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Testing binomial reducibility and thermal scaling in hadron-induced multifragmentation

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A binomial reducibility and thermal scaling analysis is performed on well-chacracterized thermal-like sources formed in 8 GeV/c $\pi^- + ^{197}$Au reactions. The fragment probability distributions are shown to be binomial when plotted as a function of the measured excitation energy E^* and the binomial elementary probability p is shown to follow the expected Boltzmann factor: $\ln(p) \propto \exp(-B/\sqrt{E^*/A})$. Binomial reducibility and thermal scaling are explored also using global variables other than E^*, and the effect of source size on the binomial parameter p and m is shown. Finally, the extracted probability p is found to be correlated with the experimentally deduced fragment emission time up to about 6A MeV of excitation energy, hinting at a possible transition in decay mechanism above that excitation energy.

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Multifragmentation is a phenomenon by which a nuclear system decays to a final configuration that contains multiple intermediate mass fragments (IMF) of charge $3 \leq Z_{\rm IMF} \leq 20$. The underlying process that leads to multifragmentation is the subject of intense research from both theoretical [1–4] and experimental points of view [5–10]. Of primary concern is whether multifragmentation is the result of a liquid-gas phase transition or of dynamical processes [3,11].

Recently, it was found in many different data sets [12–14] that when sorted as a function of the transverse energy E_t of all detected charged particles, where

$$E_t = \sum_i E_i \sin^2 \theta_i, \tag{1}$$

the IMF emission probability distributions were well described in term of a binomial distribution,

$$P_n^m(E_t) = \frac{m!}{n!(m-n)!} p^n (1-p)^{m-n}. \tag{2}$$

Here n is the IMF multiplicity, m is the number of chances to emit an IMF, and p is the binary elementary probability. In this description, the emission of n IMFs is a simple convolution of p. The parameter m acts as a constraint on the system, i.e. it insures charge conservation. The values of p and m can be extracted experimentally from the average multiplicity $\langle n \rangle$ and its variance as

$$\langle n \rangle = mp \quad \text{and} \quad \sigma_n^2 = \langle n \rangle (1-p). \tag{3}$$

The immediate implication of this result is the stochasticity, or independence, of the emission process in multifragmentation, i.e., the emission probability of the second IMF is not influenced by the first, other than by the number of tries. This has far reachinig consequences not only because of the ''natural'' statistical interpretation taken by the authors [12], but from the potential dynamical processes that may lead to stochasticity.

The rationale by the authors [12] for a statistical interpretation was based on the experimental observation that a linear relationship results when $\ln 1/p$ is plotted as a function $1/\sqrt{E_t}$. The assumption is made that E_t is proportional to the excitation energy E^*, and therefore the temperature $T \propto \sqrt{E_t}$. The linearity of such plots suggests that p has the Boltzmann form $p \propto \exp(-B/T)$ where B is the emission barrier. The plots are called Arrhenius plots and convey the notion of thermal scaling.

The simplicity of this approach was later extended to charge distributions [15,16] and angular distributions [17], where the same reducible and thermal characters were found. Finally, by restricting the IMF definition to a single Z value, it was shown that the probability distributions follow a Poisson distribution instead of a binomial distribution [18]. This is consistent with the interpretation of m as a constraint that is relaxed by the new definition of an IMF. This behavior of the system under the ''removal'' of a constraint is similar to the change in statistical mechanics from the canonical ensemble (binomial statistics) to the grand canonical ensemble (Poisson statistics). Recently, Poisson reducibility and thermal scaling have been shown to exhibit common behavior in the EOS data set, percolation, and the Fisher droplet model [19].

The binomial reducibility concept, and thermal-scaling interpretation resulting from it, has been criticized on several grounds: autocorrelation effects [20,21], self-correlations [22], source size effects [23], and the width of the relationship between E_t and the real excitation energy [24] ([25,26]

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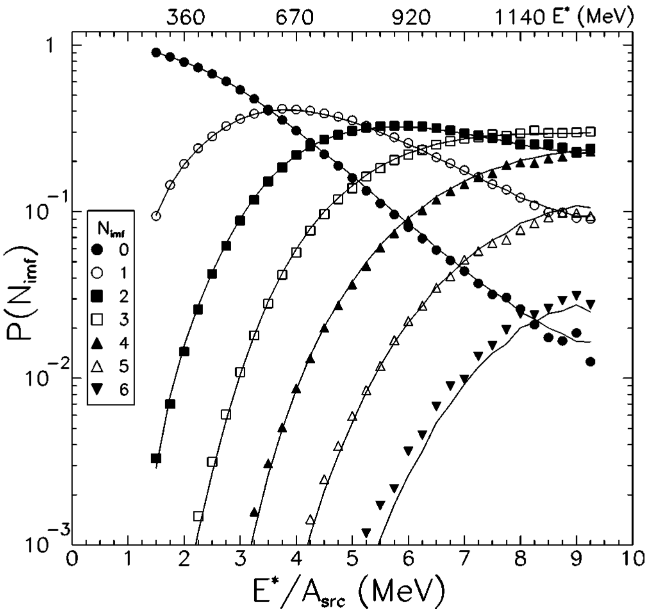

FIG. 1. The experimental (symbols) and calculated (lines) *n*-fold IMF probability distributions as a function of E^*/A. The lines assume a binomial probability distribution according to Eq. (2) and p and m were extracted from Eq. (3).

