Practical approximation scheme for the pion dynamics in the three-nucleon system

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We discuss a working approximation scheme to a recently developed formulation of the coupled πNNN-NNN problem. The approximation scheme is based on the physical assumption that, at low energies, the $2N$-subsystem dynamics in the elastic channel is conveniently described by the usual $2N$-potential approach, while the explicit pion dynamics describes small, correction-type effects. Using the standard separable-expansion method, we obtain a dynamical equation of the Alt-Grassberger-Sandhas (AGS) type. This is an important result, because the computational techniques used for solving the normal AGS equation can also be used to describe the pion dynamics in the $3N$ system once the matrix dimension is increased by one component. We have also shown that this approximation scheme treats the conventional $3N$ problem once the pion degrees of freedom are projected out. Then, the $3N$ system is described with an extended AGS-type equation where the spin-off of the pion dynamics (beyond the $2N$ potential) is taken into account in additional contributions to the driving term. These new terms are shown to reproduce the diagrams leading to modern $3N$-force models. We also recover two sets of irreducible diagrams that are commonly neglected in $3N$-force discussions, and conclude that these sets should be further investigated.

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I. INTRODUCTION

Considerable progress has been made in understanding the coupled system of two nucleons and (at least) one pion. This enables the study of pion absorption and production processes on very light nuclei, and is a next step of including explicit pion degrees of freedom, beyond the standard nuclear picture where mesonic degrees of freedom are ''frozen out,'' into nucleon-nucleon potentials. We are in particular interested in a set of theories which can be classified by the acronym TRABAM (Thomas-Rinat, Afnan-Blankleider, Avishai-Mizutani), Ref. [1]. In recent years, a fair amount of effort has been made to extend this theory to the pion-three-nucleon domain, where a richer range of phenomena is possible. In a sequence of papers [2], this system has been explored in the attempt to arrive at a consistent and connected theory of the coupled πNNN-NNN system. This effort has required the blending of the standard three- and four-body theories of Alt, Grassberger, and Sandhas (AGS) [3], with the possibility that a pion can appear and disappear anywhere in the system, and in any of its subsystems.

In a recent paper, one of the authors [4] elucidated a connected, coupled scheme for the combined πNNN-NNN dynamics. Furthermore, he derived, by the use of the quasiparticle formalism at three-cluster and two-cluster levels, an equation for the coupled πNNN-NNN system, that has the appearance of a coupled set of Lippmann-Schwinger type equations:

$$X^{(2)}_{ss'}=Z^{(2)}_{ss'}+\sum_{s''} Z^{(2)}_{ss''}\tau^{(2)}_{s''}X^{(2)}_{s''s'} . \qquad (1.1)$$

An equation of this type was first given by Lovelace [5] in the standard three-particle problem with separable interactions, using the original theory by Faddeev [6]. It was also derived by AGS [3] who applied the quasiparticle approximation to their three-body equations. Equation (1.1), like the Faddeev-Lovelace-AGS equation, is a coupled set of integral equations in one intercluster momentum variable, but here the labels $s(s',s'')$ run over four values, rather than three: $s=0$ representing the configuration consisting of a three-nucleon cluster with the pion separate, and $s=1$, 2, 3 representing the three possible arrangements of the three nucleons in a pair and a separated nucleon (the usual AGS-Lovelace scheme) with the pion associated with either the pair or the single nucleon, or absent. (See Table II in [4].)

The driving term in this equation is given in terms of quantities associated with the two levels of separable approximations needed when the quasiparticle method is applied [4] to the full πNNN-NNN system of equations, as follows:

$$\begin{aligned} Z^{(2)}_{ss'}&=\langle (s^{(2)})_-|g_0|(s'^{(2)})_-\rangle \overline{\delta}_{ss'}\\ &\quad+\sum_{a'(\subset s)}\ \sum_{b'(\subset s')}\ \sum_{a(\subset a',b')} \langle (s^{(2)})_{a'a}|\tau^{(3)}_a|(s'^{(2)})_{b'a}\rangle\\ &\quad\times(\overline{\delta}_{ss'}+\delta_{ss'}\overline{\delta}_{a'b'}), \end{aligned} \qquad (1.2)$$

and these contributions to the Z term will have an effect on both the inhomogeneous term and the kernel of the integral equation Eq. (1.1).

This expression looks rather complicated with the multiple sums and inclusion rules, but, in fact, it turns out to be rather simple, once all these rules are applied and the delta-functions invoked. We will define in detail symbols in this equation below, but let us first look at the structure of this driving term. The first term contains only components from the three-nucleon (no-pion) sector, and therefore cannot contribute at all if $s=0$ or $s'=0$. Therefore, the only contributions to this term can come from the case $s \neq 0$ and $s' \neq 0$. In this case the first term of Eq. (1.2) does not contribute on the diagonal (i.e., $s=s'$) because of the anti-delta function. Off the diagonal, since this contains only nucleon degrees of freedom, this term can properly be identified with the driving term of the traditional AGS-Lovelace approach to the standard three-nucleon problem.

For $s=s'=0$ there is no contribution to $Z^{(2)}_{ss'}$ also from the second term, because the inclusion prescriptions in the sums cannot be satisfied if $s=s'=0$. More explicitly, since $s=s'$, only the last term in the delta-function structure survives, but this requires $a' \neq b'$. However, for $s=0$, there is only one two-cluster state, namely $\pi(N_1N_2N_3)$, so the condition $a' \neq b'$ cannot be satisfied. Therefore, the driving terms do not contribute at all in the case $s=s'=0$, either in the first, or in the second term.

The second term does have nonzero values in the diagonal case $s=s' \neq 0$, and this provides additional diagonal terms to Eq. (1.1) due to the coupling to the pion degrees of freedom. Even if small, such contributions compare to zero and therefore can hardly be neglected. The second term contributes also to the off-diagonal elements, thus providing corrections to the dominant terms of the standard three-nucleon problem.

