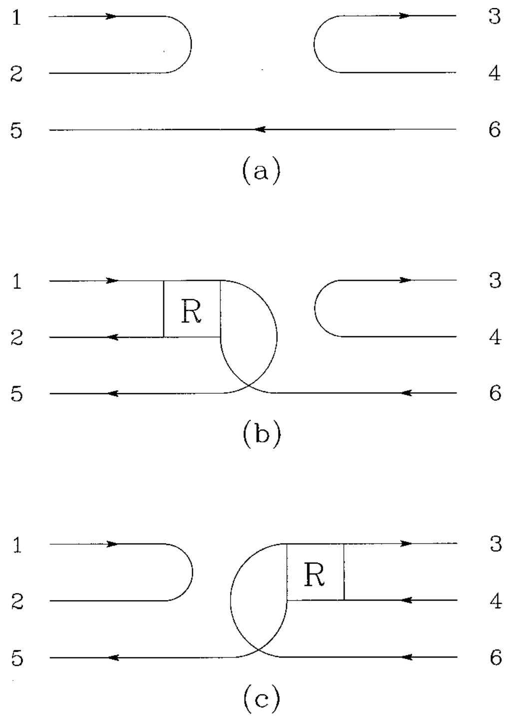

FIG. 1. Diagrammatic representation of the disconnected part $G^{(3)\ \mathrm{disc}}_{235,146}$ (a)–(c) of the three-particle Green function, defined by Eq. (2.11).

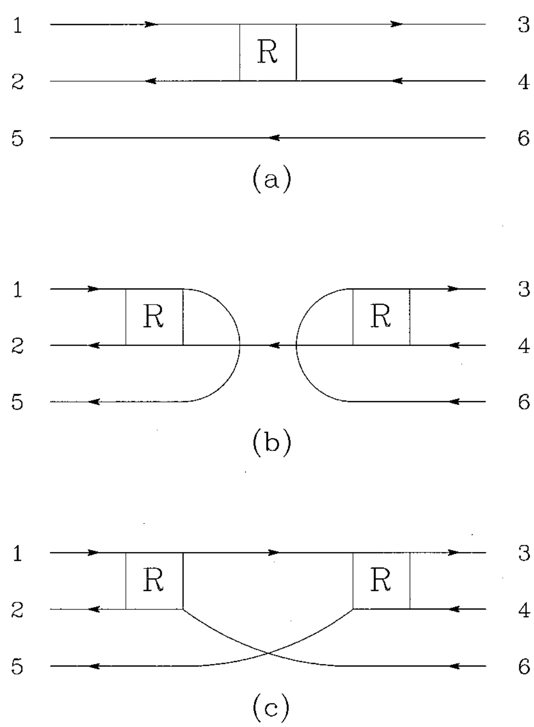

FIG. 2. Diagrammatic representation of the ''core'' part $G^{(3)\ \mathrm{core}}_{235,146}$ (a) and of the ''single-particle'' part $G^{(3)\ \mathrm{SP}}_{235,146}$ (b), (c) of the three-particle Green function which are defined by Eqs. (2.12),(2.13).

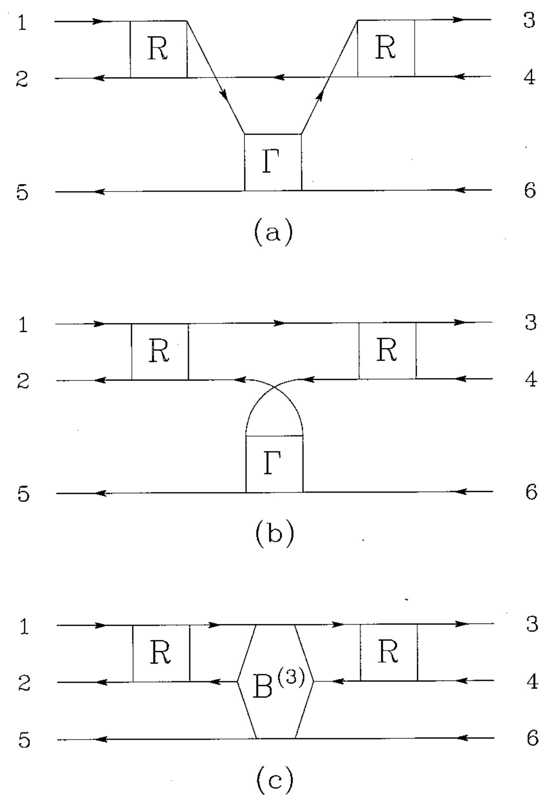

FIG. 3. Diagrammatic representation of the ''rest'' part $G^{(3)\ \mathrm{rest}}_{235,146}$ (a)–(c) of the three-particle Green function, defined by Eq. (2.14).

section. It should be noted that the formula (2.10) can be derived also by solving Eq. (15.4) from Ref. [8].

2. Projection onto the states of the odd nucleus

In what follows we suppose that $|0\rangle$ in Eqs. (2.2)–(2.4) is the ground state of the even-even nucleus, which contains N nucleons. Let us introduce notations: $|s\rangle$ is the eigenstate of the exact many-body Hamiltonian H [Eq. (2.1)] corresponding to the odd nucleus (more precisely: to the odd-even nucleus) which contains $N+\sigma_s$ nucleons, where $\sigma_s=\pm 1$ is index of the state $|s\rangle$. $|n\rangle$ is arbitrary eigenstate of H corresponding to the nucleus (even-even or odd-odd one) which contains N nucleons. In particular it may be $|n\rangle=|0\rangle$. $E_n \equiv E_n(N)$ and $E_s \equiv E_s(N+\sigma_s)$ are eigenvalues of H in the states $|n\rangle$ and $|s\rangle$:

$$H|n\rangle = E_n|n\rangle, \quad H|s\rangle = E_s|s\rangle. \tag{2.17}$$

Further,

$$\eta_1^s = \delta_{\sigma_s,+1}\langle 0|\, a_1\,|s\rangle + \delta_{\sigma_s,-1}\langle s|\, a_1\,|0\rangle, \tag{2.18}$$

$$\rho_{12}^{ss'} = \langle s|\, a_1^\dagger a_2\,|s'\rangle, \quad \rho_{12}^{nn'} = \langle n|\, a_1^\dagger a_2\,|n'\rangle, \tag{2.19}$$

$$\varepsilon_s = \sigma_s[E_s(N+\sigma_s) - E_0(N)], \quad \omega_n = E_n(N) - E_0(N). \tag{2.20}$$

Let us define using these notations

$$G^{(3)ss'}_{12,34} = -\delta_{\sigma_s,+1}\,\delta_{\sigma_{s'},+1}\langle s'|\,T\,\psi_1^\dagger\,\psi_2\,\psi_4^\dagger\,\psi_3\,|s\rangle - \delta_{\sigma_s,-1}\,\delta_{\sigma_{s'},-1}\langle s|\,T\,\psi_1^\dagger\,\psi_2\,\psi_4^\dagger\,\psi_3\,|s'\rangle. \tag{2.21}$$

The next step of our derivation is to extract the quantity $G^{(3)ss'}_{12,34}$ from the three-particle Green function. It will be shown that the Fourier transform of this quantity defines the response function of the odd nucleus we are interested in. In this subsection we present the projection technique which is most suitable for the extraction of the above quantity in our case and which is similar to the technique described in Ref. [10].

Let us define the projection operator acting in the space of the time variables

$$\Theta^t(t_1,t_2,t_3,t_4) = \theta(t-|t_1|)\,\theta(t-|t_2|)\times\theta(t-|t_3|)\,\theta(t-|t_4|), \tag{2.22}$$

where θ is the step function. Let us introduce the notation

$$G^{(3)t}_{123,456}(\alpha,\alpha';\sigma) = \Theta^t(t_1,t_2,t_4,t_5)\times G^{(3)}_{123,456}(t_3=\alpha'\sigma t,\, t_6=-\alpha\sigma t), \tag{2.23}$$

where $\alpha \geqslant 1$, $\alpha' \geqslant 1$, $\sigma = \pm 1$. Then using the definitions of Eqs. (2.4), (2.17), (2.18), (2.20), and (2.21) and introducing a complete set of intermediate states of the odd nucleus we get

$$G^{(3)t}_{123,456}(\alpha,\alpha';\sigma) = \Theta^t(t_1,t_2,t_4,t_5)\, i \times\sum_{ss'}\delta_{\sigma_s,\sigma}\,\delta_{\sigma_{s'},\sigma}\,\sigma_s\,\eta_6^{s*}\,\eta_3^{s'}\,G^{(3)ss'}_{41,25}\times\exp[-i(\alpha\,\varepsilon_s+\alpha'\,\varepsilon_{s'})\sigma t]. \tag{2.24}$$

