

Point-form analysis of elastic deuteron form factors

T. W. Allen, W. H. Klink, and W. N. Polyzou

Department of Physics and Astronomy, The University of Iowa, Iowa City, Iowa 52242

(Received 17 May 2000; published 31 January 2001)

Point-form relativistic quantum mechanics is applied to elastic electron-deuteron scattering. The deuteron is modeled using relativistic interactions that are scattering-equivalent to the nonrelativistic Argonne v_{18} and Reid '93 interactions. A point-form spectator approximation (PFSA) is introduced to define a conserved covariant current in terms of single-nucleon form factors. The PFSA is shown to provide an accurate description of data up to momentum transfers of 0.5 GeV², but falls below the data at higher momentum transfers. Results are sensitive to the nucleon form factor parametrization chosen, particularly to the neutron electric form factor.

DOI: 10.1103/PhysRevC.63.034002

PACS number(s): 21.45.+v, 21.30.-x, 24.10.Jv, 25.30.Bf

I. INTRODUCTION

Electron scattering is considered to be an ideal tool to study the electromagnetic structure of hadronic systems. Relativity cannot be ignored for momentum transfers that provide information about the structure of the hadrons at the scale of a few tenths of a fermi. In order to understand hadronic systems at this scale, consistent relativistic models of both the hadronic dynamics and the hadronic electromagnetic current operator are required. If the dynamics and the current operator satisfy cluster properties [1,2], then the information learned about the structure of the simplest two- and three-body systems provides the essential components of models needed to treat complex targets.

Elastic electron-deuteron scattering is the simplest reaction that must be accurately modeled in order to constrain the dynamical generators and current operators that are needed to model complex systems. Because the deuteron is an isoscalar target, it might be expected that it can be accurately described by a pure impulse approximation. Unfortunately, pure impulse approximations are not consistent with current covariance and current conservation. An important goal is to find a physically motivated extension of the impulse approximation that is consistent with current conservation and covariance and is also qualitatively consistent with experiment.

In the one-photon-exchange approximation the experimental observables can be expressed in terms of matrix elements of the hadronic electromagnetic current operator between the initial and final eigenstates of the hadronic Hamiltonian. The general form of these matrix elements is

$$\langle p', j', \mu'_j | \hat{J}^\mu(0) | p, j, \mu_j \rangle, \quad (1)$$

where $|p', j', \mu'_j\rangle$ and $|p, j, \mu_j\rangle$ are eigenstates of the four-momentum, spin, and three-component of the spin, in reference frames related by a boost with momentum transfer $Q = p' - p$. $\hat{J}^\mu(0)$ is the hadronic current density at $x=0$.

The dynamical constraints on the current are current conservation,

$$[\hat{P}_\mu, \hat{J}^\mu(0)] = 0, \quad (2)$$

and current covariance,

$$U(\Lambda, a) \hat{J}^\mu(x) U^\dagger(\Lambda, a) = (\Lambda^{-1})^\mu_\nu \hat{J}^\nu(\Lambda x + a). \quad (3)$$

Here $U(\Lambda, a)$ is the unitary representation of the inhomogeneous Lorentz group, whose existence is required by relativistic invariance [3]. \hat{P}^μ is the four-momentum operator, with $U(I, a) = e^{i\hat{P} \cdot a}$.

In applications there are essentially two approaches used to compute the hadronic current matrix elements. These are the covariant [4–8] and direct interaction [9–17] approaches. Each approach has its own advantages and disadvantages.

Covariant approaches assume that the underlying theory is a local quantum field theory. For the case of the deuteron the input is a covariant current vertex of the form

$$\langle 0 | T(\Psi(x_1) \Psi(x_2) \hat{J}^\mu(x) \bar{\Psi}(y_1) \bar{\Psi}(y_2)) | 0 \rangle, \quad (4)$$

which is the vacuum expectation value of a time-ordered product of nucleon and current fields. Assuming the existence of an underlying quantum theory, Mandelstam [18] showed how to extract the desired current matrix elements from the vertex. The Fourier transform of the vertex has pole terms on the deuteron mass shell. The residue includes a pair of Bethe-Salpeter amplitudes and a current matrix element. The Bethe-Salpeter normalization condition [19] can be used with the solution of the homogeneous Bethe-Salpeter equation to remove the amplitudes from the residue. What remains is the desired current matrix element.

Quasipotential equations [4–6] are based on these same concepts, but they introduce constraints designed to preserve the physical singularities. Current matrix elements are extracted using the constrained amplitudes and vertex functions.

Covariant methods are appealing because of their formal connection to a quantum field theory; however in most applications it is necessary to model the vertex and Bethe-Salpeter kernel, and to replace the two-point Green's function with the free two-point function. Quasipotential methods lead to simpler calculations [7] than the full Bethe-Salpeter approach [8], but the reductions complicate cluster properties.

Direct interaction approaches attempt to construct consistent models of $U(\Lambda, a)$ and $\hat{J}^\mu(x)$ directly. The transforma-

tion properties require that both $U(\Lambda, a)$ and $\hat{J}^\mu(x)$ have an interaction dependence. Dirac [20] addressed the problem of constructing $U(\Lambda, a)$ by including interactions in some of the infinitesimal generators of $U(\Lambda, a)$. He introduced the notion of forms of dynamics which minimize the number of interaction-dependent generators. The three main forms are: the instant form, where the interactions are in the Hamiltonian and Lorentz boost generators; the point form, where the interactions are in the four-momentum; and the front form, where interactions appear in the operators that generate transformations transverse to a fixed light front (a three-dimensional hyperplane tangent to the light cone.) But while Dirac identified the different possibilities for putting interactions in selected generators, he did not show how to actually construct the Poincaré generators with interactions.

The first exact construction of Poincaré generators with interactions was due to Bakamjian and Thomas [21] using Dirac's instant form. There are Bakamjian-Thomas-like constructions in each of the forms of dynamics [22], and they are scattering equivalent.

While explicit dynamical models of current operators are difficult to construct, consistent current matrix elements can be obtained by prescriptions that evaluate selected independent matrix elements using single nucleon currents. The remaining current matrix elements can then be determined by using covariance, current conservation and discrete symmetries. These generate the needed dynamical contributions to the current matrix elements.

Direct interaction approaches provide an exact treatment of the symmetries associated with special relativity, but are not directly related to an underlying field theory. A number of direct interaction applications to elastic electron-deuteron scattering exist in the literature. To date most applications have used Dirac's instant- [17] or front-forms [23,15,24–26] of the dynamics. The point form of relativistic quantum mechanics has important simplifying features that are useful in modeling electron scattering. The purpose of this paper is to investigate the hadronic current operator in Dirac's point form of dynamics.

In Sec. II we discuss some of the features of point-form dynamics and construct a mass operator for the deuteron. Section III deals with current operators, their relation to observables, and the point-form spectator approximation. Then, in Sec. IV the numerical results are discussed and compared with other methods. Section V presents our conclusions.

II. POINT-FORM RELATIVISTIC QUANTUM MECHANICS

Unlike nonrelativistic quantum mechanics, where all the interactions can be put in the Hamiltonian operator, for relativistic quantum mechanics it is necessary that at least three generators contain interactions. This can already be seen by examining the commutator of the Lorentz boost generators with the momentum generators. Such a commutator produces the Hamiltonian; if the Hamiltonian contains interactions, then some combination of boost and momentum generators must also contain interactions. In the instant form additional interactions are put in the boost generators, leav-

ing the momentum generators free of interactions, while in the point form the additional interactions are in the momentum generators, with the boost generators free of interactions. The front form puts interactions in a mixture of Lorentz and momentum generators.

Even though all forms of dynamics are scattering equivalent, each has certain advantages that are useful for specific applications. The goal of this paper is to analyze elastic deuteron form factors using the point form. The point form has a number of features that set it aside from the other forms. First, all of the interactions are in the Hamiltonian and momentum generators, that is, the four-momentum operator. Since there are no interactions in the boost or angular momentum generators, the Lorentz generators are all kinematic and the theory is manifestly Lorentz covariant. It is convenient to write the Poincaré commutation relations not in terms of the ten generators, but rather in terms of the four-momentum operators that contain the interactions, and global kinematic Lorentz transformations:

$$[\hat{P}_\mu, \hat{P}_\nu] = 0; \quad (5)$$

$$U_\Lambda \hat{P}_\mu U_\Lambda^{-1} = (\Lambda^{-1})_\mu{}^\nu \hat{P}_\nu; \quad (6)$$

where $U_\Lambda \equiv U_\Lambda(\Lambda, 0)$ is a unitary operator representing the Lorentz transformation Λ . These rewritten Poincaré relations will be called the point-form equations, and are the fundamental equations that have to be satisfied for the system of interest. The mass operator is given by $\hat{M} = \sqrt{\hat{P} \cdot \hat{P}}$ and must have a spectrum that is bounded from below.

Since the interactions are all in the four-momentum operators, which are the generators of space-time translations, the nonrelativistic Schrödinger equation can be generalized to a Lorentz covariant relativistic Schrödinger equation, namely

$$i \partial \Psi_x / \partial x^\mu = \hat{P}_\mu \Psi_x, \quad (7)$$

where $x = x^\mu$ is the four-vector space-time point. If the four-momentum operator does not depend explicitly on space-time, this equation becomes the eigenvalue equation

$$\hat{P}_\mu \Phi = p_\mu \Phi. \quad (8)$$

Finally, as will be shown in the following paragraphs, it is possible to define states with the property that angular momentum can be coupled in exactly the same way as is done in nonrelativistic quantum mechanics.