In Eq. (1.2), $|(s^{(2)})_{a'a}\rangle, |(s^{(2)})_{-}\rangle$ are components of the form-factor vector coming from the separable approximation at the two-cluster level, the subscript ‘‘$-$’’ representing the no-pion sector, the subscripts $a'a$ being the Yakubovskiĭ [7] chain labels for the chains of partition of the (conserved) four-body system, πNNN. The quantity g_0 is the free propagator for three nucleons, in the absence of pions; the quantity $\tau^{(3)}_a$ denotes the intermediate propagation of the possible three-cluster structures of the πNNN system. In other words,

TABLE I. Chain-labeled four-body components of the form factor $|(\tilde{s}^{(2)})_{a'a}\rangle$ for $s=1$. The last column represents how this form factor is constructed in terms of the elementary π interaction with nucleon i, by means of the πNN vertex f_i.

a'	a	$(f_1)_{a'a}$
$(N_2N_3\pi)N_1$	$(N_2N_3)\pi N_1$	f_2+f_3
$(N_2N_3\pi)N_1$	$(N_2\pi)N_3N_1$	f_3
$(N_2N_3\pi)N_1$	$(N_3\pi)N_2N_1$	f_2
$(N_2N_3)(\pi N_1)$	$(N_2N_3)\pi N_1$	f_1
$(N_2N_3)(\pi N_1)$	$N_2N_3(\pi N_1)$	

the separable representation of the two-particle t matrix, according to

$$t_a(z) \cong |(a^{(3)}(z))\rangle \tau^{(3)}_a(z) \langle (a^{(3)}(z))|, \tag{1.3}$$

describes effectively all the elastic two-body processes in the four-body space, with t_a representing either the NN or the πN two-body t matrix. The ‘‘anti-delta’’ function is $\overline{\delta}_{ss'}=1-\delta_{ss'}$. The inclusions under the summation symbols are intended in the usual sense of Yakubovskiĭ chain inclusions. For example, for $s=1$, a' is one of the two partitions $N_1(N_2N_3\pi)$, $(\pi N_1)(N_2N_3)$, and a are all the possible three-cluster partitions that can be obtained by breaking one cluster in each of the above two-cluster partitions. As an example of how the complicated-looking structure of Eq. (1.2) simplifies, we show two particular contributions to the driving term, where we specify the form-factor vectors by the chains of partition, and leave off the superscripts (2) and (3) that occur in Eq. (1.2); a typical off-diagonal element:

$$\begin{aligned} Z_{12} = &\langle N_1(N_2N_3)|g_0|N_2(N_3N_1)\rangle + \langle N_1(N_2N_3\pi);N_1N_2(N_3\pi)|\tau_{(N_3\pi)}|N_2(N_3N_1\pi);N_1N_2(N_3\pi)\rangle \\ &+ \langle N_1(N_2N_3\pi);N_1N_3(N_2\pi)|\tau_{(N_2\pi)}|(\pi N_2)(N_3N_1);N_1N_3(N_2\pi)\rangle \\ &+ \langle (\pi N_1)(N_2N_3);N_2N_3(N_1\pi)|\tau_{(N_1\pi)}|N_2(N_3N_1\pi);N_2N_3(N_1\pi)\rangle, \end{aligned} \tag{1.2a}$$

and a diagonal element:

$$\begin{aligned} Z_{11} = &\langle N_1(N_2N_3\pi);N_1(N_2N_3)\pi|\tau_{(N_2N_3)}|(N_1\pi)(N_2N_3);N_1(N_2N_3)\pi\rangle \\ &+ \langle (N_1\pi)(N_2N_3);N_1(N_2N_3)\pi|\tau_{(N_2N_3)}|N_1(N_2N_3\pi);N_1(N_2N_3)\pi\rangle. \end{aligned} \tag{1.2b}$$

For $s=1$, as can be immediately deduced by comparing Eq. (1.2) with Eq. (1.2a), the state $|(s)_{-}\rangle$ denotes the Faddeev component $|N_1(N_2N_3)\rangle$, while with $|(s)_{a'a}\rangle$ we denote the relevant Yakubovskiĭ components, such as $|N_1(N_2N_3\pi);N_1N_2(N_3\pi)\rangle$ for instance. All possible four-body components for $s=1$ are listed in Table I.

This structure for the driving term has been diagrammatically represented in Fig. 3 of Ref. [4], and is reported here in Fig. 1 for the sake of clarity. The left panel exhibits the four terms discussed in Eq. (1.2a) for the off-diagonal elements, while the two diagonal elements in Eq. (1.2b) are represented in the right-bottom panel. The right-top panel reports also the remaining two diagrams which provide the coupling to the fourth component (i.e., $0=s \neq s'$). Thus this figure exhibits in full the structure of the diagrams which are needed for setting the dynamical equation of the πNNN system.

To solve Eq. (1.1) one must first define and construct the states $|(s^{(2)})_{a'a}\rangle$, $|(s^{(2)})_{-}\rangle$, and the two-cluster Green's function $\tau^{(2)}_s$. It is precisely at this point that we propose herein a workable approximation scheme. However, before discussing the approximation, we recall for clarity the rigorous result obtained in Ref. [4]. For complete details we refer to that work.

Starting from the two-particle representation Eq. (1.3), one can derive the dynamical sub-amplitude containing the interactions internal to the coupled set of partitions $(N_1)(N_2N_3)$, $(N_1\pi)(N_2N_3)$, and $(N_1)(N_2N_3\pi)$. This is denoted by $(\mathbf{x})_{(s=1)}$ where the $s=1$ index should be interpreted with the fact that nucleon ''1'' is set apart from the other two: obviously, the other cases with $s=2,3$ are obtained by cyclic permutations of the nucleons. The $s=0$ amplitude instead contains all the interactions internal to the single $\pi(N_1N_2N_3)$ partition, hence the pion is set apart from the three nucleons here.

The dynamical equations for these four subamplitudes are given by [see Eqs. (3.21)–(3.24) of Ref. [4]]

$$(x_s)_{a'a,b'b}=\langle a^{(3)}|G_0|b^{(3)}\rangle\overline{\delta}_{ab}\delta_{a'b'}+\sum_{c'(\subset s)}\sum_{c(\subset c')}\langle a^{(3)}|G_0|c^{(3)}\rangle\overline{\delta}_{ac}\delta_{a'c'}\tau_c^{(3)}(x_s)_{c'c,b'b}+\langle a^{(3)}|G_0(f_s)_{a'a}g_0(x_s{}^{\dagger})_{-,b'b}, \tag{1.4}$$

$$(x_s{}^{\dagger})_{-,b'b}=(f_s^{\dagger})_{b'b}G_0|b^{(3)}\rangle+\sum_{c'(\subset s)}\sum_{c(\subset c')}(f_s^{\dagger})_{c'c}G_0|c^{(3)}\rangle\tau_c^{(3)}(x_s)_{c'c,b'b}+\mathcal{V}_sg_0(x_s{}^{\dagger})_{-,b'b}, \tag{1.5}$$