Now let us note that the function

$$f(\omega,t) = \int_1^2 d\alpha\, e^{i\alpha\omega t} = \frac{2}{\omega t}\sin\!\left(\frac{\omega t}{2}\right) e^{i(3/2)\omega t} \tag{2.25}$$

has the properties

$$f(0,t)=1, \quad \forall\, t,$$

$$f(\omega,t) \underset{\omega t\to\pm\infty}{\longrightarrow} 0. \tag{2.26}$$

Then from Eq. (2.24) and from the obvious property

$$\Theta^t(t_1,t_2,t_3,t_4) \underset{t\to+\infty}{\longrightarrow} 1, \tag{2.27}$$

we get the final projection formula

$$G^{(3)ss'}_{12,34} = \sigma_s(i\langle\eta^s|\,\eta^s\rangle\langle\eta^{s'}|\,\eta^{s'}\rangle)^{-1}\sum_{56}\eta_6^s\,\eta_5^{s'*}\lim_{t\to+\infty}\times\int_1^2 d\alpha\, d\alpha'\, e^{i(\alpha\,\varepsilon_s+\alpha'\,\varepsilon_{s'})\sigma_s t}\, G^{(3)t}_{235,146}(\alpha,\alpha';\sigma_s). \tag{2.28}$$

Here

$$\langle\eta^s|\,\eta^{s'}\rangle = \sum_1 \eta_1^{s*}\,\eta_1^{s'} \tag{2.29}$$

and we adopt the natural assumption that the overlap integrals $\langle\eta^s|\,\eta^{s'}\rangle$ for the states with $\varepsilon_s=\varepsilon_{s'}$ form the diagonal matrix with indices s, s'.

3. Exact formula for the response function of the odd nucleus

Applying the projection technique of the previous subsection, namely, Eq. (2.28), to both parts of Eq. (2.10) we obtain the formula for quantity $G^{(3)ss'}_{12,34}$ in the time representation. Further we put $t_1=t_2+0$, $t_4=t_3+0$, $t_1+t_3=0$, $\tau=t_3-t_1$ and carry out Fourier transformation according to the following definition:

$$G^{(3)ss'}_{12,34}(\omega) = -i\int_{-\infty}^{\infty} d\tau \times\exp\!\left[i\left(\omega+\frac{1}{2}(\varepsilon_s-\varepsilon_{s'})\right)\tau\right] G^{(3)ss'}_{12,34}(\tau). \tag{2.30}$$

Then we transfer quantity $G^{(3)\ \mathrm{disc}}$ to the left-hand side (LHS) of the transformed Eq. (2.10) and obtain the following equality:

$$R^{\mathrm{odd}\ (ss')}_{12,34}(\omega) = R^{\mathrm{core}\ (ss')}_{12,34}(\omega) + R^{\mathrm{SP}\ (ss')}_{12,34}(\omega) + R^{\mathrm{rest}\ (ss')}_{12,34}(\omega). \tag{2.31}$$

Here quantity $R^{\mathrm{odd}\ (ss')}_{12,34}(\omega)$ is the result of subtracting the transformed disconnected part $G^{(3)\ \mathrm{disc}}$ from $G^{(3)ss'}_{12,34}(\omega)$. The physical sense of $R^{\mathrm{odd}\ (ss')}_{12,34}(\omega)$ is clarified from the spectral expansion of this quantity which is obtained by the method being analogous with the usual methods of the GF formalism (see Refs. [5,8]). After cumbersome transformations we get

$$R^{\mathrm{odd}\ (ss')}_{12,34}(\omega) = \sigma_s\sum_{s''}\times\left(\frac{\bar\rho^{s's''*}_{21}\,\bar\rho^{ss''}_{43}}{\omega+\varepsilon_{s''s'}-i\,\sigma_{s''}\times 0} - \frac{\bar\rho^{ss''}_{12}\,\bar\rho^{s's''*}_{34}}{\omega-\varepsilon_{s''s}+i\,\sigma_{s''}\times 0}\right), \tag{2.32}$$

$$\overline{\rho}_{12}^{ss'}=\delta_{\sigma_s,\sigma_{s'}}\,\sigma_s\,(\delta_{\sigma_s,+1}\rho_{12}^{s's}+\delta_{\sigma_s,-1}\rho_{12}^{ss'}-\delta_{ss'}\,\rho_{12}^{00}), \tag{2.33}$$

$$\varepsilon_{ss'}=\varepsilon_s-\varepsilon_{s'}. \tag{2.34}$$

Actually, $R_{12,34}^{\mathrm{odd}\ (ss')}(\omega)$ is the matrix response function of the odd nucleus, the matrix indices of this function being the indices s, s' marking the eigenstates of the exact many-body Hamiltonian H. The quantities $\overline{\rho}_{12}^{ss'}$ in Eq. (2.32) are the relative transition densities of the odd nucleus.

In the application of this formalism to description of the odd nuclei excitations it is sufficient to consider the response function $R_{12,34}^{\mathrm{odd}\ (ss')}(\omega)$ with the indices s, s' corresponding to the (degenerate) ground state of the given odd nucleus. In this case $\varepsilon_s=\varepsilon_{s'}$ and, according to Eq. (2.32) the poles of the function $R_{12,34}^{\mathrm{odd}\ (ss')}(\omega)$ coincide with the excitation energies, while the residues determine the transition probabilities. Thus in the following text we put $\varepsilon_s=\varepsilon_{s'}$ in all terms of Eq. (2.31).

Further, the quantity $R^{\mathrm{core}\ (ss')}(\omega)$ in Eq. (2.31) is defined by the equality

$$R_{12,34}^{\mathrm{core}\ (ss')}(\omega)=\delta_{ss'}\,R_{12,34}(\omega), \tag{2.35}$$

where $R(\omega)$ is the usual particle-hole response function of the even-even nucleus which was defined by Eq. (2.7). Its spectral expansion is as follows:

$$R_{12,34}(\omega)=\sum_{n\neq 0}\left(\frac{\rho_{21}^{n0*}\rho_{43}^{n0}}{\omega+\omega_n-i\times 0}-\frac{\rho_{12}^{n0}\rho_{34}^{n0*}}{\omega-\omega_n+i\times 0}\right). \tag{2.36}$$

It is convenient to write other quantities in Eq. (2.31) by introducing the (exact) vertex operator $\mathcal{T}$ in accordance with the definition of Ref. [11]. Namely, in the time representation we put

$$R_{12,34}=-\int d\,5\,d\,6\,G_{51}G_{26}\mathcal{T}_{56,34}. \tag{2.37}$$

The energy representation (Fourier transformation) of $\mathcal{T}$ is defined as

$$\mathcal{T}_{12,34}(\omega,\varepsilon)=\int_{-\infty}^{\infty}d\,\tau_1\,d\,\tau_2\,e^{i(\omega\tau_1+\varepsilon\tau_2)}\mathcal{T}_{12,34}(\tau_1,\tau_2),$$

$$\tau_1=t_3-t_1,\quad \tau_2=t_2-t_1,\quad t_4=t_3+0, \tag{2.38}$$

$$\mathcal{T}_{12,34}^{T}(\omega,\varepsilon)=\mathcal{T}_{43,21}(-\omega,\varepsilon+\omega). \tag{2.39}$$

In addition, we define the Fourier transforms of the quantities Γ and $B^{(3)}$ as

$$\Gamma_{12,34}(\omega,\varepsilon,\varepsilon')=\int_{-\infty}^{\infty}d\,\tau_1\,d\,\tau_2\,d\,\tau_3\,e^{i(\omega\tau_1+\varepsilon\tau_2+\varepsilon'\tau_3)}\times\Gamma_{12,34}(\tau_1,\tau_2,\tau_3), \tag{2.40}$$

$$B_{12,34;56}^{(3)}(\omega,\varepsilon,\varepsilon',\varepsilon'',\omega')=i\int_{-\infty}^{\infty}d\,\tau_1\,d\,\tau_2\,d\,\tau_3\,d\,\tau_4\,d\,\tau_5\times e^{i(\omega\tau_1+\varepsilon\tau_2+\varepsilon'\tau_3+\varepsilon''\tau_4+\omega'\tau_5)}\times B_{12,34;56}^{(3)}(\tau_1,\tau_2,\tau_3,\tau_4,\tau_5),$$

$$\tau_1=t_3-t_1,\quad \tau_2=t_2-t_1,\quad \tau_3=t_3-t_4,$$

$$\tau_4=t_5-t_6,\quad \tau_5=t_1-t_5. \tag{2.41}$$

With these definitions we have $(\varepsilon_s=\varepsilon_{s'})$:

$$R_{12,34}^{\mathrm{SP}\ (ss')}(\omega)=-\sigma_s\sum_{1'2'3'4'}\eta_{3'}^{s}\,\eta_{1'}^{s'*}\,G_{2'4'}(\varepsilon_s-\omega)\mathcal{T}_{12,1'2'}^{T}(\omega,\varepsilon_s-\omega)\,\mathcal{T}_{3'4',34}(\omega,\varepsilon_s-\omega)$$
$$-\sigma_s\sum_{1'2'3'4'}\eta_{2'}^{s}\,\eta_{4'}^{s'*}\,G_{3'1'}(\varepsilon_s+\omega)\mathcal{T}_{12,1'2'}^{T}(\omega,\varepsilon_s)\,\mathcal{T}_{3'4',34}(\omega,\varepsilon_s), \tag{2.42}$$