The simplest example of a system satisfying the point form equations is a one-particle system with mass m and spin j . If $|p, \sigma\rangle$ is an eigenstate of four-momentum p (with $p \cdot p = m^2$) and spin projection σ , then

$$\hat{P}_\mu |p, \sigma\rangle = p_\mu |p, \sigma\rangle, \quad (9)$$

$$U_\Lambda |p, \sigma\rangle = \sum_{\sigma'} |\Lambda p, \sigma'\rangle D_{\sigma', \sigma}^j(R_W) \sqrt{\frac{v_0'}{v_0}}, \quad (10)$$

with R_W a Wigner rotation defined by $R_W = B^{-1}(\Lambda v)\Lambda B(v)$, and $B(v)$ a canonical spin (rotationless) boost (see Ref. [27]) with argument $v = p/m$. $D_{\sigma'\sigma}^j(R_W)$ is a Wigner D function, and the eigenstates are normalized to

$$\langle p', \sigma' | p, \sigma \rangle = \delta^3(\mathbf{p}' - \mathbf{p}) \delta_{\sigma'\sigma}, \quad (11)$$

relativity requiring the $\sqrt{v'_0/v_0}$ factor.

States of many noninteracting particles are tensor products of one-particle states; however, a problem arises when such many-particle states are Lorentz transformed. As can be seen from Eq. (10) each state is Lorentz transformed by its own Wigner rotation, which in general are different. This means that these multiparticle states cannot be directly coupled together as is the case nonrelativistically. Such a problem is resolved by tensoring the single-particle states in the overall rest frame and boosting. It is convenient to label the state by the system's four-velocity v :

$$\begin{aligned} |v, \mathbf{k}_i, \mu_i\rangle &:= U_{B(v)}(|k_1, \mu_1\rangle \dots |k_n, \mu_n\rangle) \\ &= \sum \left(|p_1, \sigma_1\rangle \dots |p_n, \sigma_n\rangle \right. \\ &\quad \left. \times \prod_i \left[D_{\sigma_i, \mu_i}^{j_i}(R_{W_i}) \sqrt{\frac{(v'_i)_0}{(v_i)_0}} \right] \right), \quad (12) \end{aligned}$$

where $p_i = B(v)k_i$, $\Sigma \mathbf{k}_i = \mathbf{0}$, and $R_{W_i} = B^{-1}(p_i/m)B(v)B(k_i/m)$. Under Lorentz transformations, using the definition, Eq. (12), such velocity states transform as

$$U_\Lambda |v, \mathbf{k}_i, \mu_i\rangle = |\Lambda v, R_W \mathbf{k}_i, \mu'_i\rangle \prod_i \left[D_{\mu'_i, \mu_i}^{j_i}(R_W) \sqrt{\frac{(v'_i)_0}{(v_i)_0}} \right], \quad (13)$$

where the Wigner rotation $R_W = B^{-1}(\Lambda v)\Lambda B(v)$ is the same in all the arguments of the D functions and all the internal momentum vectors \mathbf{k}_i . That means all the spins as well as the orbital angular momenta can be coupled together exactly as is done nonrelativistically. This property will be used in the following paragraphs for coupling the nucleon spins together with the relative orbital angular momentum to get the spin of the deuteron. From the relation between external and internal momenta, it follows that the velocity states defined in Eq. (12) are eigenstates of the noninteracting mass operator \hat{M}_{free} and free four-velocity operator \hat{V}_μ :

$$\hat{M}_{\text{free}} |v, \mathbf{k}_i, \mu_i\rangle = \sum_i \sqrt{m_i^2 + \mathbf{k}_i^2} |v, \mathbf{k}_i, \mu_i\rangle; \quad (14)$$

$$\hat{V}_\mu |v, \mathbf{k}_i, \mu_i\rangle = v_\mu |v, \mathbf{k}_i, \mu_i\rangle. \quad (15)$$

The Bakamjian-Thomas procedure is implemented in the point form by writing $\hat{P}_\mu = \hat{M} \hat{V}_\mu$, where now \hat{M} is the sum of free and interacting mass operators, $\hat{M} = \hat{M}_{\text{free}} + \hat{M}_{\text{int}}$. \hat{M} takes the place of the center of momentum Hamiltonian $\hat{h} = \hat{H} - \hat{P}^2/2M$ in nonrelativistic quantum mechanics; note

however that even though there is only one operator containing interactions, namely the mass operator, that nevertheless there are interactions in all four components of the four-momentum operator.

In order that the four-momentum operator satisfy the point-form equations, Eqs. (5) and (6), the interacting mass operator must satisfy certain conditions. To satisfy Eq. (5), the mass operator must commute with the four-velocity operator, defined in Eq. (15):

$$[\hat{M}, \hat{V}^\mu] = 0. \quad (16)$$

This has the consequence that mass and four-velocity can be simultaneously diagonalized. Eigenstates of the four-momentum operator can thus be written as the mass times the four-velocity. Since the four-velocity is purely kinematic, it can be factored from the wave function leaving the covariant Schrödinger equation, Eq. (8), to become a mass operator eigenvalue equation,

$$\hat{M} \Phi = \lambda \Phi. \quad (17)$$

Moreover, even though the four-momentum is conserved in reactions, the total four-momentum is not the sum of the four-momenta of the individual particles. Rather what is conserved is the overall four-velocity of the individual particles, and the mass is then ‘‘off-shell,’’ not unlike the situation with Feynman diagrams. This is to be contrasted with the instant form, where the three-momentum of all the individual particles gives the total three-momentum of the system, while the energy is ‘‘off-shell.’’

The mass operator must also satisfy the other point form equation, Eq. (6), implying the mass operator is a Lorentz scalar. On velocity states this means the kernel of the mass operator must be rotationally invariant and independent of \mathbf{v}^2 , exactly the condition put on nonrelativistic Hamiltonians in order that they be Galilei invariant.

For a two-body system such as the deuteron, the relevant Hilbert space is the tensor product of proton and neutron Hilbert spaces, $H = H_p \otimes H_n$. In that case the velocity states can be written as $|v, \mathbf{k}, \mu_p, \mu_n\rangle$, where $\mathbf{k} = \mathbf{k}_1 = -\mathbf{k}_2$, and μ_p and μ_n are the eigenvalues of the three-components of the canonical spins of the proton and neutron, respectively. Because with velocity states the angular momenta can all be coupled together, these states can also be written as $|v, |\mathbf{k}|, j, \mu_j, l, s\rangle$, as in the nonrelativistic case. The mass of the two particle state, from Eq. (14), is $2\sqrt{m^2 + \mathbf{k}^2}$; j is the total angular momentum, while l and s are the orbital and spin angular momentum, respectively.

It is advantageous to express the interacting mass operator in terms of a mass squared operator with matrix elements:

$$\begin{aligned} \langle v, |\mathbf{k}|, j, \mu_j, l, s | \hat{M}_I^2 | v', |\mathbf{k}'|, j', \mu'_j, l', s' \rangle \\ = \delta(v - v') \delta_{\mu_j \mu'_j} \delta_{jj'} \langle k, l, s | (m_I^j)^2 | k', l', s' \rangle. \quad (18) \end{aligned}$$

A mass operator with a kernel of the form Eq. (18) will satisfy Eq. (16) and thus the Poincaré commutation relations, Eqs. (5) and (6). The kernel of \hat{M}_I^2 is taken to be

$$\langle k, l, s \| (m_j^j)^2 \| k', l', s' \rangle := 4m \langle k, l, s \| v_{nn}^j \| k', l', s' \rangle, \quad (19)$$

where v_{nn}^j is a nucleon-nucleon interaction. The mass is then defined by

$$\hat{M} := \sqrt{\hat{M}^2}; \quad \hat{M}^2 := 4(\mathbf{k}^2 + m^2) + \hat{M}_I^2. \quad (20)$$

Denoting the eigenvalue of the interacting mass operator by λ^2 , the equation

$$\hat{M}^2 \Phi = (4m^2 + 4\mathbf{k}^2 + 4mv_{nn}^j) \Phi = \lambda^2 \Phi \quad (21)$$

can be rewritten [24] in the form of the nonrelativistic Schrödinger equation,

$$\left(\frac{\mathbf{k}^2}{m} + v_{nn}^j \right) \Phi = \left(\frac{\lambda^2}{4m} - m \right) \Phi. \quad (22)$$

This defines a relativistic model of the two-nucleon system. It can be shown [22] that this model leads to a small correction to the nonrelativistic binding energy and has scattering observables identical to the corresponding nonrelativistic model. Equation (18) shows that the solution of Eq. (22) leads to simultaneous eigenstates of the mass, velocity, spin, and z component of spin. The Poincaré transformation properties of the deuteron eigenstates are given by

$$\hat{P}^\mu |v, m_D, j, \mu_j\rangle = \lambda v^\mu |v, m_D, j, \mu_j\rangle \quad (23)$$

and

$$U_\Lambda |v, m_D, j, \mu_j\rangle = \sum_{\mu'_j} |\Lambda v, m_D, j, \mu'_j\rangle \times D_{\mu'_j \mu_j}^j(R_W(\Lambda, v)) \sqrt{\frac{(\Lambda v)_0}{v_0}}, \quad (24)$$

where

$$\langle v, |\mathbf{k}|, j, \mu_j, l, s | v', m_D, j', \mu'_j \rangle = \delta(v - v') \delta_{\mu'_j \mu_j} \delta_{j' j} \Psi_{ls}^j(|\mathbf{k}|). \quad (25)$$

$\Psi_{ls}^j(|\mathbf{k}|)$ is the nonrelativistic wave function associated with one of the two chosen nonrelativistic potentials. (There are analogous formulas for the scattering states.) This provides the desired point-form dynamics.