$$(x_s)_{a'a,-}=\langle a^{(3)}|G_0(f_s)_{a'a}+\sum_{c'(\subset s)}\sum_{c(\subset c')}\overline{\delta}_{ac}\delta_{a'c'}\langle a^{(3)}|G_0|c^{(3)}\rangle\tau_c^{(3)}(x_s)_{c'c,-}+\langle a^{(3)}|G_0(f_s)_{a'a}g_0(x_s)_{-,-}, \tag{1.6}$$

$$(x_s)_{-,-}=\mathcal{V}_s+\mathcal{V}_sg_0(x_s)_{-,-}+\sum_{c'(\subset s)}\sum_{c(\subset c')}(f_s^{\dagger})_{c'c}G_0|c^{(3)}\rangle\tau_c^{(3)}(x_s)_{c'c,-}, \tag{1.7}$$

with $a\subset a'\subset s$ and $b\subset b'\subset s$. For each s, the subamplitudes $(\mathbf{x}_s)$ have components labeled by the Yakubovskiĭ chain labels $(a'a)$, or by the symbol ''$-$'' in the case of the no-pion sector, where the index s specifies a unique physical partition (last column of Table II in [4]). Only when $s=0$, all couplings to the no-pion sector are vanishing. These coupled equations are obtained once the representation Eq. (1.3) has been assumed, and the amplitudes are expressed in the four-body space in the corresponding quasiparticle representation. G_0 is the full four-body free propagator, g_0 is the already mentioned free three-nucleon propagator in the no-pion sector, $\mathcal{V}_s$ is the pair potential between the two interacting nucleons in the partitions denoted by s and, in addition to the short-range part of the *2N* potential, it includes explicitly the OPE diagram. Finally, f_s $(f_s^{\dagger})$ is the elementary creation (annihilation) vertex for a pion into (from) the appropriate subsystem denoted by the chain-label subscript. An important aspect of these subamplitudes is that it is always possible to factor out a Dirac δ function in momentum space for the ''spectator'' nucleon (or pion, for $s=0$). The new aspect obtained in Ref. [4] is that this factorization property has been maintained when there is a pion associated with either the nucleon pair or the spectator nucleon, or when there is no pion at all.

The basic assumption for the x_s subamplitudes consists in the finite-rank representation

$$(x_s)_{a'a,b'b}=|(s^{(2)})_{a'a}\rangle\tau_s^{(2)}\langle(s^{(2)})_{b'b}|, \tag{1.8}$$

$$(x_s^{\dagger})_{-,b'b}=|(s^{(2)})_{-}\rangle\tau_s^{(2)}\langle(s^{(2)})_{b'b}|, \tag{1.9}$$

$$(x_s)_{a'a,-}=|(s^{(2)})_{a'a}\rangle\tau_s^{(2)}\langle(s^{(2)})_{-}|, \tag{1.10}$$

$$(x_s)_{-,-}=|(s^{(2)})_{-}\rangle\tau_s^{(2)}\langle(s^{(2)})_{-}|. \tag{1.11}$$

This representation provides all the ingredients needed to construct the connected dynamical equation (1.1). We have limited here the discussion to the case of one separable term, but the algebraic generalization of Eq. (1.1) to more separable terms is straightforward.

II. APPROXIMATION SCHEME

Although this is a very relevant issue, we will not concentrate here on the mathematical aspects and general constraints needed to obtain a mathematically converging separable expansion for the subamplitudes $\mathbf{x}_s$. A very clear explanation about the general methods required for considering such questions can be found in Ref. [8].

We will instead concentrate on the development of an approximate scheme for the separable representation of such subamplitudes. The proposed approximation scheme is based on the physical assumption that in standard nuclear physics the picture of nucleons interacting via realistic nucleon-nucleon potentials, with the pion degrees of freedom ''frozen out,'' provides an acceptable first-order description of the low-energy/low-momenta dynamics. The effects of including explicit pionic degrees of freedom beyond that picture should then be considered only as dynamical ''corrections.''

If we consider Eq. (1.7), we observe that the last term accounts in fact for the one-pion dynamics in the *2N* subsystem $(s\neq 0)$, once the OPE diagram has been taken out (because it is already included in $\mathcal{V}_s$). In particular $(x_s)_{-,-}$

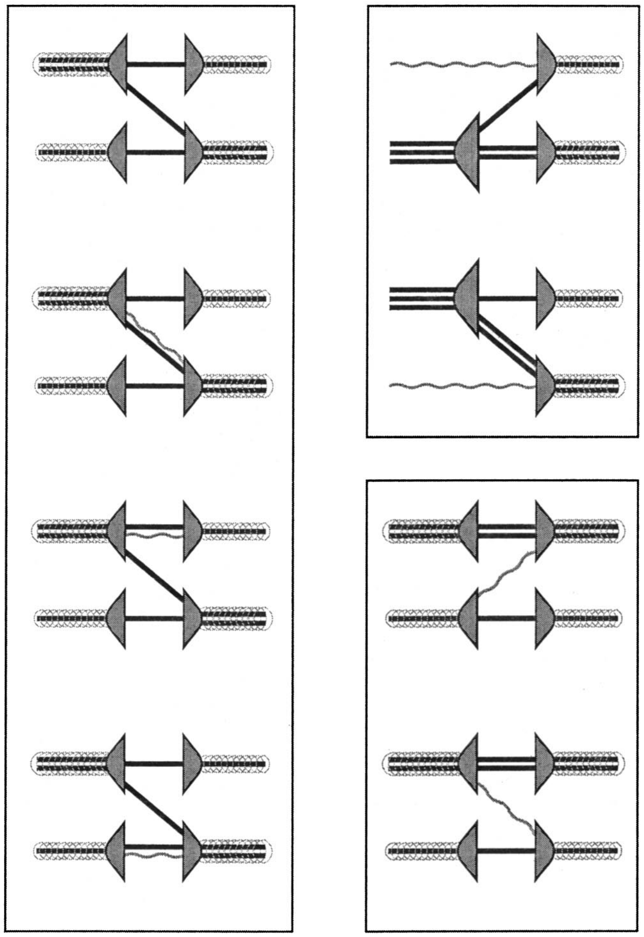

FIG. 1. Exchange diagrams contributing to the extended *3N* equation with one pion. The left panel shows diagrams contributing to $Z_{12}^{(2)}$ (off the diagonal), the bottom-right panel denotes contributions to the diagonal element $Z_{11}^{(2)}$, and finally the diagrams coupling the three Faddeev components with the fourth component (with $s=0$) are shown on the top-right panel. The straight solid lines represent nucleons, while the pale wavy line denotes the pion. When $s\neq 0$, the pion line surrounds the two-cluster partition, since the pion may couple with the nucleon pair, or with the spectator, or be absent.