$$R_{12,34}^{\mathrm{rest}\ (ss')}(\omega)=-\sigma_s\sum_{56}\eta_6^{s}\,\eta_5^{s'*}\sum_{1'2'3'4'}\Bigg\{\int_{-\infty}^{\infty}\frac{d\,\varepsilon}{2\pi i}\mathcal{T}_{12,1'2'}^{T}(\omega,\varepsilon)\,\mathcal{T}_{3'4',34}(\omega,\varepsilon)\sum_{5'6'}(\Gamma_{5'6',56}(0,\varepsilon+\omega,\varepsilon_s)$$
$$\times G_{3'6'}(\varepsilon+\omega)\,G_{5'1'}(\varepsilon+\omega)\,G_{2'4'}(\varepsilon)+\Gamma_{5'6',56}(0,\varepsilon,\varepsilon_s)\,G_{3'1'}(\varepsilon+\omega)\,G_{2'6'}(\varepsilon)\,G_{5'4'}(\varepsilon))$$
$$+\int_{-\infty}^{\infty}\frac{d\,\varepsilon\,d\,\varepsilon'}{(2\pi i)^2}\mathcal{T}_{12,1'2'}^{T}(\omega,\varepsilon)\,\mathcal{T}_{3'4',34}(\omega,\varepsilon')\sum_{5'6'7'8'}B_{5'6',7'8';56}^{(3)}(\omega,\varepsilon,\varepsilon',\varepsilon_s,0)$$
$$\times G_{5'1'}(\varepsilon+\omega)\,G_{2'6'}(\varepsilon)\,G_{3'7'}(\varepsilon'+\omega)\,G_{8'4'}(\varepsilon')\Bigg\}. \tag{2.43}$$

Single-particle Green function $G(\varepsilon)$ entering these formulas is Fourier transform of the quantity defined by Eq. (2.2) (see, for example, Ref. [5]). It can be represented as the solution of the Dyson equation

$$G_{12}(\varepsilon)=\tilde{G}_{12}(\varepsilon)+\sum_{34}\tilde{G}_{13}(\varepsilon)\Sigma^{e}_{34}(\varepsilon)G_{42}(\varepsilon). \quad (2.44)$$

Here $\tilde{G}(\varepsilon)$ is the Green function which describes the motion of the nucleon in some mean field (nonlocal in general case). It satisfies the equation

$$\tilde{G}_{12}(\varepsilon)=G^{0}_{12}(\varepsilon)+\sum_{34}G^{0}_{13}(\varepsilon)\tilde{\Sigma}_{34}\tilde{G}_{42}(\varepsilon), \quad (2.45)$$

where $G^{0}(\varepsilon)=(\varepsilon-h^{0})^{-1}$ is the free Green function, $\tilde{\Sigma}$ is the energy-independent part of the exact mass operator (in particular it can have the form of the phenomenological or Hartree-Fock potential). In terms of the eigenfunctions $\varphi_{\lambda}(1)$ and eigenvalues ε_{λ} of the single-particle Hamiltonian $h_{12}=h^{0}_{12}+\tilde{\Sigma}_{12}$, the spectral expansion of $\tilde{G}(\varepsilon)$ reads

$$\tilde{G}_{12}(\varepsilon)=\sum_{\lambda}\frac{\varphi_{\lambda}(1)\varphi^{*}_{\lambda}(2)}{\varepsilon-\varepsilon_{\lambda}+i\,\sigma_{\lambda}\times 0}, \quad (2.46)$$

where $\sigma_{\lambda}=1-2n_{\lambda}$, $n_{\lambda}=0$ or 1 is the occupation number. Quantity $\Sigma^{e}(\varepsilon)$ in Eq. (2.44) is the energy-dependent part of the exact mass operator Σ [entering Eq. (2.9)] which describes the coupling of the single-particle and collective motions. In these notations, $\Sigma(\varepsilon)=\tilde{\Sigma}+\Sigma^{e}(\varepsilon)$. It is important to note that $\Sigma^{e}(\varepsilon)$ is the quantity of the second order in the interaction.

B. Random phase approximation for odd nuclei

1. Approximation for the response function of the odd nucleus

Now let us consider approximations. As has been noted in the Introduction, one of the aims of this work is to construct the model which would take into account specific features of the odd nucleus on one hand and which would be similar to the RPA for even-even nuclei, on the other. The main problem is that the straightforward application of the RPA to the description of odd nuclei excitations faces some difficulties due to the fact that the ground state of the odd nucleus is degenerate (if we give up its averaged treatment) and is not particle-hole phonon vacuum. In this connection it is important that the above-obtained exact formulas take into account these circumstances completely. On the other hand, it is important that RPA in fact is the first order theory. This means that only the first order contributions in the interaction are taken into account correctly in the RPA. The higher order contributions are incorporated only in part. We will use this property of the RPA as the main principle in the construction of our model. The equations of the preceding subsection enable us to do it because all of the first order contributions can be easily extracted from the terms in these equations. Nevertheless, we retain all higher order contributions of the RPA type in the response function of the even-even nucleus R and in the related quantities $\mathcal{T}$, Γ, and $B^{(3)}$. So, we adopt the following approximations within the accuracy of the first order in the interaction:

(i) The exact single-particle Green function G is replaced by the Green function $\tilde{G}$, exact amplitudes η^{s}_{1} and energies ε_{s} are replaced by single-particle wave functions $\varphi_{\lambda}(1)$ and energies ε_{λ}. So in the following we shall use single-particle index λ as the index of the odd nucleus state (instead of s).

(ii) The response function R and the vertex operator $\mathcal{T}$ are defined within the RPA: functions R and $\mathcal{T}$ are replaced by functions $\tilde{R}$ and $\tilde{\mathcal{T}}$, where $\tilde{R}$ is the solution of the RPA equation

$$\tilde{R}_{12,34}(\omega)=A_{12,34}(\omega)-\sum_{5678}A_{12,56}(\omega)\,\mathcal{F}_{56,78}\,\tilde{R}_{78,34}(\omega), \quad (2.47)$$

$\tilde{\mathcal{T}}$ is defined by the formulas

$$\tilde{\mathcal{T}}_{12,34}(\omega)=\delta_{13}\delta_{24}-\sum_{56}\mathcal{F}_{12,56}\,\tilde{R}_{56,34}(\omega), \quad (2.48)$$

$$\tilde{\mathcal{T}}^{T}_{12,34}(\omega)=\delta_{13}\delta_{24}-\sum_{56}\tilde{R}_{12,56}(\omega)\,\mathcal{F}_{56,34}. \quad (2.49)$$

Quantity $A(\omega)$ in Eq. (2.47) is the particle-hole propagator in the RPA. It is defined by the ansatz

$$A_{12,34}(\omega)=-\int_{-\infty}^{\infty}\frac{d\,\varepsilon}{2\pi i}\tilde{G}_{31}(\varepsilon+\omega)\,\tilde{G}_{24}(\varepsilon)$$
$$=\sum_{\lambda\lambda'}\varphi^{*}_{\lambda}(1)\varphi_{\lambda'}(2)\varphi_{\lambda}(3)\varphi^{*}_{\lambda'}(4)$$
$$\times\frac{n_{\lambda}-n_{\lambda'}}{\omega-\varepsilon_{\lambda}+\varepsilon_{\lambda'}+i\,\sigma_{\lambda}\cdot 0}. \quad (2.50)$$

Energy-independent amplitude $\mathcal{F}$ is the approximation for the energy-dependent amplitude $\mathcal{U}$ and will be specified in the following.