III. CURRENT OPERATORS, FORM FACTORS, AND ELASTIC OBSERVABLES

The second key element in a theoretical description of electron-scattering is a conserved, covariant hadronic current density $\hat{J}^\mu(x)$. In the point form the dynamical Poincaré transformations are the space-time translations. Translational covariance can be realized by using the dynamical translation operators to define $\hat{J}^\mu(x)$ in terms of $\hat{J}^\mu(0)$:

$$\hat{J}^\mu(x) := e^{i\hat{P} \cdot x} \hat{J}^\mu(0) e^{-i\hat{P} \cdot x}. \quad (26)$$

The density $\hat{J}^\mu(0)$ is assumed to transform as a four-vector [14,28] with respect to the free Lorentz transformations.

We now want to define the model current operator in terms of measured one-body current operators. This is done as follows. The deuteron matrix elements of $\hat{J}^\mu(0)$ are defined in terms of their Breit-frame values with Q in the \hat{z} direction:

$$\langle \mathbf{Q}/2, 1, \mu'_j | \hat{J}^\mu(0) | -\mathbf{Q}/2, 1, \mu_j \rangle. \quad (27)$$

For $\mu = 0, 1, 2$ the current matrix elements are defined in terms of the single-nucleon current matrix elements:

$$\begin{aligned} \langle \mathbf{Q}/2, 1, \mu'_j | \hat{J}^\mu(0) | -\mathbf{Q}/2, 1, \mu_j \rangle \\ = \langle \mathbf{Q}/2, 1, \mu'_j | (\hat{J}_p^\mu(0) \otimes \hat{I}_n + \hat{I}_p \otimes \hat{J}_n^\mu(0)) | -\mathbf{Q}/2, 1, \mu_j \rangle. \end{aligned} \quad (28)$$

Current conservation requires that

$$\sum_{\mu} Q_{\mu} \langle \mathbf{Q}/2, 1, \mu'_j | \hat{J}^\mu(0) | -\mathbf{Q}/2, 1, \mu_j \rangle = 0, \quad (29)$$

which generates a dynamical contribution $\hat{J}_{pn}^\mu(0)$ to the \hat{z} component of the current:

$$\begin{aligned} \langle \mathbf{Q}/2, 1, \mu'_j | \hat{J}_{pn}^3(0) | -\mathbf{Q}/2, 1, \mu_j \rangle \\ = -\langle \mathbf{Q}/2, 1, \mu'_j | (\hat{J}_p^3(0) \otimes \hat{I}_n + \hat{I}_p \otimes \hat{J}_n^3(0)) | -\mathbf{Q}/2, 1, \mu_j \rangle. \end{aligned} \quad (30)$$

These relations define the components of the Breit-frame matrix elements of $\hat{J}^\mu(0)$. The remaining deuteron matrix elements of $\hat{J}^\mu(x)$ are fixed by kinematic Lorentz covariance and dynamical space-time translational covariance. Although the current matrix element is defined in the Breit frame, the expression for the general current matrix element is Lorentz covariant, as can be seen in Ref. [28], Eq. (3.31).

The computation of the matrix elements is carried out by inserting single-particle intermediate states in the velocity basis that was used to formulate the dynamical model in the previous section. The deuteron wave function in the basis Eq. (25) has the form

$$\begin{aligned} \langle v, |\mathbf{k}|, j, \mu_j, l, s | v', m_D, j, \mu'_j \rangle \\ = \delta(v - v') \delta_{\mu'_j \mu_j} \delta_{j_1} \delta_{s_1} [\delta_{l_0} u_0(k) + \delta_{l_2} u_2(k)], \end{aligned} \quad (31)$$

where $u_0(k)$ and $u_2(k)$ are the nonrelativistic S and D state deuteron wave functions. Transformation coefficients [22,27] are used to express this in terms of single-particle basis elements:

$$\begin{aligned}
\langle v_1, \mu_1, v_2, \mu_2 | v', \mu'_j, m_D \rangle &= \delta^3[\mathbf{v}' - \mathbf{v}(\mathbf{v}_1, \mathbf{v}_2)] \frac{\delta[k - k(\mathbf{v}_1, \mathbf{v}_2)]}{k^2} \left| \frac{\partial(\mathbf{v}, \mathbf{k})}{\partial(\mathbf{v}_1, \mathbf{v}_2)} \right|^{1/2} \\
&\times D_{\mu_1 \mu'_1}^{1/2} \left[B^{-1}(v_1) B(v) B\left(\frac{k_1}{m}\right) \right] D_{\mu_2 \mu'_2}^{1/2} \left[B^{-1}(v_2) B(v) B\left(\frac{k_2}{m}\right) \right] Y_{l \mu_l}[\hat{\mathbf{k}}_1(\mathbf{v}_1, \mathbf{v}_2)] \\
&\times C_{\mu_s \mu_1 \mu_2}^{1/2} C_{\mu'_j \mu_1 \mu_s}^{1/2} \{ \delta_{l0} u_0[k(\mathbf{v}_1, \mathbf{v}_2)] + \delta_{l2} u_2[k(\mathbf{v}_1, \mathbf{v}_2)] \}. \quad (32)
\end{aligned}$$

These expressions can be used to compute the current matrix element

$$\begin{aligned}
\langle v, m_D, 1, \mu_j | \hat{J}_{SA}^\mu(0) | v', m_D, 1, \mu'_j \rangle &= \int \langle v, m_D, 1, \mu_j | v_1, \mu_1, v_2, \mu_2 \rangle \langle v'_1, \mu'_1, v'_2, \mu'_2 | v', m_D, 1, \mu'_j \rangle \\
&\times [\langle v_1, \mu_1 | \hat{J}_1^\mu(0) | v'_1, \mu'_1 \rangle \delta^3(\mathbf{v}'_2 - \mathbf{v}_2) \delta_{\mu'_2 \mu_2} + \langle v_2, \mu_2 | \hat{J}_2^\mu(0) | v'_2, \mu'_2 \rangle \delta^3(\mathbf{v}'_1 - \mathbf{v}_1) \delta_{\mu'_1 \mu_1}], \quad (33)
\end{aligned}$$

where the nucleon current matrix elements are given by Eq. (28). After integrating out the delta functions, one is left with a final three-dimensional integral:

$$\begin{aligned}
\langle v', m_D, 1, \mu'_j | \hat{J}^\mu(0) | v, m_D, 1, \mu_j \rangle &= \sum_{\mu'_1 \mu_1} \sum_{\mu'_2 \mu_2} \sum_{\mu'_s \mu_s} \sum_{l' l} \sum_{\mu'_1 \mu_1} \sum_{\sigma'_1 \sigma_1} \int d^3 k C_{\mu'_s \mu_1 \mu_2}^{11/2} C_{\mu_s \mu_1 \mu_2}^{11/2} C_{\mu'_j \mu_1 \mu_s}^{11'1} C_{\mu_j \mu_1 \mu_s}^{11} Y_{l' \mu_{l'}}^*(\theta', \phi') u_{l'}(|\mathbf{k}'|) \\
&\times Y_{l \mu_l}(\theta, \phi) u_l(|\mathbf{k}|) D_{\mu'_1 \sigma'_1}^{*1/2} \{ R_W^{-1}[k'_1, B(v')] \} D_{\mu'_2 \sigma'_2}^{*1/2} \{ R_W^{-1}[k'_2, B(v')] \} D_{\sigma_1 \mu_1}^{1/2} \{ R_W[k_1, B(v)] \} \\
&\times D_{\sigma_2 \mu_2}^{1/2} \{ R_W[k_2, B(v)] \} \bar{u}(p'_1 \sigma'_1) \left\{ \gamma^\mu F_1[(p'_1 - p_1)^2] + i \sum_\nu \sigma^{\mu\nu} \frac{(p'_1 - p_1)_\nu}{2m_N} \right. \\
&\left. \times F_2[(p'_1 - p_1)^2] \right\} u(p_1 \sigma_1) + \{1 \leftrightarrow 2\}; \quad (34)
\end{aligned}$$

where the C 's are $SU(2)$ Clebsch-Gordan coefficients and the conventions for the spinors, gamma, and sigma matrices are those of Bjorken and Drell [29].

In this form it can be seen that the momentum of the unstruck particle (the spectator) is unchanged, while the struck particle's momentum is changed, but the impulse given to the struck particle is *not* the impulse given to the deuteron. For this reason we call this the point-form spectator approximation (PFSA). It should not be confused with the use of the term spectator approximation in, for example, Ref. [4]. The practical advantage of the PFSA is that the steps above can be generalized to any hadronic target. Moreover, the current matrix element is generally Lorentz covariant and can be evaluated in any frame.