represents the complete, elastic *2N* amplitude in presence of a spectator nucleon. Obviously, $(x_s)_{-,-}$ should be obtained from the coupled set of Eqs. (1.4)–(1.7), but we will identify instead $(x_s)_{-,-}$ with the conventional *2N t* matrix, derived by the solution of the standard *2N* Lippmann-Schwinger equation (in the presence of a spectator nucleon) using as input the phenomenological *NN* potential. In other words, we set to zero the last term of Eq. (1.7) and use for $\mathcal{V}_s$ the conventional *NN* potential which includes in an effective way the contributions from the pion-*2N* dynamics. Note that there will be a price to pay for this; namely, all *disconnected* dispersive effects to the *3N* dynamics, originated in the *2N* subsystems by the dynamical equations (1.4)–(1.7), will be approximated to zero. This is a consequence of the fact that the dynamical description of the pion degrees of freedom implied by these equations has been replaced with an istantaneous, effective *2N* potential. Implicitly, this approximation is assumed in all potential approaches to the *3N* problem, but it has not really been tested. The exception is in Ref. [9], where these dispersive effects of the *2N* subsystem have been sized in the extreme situation where the πN interaction is entirely represented by its coupling through a forward propagating Δ isobar. Interestingly, the Δ-mediated, disconnected dispersive effects in the *3N* system turned out to be not negligible. It is clear that this problem should be investigated further; nevertheless we will not do this here since our aim is to follow in this respect the standard potential approach to the *3N* problem, where the *3N* dispersive effects generated in the *2N* subsystems are completely ignored.

Once we have accepted that the *2N* dynamics in the *elastic* channel is described by means of a phenomenological nucleon-nucleon potential, our description can be closely compared to the usual, potential-based, quantum-mechanical *3N* approaches since the dynamical input of the two approaches appears to be identical. Then, if we consider Eq. (1.11), this reduces to the well-known, standard separable representation of the two-nucleon *t* matrix, which we express in the polar form

$$(x_s)_{-,-}\approx|\tilde{s}^{(2)}\rangle\tilde{\tau}_s^{(2)}\langle\tilde{s}^{(2)}|, \tag{2.1}$$

and this already gives the approximations $|(s^{(2)})_-\rangle\approx|\tilde{s}^{(2)}\rangle$, and $\tau_s^{(2)}\approx\tilde{\tau}_s^{(2)}$ which are needed for the determination of the driving term (first contribution) and for the kernel of Eq. (1.1). It should be noted that the form factors $|\tilde{s}^{(2)}\rangle$ for *NN* interactions are, in this approximation, not distinct from the form factors $|a^{(3)}\rangle$ of the quasiparticle approximation at the three-cluster level, Eq.(1.3). Both come, in fact, from the pole approximation of the elastic two-nucleon *t* matrix. The only difference is that the $|\tilde{s}^{(2)}\rangle$ factor refers only to the *2N t* matrices and is expressed in the $(NN)+N$ two-cluster space, with one Jacobi coordinate removed already. The form factor $|a^{(3)}\rangle$, on the other hand, refers to all the two-body *t*-matrices in the four-body space, and this includes also the πN *t*-matrices in addition to the *NN* ones. Thus, a convenient feature emerges from the approach herein discussed: only one standard pole approximation (or expansion, in the more general case of higher ranks) has to be made for the two-nucleon interaction, to be used for the *2N t* matrices in both four-body and *3N* spaces.

At this point one fact must be stressed: namely, if we do not consider further contributions, our description precisely collapses into the quantum-mechanical approach to the *3N* system in terms of a *2N* potential, since the resulting *3N* equations have *exactly* the AGS form. This is because all couplings to πNNN state are made via the components $|(s^{(2)})_{a'a}\rangle$ which would be set to zero in this case. Thus, these components, together with the $s=0$ partition, are fundamental for an explicit treatment of the pion degrees of freedom in the *3N* system, and we get an approximation of the $|(s^{(2)})_{a'a}\rangle$ factors, to the lowest order, by considering in particular Eq. (1.6). Assuming that the leading contributions to $(x_s)_{a'a,-}$ are dominated by the pole structures [see Eq. (2.1)] of $(x_s)_{-,-}$ in the last term of Eq. (1.6), we obtain

$$(x_s)_{a'a,-} \approx \langle a^{(3)}|G_0(f_s)_{a'a}g_0|\tilde{s}^{(2)}\rangle\tilde{\tau}_s^{(2)}\langle\tilde{s}^{(2)}|,$$

and considering Eq. (1.10),

$$|(s^{(2)})_{a'a}\rangle \approx |(\tilde{s}^{(2)})_{a'a}\rangle \equiv \langle a^{(3)}|G_0(f_s)_{a'a}g_0|\tilde{s}^{(2)}\rangle. \tag{2.2}$$

This provides the form factors needed for the second term of Eq. (1.2), which in this approximation represents the contribution entirely responsible for the explicit treatment of the pion dynamics in Eq. (1.1), beyond that effectively contained in the static *2N* potential. The $\langle a^{(3)}|$ on the left of this is needed because the quasiparticle approximation has already been made for the pair interaction in the four-body space, by Eq. (1.3).

Equation (1.2) also requires the ''bra'' vectors $\langle(s^{(2)})_-|$, $\langle(s^{(2)})_{a'a}|$. We simply construct the adjoints, by taking the corresponding ''bra'' states of the underlying quasiparticle approximation (2.1) of the standard *2N* t matrix. In other words

$$\langle(s^{(2)})_-| \approx \langle\tilde{s}^{(2)}|, \tag{2.3}$$

and

$$\langle(s^{(2)})_{a'a}| \approx \langle\tilde{s}^{(2)}|g_0(f_s^\dagger)_{a'a}G_0|a^{(3)}\rangle. \tag{2.4}$$

By means of these positions, we identify three types of new contributions which take into account, to the lowest order, the explicit pion dynamics in the extended *3N* equation, Eq. (1.1). These contributions modify the driving term $Z^{(2)}$ of the standard AGS equation in a very selective way. We will discuss the details of these contributions in the next section, while here we briefly summarize the result. The first contributions enlarge the number of components with respect to the three standard Faddeev components, with the addition of the fourth component $(s=0)$ specifying the partition when the pion is set apart from the three nucleons. Such contributions provide the new couplings between $s=0$ and $s'\neq 0$, as well as with $s\neq 0$ and $s'=0$. These link the *3N* Faddeev components with the partition consisting of a three-nucleon cluster and a separated pion $(s=0)$. A second type of terms enter in the diagonal part, $s=s'\neq 0$, of $Z_{ss'}^{(2)}$ while traditionally these elements have been assumed to be vanishing in the standard *3N* theory: hence, these terms should be considered important since they compare to zero in the standard AGS equation. Finally, the second term of Eq. (1.2) provides corrections also for $0\neq s\neq s'\neq 0$, that is, for the off-diagonal elements of the driving term. Instances of such terms are shown in the last three contributions of Eq. (1.2a). They all represent modifications that have to be added to the standard *3N* driving term, identified with the first contribution in Eq. (1.2). Thus, the new contributions coming from the explicit pion degrees of freedom, provide minimal, though important, modifications to the standard AGS formulation of the three-nucleon problem.