(iii) The exact scattering amplitude Γ is replaced by its approximation in the p-h channel $\tilde{\Gamma}$:

$$\tilde{\Gamma}_{12,34}(\omega)=\mathcal{F}_{12,34}-\sum_{5678}\mathcal{F}_{12,56}\,\tilde{R}_{56,78}(\omega)\,\mathcal{F}_{78,34}. \quad (2.51)$$

(iv) The functional derivative $\delta\mathcal{U}_{12,34}/\delta G_{65}$ in Eq. (2.16) for the amplitude $B^{(3)}$ is replaced (in the energy representation) by the quantity $\delta\mathcal{F}_{12,34}/\delta\rho_{65}$, where ρ_{12} is the approximation for the ground-state density matrix:

$$\rho_{12}=-i\,\tilde{G}_{12}(t_{1},t_{2}=t_{1}+0)=\sum_{\lambda}n_{\lambda}\,\varphi_{\lambda}(1)\varphi^{*}_{\lambda}(2). \quad (2.52)$$

Together with the approximations (i) and (iii) it leads to the following approximation $\tilde{B}^{(3)}$ for the quantity $B^{(3)}$:

$$\tilde{B}^{(3)}_{12,34;56}(\omega,\varepsilon,\varepsilon',\varepsilon'',\omega')=\sum_{5'6'}\tilde{\mathcal{T}}_{65,6'5'}(\omega')\frac{\delta\mathcal{F}_{12,34}}{\delta\rho_{6'5'}}. \tag{2.53}$$

Making use of these approximations and carrying out the integrations in Eq. (2.43) we obtain from Eq. (2.31) the formula for the response function of the odd nucleus within this approach. After a series of algebraic transformations it can be brought to the following ansatz:

$$\tilde{R}^{\mathrm{odd}\ (\lambda\lambda')}_{12,34}(\omega)=\tilde{R}^{\mathrm{core}\ (\lambda\lambda')}_{12,34}(\omega)+\tilde{R}^{\mathrm{SP}\ (\lambda\lambda')}_{12,34}(\omega)+\tilde{R}^{\mathrm{rest}\ (\lambda\lambda')}_{12,34}(\omega), \tag{2.54}$$

where $\varepsilon_{\lambda}=\varepsilon_{\lambda'}$ and

$$\tilde{R}^{\mathrm{core}\ (\lambda\lambda')}_{12,34}(\omega)=\delta_{\lambda\lambda'}\tilde{R}_{12,34}(\omega), \tag{2.55}$$

$$\tilde{R}^{\mathrm{SP}\ (\lambda\lambda')}_{12,34}(\omega)=\sum_{5678}\tilde{\mathcal{T}}^{T}_{12,56}(\omega)A^{\mathrm{SP}\ (\lambda\lambda')}_{56,78}(\omega)\tilde{\mathcal{T}}_{78,34}(\omega), \tag{2.56}$$

$$\begin{aligned}\tilde{R}^{\mathrm{rest}\ (\lambda\lambda')}_{12,34}(\omega)=\sum_{5678}[&\tilde{\mathcal{T}}^{T}_{12,56}(\omega)\,Q^{(\lambda\lambda')^{\dagger}}_{56,78}\,\tilde{R}_{78,34}(\omega)\\&+\tilde{R}_{12,56}(\omega)\,Q^{(\lambda\lambda')}_{56,78}\,\tilde{\mathcal{T}}_{78,34}(\omega)\\&-\tilde{R}_{12,56}(\omega)\,\Phi^{(\lambda\lambda')}_{56,78}\,\tilde{R}_{78,34}(\omega)],\end{aligned} \tag{2.57}$$

$$\begin{aligned}A^{\mathrm{SP}\ (\lambda\lambda')}_{12,34}(\omega)=-\sigma_{\lambda}[&\varphi_{\lambda}(2)\varphi^{*}_{\lambda'}(4)\tilde{G}_{31}(\varepsilon_{\lambda}+\omega)\\&+\varphi_{\lambda}(3)\varphi^{*}_{\lambda'}(1)\tilde{G}_{24}(\varepsilon_{\lambda}-\omega)],\end{aligned} \tag{2.58}$$

$$Q^{(\lambda\lambda')}_{12,34}=\delta_{31}[\rho,\overline{\rho}^{\mathrm{odd}\ (\lambda\lambda')}]_{24}-\delta_{24}[\rho,\overline{\rho}^{\mathrm{odd}\ (\lambda\lambda')}]_{31}, \tag{2.59}$$

$$Q^{(\lambda\lambda')^{\dagger}}_{12,34}=-Q^{(\lambda\lambda')}_{12,34}, \tag{2.60}$$

$$\begin{aligned}\Phi^{(\lambda\lambda')}_{12,34}=\sum_{56}&\frac{\delta\mathcal{F}_{12,34}}{\delta\rho_{56}}\overline{\rho}^{\mathrm{odd}\ (\lambda\lambda')}_{56}+(2\rho-1)_{31}\Sigma^{\mathrm{odd}\ (\lambda\lambda')}_{24}\\&+\Sigma^{\mathrm{odd}\ (\lambda\lambda')}_{31}(2\rho-1)_{24},\end{aligned} \tag{2.61}$$

$$\Sigma^{\mathrm{odd}\ (\lambda\lambda')}_{12}=\sum_{34}\overline{\rho}^{\mathrm{odd}\ (\lambda\lambda')}_{34}\mathcal{F}_{34,12}, \tag{2.62}$$

$$\overline{\rho}^{\mathrm{odd}\ (\lambda\lambda')}_{12}=\sigma_{\lambda}\sum_{34}\varphi_{\lambda}(3)\varphi^{*}_{\lambda'}(4)\tilde{\mathcal{T}}_{34,12}(0). \tag{2.63}$$

In the symbolic notations Eq. (2.54) can be rewritten in the following form:

$$\tilde{R}^{\mathrm{odd}}=\tilde{R}+Q^{\dagger}\tilde{R}+\tilde{R}\,Q+A^{\mathrm{SP}}-\tilde{R}\,\mathcal{F}A^{\mathrm{SP}}-A^{\mathrm{SP}}\mathcal{F}\tilde{R}-\tilde{R}\,\tilde{\mathcal{F}}^{\mathrm{odd}}\tilde{R}, \tag{2.64}$$

where

$$\tilde{\mathcal{F}}^{\mathrm{odd}\ (\lambda\lambda')}_{12,34}(\omega)=\tilde{\Phi}^{(\lambda\lambda')}_{12,34}-\sum_{5678}\mathcal{F}_{12,56}A^{\mathrm{SP}\ (\lambda\lambda')}_{56,78}(\omega)\mathcal{F}_{78,34}, \tag{2.65}$$

$$\tilde{\Phi}^{(\lambda\lambda')}_{12,34}=\Phi^{(\lambda\lambda')}_{12,34}+\sum_{56}(Q^{(\lambda\lambda')}_{12,56}\mathcal{F}_{56,34}+\mathcal{F}_{12,56}Q^{(\lambda\lambda')^{\dagger}}_{56,34}). \tag{2.66}$$

Despite our formal limitation of the first order in the interaction, the function $\tilde{R}^{\mathrm{odd}}(\omega)$ contains the higher order contributions owing to the diagram summation within the RPA in the right-hand side (RHS) terms of Eq. (2.64). Further, it is easy to see that function $\tilde{R}^{\mathrm{odd}}(\omega)$ defined by Eq. (2.64) contains second-order poles in term $\tilde{R}\,\tilde{\mathcal{F}}^{\mathrm{odd}}\tilde{R}$. One can get rid of these poles, that have no physical sense, by means of shifting the poles of function $\tilde{R}(\omega)$ which has the spectral expansion analogous to Eq. (2.36). This shift remains within the accuracy of the first order in the interaction and is determined by the matrix

$$\tilde{\mathcal{F}}^{\mathrm{odd}\ (\lambda\lambda')}_{nn'}=\sum_{1234}\tilde{\rho}^{n0*}_{12}\tilde{\mathcal{F}}^{\mathrm{odd}\ (\lambda\lambda')}_{12,34}(\omega_n)\tilde{\rho}^{n'0}_{34}, \tag{2.67}$$

where $\omega_n=\omega_{n'}$ is the energy of the excitation (phonon), $\tilde{\rho}^{n0}_{12}$ is its transition density. In the case of spherical symmetry of the core, the angular momentum coupling in the states $\lambda\otimes n$ and $\lambda'\otimes n'$ leads to different shifts for different multiplet members in the odd nucleus. A similar method of the multiplet splitting evaluation was developed and used in Ref. [4] with a different technique. However, the said method, if considered as that for the second order poles removal, is not suitable for the description of giant resonances, primarily due to presence of continuum. So we will consider another model in the following subsections.

Let us note that formula (2.67) coincides with the result for the multiplet splitting of Ref. [4]. The result of the particle-vibration coupling model is obtained from Eq. (2.67) if we put $\tilde{\Phi}=0$ in Eq. (2.65). The meaning of the corrections introduced by the additional quantity $\tilde{\Phi}$ (contributions of the many-particle diagrams) is discussed in Ref. [4].

2. Self-consistent approach

The response function of the odd nucleus $\tilde{R}^{\mathrm{odd}\ (\lambda\lambda')}_{12,34}(\omega)$ and other quantities in Eqs. (2.54)–(2.66) are determined completely if quantities $\tilde{\Sigma}_{12}$ and $\mathcal{F}_{12,34}$ are specified in some way. Let us remind the reader that the mass operator $\tilde{\Sigma}$ determines single-particle basis $\{\varphi_{\lambda},\varepsilon_{\lambda}\}$. Quantity $\mathcal{F}$ is the amplitude of the effective nucleon-nucleon interaction in the *p-h* channel. The simplest and at the same time reasonable way to define $\tilde{\Sigma}$ and $\mathcal{F}$ is a phenomenological one. It is used in the standard variant of TFFS [5] and in the applications of our model to be represented in the following section.