Because the interactions in the point form are in the four-momentum, in the PFSA the momentum transfer seen by the scattered nucleon is not the same as the momentum transfer seen by the nucleus. In Appendix A we show that the relationship between the momentum Q transferred to the deuteron and the momentum transferred to the interacting nucleon is

$$|(p'_1 - p_1)^2| = Q^2 \frac{4(m_N^2 + \mathbf{k}_\perp^2)}{m_D^2} \left(1 + \frac{Q^2}{4m_D^2} \right) \quad (35)$$

$$> Q^2 \frac{4m_N^2}{m_D^2} \left(1 + \frac{Q^2}{4m_D^2} \right) > Q^2. \quad (36)$$

That is, the point-form momentum transfer seen by an individual nucleon will be greater in magnitude than the total deuteron momentum transfer Q^2 .

Two important implications follow from Eqs. (35) and (36) above. First, the PFSA momentum transfer depends on the internal momentum \mathbf{k} , which is a variable of integration. Thus in the PFSA, form factors depending on $(p'_i - p_i)^2$ must remain inside the integral. Second, since $|(p'_i - p_i)^2| > Q^2$, the deuteron form factors will fall off faster in the point-form calculations than in forms where $|(p'_i - p_i)^2| = Q^2$.

The input to the PFSA are single-nucleon current operators. The general structure of these operators follows from covariance, parity, Hermiticity, and time-reversal symmetry. For a spin-1/2 target the conditions imply that all matrix elements can be expressed in terms of the Dirac form factors, $F_1(Q^2)$ and $F_2(Q^2)$. The general expression has the form [29]

$$\begin{aligned} \langle p, \nu | J^\mu(0) | p', \nu' \rangle = & \bar{u}_{\nu'}(p') \left[F_1(Q^2) \gamma^\mu \right. \\ & \left. + F_2(Q^2) \sum_\alpha \frac{i Q_\alpha \sigma^{\mu\alpha}}{2m} \right] u_\nu(p), \end{aligned} \quad (37)$$

where $u_\nu(p)$ and $\bar{u}_{\nu'}(p')$ are Dirac spinors. In this form the one-body matrix elements are easily evaluated in any kinematic frame.

The Sachs electric and magnetic form factors of the nucleons are

$$\begin{aligned} G_E(Q^2) &= \sqrt{1+\tau} \left\langle \mathbf{Q}/2, \frac{1}{2}, \frac{1}{2} | \hat{J}^0(0) | -\mathbf{Q}/2, \frac{1}{2}, \frac{1}{2} \right\rangle; \quad (38) \\ G_M(Q^2) &= \sqrt{\frac{1+\tau}{\tau}} \left\langle \mathbf{Q}/2, \frac{1}{2}, \frac{1}{2} | \hat{J}^1(0) | -\mathbf{Q}/2, \frac{1}{2}, -\frac{1}{2} \right\rangle; \quad (39) \end{aligned}$$

where $\tau = Q^2/4m^2$. (Here the standard frame is the Breit frame, where the nucleon enters with momentum $-Q/2$ and exits with momentum $Q/2$, both along the z axis, which is also the axis along which the spin projection is measured.) These Dirac and Sachs form factors are related by

$$\begin{aligned} F_1(Q^2) &= \frac{1}{1+\tau} [G_E(Q^2) + \tau G_M(Q^2)]; \quad (40) \\ F_2(Q^2) &= \frac{1}{1+\tau} [G_M(Q^2) - G_E(Q^2)]. \quad (41) \end{aligned}$$

The input we use to define the model PFSA current is the single-nucleon form factor parametrizations of Gari-Krümpelmann [30] and Mergell-Meissner-Drechsel [31].

The experimental observables for the deuteron and nucleon are well known. The elastic observables $A(Q^2)$ and $B(Q^2)$ are extracted from the Rosenbluth formula for the cross section of unpolarized scattering in the lab frame,

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)} \frac{E'}{E} [A(Q^2) + B(Q^2) \tan^2(\theta/2)], \quad (42)$$

where α is the fine-structure constant, θ the scattering angle, and E and E' the initial and final energies. For the nucleons, it can be shown [32] that

$$A(Q^2) = \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau}; \quad (43)$$

$$B(Q^2) = 2\tau G_M^2(Q^2). \quad (44)$$

For spin-1/2 particles, measurements of $A(Q^2)$ and $B(Q^2)$ suffice to determine G_E and G_M . Various models of the nucleon form factors [30,31,33–35] have been constructed.

The deuteron has three independent form factors. A common classification is to denote them as the charge monopole

G_E , magnetic dipole G_M , and electric quadrupole G_Q form factors. As current matrix elements, these are defined in the Breit frame:

$$\begin{aligned} G_E &= \frac{1}{3} \langle \mathbf{Q}/2, 1, 0 | \hat{J}^0(0) | -\mathbf{Q}/2, 1, 0 \rangle \\ &+ \frac{2}{3} \langle \mathbf{Q}/2, 1, 1 | \hat{J}^0(0) | -\mathbf{Q}/2, 1, 1 \rangle; \quad (45) \end{aligned}$$

$$G_M = \sqrt{\frac{2}{\eta}} \langle \mathbf{Q}/2, 1, 1 | \hat{J}^1(0) | -\mathbf{Q}/2, 1, 0 \rangle; \quad (46)$$

$$\begin{aligned} G_Q &= \frac{1}{2\eta} [\langle \mathbf{Q}/2, 1, 0 | \hat{J}^0(0) | -\mathbf{Q}/2, 1, 0 \rangle \\ &- \langle \mathbf{Q}/2, 1, 1 | \hat{J}^0(0) | -\mathbf{Q}/2, 1, 1 \rangle]; \quad (47) \end{aligned}$$

where $\eta = Q^2/4m_D^2$. These form factors have the static limits

$$G_E(0) = e; \quad (48)$$

$$\lim_{Q^2 \rightarrow 0} G_M(Q^2) = e \frac{m_D}{m_N} \mu_D; \quad (49)$$

$$\lim_{Q^2 \rightarrow 0} G_Q(Q^2) = e m_D^2 Q_D; \quad (50)$$

where e is the charge, μ_D the magnetic dipole moment, and Q_D the electric quadrupole moment of the deuteron.

The Rosenbluth formula alone cannot determine all three of the deuteron's form factors. The other independent observable normally measured is the deuteron tensor polarization T_{20} , defined as

$$T_{20} := \sqrt{2} \frac{d\sigma^1 - d\sigma^0}{d\sigma}, \quad (51)$$

where $d\sigma^\mu$ refers to the differential cross section with helicity μ . Conventionally it is displayed at a 70° angle in the lab frame. The deuteron elastic observables are

$$A(Q^2) = G_E^2 + \frac{8}{9} \eta^2 G_Q^2 + \frac{2}{3} \eta G_M^2; \quad (52)$$

$$B(Q^2) = \frac{4}{3} \eta (1 + \eta) G_M^2; \quad (53)$$

$$T_{20}(Q^2) = -\sqrt{2} \eta \frac{\frac{4}{9} \eta G_Q^2 + \frac{4}{3} G_Q G_E + \frac{1}{3} f G_M^2}{A(Q^2) + B(Q^2) \tan^2(\theta/2)}; \quad (54)$$

where $f = 1/2 + (1 + \eta) \tan^2(\theta/2)$.

IV. NUMERICAL RESULTS AND COMPARISONS

One purpose of this work was to test the point-form spectator approximation on the simplest nucleus, the deuteron, where realistic interactions and nucleon form factors are available. Comparisons with other models are then an indi-

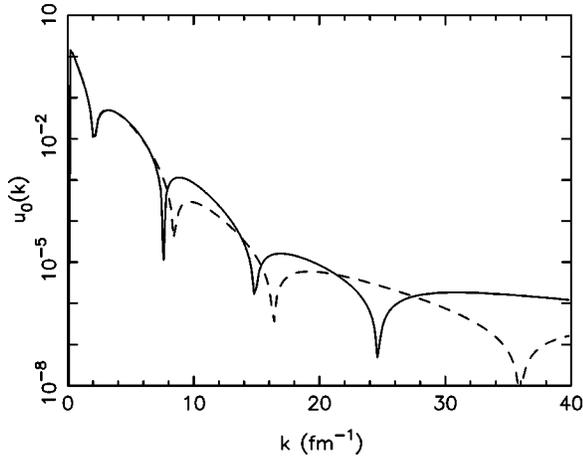


FIG. 1. The bound state S ($l=0$) deuteron wave function, using the Argonne (solid) and Reid (dashed) potentials.

cation of the relative size of the required two-body currents.

For the nucleon-nucleon interaction, we have used the Argonne v_{18} [36] and Reid '93 [37] potentials to construct a mass operator \hat{M} . The S ($l=0$) and D ($l=2$) state momentum space wave functions are plotted in Figs. 1 and 2. The only significant differences between the wave functions these potentials produce in configuration space occur below 0.4 fm for the S wave and below 1.0 fm for the D wave. In momentum space the wave functions do not exhibit significant differences up to 5 fm^{-1} , about 1 GeV, above which they do differ noticeably. The effects on the choice of interaction may therefore be expected to be relevant at higher momentum transfers, but as will be seen, these high-momentum differences in the wave function make only slight differences in the calculations.