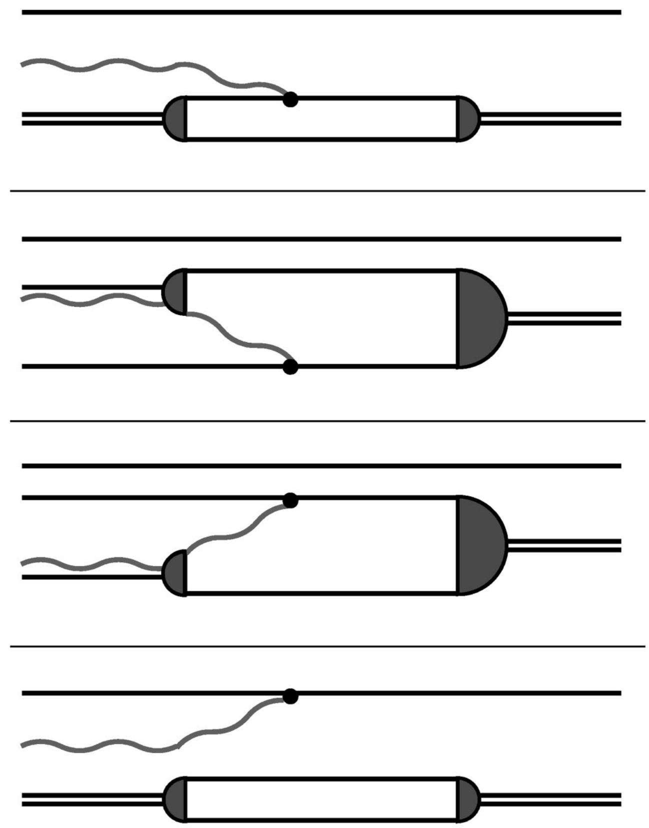

FIG. 2. Diagrammatic representation of the form factors $|(\tilde{s}^{(2)})_{a'a}\rangle$ in the Yakubovskiǐ chain-labeled space.

III. DISCUSSION

In the previous section we have shown that it is possible to treat approximately the pion dynamics in the *3N* equations by introducing the new form factor

$$|(\tilde{s}^{(2)})_{a'a}\rangle \equiv \langle a^{(3)}|G_0(f_s)_{a'a}g_0|\tilde{s}^{(2)}\rangle. \tag{3.1}$$

The operator $(f_s)_{a'a}$ has been defined in Ref. [4] in terms of the renormalized elementary πNN vertex by means of the inclusion prescription

$$(f_s)_{a'a} = \sum_{i=1}^{3} f_i \overline{\delta_{ia}}\delta_{i,a\subset a'}\delta_{a'\subset s}. \tag{3.2}$$

Here, the label ''i'' represents the πN pair interacting via the vertex f_i.

In view of the key role played by this new ingredient in the extended *3N* equation we provide its detailed diagrammatic interpretation. To fix the ideas, we choose $s=1$ and consider the various components depending on the Yakubovskiǐ chain-of-partition subscript $a'a$, listed in Table I. By applying the inclusion prescription for the vertex operator, we obtain the set of diagrams drawn in Fig. 2. Note that there are only four diagrams in the figure while there are five components in the table. This is because the inclusion prescription automatically set to zero the contribution corresponding to the last Yakubovskiǐ component, as shown in Table I. Moreover, there is another diagram (omitted in Fig.

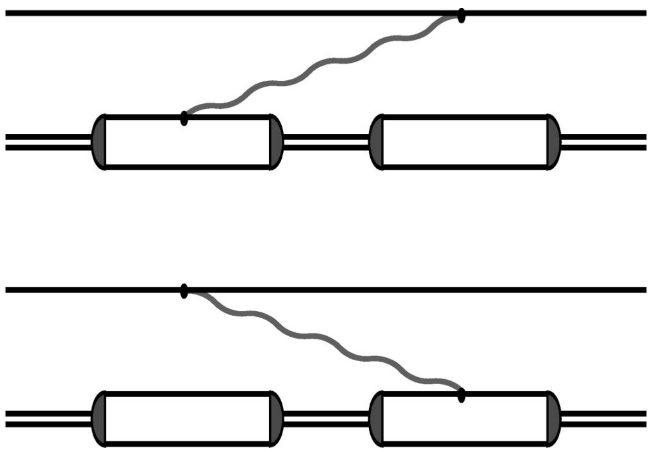

FIG. 3. Diagrams contributing to $Z_{11}^{(2)}$. In conventional *3N* theory $Z_{11}^{(2)}$ is zero.

2) which has to be added to the first diagram shown in the figure, obtained by interchanging the two nucleons ''2'' and ''3'' within the pair. This sum is evidenced in the first row of Table I. All these contributions represent the proper quasiparticle generalization of the standard *2N* form factor, in presence of a spectator nucleon, to the pion inelastic channel.