However, from a theoretical point of view, the self-consistent definition of $\tilde{\Sigma}$ and $\mathcal{F}$ is more preferable. Let us consider the main features of this approach. First of all we suppose that an energy functional $E[\rho]$ is given which depends on the density matrix ρ_{12} [defined by Eq. (2.52)] and describes in a reasonable approximation the ground-state energies of even-even nuclei. It may be for example the Skyrme-type functional [12] or the energy functional of a more general type [13,14].

In this case we can define quantities $\tilde{\tilde{\Sigma}}$ and $\mathcal{F}$ by the following relations:

$$h_{12}=\frac{\delta E[\rho]}{\delta\rho_{21}}=h^{0}_{12}+\tilde{\tilde{\Sigma}}_{12}, \quad \mathcal{F}_{12,34}=\frac{\delta^{2}E[\rho]}{\delta\rho_{12}\delta\rho_{43}}, \tag{2.68}$$

where h^0 is free single-particle Hamiltonian of Eq. (2.1). These definitions imply a self-consistency procedure in which density matrix ρ is defined by Eq. (2.52) with the eigenfunctions φ_λ of the Hamiltonian h from Eq. (2.68).

Let us suppose that functional $E[\rho]$ is invariant under the following transformation of density matrix:

$$E[e^{-i\alpha q}\rho e^{i\alpha q}]=E[\rho], \quad \forall\ \alpha, \tag{2.69}$$

where α is a real number, q is some single-particle operator and $q^{\dagger}=q$. In particular, Eq. (2.69) must be satisfied for operator q of any conserving quantity, for example, if q is the component of the momentum operator $\mathbf{p}$. Differentiating Eq. (2.69) with respect to α and putting $\alpha=0$, we obtain

$$Sp\left([q,\rho]\frac{\delta E[\rho]}{\delta\rho}\right)=0, \quad \forall\ \rho. \tag{2.70}$$

Functional differentiation of this identity with respect to ρ leads to the following equalities in the equilibrium point with account of Eqs. (2.68):

$$\sum_{34}\mathcal{F}_{12,34}[q,\rho]_{43}=[q,h]_{21}, \tag{2.71}$$

$$\sum_{1'}q_{1'1}\mathcal{F}_{1'2,34}-\sum_{2'}q_{22'}\mathcal{F}_{12',34}-\sum_{3'}\mathcal{F}_{12,3'4}q_{33'}+\sum_{4'}\mathcal{F}_{12,34'}q_{4'4}=\sum_{56}\frac{\delta\mathcal{F}_{12,34}}{\delta\rho_{56}}[\rho,q]_{56}. \tag{2.72}$$

Making use of Eqs. (2.58)–(2.63), (2.65), (2.66), (2.71), (2.72), we obtain, after a lengthy series of transformations, the following important result for any single-particle operators q and q' which satisfy the condition (2.69):

$$\sum_{1234}[q',\rho]_{12}\tilde{\mathcal{F}}^{\mathrm{odd}\ (\lambda\lambda')}_{12,34}(0)[q,\rho]_{43}=0. \tag{2.73}$$

The fulfillment of this equality is necessary for the absence of spurious states energy splitting and shift in the multiplets calculations in odd nuclei [see Eq. (2.67) and Ref. [4]].

3. Strength function and the analysis of approximations

Let us come back to the exact response function $R^{\mathrm{odd}\ (ss')}_{12,34}(\omega)$ defined by Eqs. (2.31), (2.32), and introduce the difference response function depending on two variables ω, ω':

$$D^{\mathrm{odd}\ (ss')}_{12,34}(\omega,\omega')=R^{\mathrm{odd}\ (ss')}_{12,34}(\omega)-R^{\mathrm{odd}\ (ss')}_{12,34}(\omega'). \tag{2.74}$$

This quantity determines the strength function $S^{\mathrm{odd}\ (s)}$ of odd nucleus which describes the strength distribution of excitations caused by an external field $V^{(0)}$. In the case of spherical symmetry we have

$$S^{\mathrm{odd}\ (s)}(E,\Delta)=\frac{1}{2\pi i}\frac{2J+1}{2j_s+1}\times\sum_{m_s}\sum_{1234}V^{(0)*}_{21}D^{\mathrm{odd}\ (ss)}_{12,34}(\omega,\omega^{*})V^{(0)}_{43}, \tag{2.75}$$

where $\omega=\sigma_s(E+i\Delta)$, $\sigma_s=+1$ for odd nucleus with added nucleon, $\sigma_s=-1$ for odd nucleus with removed nucleon, j_s and m_s are the angular momentum and its projection for odd nucleus ground state $|s\rangle$, J is the angular momentum of an external field $V^{(0)}$, Δ is the smearing parameter. Making use of the spectral expansion (2.32), it is easy to verify that at $\Delta\to+0$ and $E>0$ Eqs. (2.74), (2.75) coincide with the usual definition of the strength function

$$S(E)=\sum_{\nu}B_{\nu}\,\delta(E-\Omega_{\nu}), \tag{2.76}$$

where B_ν and Ω_ν are the reduced probabilities and the energies of excitations.

The aim of this and the next subsections is to obtain a formula for the function $D^{\mathrm{odd}\ (ss')}_{12,34}(\omega,\omega')$ and consequently for the strength function of odd nucleus within the aboveassumed approximations. As has been already mentioned above, the direct substitution for function $\tilde{R}^{\mathrm{odd}}(\omega)$ instead of exact function $R^{\mathrm{odd}}(\omega)$ in Eq. (2.74) is impossible because of the second-order poles problem. It can be shown that the strength function can take negative values in the vicinity of these poles. Nevertheless it appears easier to solve this problem just for the difference response function $D^{\mathrm{odd}\ (ss')}$, than for self-response function $R^{\mathrm{odd}\ (ss')}$. Before writing the resulting formulas, some notes should be made.

(i) All of the quantities both in Eq. (2.64) and in the following similar equations are supposed to be the matrix functions with the matrix multi-indices M,M', where $M=\{12,\lambda\}$. The quantities $\tilde{R}$ and $\mathcal{F}$ depend on λ as $\delta_{\lambda\lambda'}$. The production of the quantities implies the summation (integration) over intermediate multiindex M including index λ.

(ii) Let us divide quantity A^{SP} in Eq. (2.64) into two parts:

$$A^{\mathrm{SP}}=A^{(+)\mathrm{SP}}+A^{(-)\mathrm{SP}}, \tag{2.77}$$

where $(\varepsilon_\lambda=\varepsilon_{\lambda'})$

$$A^{(-)\mathrm{SP}(\lambda\lambda')}_{12,34}(\omega)=\sum_{56} P^{(\lambda\lambda')}_{12,56}A_{56,34}(\omega)=\sum_{56} A_{12,56}(\omega)P^{(\lambda\lambda')}_{56,34}, \tag{2.78}$$

$$P^{(\lambda\lambda')}_{12,34}=-[\delta_{31}\varphi_\lambda(2)\varphi^*_{\lambda'}(4)+\delta_{24}\varphi_\lambda(3)\varphi^*_{\lambda'}(1)], \tag{2.79}$$

quantity A is p-h propagator defined by Eq. (2.50), propagator $A^{(+)\mathrm{SP}}$ is defined as the difference: $A^{(+)\mathrm{SP}}=A^{\mathrm{SP}}-A^{(-)\mathrm{SP}}$ using Eqs. (2.58), (2.78). As is seen from Eqs. (2.78), function $A^{(-)\mathrm{SP}}(\omega)$ contains only p-h poles being the part of the poles of p-h propagator $A(\omega)$. The function $A^{(+)\mathrm{SP}}(\omega)$ contains only particle-particle poles of the type $\omega=\pm(\varepsilon_{\lambda''}-\varepsilon_\lambda)$, $n_\lambda=n_{\lambda''}=0$ if the odd nucleon is added, and contains only hole-hole poles of the type $\omega=\pm(\varepsilon_\lambda-\varepsilon_{\lambda''})$, $n_\lambda=n_{\lambda''}=1$ if the odd nucleon is removed. Using these definitions and Eqs. (2.47), (2.64), one can show that the function $\tilde{R}^{\mathrm{odd}}(\omega)$ contains the poles of only two types: the poles of response function $\tilde{R}(\omega)$, which coincide with the core excitation energies and represent the collective branch of excitations, and the poles of the propagator $A^{(+)\mathrm{SP}}(\omega)$, which coincide with the single-particle transitions energies and represent the single-particle branch of excitations. The poles of the propagator $A^{(-)\mathrm{SP}}(\omega)$ disappear in the function $\tilde{R}^{\mathrm{odd}}(\omega)$. In fact the propagator $A^{(-)\mathrm{SP}}(\omega)$ is a correction to the p-h propagator A in Eq. (2.47) caused by the Pauli principle. This correction is characterized by the small parameter $1/N$, where N is the number of particles in the fermion system. So further we shall neglect the Pauli principle corrections of the higher order $1/N^2$ in the response function part describing the collective branch of the odd nucleus excitations.