The PFSA currents are constructed using the Gari-Krümpelmann [30] and Mergell-Meissner-Drechsel [31] parametrizations of the nucleon form factors. At the range of momentum transfer under consideration, the parametrizations give very similar results for the proton form factors and the neutron magnetic form factor. The neutron electric form factor, however, varies significantly between the two.

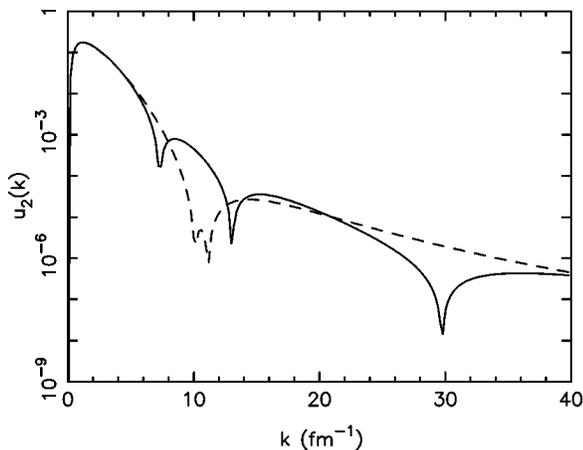


FIG. 2. The bound state D ($l=2$) deuteron wave function, using the Argonne (solid) and Reid (dashed) potentials.

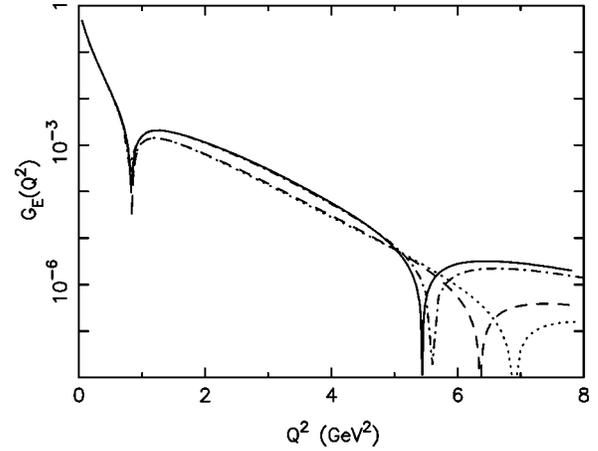


FIG. 3. $G_E(Q^2)$ for the Argonne potential with G-K (solid) and MMD (dashed) form factors, and for the Reid potential with G-K (dash-dot) and MMD (dotted) form factors.

The deuteron form factor G_E has been calculated using both form factor parametrizations and both nucleon-nucleon potentials. The absolute values of the results are displayed in Fig. 3. The results are independent of the nucleon-nucleon potential used, except for small variations at high momenta. The primary differences in G_E and G_Q are due to the nucleon form factor parametrizations. (Figure 4 compares the G-K and MMD neutron form factors. Note that the major difference is in the parametrization of the neutron electric form factor; the neutron magnetic form factor parametrizations, and the proton parametrizations as well, are very similar.) For G_E , both the Gari-Krümpelmann and the Mergell-Meissner-Drechsel form factors agree at low momentum transfers and have zeros near 0.8 GeV^2 . The G-K form factors predict a second zero near 5.5 GeV^2 while MMD predicts a second zero between 6 and 7 GeV^2 . Because the two form factors are almost identical except for the parametrization of the neutron electric form factor, this would suggest that the neutron form factor is the dominant cause of the differences in the calculations of G_E .

Figure 5 illustrates the dependence of the magnitude of the form factor G_M on the potential and on the nucleon form

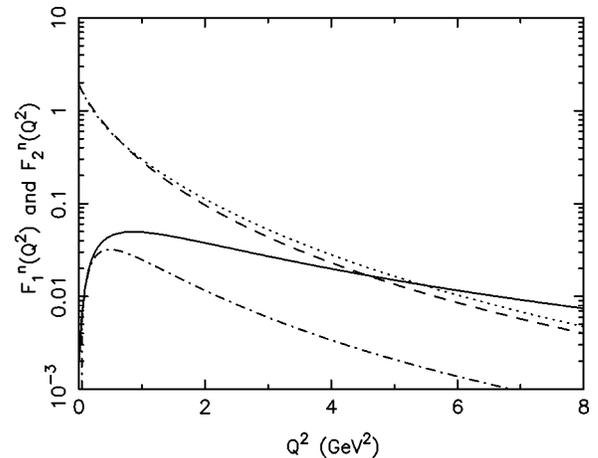


FIG. 4. The neutron magnetic and electric form factors of G-K (dotted and dash-dot), and of MMD (dashed and solid).

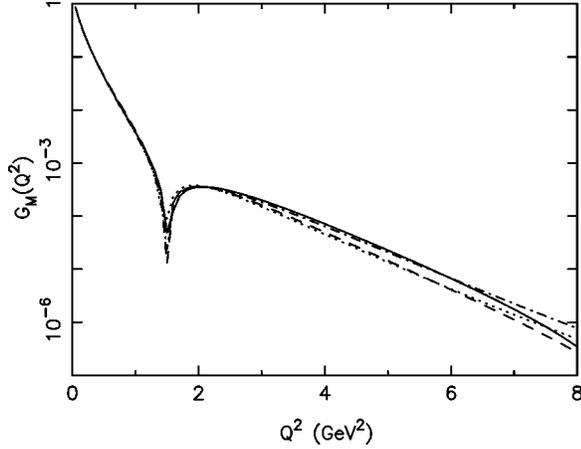


FIG. 5. $G_M(Q^2)$ for the Argonne potential with G-K (solid) and MMD (dashed) form factors, and for the Reid potential with G-K (dash-dot) and MMD (dotted) form factors.

factors used. Both parametrizations predict the same behavior up to the first zero, this time at 1.6 GeV^2 , and within the range studied, fall off with almost identical behavior. Comparison with Fig. 3 would suggest that the neutron electric form factor has little effect on the calculation of G_M . A further comparison to experimental data can be made by examining the static limit of G_M . Equation (49) relates this limit to the deuteron magnetic moment, and Table I displays the results. In the static limit, the parametrization of the nucleon form factors does not affect the results, while the choice of nucleon-nucleon interaction does. This is expected, as the form factors must approach precise limits as $Q^2 \rightarrow 0$, while the momentum-space wave functions have no such constraints.

This procedure is repeated for G_Q in Fig. 6. As was the case with G_E , there is little difference due to the potential used, but a noticeable difference between the predictions of the G-K and MMD parametrizations. The G-K form factors show a zero between 4.5 and 5.0 GeV^2 , while the MMD form factors produce a zero approximately 1 GeV^2 higher.

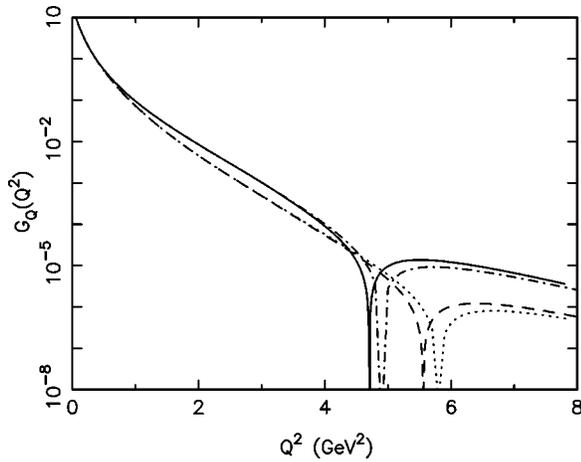


FIG. 6. $G_Q(Q^2)$ for the Argonne potential with G-K (solid) and MMD (dashed) form factors, and for the Reid potential with G-K (dash-dot) and MMD (dotted) form factors.

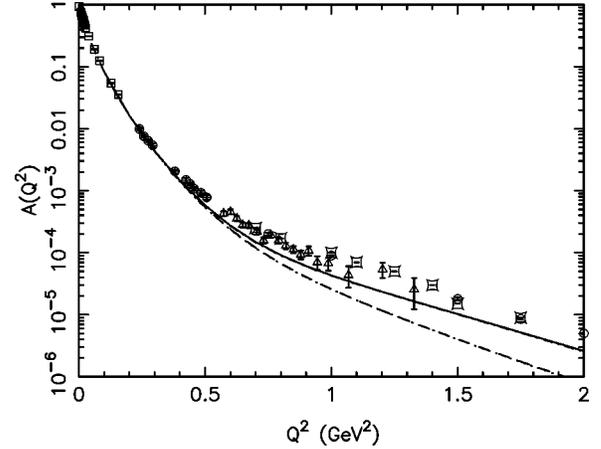


FIG. 7. $A(Q^2)$ for the Argonne potential with G-K (solid) and MMD (dashed) form factors, and for the Reid potential with G-K (dash-dot) and MMD (dotted) form factors. The data come from Refs. [41] (squares), [42] (crossed circles), [43] (triangles), [44] (open circles), and [56] (curved squares).

Further, the magnitude of the G-K results is greater than that of MMD almost everywhere throughout. The different neutron electric form factor parametrizations is the primary cause of the differences in the results. The deuteron electric quadrupole moment [Eq. (50)], displayed in Table I, differs from the experimental result, the calculated values approaching about 90% of the experimental value, as opposed to 99% for the magnetic moment calculations. This is consistent with other models [11,17,34,36,38–40].