Once we have illustrated the form factor diagrams, we can discuss the detailed structure of the new contributions to the driving term of the extended *3N* equation. Indeed, we can interpret the main result obtained in Sec. III of Ref. [4], and reported here in Eqs. (1.1) and (1.2), as a simple prescription to include the pion dynamics in the AGS equation,

$$Z_{ss'}=Z_{ss'}^{AGS}+Z_{ss'}^{\pi}, \tag{3.3}$$

and the main result of the previous section is a practical approximation scheme to get the part of the driving term which handles the pion dynamics when $s,s'\neq 0$

$$Z_{ss'}^{\pi}=\sum_{a,a',b'}\langle\tilde{s}^{(2)}|g_0(f_s^{\dagger})_{a'a}G_0|a^{(3)}\rangle\tau_a^{(3)} \times\langle a^{(3)}|G_0(f_{s'})_{b'a}g_0|\tilde{s}'^{(2)}\rangle(\overline{\delta_{ss'}}+\delta_{ss'}\overline{\delta_{a'b'}}). \tag{3.4}$$

We begin with the contributions to Eq. (3.4) in the $s=s'=1$ case. Here, only the term with $\delta_{ss'}$ survives in Eq. (1.2), which implies $a'\neq b'$, and also that only the (N_2N_3) pair can be formed in the intermediate state. Consequently, the only possible diagrams that can be constructed are obtained from only the first and last diagrams in Fig. 2 yielding the diagrams shown in Fig. 3, plus obviously those obtained by interchanging the pairing nucleons ''2'' and ''3,'' for a total of four time-ordered diagrams. Clearly, these are irreducible contributions that cannot be represented in conventional *2N* potential theory.

We then consider the contributions to the *Z*-pionic term when $0\neq s\neq s'\neq 0$, e.g., when $s=1$ and $s'=2$. In this case only the $\overline{\delta_{ss'}}$ contribution survives in the second term of Eq. (1.2); here the intermediate pair can only be formed with the

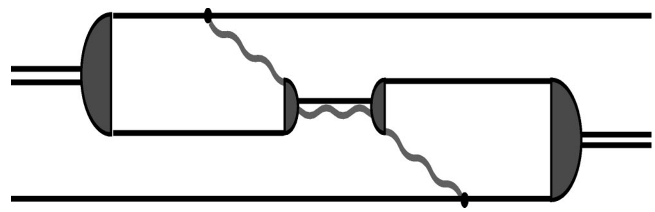

FIG. 4. The additional contribution to $Z_{12}^{(2)}$ due to the treatment of the pion dynamics.

pion and the nucleon N_3, which is uniquely defined since it does not act as spectator in both components $s=1$ and $s'=2$. In this case, one gets the diagram shown in Fig. 4. Note however that this is an approximated result: Once we have approximated the inelastic form factors in Eqs. (1.8) and (1.9) by the leading expressions (2.2) and (2.4), then the last two terms in Eq. (1.2a) vanish, because the last row in Table I is empty in this case. However, in the more general approach of Ref. [4] there are additional contributions (shown in Fig. 3 of Ref. [4]) which originate from the last two terms in Eq. (1.2a). Physically, these additional terms represent four-body multiple-rescattering contributions.

So far, we have discussed the additional contributions one must include in the AGS equation to take minimally into account the pion dynamics beyond the OPE term. We observe that in both cases these diagrams represent, in fact, contributions that can be reinterpreted as irreducible *3N* potentials. In particular, the diagram of Fig. 4 has the topological structure of the well-known Fujita-Miyazawa [10] diagram. Similar forms (where the exchanged pion rescatters before being absorbed), are the basic ansatz for building the pion part of the irreducible *3N* forces, such as the Tucson-Melbourne [11], Ruhr [12], Brazil [13], or Texas [14] *3N* interactions. However, this scheme provides at the same time also another set of irreducible *3N* diagrams which must be considered as well; these are given in Fig. 3 and represent a totally different structure from that of Fig. 4. These irreducible diagrams have been proposed by Brueckner *et al.* [15], as early as 1954, and have been investigated quantitatively for the triton binding energy by Pask [16], who finds they give a large contribution. Since that time, it has been pointed out (see Refs. [12,17,18]) that these terms cancel out against relativistic corrections to the iterated one-pion exchange term. However, the effect of this cancellation has been studied recently [19], and the cancellation turned out to be remarkably incomplete, with a breaking effect of 15–30 % in the case of realistic interactions. The reason that breaks the cancellation is dynamical, and can be evidenced with an approach preserving the cluster substructures of the multinucleon system, while describing the pion-exchange dynamics. This is, we believe, one of the advantages of using the formulation discussed here. From the diagrams of Fig. 3 a new contribution to the *3N* force has been extracted [19], and by means of this contribution the third nucleon affects in particular the triplet-odd waves of the *2N* subsystem, with consequences for the *N*-*d* vector analyzing powers, as has been demonstrated in Ref. [20].

Finally, we consider the contributions of the *Z*-pionic term when $s=0$ and $s'\neq 0$. Here, the pionic contributions

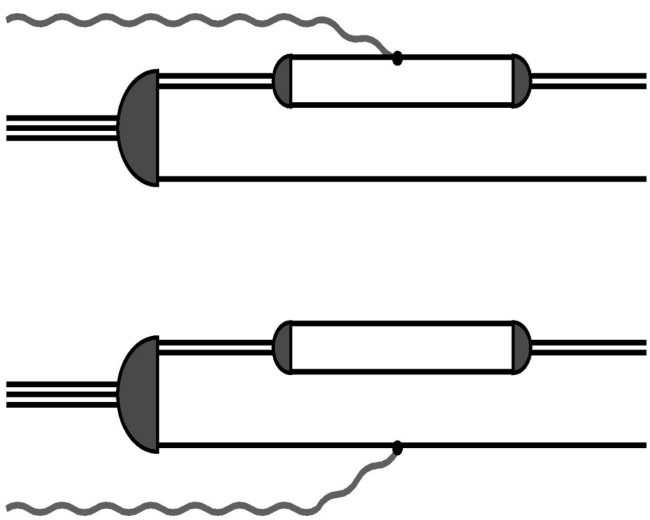

FIG. 5. Processes contributing to $Z_{01}^{(2)}$ due to the treatment of the pion dynamics.

are crucial in providing the couplings to the new, fourth component. In such a case, one has to modify Eq. (3.4) since the approximation discussed in the previous section concerns the structure of the $|s^{(2)}\rangle$ form factors only when $s\neq 0$, while nothing is said otherwise. However, the $s=0$ case has been discussed in Ref. [4], see Eqs. (2.16)–(2.18) therein, and it turns out that for $s=0$ the only nonvanishing subamplitude is given by the equation

$$(x_0)_{a'a,a'b}=\langle a^{(3)}|G_0|b^{(3)}\rangle\overline{\delta}_{ab}+\sum_{c(\subset a')}\langle a^{(3)}|G_0|c^{(3)}\rangle\overline{\delta}_{ac}\tau_c^{(3)}(x_0)_{a'c,a'b}, \tag{3.5}$$

which represents a Faddeev equation for three interacting nucleons in presence of a spectator pion, since $a'=(NNN)\pi$. In the vicinity of its poles, $\mathbf{x}_0$ becomes

$$(x_0)_{a'a,a'b}\simeq|(s^{(2)})_{a'a}\rangle\tau_s^{(2)}\langle(s^{(2)})_{a'b}|. \tag{3.6}$$