4. Model

Taking into account these notes let us come back to Eq. (2.74) and define the following function:

$$\tilde{D}^{\mathrm{odd}}(\omega,\omega')=\mathcal{T}^{(-)^T}(\omega)[A^{(-)}(\omega)-A^{(-)}(\omega')]\mathcal{T}^{(-)}(\omega')+\mathcal{T}^{(+)^T}(\omega)[A^{(+)}(\omega)-A^{(+)}(\omega')]\mathcal{T}^{(+)}(\omega'), \tag{2.80}$$

where

$$A^{(-)}=A+A^{(-)\mathrm{SP}},\quad A^{(+)}=A^{(+)\mathrm{SP}}, \tag{2.81}$$

$$\mathcal{T}^{(-)}=(1+\Lambda)^{-1}(1+Q-\mathcal{F}A^{(+)\mathrm{SP}}), \tag{2.82}$$

$$\mathcal{T}^{(-)^T}=(1+Q^\dagger-A^{(+)\mathrm{SP}}\mathcal{F})(1+\Lambda^T)^{-1}, \tag{2.83}$$

$$\Lambda=(\mathcal{F}+\tilde{\Phi}-\mathcal{F}A^{(+)\mathrm{SP}}\mathcal{F})A+\mathcal{F}A^{(-)\mathrm{SP}}, \tag{2.84}$$

$$\Lambda^T=A(\mathcal{F}+\tilde{\Phi}-\mathcal{F}A^{(+)\mathrm{SP}}\mathcal{F})+A^{(-)\mathrm{SP}}\mathcal{F}, \tag{2.85}$$

$$\mathcal{T}^{(+)}=1-\mathcal{F}A\mathcal{T}^{(-)}, \tag{2.86}$$

$$\mathcal{T}^{(+)^T}=1-\mathcal{T}^{(-)^T}A\mathcal{F}. \tag{2.87}$$

After a series of transformations one can prove that the function $\tilde{D}^{\mathrm{odd}}(\omega,\omega')$ differs from the result of the direct substitution for function $\tilde{R}^{\mathrm{odd}}(\omega)$ in Eq. (2.74) by the terms which are beyond accuracy of above-mentioned approximations. In this sense the method of the constructing of function $\tilde{D}^{\mathrm{odd}}$ is based on the same ideology as the method of matrix Padé approximations. The meaning of introducing the function $\tilde{D}^{\mathrm{odd}}(\omega,\omega')$ is as follows: it does not have second-order poles and leads to the positive-definite strength function. We shall consider the quantity $\tilde{D}^{\mathrm{odd}}$ as the approximation to the exact difference response function in our model. Since all preceding considerations have been based on the RPA for even-even core which is corrected owing to adding of the odd particle (hole), we shall denote this model as the odd random phase approximation (ORPA).

Let us make two notes. First, it is easy to prove after some algebra that if we neglect the influence of the odd particle (or hole) putting the quantities $A^{(+)\mathrm{SP}}$, $A^{(-)\mathrm{SP}}$, Q and $\tilde{\Phi}$ equal to zero in Eqs. (2.81)–(2.87), then Eq. (2.80) yields the RPA result for the difference response function:

$$\tilde{D}^{\mathrm{odd}\ (\lambda\lambda')}_{12,34}(\omega,\omega')=\delta_{\lambda\lambda'}[\tilde{R}_{12,34}(\omega)-\tilde{R}_{12,34}(\omega')]. \tag{2.88}$$

Second, substituting Eq. (2.78) into definition (2.84) of quantity Λ we get

$$\Lambda(\omega)=[\mathcal{F}+\mathcal{F}^{\mathrm{odd}}(\omega)]A(\omega), \tag{2.89}$$

where

$$\mathcal{F}^{\mathrm{odd}}(\omega)=\tilde{\Phi}+\mathcal{F}P-\mathcal{F}A^{(+)\mathrm{SP}}(\omega)\mathcal{F}. \tag{2.90}$$

Making use of the equation for the transition densities in RPA: $\tilde{\rho}^{n0}=-A(\omega_n)\mathcal{F}\tilde{\rho}^{n0}$, we obtain the following equality from Eqs. (2.65), (2.67), (2.78), and (2.90):

$$\tilde{\mathcal{F}}^{\mathrm{odd}\ (\lambda\lambda')}_{nn'}=\mathcal{F}^{\mathrm{odd}\ (\lambda\lambda')}_{nn'}=\sum_{1234}\tilde{\rho}^{n0*}_{12}\mathcal{F}^{\mathrm{odd}\ (\lambda\lambda')}_{12,34}(\omega_n)\tilde{\rho}^{n'0}_{34},$$

$$\omega_n=\omega_{n'}. \tag{2.91}$$

This means that in the first order in the interaction amplitude $\mathcal{F}^{\mathrm{odd}}$ the ORPA yields the same result for the multiplet splitting as the self-consistent TFFS [4].

The strength function of odd nucleus $\tilde{S}^{\mathrm{odd}\ (\lambda)}(E,\Delta)$ is defined in ORPA according to Eq. (2.75) with the substitution for function $\tilde{D}^{\mathrm{odd}}$ instead of exact function D^{odd}. The principal equation which we have to solve for calculating the strength function in ORPA is that for the effective external field $V^{\mathrm{odd}}(\omega)=\mathcal{T}^{(-)}(\omega)V^{(0)}$. It follows from the definition (2.82) that

$$V^{\mathrm{odd}}(\omega)=[1+Q-\mathcal{F}A^{(+)\mathrm{SP}}(\omega)]V^{(0)}-\Lambda(\omega)V^{\mathrm{odd}}(\omega). \tag{2.92}$$

Making use of the definition (2.89) one can see that excepting ''odd corrections,'' introduced by the quantities Q, $A^{(+)\mathrm{SP}}$, and $\mathcal{F}^{\mathrm{odd}}$, this equation coincides with that for effective field in TFFS [5] describing excitations in even-even nuclei. But in contrast to TFFS equation, Eq. (2.92) is more complex because it is written in the extended configurational space $\{p\otimes h\otimes\lambda\}$ determined by the multi-index $M=\{12,\lambda\}$ where λ is the index of the subspace of odd nucleus ground state wave functions with the fixed ε_λ.

In case of spherical symmetry we have to solve Eq. (2.92) and to calculate strength function $\tilde{S}^{\text{odd}\ (\lambda)}$ separately for each total angular moment j^* of the odd nucleus excitation, which satisfies the selection rules for a given external field $V^{(0)}$. The total strength function as defined by Eq. (2.75) is a sum of partial components. This results in a splitting of core excited states in odd nucleus, i.e., to the multiplet appearance.

On the other hand, the enlarging of the configurational space in Eq. (2.92) leads to the coupling of different channels of the core p-h excitations characterized by a different total angular moment L of the p-h pair which satisfies the triangle rule $\triangle(Lj_0j^*)$, where j_0 is the total angular moment of the odd nucleus ground state. So in the case of spherical symmetry the dimension of the equation system (2.92) is increased compared to TFFS in the number of times equal to the number of incorporated channels of core excitations.

III. CALCULATIONS OF THE $E1$ RESONANCES IN ^{17}O AND ^{16}O

A. Numerical details

As a demonstration of the ORPA application we will consider the calculation of $E1$ excitations in ^{17}O. The $E1$ photoabsorption cross section in ^{16}O being the even-even core for the nucleus ^{17}O was calculated within the self-consistent RPA, i.e., with a complete account for the single-particle continuum, in Refs. [15,16] (see also references therein). But to our knowledge, a consistent account for this continuum and other effects associated with the odd nucleon for ^{17}O is realized here for the first time. It is of interest to calculate the $E1$ photoabsorption cross section for light nuclei ^{17}O and ^{16}O where the role of the continuum is very important. It is also of great interest to do it in the framework of the same calculational scheme, particularly because, as was noticed in the recent detailed experimental study of the $E1$ resonance in ^{17}O [17], the relatively low (γ,tot) strength for ^{17}O remains unexplained (the authors [17] obtained the $\sigma(\gamma,\text{tot})$ as the sum of partial cross sections). See also references and other information dealing with experimental and theoretical studies of the $E1$ resonance in the ^{17}O nucleus in Ref. [17].

Here we use a calculation scheme with ''forced consistency.'' The variant of this approach intended for RPA calculations in even-even nuclei was developed in Ref. [18]. In ORPA we put in the previous formulas (2.82)–(2.85): $Q=0$, $\tilde{\Phi}=\mathcal{F}^{\text{rest}}$, where $\mathcal{F}^{\text{rest}}$ is a restoring amplitude which is taken in the separable form, adjusted in order to set the spurious state in the even-even core at exactly zero energy (see Ref. [18] for more details). The ''forced consistency'' enables one to define the mass operator $\tilde{\Sigma}$ and interaction amplitude $\mathcal{F}$ as independent quantities keeping zero energy of the spurious state.