To summarize, these point-form calculations imply that the deuteron form factors are essentially independent of the potential (Argonne v_{18} or Reid '93) used, but depend more significantly on the parametrizations of the form factors, and in particular on the neutron electric form factor, as this is the only substantial difference between the G-K and MMD parametrizations. The static moments are similar to predictions in other realistic models with the electric quadrupole

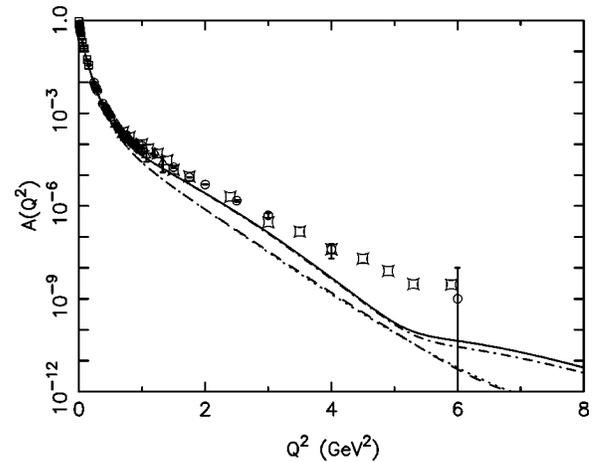


FIG. 8. $A(Q^2)$ for the Argonne potential with G-K (solid) and MMD (dashed) form factors, and for the Reid potential with G-K (dash-dot) and MMD (dotted) form factors. The legend for the data is the same as in Fig. 7.

TABLE I. The magnetic dipole and electric quadrupole moments, computed as $Q^2 \rightarrow 0$.

Moment	Units	Experimental	Argonne v_{18}	Argonne v_{18}	Reid '93	Reid '93
			G-K	MMD	G-K	MMD
μ_D	$e\mu_N$	0.85741	0.8613	0.8623	0.8615	0.8625
Q_D	e/GeV^2	7.3422	6.6	6.6	6.6	6.6

moment differing with experiment by about 10%.

Figure 7 displays the results for $A(Q^2)$ up to 2 GeV^2 for both potentials and both form factor parametrizations; Fig. 8 extends the calculations to 8 GeV^2 . The data come from Refs. [41–44]. Differences among the various calculations begin to appear at intermediate momentum transfers. For $A(Q^2)$, with Q^2 between 0.5 and 3 GeV^2 , the PFSA combined with the Gari-Krümpelmann form factors fit the data fairly closely, while the Mergell-Meissner-Drechsel form factors produce results that fall short by an order of magnitude. This pattern occurs in other impulse and spectator calculations as well. In the front-form calculations of Chung *et al.* [15], the fit for an earlier G-K parametrization (using the Argonne v_{14} potential) is even better, while the Höhler (on which the newer MMD form factors were based) calculations again fall an order of magnitude short. In Lev *et al.* [11] the two curves are closer, though the G-K remains higher and fits the data out to 2.0 GeV^2 . In the nonrelativistic calculations of Carlson and Schiavilla [39], which cover the range 0 – 2.4 GeV^2 and use only the Höhler form factors, impulse approximations using various potentials (including the Argonne v_{18}) all fall nearly an order of magnitude short in the intermediate range. In the work of Van Orden *et al.* [7], which contains an impulse approximation that fits most of the data for all three form factors quite closely, the variation from the $A(Q^2)$ data starts at 2.0 GeV^2 and is an order of magnitude short at high (8 GeV^2) momentum transfers.

The elastic observable $B(Q^2)$, related directly to the mag-

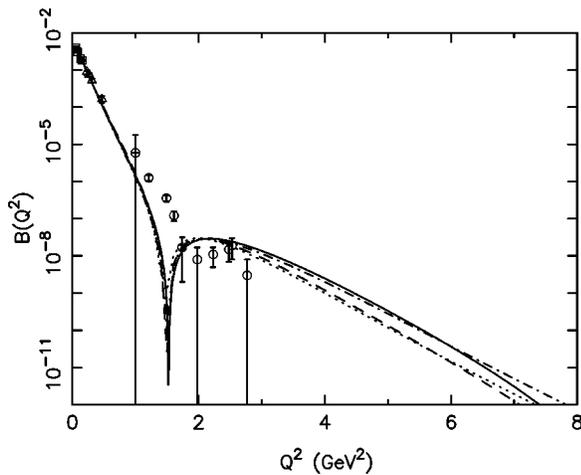


FIG. 9. $B(Q^2)$ for the Argonne potential with G-K (solid) and MMD (dashed) form factors, and for the Reid potential with G-K (dash-dot) and MMD (dotted) form factors. The data come from Refs. [41] (squares), [45] (triangles), [46] (crossed circle), and [47] (open circles).

netic form factor G_M , is displayed in Fig. 9 up to 8 GeV^2 using both potentials and both sets of parametrizations. The data come from Refs. [41,45–47]. The PFSA calculation of $B(Q^2)$ in the intermediate region 0.5 – 3 GeV^2 , a region which contains all presently available data, fits that data poorly, though the differences between the two form factor parametrizations are less marked. Both parametrizations exhibit a zero at $1.5 \pm 0.1 \text{ GeV}^2$, causing a wide discrepancy with experiment; data suggest a zero nearer 1.9 GeV^2 . Again Chung *et al.* and Van Orden *et al.* fit the $B(Q^2)$ data quite well. In contrast, Chung *et al.*, using the Paris and the Bonn wave functions instead of the Argonne v_{14} , produce results quite similar to the point form's. Carlson and Schiavilla obtain a zero at 2.2 GeV^2 only with the Nijmegen potential; their other potentials reproduce the zero at 1.6 GeV^2 . Lev *et al.* produce zeros between 1 – 2 GeV^2 in the 0 – 4 GeV^2 region using various potentials and parametrizations. The position of the zero in all these forms seems to be the most salient feature of calculations of $B(Q^2)$, affecting as it does the deviation from data in the 1 – 2 GeV^2 range.

The tensor polarization, $T_{20}(Q^2)$, is displayed in Fig. 10, with data from Refs. [48–55]. As with $B(Q^2)$, measurements of $T_{20}(Q^2)$ have only investigated the low (0 – 0.5 GeV) and intermediate (0.5 – 1.5 GeV) ranges of momentum transfer. In both ranges, the MMD and G-K parametrizations produce identical results in the PFSA. In the intermediate range, the results fall slightly below the data. The impulse approximations of Carlson and Schiavilla as well as Lev *et al.* do this too, while Van Orden *et al.* and Chung *et al.* produce curves that fit modern data quite closely.

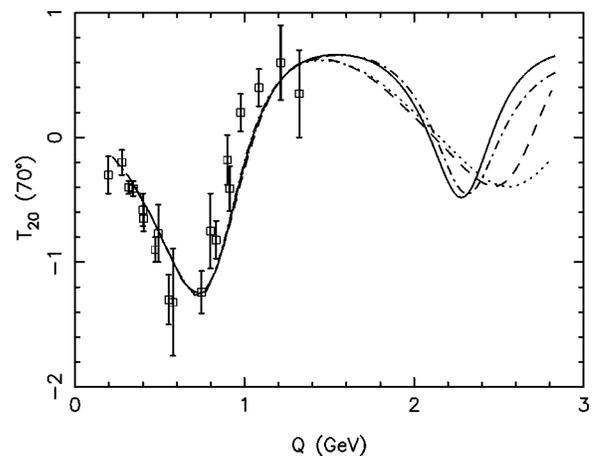


FIG. 10. $T_{20}(Q^2)$ for the Argonne potential with G-K (solid) and MMD (dashed) form factors, and for the Reid potential with G-K (dash-dot) and MMD (dotted) form factors. The data are compiled from Refs. [48–55].

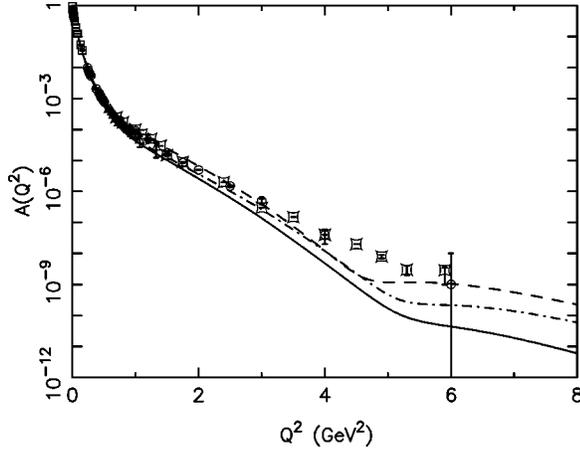


FIG. 11. Point-form (solid), nonrelativistic (dashed), and constant- Q^2 (dash-dot) results for $A(Q^2)$ using the Argonne potential and the G-K form factors.

Finally, it is instructive to compare the results obtained in point-form dynamics to the results of nonrelativistic impulse calculations to get some idea of the nature of the relativistic effects; and to compare the results in the point form to the same relativistic calculations done assuming $|(p'_1 - p_1)^2| = Q^2$ (that is, pulling the nucleon form factors outside of the integral) to examine how the point-form momentum transfer affects the results.