The $s=0$ form factors are solutions of the homogeneous equation associated to Eq. (3.5), and represent the virtual decay of a *3N* interacting cluster into a correlated *2N* pair, *a*, plus a nucleon, in presence of the spectator pion. For $s=0$, the form factor is obviously vanishing in the pure *3N* sector, since there is always the presence of the spectator pion. Hence, taking $s=0$ and $s'=1$ for instance, the *Z*-pionic term becomes

$$Z_{ss'}^{\pi}=\sum_{a,a',b'}\langle(s^{(2)})_{a'a}|\tau_a^{(3)}\langle a^{(3)}|G_0(f_{s'})_{b'a}g_0|\tilde{s}'^{(2)}\rangle, \tag{3.7}$$

once the complicated δ structure in Eq. (1.2) has been properly taken into account. We observe that for $s\neq s'$ it is $a'\neq b'$ always. Furthermore, since a' identifies the $s=0$ partition, *a* may represent only a *NN* pair, which means that the term in Eq. (3.7) selects only the first and the fourth Yakubovskiǐ components (in Table I), and this specifies completely the diagrams contributing to Z_{01}^{π}, as shown in Fig. 5. In the figure the first diagram should include an additional contribution obtained by interchanging the pairing nucleons, as usual.

At this point, we have achieved the main goal of this work; indeed, starting from the more general approach of Ref. [4], we derived a practical, approximated scheme for the treatment of the pion dynamics in the *3N* system. In particular, we obtained a set of dynamical equations where the main input is given by the *2N t* matrix generated by phenomenological *NN* potentials (hence, constrained by phase-shift analysis). The additional inputs required for this description are the πN *t* matrix, once its polar part has been subtracted, and the nonrelativistic πNN vertex, needed for the construction of new form factors expressed by Eq. (3.1).

This set of equations can be considered a natural, approximated extension of *3N* AGS equations for the explicit treatment of the pionic channel. We have identified the modifications implied by this treatment: they consist in additional terms, $Z_{ss'}^{\pi}$, representing corrections to the standard AGS driving term $Z_{ss'}^{AGS}$. We have classified these corrections by the structure of the underlying diagrams and found they all correspond to irreducible *3N*-force diagrams. We have also pointed out the presence of a fourth, pionic, component representing the $\pi+(NNN)$ partition, and described how this is coupled to the other three Faddeev components. Finally, we have discussed the main limitation implied by the approach, which consists in ignoring all *3N* disconnected (or reducible) dispersive effects: this is unavoidable if we use as input a potential-based description for the dynamics of the *2N* subsystem; indeed, such an approximation is implicitly assumed in all conventional quantum-mechanical descriptions of the *3N* system based on the potential approach. To overcome this limitation, one should consider the more general approach of Refs. [1,4], wherein the input interactions cannot be defined in a simple manner. With respect to this point, we observe that, since the *2N*-subsystem dynamics is described in this work in terms of the standard *2N* potential approach, the mass of the nucleon in the no-pion sector is not generated dynamically, but is static, and hence these approximated equations are not plagued by the nucleon-renormalization problem [21]. On the other hand, above the pion threshold for the *NN* subsystem, the approach presented herein cannot be considered unitary, since the input *2N t* matrices are not unitary either.

It might be surprising to find out that the dynamical equations analyzed here as well as in Ref. [4] lead to another class of irreducible *3N* diagrams. Such a class of diagrams is present but still hidden in these dynamical equations since these represent formulations suited for the *3N* problem above the pion threshold. If we consider the *3N* system at lower energies, then it is more convenient to eliminate from these formulations the fourth Faddeev component, $s=0$, related to the mesonic channel. This can be accomplished by using in a very straightforward way Feshbach's projection technique. The resulting dynamical equations have only three Faddeev components, just like the standard AGS formalism, while the

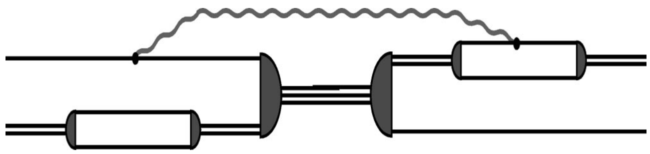

FIG. 6. Intermediate *3N* cluster formation while the meson is in flight. Processes contributing to $Z'_{ss'}$ and generated by (projecting out) the fourth Faddeev component.

mesonic channel has been projected out from Eq. (1.1). This recasting of the dynamics (which represents an exact result) leads to an additional modification of the driving term $Z_{ss'}$, due to the effects of the coupling to the fourth component,

$$Z_{ss'} = Z^{AGS}_{ss'} + Z^{\pi}_{ss'} + Z'_{ss'} . \tag{3.8}$$

(Here and in the following, it must be assumed that the indices s, and s' span only from 1 to 3, since the mesonic channel has been projected out.) The additional term, $Z'_{ss'}$, is given by

$$Z'_{ss'} = Z^{\pi}_{s0}\tau^{(2)}_0 Z^{\pi}_{0s'} , \tag{3.9}$$

while Z^{π}_{00} is identically zero as has been pointed out in the Introduction.

We consider this new additional contribution to the driving term, $Z'_{ss'}$, and discuss what kind of irreducible *3N* diagrams are involved. This can be accomplished by considering the definition of Z^{π}_{0s} given in Eq. (3.7), and taking into account the fact that $\tau^{(2)}_0$ represents the strength of the connected *3N* correlations while the pion is ''in flight,'' as discussed in Ref. [4]. A strong *3N* correlation would correspond to a pole-like structure for $\tau^{(2)}_0$, shifted by the energy carried by the exchanged pion. Using the result expressed in Eq. (3.7) one can show that the term $Z'_{ss'}$ can be written as

$$Z'_{ss'} = \sum_{b,c,b',c'} \langle \tilde{s}^{(2)} | g_0 (f^{\dagger}_s)_{b'b} G_0 | b^{(3)} \rangle \tau^{(3)}_b | (0^{(2)})_{a'b} \rangle \tau^{(2)}_0 \times \langle (0^{(2)})_{a'c} | \tau^{(3)}_c \langle c^{(3)} | G_0 (f_{s'})_{c'c} g_0 | \tilde{s}'^{(2)} \rangle , \tag{3.10}$$

and the diagram shown in Fig. 6 is just one contribution to this structure. (Similar diagrams for the *3N* force have been observed also in Ref. [22], within a relativistic approach to the *3N* problem based on a cluster description, instead of using a microscopic description based on individual particles.) Note that the pion can couple any of the three ingoing nucleon lines with each one of the three outgoing lines. Moreover this occurs with all possible recombinations in intermediate *2N* pairs.