In our calculations the mass operator $\tilde{\Sigma}$ was taken in the form of a realistic Woods-Saxon potential [19] (plus spin-orbital potential, plus Coulomb potential for protons). The fitting procedure was applied in order to obtain agreement between the experimental single-particle energies and the calculated levels that lie near the Fermi surface in the nucleus ^{16}O. The procedure consists of changing the well depth U_{jl} so that to get $\varepsilon_\lambda=\varepsilon_\lambda^{\text{exp}}$ for the given quantum numbers j,l and for $\varepsilon_\lambda^{\text{exp}}$ determined by Eq. (2.20) with quantities $E_0(^{16}\text{O})$, $E_\lambda(^{16}\text{O}\pm 1\ \text{nucleon})$ taken from experiment.

The Landau-Migdal interaction [5] was taken as $\mathcal{F}$ with parameters

$$f_{\text{ex}}=-2.373,\quad f_{\text{in}}=-0.002,\quad f'_{\text{ex}}=2.30,\quad f'_{\text{in}}=0.76,$$
$$g=0.05,\quad g'=0.96,\quad C_0=300\ \text{MeV fm}^3. \tag{3.1}$$

This parameter set coincides with that used in our previous calculations (see, e.g., Refs. [20,21,18]) except for parameter f_{ex} which was obtained from fitting the energy of the 3_1^- level in ^{16}O to the experimental value of 6.13 MeV. The fitted value $f_{\text{ex}}=-2.373$ also gives complete coincidence between the theoretical and the experimental value of $B(E3)=1.5\times 10^3\ e^2\ \text{fm}^6$ for this level. The characteristics of the 3_1^- level were calculated within the continuum RPA using the coordinate representation technique [22,15]. The ground state nuclear density in the interpolation formula of the interaction [5] was obtained making use of our Woods-Saxon single-particle wave functions which turned to be important for light nuclei (see Ref. [21]). It should be pointed out that the interaction parameter set (3.1) and the standard parameters of the Woods-Saxon potential [19] were used in all calculations. In addition, the above-described fitting procedure for single-particle levels was applied.

B. Results and discussion

1. Isovector E1 resonance

With the definitions described above we have calculated within ORPA the strength function $\tilde{S}^{\text{odd}\ (\lambda)}(E,\Delta)$ for isovector $E1$ excitations in ^{17}O and the $E1$ photoabsorption cross section σ_{E1} according to the formula

$$\sigma_{E1}(E)=\frac{16\pi^3}{9}\frac{e^2}{\hbar c}E\tilde{S}^{\text{odd}\ (\lambda)}(E,\Delta). \tag{3.2}$$

The external field operator in the case under consideration is defined by

$$V_{12}^{(0)}=e_{\tau_1}\,\delta_{\sigma_1,\sigma_2}\delta_{\tau_1,\tau_2}\delta(\mathbf{r}_1-\mathbf{r}_2)r_1Y_{1\,\mu}\left(\frac{\mathbf{r}_1}{r_1}\right), \tag{3.3}$$

where e_τ are the nucleon effective charges in the center-of-mass reference frame $e_p=N/A$, $e_n=-Z/A$. Equation (2.92) for the effective field $V^{\text{odd}}(\omega)$ was solved in the coordinate representation so as to take into account the single-particle continuum completely. In order to simulate the experimental resolution the nonzero value of the smearing parameter Δ should be used in the calculations (in addition, this parameter

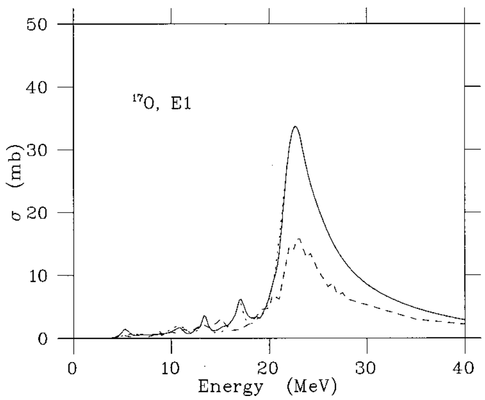

FIG. 4. The $E1$ photoabsorption cross section for ^{17}O nucleus calculated in ORPA with three channels of core excitations 1^-, 3^-, and 5^- (solid line) and with one entrance channel 1^- (dotted line). The smearing parameter Δ is equal to 300 keV. The dashed line presents experimental data from Ref. [17].

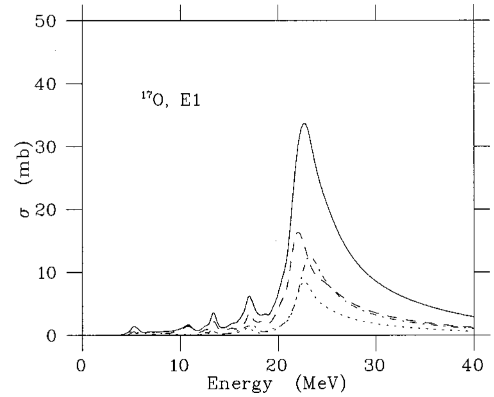

FIG. 5. Same as Fig. 4, but the dashed line presents the $\frac{7}{2}^-$ partial component of the calculated cross section, dash-dotted line, $\frac{5}{2}^-$, and dotted line, $\frac{3}{2}^-$.

imitates contribution of complex configurations and its introduction decreases strongly the numerical difficulties). In all calculations of the isovector $E1$ excitations we took the value $\Delta = 300$ keV which is approximately equal to the experimental resolution used in the measurements of ^{17}O [17].

Because the quantum numbers j^π of the ^{17}O ground state are $\frac{5}{2}^+$ (single-particle orbital $1d_{5/2}$), there exist three partial components of $E1$ strength function $\frac{3}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$. Consequently there are the following possible channels of core excitations with normal parity which satisfies the triangle rule $\triangle(Lj_0j^*)$: 1^- and 3^- for $\frac{3}{2}^-$ component, 1^-, 3^- and 5^- for $\frac{5}{2}^-$ and $\frac{7}{2}^-$ components. Two cases were compared in the calculations: (i) full calculation with incorporating of all three possible channels of electric type, (ii) calculation with incorporating of only entrance channel 1^-. The result is presented in Fig. 4. The effect of channel coupling turns to be negligible for this calculation. But this is a consequence of the isovector nature of the external field because the additional channels 3^- and 5^- are important mainly for isoscalar excitations (see below). Partial decomposition of the calculated total photoabsorption cross section in ^{17}O is shown in Fig. 5. In fact there is triplet of the giant resonances but the members of this triplet cannot be resolved in the total cross section in view of its large widths.

The available experimental data for $E1$ photoabsorption in ^{17}O are presented also in Fig. 4 [the experimental curve is sketched using Fig. 6(b) of Ref. [17]]. As can be seen, the discrepancy between our theory and the experiment is very large. Mainly this concerns the value of $\sigma_{\max}$ because the positions of the resonance centroids are close. In order to achieve a better understanding of the situation we carried out the calculation of the $E1$ photoabsorption cross section in ^{16}O nucleus within the RPA making use of the above described calculation scheme. The result is shown in Fig. 6. The experimental data in this figure is the result of multi-Lorentzian parametrization of the cross section from Ref. [23]. In this case the agreement with the experiment is much more satisfactory than for ^{17}O. It can be seen that RPA cannot yield a fine structure in the region of giant resonance caused by the complex configurations but on the average the envelope is reproduced fairly well. So one can think that an improvement of the ORPA by means of taking into account additional complex configurations will not change the picture radically, especially as our results show that the effect of channel coupling in ORPA is small for isovector excitations in ^{17}O.

So, we suppose that the reason for a large discrepancy between the ORPA and the experiment in the case under consideration can be in the experimental technique used in Ref. [17]. In Ref. [23] it was pointed out that the method of partial cross sections summing, which was used in Ref. [17], yields systematically smaller integrated cross sections as compared with other methods (see Table I from Ref. [23]). The results, related with our discussion, are represented in Table I of our paper. In this table elastic photon scattering data for ^{16}O nucleus from Ref. [23] is represented. These data are in a reasonable agreement with our calculations. Unfortunately, we do not know any analogous data for ^{17}O nucleus. The situation might be clarified by further experimenting.