The results (using the Argonne v_{18} potential and the Gari-Krumpelmann form factors) for $A(Q^2)$ are displayed in Fig. 11. For low momentum transfers, all three agree. In the range of 1.0–5.0 GeV^2 , the nonrelativistic and PFSA calculations decrease similarly, the nonrelativistic curve consistently higher. The constant- Q^2 calculation in this region gradually rises from the PFSA to the nonrelativistic curve. Above 5.0 GeV^2 , all three curves systematically decrease, nonrelativistic above constant- Q^2 above PFSA. While a relativistic treatment is needed as a matter of principle at high momentum transfers, it is clear that in these calculations the effects of combining the PFSA with point-form quantum models has a tendency to reduce the structure function $A(Q^2)$ at high Q^2 . One clear cause of this is that

$$|(p'_i - p_i)^2| > Q^2; \quad (55)$$

the magnitude of the point-form momentum transfer is greater than the magnitude of the nonrelativistic momentum transfer. Because the form factors depend on the magnitude of the momentum transfer, they therefore drop off more quickly in the point form. This reduces the point-form results in comparison to the constant- Q^2 calculations, as Fig. 11 shows: as the momentum transfer increases, the two calculations diverge further from each other.

The point-form and nonrelativistic results for $B(Q^2)$ and $T_{20}(Q^2)$ do not exhibit as dramatic differences as they did for $A(Q^2)$. Again in the graphs of $B(Q^2)$ and $T_{20}(Q^2)$, Figs. 12 and 13, one sees the similar but increasingly divergent results obtained from the constant- Q^2 and the PFSA methods.

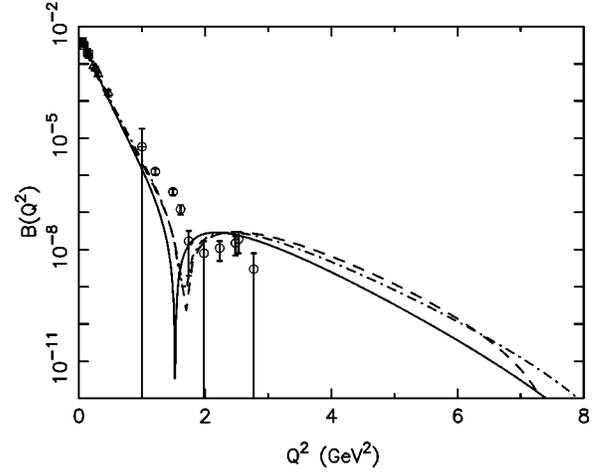


FIG. 12. Point-form (solid), nonrelativistic (dashed), and constant- Q^2 (dash-dot) results for $B(Q^2)$ using the Argonne potential and the G-K form factors.

V. CONCLUSION

This work has used the point form of relativistic dynamics to calculate elastic deuteron form factors. The point form stands somewhat between the covariant approaches and direct-interaction approaches mentioned in the introduction in that it is on the one hand, manifestly covariant (because the Lorentz generators are kinematic) and it is the mass that is “off-shell” (rather than the energy as is the case with the instant form.) On the other hand, the point form is one of the forms of dynamics listed by Dirac, in which all of the interactions are in the four-momentum generators. Moreover there is a natural way in which one-body currents can be introduced in the point form (called the point-form spectator approximation) that satisfies the correct Poincaré and charge conservation properties.

We have shown that the PFSA produces results consistent with other impulse and spectator approximations. Within the range Schiavilla and Riska examined, for example, their impulse approximation and the PFSA (using G-K form factors)

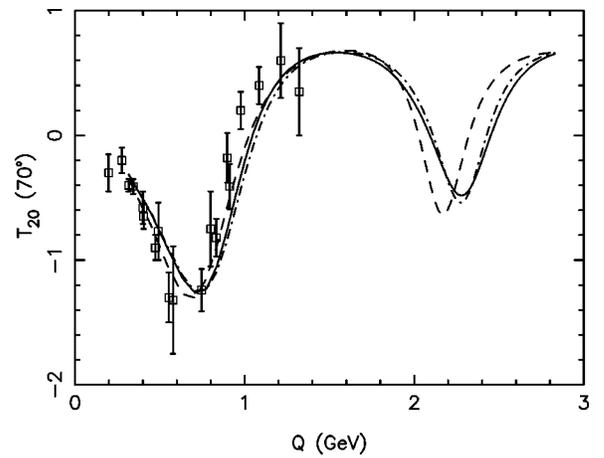


FIG. 13. Point-form (solid), nonrelativistic (dashed), and constant- Q^2 (dash-dot) results for $T_{20}(Q^2)$ using the Argonne potential and the G-K form factors.

predict nearly identical results for $A(Q^2)$ and $T_{20}(Q^2)$; and though the zero they predict for $B(Q^2)$ falls near 2.0 GeV^2 rather than the PFSA's 1.6 , the fall-off from the data begins near 0.5 GeV^2 in both. The calculations of Kobushkin and Syantomov [38] (before using their approach of reduced transition amplitudes) and the PFSA (G-K) results for $A(Q^2)$ and $B(Q^2)$ nearly duplicate each other, as do the results for $T_{20}(Q^2)$ except in the high-momentum range, where no data are currently available. And although the calculations of Chung *et al.* with earlier G-K form factors and the Argonne v_{14} potential fall quite close to the data for all three observables, their calculations using Höhler [35] form factors and potentials show the same salient points: the deviation from data beginning in $A(Q^2)$ near 1 GeV^2 and in $B(Q^2)$ near 0.5 GeV^2 ; the location of the first zero in $B(Q^2)$ between $1.5\text{--}2.0 \text{ GeV}^2$; and the first minimum in $T_{20}(Q^2)$ around 0.8 GeV^2 . The work of Van Orden *et al.* gives results similar to Chung *et al.*, except that the high- Q^2 behavior of $A(Q^2)$ and $B(Q^2)$ is nearly level at 10^{-8} , while Chung *et al.* and the PFSA show gradual decreases at 10^{-11} for $A(Q^2)$ and at 10^{-8} for $B(Q^2)$. In contrast, Lev *et al.* calculate that the Höhler form factors produce results for $A(Q^2)$ that lie close to the data, while the G-K form factors fit $A(Q^2)$ up to 2 GeV^2 but produce results increasingly higher than the data thereafter. Their results for $B(Q^2)$ and $T_{20}(Q^2)$ are similar to those of Van Orden *et al.* and Chung *et al.*

This work also addressed the sensitivity of PFSA results to different nucleon-nucleon interactions and different parametrizations of the nucleon form factors. In almost every instance it was found that the two nucleon-nucleon potentials produced only slight, if any, differences in the form factors and elastic observables. This may not be surprising considering that the Argonne v_{18} and Reid '93 nucleon-nucleon interactions produce nearly identical momentum-space wave functions on the momentum scale of interest.

Much more pronounced were the differences between the Gari-Krümpelmann and the Mergell-Meissner-Drechsel parametrizations of the nucleon form factors. As the momentum transfers become higher, the two often predict significantly different results. These are most notable in the deuteron form factors G_E and G_Q , which are sensitive to the neutron electric form factor; the G-K parametrization, whose neutron electric form factor falls off markedly more rapidly than the MMD, produces deuteron form factor zeros in the intermediate range that occur at higher momentum transfers in the MMD results. That this phenomenon is due to the neutron electric form factor is supported by the similarity of results for G_M , where the nucleon magnetic form factors dominate the calculations.

Aside from differences due to varying potentials and nucleon form factors, the consequences of the point form's nontrivial momentum transfer have also been examined. We have shown that the point-form momentum transferred to a nucleon is greater than the Q^2 transferred to the deuteron, and that its deviation increases with increasing Q^2 . This results in a lowering of the deuteron form factors and elastic scattering observables compared to nonrelativistic calculations. The greater magnitude of the point-form momentum transfer causes a quicker fall-off of the nucleon form factors,

and some deviation from nonrelativistic calculations was attributed to this.

Additional dynamically consistent two-body currents must be added to the PFSA in order to bring the calculations into agreement with data at intermediate to high momentum transfers. Such currents have not been considered in this preliminary work; but other calculations that include dynamical two-body currents (see Refs. [8,11,15,17]) suggest that the addition of dynamical currents is capable of reconciling the differences between various impulse or spectator approximations with data.

In the point form it is quite easy to interpret Feynman diagrams for nucleon-nucleon scattering with the production of a photon as a current matrix element satisfying the requirements given in Sec. III. However, while the addition of such current matrix elements may produce better agreement with data, it does not provide a systematic procedure for constructing two-body currents. What is needed is a procedure for constructing conserved currents from one-body currents and the dynamical mass operator. Models based on such a procedure are being developed.

ACKNOWLEDGMENTS

The author thank Fritz Coester and Gerald Payne for advice and many helpful suggestions.