This set of diagrams represent another, new class of irreducible *3N* mechanisms contributing to the construction of the *3N* force. Physically, these diagrams represent all possible *connected* correlations among the three nucleons while the exchanged pion is in flight. On the contrary, the diagrams of Fig. 3 represent all possible *disconnected 2N* correlations while the pion is in flight.

IV. SUMMARY AND CONCLUSIONS

Recently, a new approach for the explicit treatment of the pion dynamics in the *3N* system has been obtained [4]. Herein, we have shown how to derive from this formulation a practical calculation scheme which is phenomenologically sound. The approximation is based on the assumption that the elastic *2N* subamplitudes can be conveniently described with the phenomenological *2N* potential approach. In other words, instead of treating explicitly the pion dynamics in full, in the *3N* system we describe explicitly only those aspects of the pion dynamics which cannot be buried into the all-comprehensive, phenomenological *2N* potential. Thus the procedure could be viewed as a method to cool down (or to gradually project out) the pion dynamics from the theory of Ref. [4].

One advantage of making such an approximation is that this approach is not plagued with the nucleon-renormalization problem, since the dynamical equation used to construct the subsystem amplitudes is represented by the standard *2N* Lippmann-Schwinger equation, where the nucleon masses are static. However, the approach includes also the pion degrees of freedom, via the inelastic subamplitudes. These are represented as *2N* form factors defined in the Yakubovskiĭ chain-labeled space of the (four-body) πNNN system. Here, we consider only the first-order contributions to such inelastic form factors in terms of the effective πNN coupling vertex, see Table I. (It is evident that more complex rescattering mechanisms contributing to such form factors can be implemented at a later stage.) Another advantage of this approach is that the dynamical input can be constrained by the *2N* experimental data; therefore the method can be directly compared with the standard quantum-mechanical approach to the *3N* problem based upon a *2N* potential description.

A possible problem with such an approximation is the breaking of unitarity above the pion production threshold. The question of unitarity, and the level at which it breaks down, is always a delicate one in a truncated field theory such as that on which Ref. [4] is based. Since in that paper, a complete and connected Faddeev-based theory for three nucleons in the presence of at most one pion is developed, full unitarity at the four-body level can be expected, subject to the limitations thoroughly discussed in Sec. I of that paper. In the approximation scheme presented here, unitarity breaking would be introduced by, in particular, neglecting the last term of Eq. (1.7). The scheme may not be fully unitary above the pion production threshold, since that is what is accounted for by the neglected term. Two comments can be made regarding this. In the first instance, this scheme can be applied for calculating *3N* observables, where it allows us to estimate dynamical effects due to the inclusion of one possible intermediate pion. Secondly, since the theory is designed as a minimal extension to the standard, ''potential-like'' description of nuclear physics wherein the pion dynamics is frozen, it should be possible also to extend calculations to moderately low energies above the pion production threshold, where interesting new physics could be studied with this approach. Furthermore, the more complete theory of Ref. [4] can serve as a basis, and one could possibly study the effect of including additional terms from Eqs. (1.4)–(1.7), perhaps in a perturbative fashion.

By using a separable-expansion representation of the elastic *NN t* matrix, it has been possible to recast the dynamical equation into an extended AGS form, where the part of the pion dynamics not buried in the *2N* potential is treated explicitly. We discussed the three modifications implied by this extended *3N* equation with respect to the normal AGS one.

First, the 3×3 AGS driving term, defined in terms of the three Faddeev components, acquires additional contributions from the pion dynamics, and these new contributions act in both diagonal and off-diagonal matrix elements (while the standard AGS driving term is known to act only in the off-diagonal elements). We discussed these contributions also in terms of their diagrammatic interpretation, and found that the off-diagonal corrections correspond, to their lowest order, to irreducible *3N*-force diagrams where the pion, while being exchanged between two nucleons, rescatters from the third before annihilating. This rescattering mechanism is practically the only case discussed in the construction of the pionic part of the *3N* force. And the different 3NF approaches differ mainly for the model representation of the πN rescattering amplitude; there are, of course, additional short-range effects where there is much more ambiguity and where the various 3NF models differ considerably among each other. Conversely, the second modification refers to the diagonal part of the driving term, and represents irreducible *3N*-force diagrams of different topology, where it is one of the two nucleons exchanging the pion that rescatters with the third one. These 3NF diagrams are not included in the construction of modern *3N* potentials because it is usually assumed that they cancel out if one takes into account meson retardation effects in all their possible variety of time orderings. However, as has been pointed out in a recent study [19], this cancellation as a 100% effect is questionable. The reason for questioning the cancellation is due to the fact that a full *2N* subamplitude enters in this *3N*-force diagram, while the cancellation involves only the ''instantaneous'' part of the diagram. The construction of 3NF models from meson-exchange mechanisms evaluated in terms of instantaneous processes will lead inevitably to a 100% cancellation effect; however such methods do not take into account that two nucleons may clusterize while the pion is being exchanged, and this implies that an entire set of rescattering processes have to be subsummed during the pion-exchange process, thus leading to an incomplete cancellation effect.

The third and last modification with respect to the normal AGS equation implies the increasing of the matrix dimension by one unit, since the three Faddeev components are now coupled to the additional two-cluster partition $\pi+(NNN)$. We have discussed the structure of such couplings and the corresponding diagrams, thus providing details for all the ingredients of this new dynamical equation.

Finally, we have shown that it is possible to project out the effects of the coupling to this pionic channel, thus providing a description of the *3N* dynamics with explicit treatment only for the *3N* coordinates. This modification is suited for treatments of the *3N* system at lower energies, below the pion threshold. Then, the extended AGS equation involves only the three standard Faddeev components, and the effect of the $\pi+(NNN)$ channel in the intermediate states is contained into an additional, third contribution to the driving term. We have analyzed the diagrams involved and found that they correspond to a third class of irreducible 3NF diagrams, topologically different from the other, previously discussed two classes. The diagrams correspond to the variety of connected correlations among the three nucleons while the meson is in flight. In other words, these mechanisms take into account all possible *3N*-cluster effects while the meson is being exchanged. This new correction acts in both diagonal and off-diagonal matrix elements of the driving term and may possibly provide additional influence on the structure of the *3N* force.

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