2. Isoscalar E1 resonance

The results presented show that the influence of odd neutron in ^{17}O appears to be relatively small for isovector $E1$

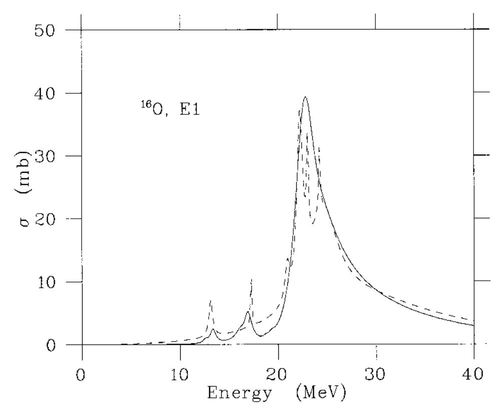

FIG. 6. The $E1$ photoabsorption cross section for ^{16}O nucleus calculated in RPA (solid line, $\Delta = 300$ keV) as compared with the experimental data (dashed line) from Ref. [23].

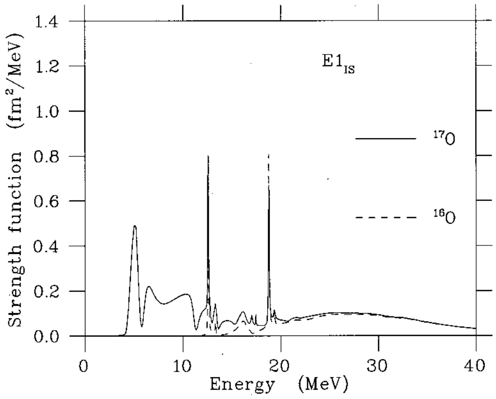

FIG. 7. The isoscalar (IS) $E1$ strength in ^{17}O nucleus calculated in ORPA with two channels of core excitations 1^- and 3^- (solid line) as compared with the $E1_{IS}$ strength in ^{16}O calculated in RPA (dashed line). The smearing parameter Δ is equal to 30 keV.

resonance. So it is of interest to study this influence for excitations of isoscalar type. The corresponding external field operator is written as

$$V_{12}^{(0)}=\frac{1}{2}\,\delta_{\sigma_1,\,\sigma_2}\delta_{\tau_1,\,\tau_2}\delta(\mathbf{r}_1-\mathbf{r}_2)f_{IS}(r_1)Y_{1\,\mu}\left(\frac{\mathbf{r}_1}{r_1}\right), \quad (3.4)$$

where $f_{IS}(r)$ is the radial form-factor for isoscalar $E1$ excitations ($E1_{IS}$). It is reasonable to determine function $f_{IS}(r)$ so as to obtain the spurious state probability $B_0(E1_{IS})$ equal to zero. In order to fulfill this condition we use the form

$$f_{IS}(r)=r\left(1-\frac{r^2}{R_{IS}^2}\right). \quad (3.5)$$

Parameter R_{IS} is determined within the ''forced consistency'' scheme by formulas

$$R_{IS}=\sqrt{\frac{I_3}{I_1}}, \quad I_k=\int_0^\infty dr\, r^{k+2}[\xi_{0n}(r)+\xi_{0p}(r)], \quad (3.6)$$

where functions $\xi_{0n}(r)$ and $\xi_{0p}(r)$ are defined in Ref. [18]. Actually, these functions are proportional to the neutron and proton components of the spurious state radial transition density. These definitions ensure the equality $B_0(\mathrm{E1}_{IS})=0$ in our approach at least for the even-even nucleus ^{16}O. The value of R_{IS} calculated by this method appeared to be close to that of $R=1.24A^{1/3}=3.12$ fm for ^{16}O nucleus which is used in the Woods-Saxon potential parametrization [19]. Namely, we have obtained $R_{IS}\simeq 1.04R$. Let us note that the method of the spurious state suppression described is similar to that in Ref. [24] [see also Ref. [25] where a form-factor similar to Eq. (3.5) was used in the RPA calculations of the isoscalar resonances].

In Fig. 7 we show the $E1_{IS}$ strength functions for ^{17}O calculated in ORPA and for ^{16}O, calculated in RPA, with the isoscalar external field defined above in both cases. In the ORPA calculation two channels of core excitations have been incorporated 1^- and 3^-. The small value of smearing parameter $\Delta=30$ keV was taken in order to exhibit the role of the single-particle continuum. The results presented indicate that $E1_{IS}$ strength in ^{17}O nucleus below 12.5 MeV is completely determined by the odd neutron contribution.

The nature of the resonances in this energy region is clarified from the analysis of partial decomposition. Here are three relatively wide peaks. The first peak with the maximum at 5.1 MeV comes from $p_{3/2}$ single-particle resonance. The second peak, strongly overlapping with the third one and having the maximum at 6.5 MeV, corresponds to the $\frac{3}{2}^-$ state of the $\{d_{5/2}\otimes 3^-\}$ sextuplet which contributes owing to the channel coupling in ORPA (this peak is absent in the strength function calculated with incorporating of only 1^- channel). Let us note that the $\frac{5}{2}^-$ and $\frac{7}{2}^-$ states of this sextuplet have too small $E1_{IS}$ strengths and are absorbed by background. And, finally, the third peak with the maximum at 10.3 MeV comes from $f_{7/2}$ single-particle resonance. It is important that the strengths of all these resonances are renormalized due to the interaction between the odd neutron and the core.

The relative contribution of these three resonances into the energy-weighted moment $m_1(E_{\mathrm{max}})$, defined by

$$m_k(E_{\mathrm{max}})=\int_0^{E_{\mathrm{max}}} dE\, E^k\, \tilde{S}^{\mathrm{odd}\ (\lambda)}(E,\Delta), \quad (3.7)$$

turns out to be considerable also. For the strength function of Fig. 7 we obtain

$$m_1(11.3\ \ \mathrm{MeV})/m_1(40\ \ \mathrm{MeV})=0.15, \quad (3.8)$$

where the value of 11.3 MeV is the energy of the minimum following the third resonance peak.

At the excitation energies above 12.5 MeV the forms of the $E1_{IS}$ strength functions of ^{17}O and ^{16}O nuclei are similar. This reminds one of the situation with isovector strength in these nuclei. It should be noted that in view of the small value of the smearing parameter $\Delta=30$ keV used in the calculations, the widths of the most resonances shown in Fig. 7 (except for several discrete states embedded into continuum) are formed predominantly by decay of excited states into the single-particle continuum.

TABLE I. Integrated photonuclear cross sections for ^{16}O and ^{17}O (percentage with respect to the corresponding Thomas-Reiche-Kuhn value $59.74\,NZ/A$ MeV mb). The integration up to 30 MeV. The partial cross sections data are taken from Ref. [17]: n denotes (γ, sn) cross section, p denotes the (γ, p) one. The elastic photon scattering (EPS) data are taken from Ref. [23].

	Experiment				Theory
	n	p	$n+p$	EPS	
^{16}O	24	44	68	88	84
^{17}O	38	9	47		85

IV. CONCLUSION

A model has been developed describing both the single-particle and collective parts of the odd nucleus excitation spectrum including splitting of particle (hole) $\otimes$ phonon multiplets on a common basis. The model takes into account the single-particle continuum including the giant resonance continuum, i.e., the source of the escape width, which is important for light and medium mass nuclei. In other words, it corresponds to the continuum RPA for even-even nuclei and, in addition, consistently accounts for the specificity of the odd nucleus under consideration. These properties of the model are of great interest for the odd nuclei without pairing which have the nucleon binding energy close to zero.

The model developed is self-consistent (see Sec. II B 2) although in the calculations presented we have used a simpler variant. The self-consistency allows us to use the model also to calculate very unstable nuclei for which there is no information for fitting parameters of the particle-hole interaction.

In the framework of the simplified variant of the model the isovector $E1$ photoabsorption cross section and the isoscalar dipole strength function in ^{17}O have been calculated. Also, in order to clarify the role of odd neutron the same quantities for ^{16}O have been calculated within the continuum RPA. We obtained the following results.

(i) The isovector $E1$ photoabsorption cross section in ^{16}O obtained in elastic photon scattering has been described reasonably by our model. The main reason is that the role of the single-particle continuum in the light ^{16}O nucleus is very significant. However, our calculations in ^{17}O did not give an agreement with the experimental data available, obtained in (γ, sn) and (γ, p) cross section measurements. Thus, one can hope that future experiments for ^{17}O will give a better agreement with our calculations. The calculations predict a reasonable value of 85% of the TRK sum rule in the interval up to 30 MeV, which is almost the same as for ^{16}O. Theoretically, it is difficult to imagine any mechanism which would reduce strongly the total isovector $E1$ strength in ^{17}O as compared with that in ^{16}O.

(ii) In contrast to the case of the isovector $E1$ resonance, for the isoscalar $E1$ resonance our calculations have given a very noticeable difference between the ^{17}O and ^{16}O nuclei. Consistent accounting for the odd neutron in ^{17}O resulted in appearance of an additional low-lying contribution of the isoscalar strength below 12.5 MeV induced by the odd neutron only. This part of the strength gives about 15% of the energy-weighted moment m_1 integrated up to 40 MeV, and it can be measured in experiment.

ACKNOWLEDGMENTS

This work was supported by the grant of the Swedish Institute within The Visby Programme.

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