APPENDIX A: POINT-FORM MOMENTUM TRANSFER

The momentum transfer ($p'_i - p_i$) seen by nucleon i can be computed following Ref. [28]. Suppose that the momentum transfer is along the z axis,

$$B(v_{\text{in}}) = \begin{pmatrix} \cosh \Delta/2 & 0 & 0 & \sinh \Delta/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sinh \Delta/2 & 0 & 0 & \cosh \Delta/2 \end{pmatrix}, \quad (\text{A1})$$

$$B(v_{\text{out}}) = \begin{pmatrix} \cosh \Delta/2 & 0 & 0 & -\sinh \Delta/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sinh \Delta/2 & 0 & 0 & \cosh \Delta/2 \end{pmatrix}, \quad (\text{A2})$$

are the boosts that take the deuteron from the center of momentum frame to the Breit frame (where $\mathbf{P}'_{\text{tot}} = -\mathbf{P}_{\text{tot}}$.) For elastic scattering,

$$\sinh \Delta/2 = \sqrt{\frac{Q^2}{4m_D^2}}. \quad (\text{A3})$$

The initial energies and momenta are then

$$\begin{aligned} E_1 &= \omega \cosh \Delta/2 + k_z \sinh \Delta/2; \\ p_{1z} &= k_z \cosh \Delta/2 + \omega \sinh \Delta/2; \\ E_2 &= \omega \cosh \Delta/2 - k_z \sinh \Delta/2; \\ p_{2z} &= -k_z \cosh \Delta/2 + \omega \sinh \Delta/2; \end{aligned} \quad (\text{A4})$$

where ω and \mathbf{k} are center of momentum variables.

In this notation, k_z refers to the relative z -axis momentum of particle one. This convention gives rise to the following relations:

$$\omega' = \omega \cosh \Delta \mp k_z \sinh \Delta; \quad (\text{A5})$$

$$k'_z = k_z \cosh \Delta \mp \omega \sinh \Delta; \quad (\text{A6})$$

where the minus signs are used when particle one is struck, the plus signs when particle two is struck. Suppose for illustration that particle one is struck. The final energies and momenta will then be

$$E'_1 = \omega \cosh 3\Delta/2 - k_z \sinh 3\Delta/2;$$

$$p'_{1z} = k_z \cosh 3\Delta/2 - \omega \sinh 3\Delta/2;$$

$$E'_2 = E_2; \quad p'_{2z} = p_{2z}. \quad (\text{A7})$$

Now some hyperbolic trigonometry reveals that

$$(p'_1 - p_1)^2 = 4(k_z^2 - \omega^2) \sinh^2 \Delta. \quad (\text{A8})$$

Since

$$\sinh \Delta = 2 \sqrt{\frac{Q^2}{4m_D^2}} \sqrt{1 + \frac{Q^2}{4m_D^2}}, \quad (\text{A9})$$

and

$$k_z^2 - \omega^2 = k_z^2 - m_n^2 - \mathbf{k}^2 = -(m_N^2 + \mathbf{k}_\perp^2), \quad (\text{A10})$$

the resulting Eq. (36) is established.

-
- [1] S. N. Sokolov, Dokl. Akad. Nauk SSSR **233**, 575 (1977).
 [2] F. Coester and W. N. Polyzou, Phys. Rev. D **26**, 1348 (1982).
 [3] E. Wigner, Ann. Math. **40**, 149 (1939).
 [4] F. Gross, Phys. Rev. C **26**, 2203 (1982).
 [5] R. Blankenbecler and R. Sugar, Phys. Rev. **142**, 1051 (1966).
 [6] A. A. Logunov and A. N. Tavkhelidze, Nuovo Cimento **29**, 380 (1963).
 [7] J. W. Van Orden, N. Devine, and F. Gross, Phys. Rev. Lett. **75**, 4369 (1995).
 [8] E. Hummel and J. A. Tjon, Phys. Rev. C **49**, 21 (1994).
 [9] J. Carbonell and V. A. Karmanov, Eur. Phys. J. A **6**, 9 (1999).
 [10] J. Carbonell, B. Desplanques, V. A. Karmanov, and J.-F. Mathiot, Phys. Rep. **300**, 215 (1998).
 [11] F. M. Lev, E. Pace, and G. Salmé, Nucl. Phys. **A663**, 365 (2000).
 [12] F. M. Lev, E. Pace, and G. Salmé, Phys. Rev. Lett. **83**, 5250 (1999).
 [13] F. M. Lev, E. Pace, and G. Salmé, Nucl. Phys. **A641**, 229 (1998).
 [14] F. M. Lev, Ann. Phys. (N.Y.) **237**, 355 (1995).
 [15] P. L. Chung, F. Coester, B. D. Keister, and W. N. Polyzou, Phys. Rev. C **37**, 2000 (1988).
 [16] L. L. Frankfurt, T. Frederico, and M. Strikman, Phys. Rev. C **48**, 2182 (1993).
 [17] R. Schiavilla and D. O. Riska, Phys. Rev. C **43**, 437 (1991).
 [18] S. Mandelstam, Proc. R. Soc. London, Ser. A **233**, 1425 (1955).
 [19] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw Hill, New York, 1980), p. 484.
 [20] P. A. M. Dirac, Rev. Mod. Phys. **21**, 302 (1949).
 [21] B. Bakamjian and L. H. Thomas, Phys. Rev. **92**, 1300 (1953).
 [22] B. D. Keister and W. N. Polyzou, in *Advances in Nuclear Physics*, Vol. 20, edited by J. W. Negele and E. Vogt (Plenum, New York, 1991), p. 225.
 [23] F. Coester and A. Ostebee, Phys. Rev. C **11**, 1836 (1975).
 [24] F. Coester, S. C. Pieper, and F. J. D. Serduke, Phys. Rev. C **11**, 1 (1975).
 [25] B. L. G. Bakker, L. A. Kondratyuk, and M. V. Terent'ev, Nucl. Phys. **B158**, 497 (1979).
 [26] M. V. Terent'ev, Yad. Fiz. **24**, 207 (1976) [Sov. J. Nucl. Phys. **24**, 106 (1976)].
 [27] W. H. Klink, Phys. Rev. C **58**, 3617 (1998).
 [28] W. H. Klink, Phys. Rev. C **58**, 3587 (1998).
 [29] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).
 [30] M. Gari and W. Krümpelmann, Phys. Lett. B **274**, 159 (1992).
 [31] P. Mergell, U.-G. Meissner, and D. Drechsel, Nucl. Phys. **A596**, 367 (1996).
 [32] M. Gourdin, Nuovo Cimento **28**, 533 (1963).
 [33] P. L. Chung and F. Coester, Phys. Rev. D **44**, 229 (1991).
 [34] E. L. Lomon, Ann. Phys. (N.Y.) **125**, 309 (1980).
 [35] G. Höhler, E. Pietarinen, I. Sabba-Stefanescu, F. Borkowski, G. G. Simon, V. H. Walther, and R. D. Wendling, Nucl. Phys. **B114**, 505 (1976).
 [36] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
 [37] V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, Phys. Rev. C **49**, 2950 (1994).
 [38] A. P. Kobushkin and A. I. Syamtomov, Phys. At. Nucl. **58**, 1477 (1994).
 [39] J. Carslon and R. Schiavilla, Rev. Mod. Phys. **70**, 743 (1998).
 [40] P. L. Chung, B. D. Keister, and F. Coester, Phys. Rev. C **39**, 1544 (1989).
 [41] G. G. Simon, Ch. Schmitt, and V. H. Walther, Nucl. Phys. **A364**, 285 (1981).
 [42] S. Galster, H. Klein, J. Moritz, K. H. Schmidt, D. Wegener, and J. Bleckwenn, Nucl. Phys. **B32**, 221 (1971).
 [43] J. E. Elias, J. I. Friedman, G. C. Hartmann, H. W. Kendall, P. N. Kirk, M. R. Sogard, L. P. Van Speybroeck, and J. K. de Pagter, Phys. Rev. **177**, 2075 (1969).
 [44] R. G. Arnold, B. T. Chertok, E. B. Dally, A. Grigorian, C. L. Jordan, W. P. Schütz, R. Zdarko, F. Martin, and B. A. Mecking, Phys. Rev. Lett. **35**, 776 (1975).
 [45] C. D. Buchanan and M. R. Yearian, Phys. Rev. Lett. **15**, 303 (1965).
 [46] F. Martin, R. G. Arnold, B. T. Chertok, E. B. Dally, A. Grigorian, C. L. Jordan, W. P. Schütz, R. Zdarko, and B. A. Mecking, Phys. Rev. Lett. **38**, 1320 (1977).
 [47] R. G. Arnold *et al.*, Phys. Rev. Lett. **58**, 1723 (1987).

- [48] M. E. Schulze *et al.*, Phys. Rev. Lett. **52**, 597 (1984).
[49] R. Gilman *et al.*, Phys. Rev. Lett. **65**, 1733 (1990).
[50] I. The *et al.*, Phys. Rev. Lett. **67**, 173 (1991).
[51] M. Garçon *et al.*, Phys. Rev. C **49**, 2516 (1994).
[52] M. Ferro-Luzzi *et al.*, Phys. Rev. Lett. **77**, 2630 (1996).
[53] V. F. Dmitriev, D. M. Nikolenko, S. G. Popov, I. A. Rachek,
Yu. M. Shatunov, D. K. Toporkov, E. P. Tsentalovich, Yu. G.
Ukrainsev, B. B. Voitsekhovskii, and V. G. Zelevinsky, Phys.
Lett. **157B**, 143 (1985).
[54] M. Bouwhuis *et al.*, Phys. Rev. Lett. **82**, 3755 (1999).
[55] D. Abbott *et al.*, Phys. Rev. Lett. **84**, 5053 (2000).
[56] L. C. Alexa *et al.*, Phys. Rev. Lett. **82**, 1374 (1999).