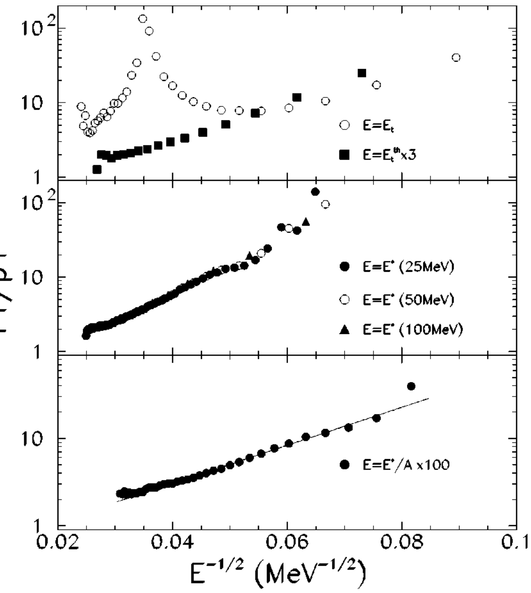

FIG. 2. Upper panel: Inverse of the single fragment emission probability p as a function of $1/\sqrt{E_t}$ and $1/\sqrt{E_t^{th}}$. Middle panel: $1/p$ as a function of $1/\sqrt{E^*}$ for bin widths in E^* of 25, 50, and 100 MeV. Bottom panel: Arrhenius plot of $1/p$ using $1/\sqrt{E^*/A}$. The solid line is a fit using Eq. (4).

reply to some of these criticisms). However, the heart of the argument goes mainly to the assumed relation between E_t and E^*, and the source size.

On the other hand, three different models of nuclear decay [21,27,28] have shown that n IMF probability distributions can be reproduced by binomial distributions when sorting is done as a function of excitation energy, and that p does scale according to a Boltzmann factor. However, support from an experimental data set as a function of E^* and source size has been lacking.

In this Rapid Communication, we establish an experimental reference frame for further discussion on the subject of reducibility and thermal scaling by ''testing'' the procedure on well-characterized equilibriumlike systems (known *a priori*) formed in 8 GeV/c $\pi^- + {}^{197}$Au reactions [29]. For such systems, the excitation energy has been evaluated on an event-by-event basis, as described in Ref. [30]. Moreover, for the first time the experimental relationship between p and the IMF emission time τ is examined.

Charged particles with kinetic energy between 1.0 and 92A MeV were detected with the ISiS 4π detector array [31] during experiment E900a at the Brookhaven National Laboratory AGS accelerator. Tagged beams of 8 GeV/c π^- were incident on a 2.0 mg/cm^2 ^{197}Au target. Fragments with charge $Z \leqslant 16$ were identified with a set of 162 gas-ion chamber/Si/CsI triple telescopes. Geometric acceptance was 74% of 4π. Grey particles up to $\sim$300–400 MeV (assumed to be protons) were also detected. Futher experimental details can be found in [29,32].

The quality of the binomial description of the IMF probability distributions is shown in Fig. 1 as function of E^*/A for the parameters p and m, which were extracted using Eq. (3). The representation is good down to $N_{\rm IMF}=5$ with some discrepencies for $N_{\rm IMF}=6$. The average observed IMF multiplicity at 9A MeV is about three (corrected multiplicity is about 30–35% higher).

In the top panel of Fig. 2, the binomial parameter p was extracted from the variance and the mean of the IMF multiplicity distribution [Eq. (3)] as in [12], and shown as a function of E_t and E_t^{th}. A divergence in the $1/p$ plot is shown when E_t for all charged particles is used. This behavior is attributed to the excessively large width of the correlation between E_t and the ''real'' (internal) excitation energy, as explained by Toke *et al.* [24]. On the other hand, in this data set the divergence does not occur at low E_t, but around 800 MeV. As soon as the first stage particles are removed, the divergence disappears, leaving an almost linear plot of $1/p$ vs $1/\sqrt{E_t^{th}}$ (except for the first point). This result seems at variance with Refs. [21,24] in which divergence is seen even when E_t from an evaporation model is used to generate an Arrhenius plot. As pointed out by Moretto *et al.* [25], since such models are unable to describe multifragmentation in the first place, it is unclear that the intrinsic widths between E^* and E_t as taken from these models are relevant. Our data set appears consistent with this point. The above result is stable with regard to our definition of thermal particles, i.e., even including some preequilibrium particles by relaxing our thermal particle definition [29], linear Arrhenius plots are still produced, as long as highly energetic particles ($\geqslant$30A MeV) are removed.

In the middle panel of Fig. 2, Arrhenius plots are generated as a function of the total excitation energy E^*. Bin widths in E^* of 25, 50, and 100 MeV are assumed in order to test the sensitivity of our event selection. All three bin-width assumptions display a common linear behavior in

TABLE I. Values of the binomial parameter m, the primary source size S_{src}, the observed charge Z_{obs}, and the Q_{value} for various E^*/A bins.

E^*/A	m	Z_{src}	Z_{obs}	Q_{value}
2.0	3.36	74.3	8.75	−160
3.0	4.82	71.2	15.2	−204
4.0	5.61	68.4	21.9	−249
5.0	6.29	65.8	28.1	−295
6.0	6.72	63.5	33.5	−341
7.0	7.67	61.1	38.1	−383
8.0	8.31	59.0	42.2	−420
9.0	7.79	57.1	45.4	−451

which $1/p$ changes by roughly a factor of 20 over the measured range. Finally, in the lower panel of Fig. 2, the dependence on E^*/A is examined. The line represents a fit to the data using the Boltzmann form $\exp(-B/\sqrt{E^*/A})$, where B is of the order 16 MeV, and a level density parameter $a=A/11$ MeV^{-1} is used. The first-chance emission barrier for $Z=3$ at high excitation energy is about 22 MeV for this reaction. It is important to stress that because of the thermal nature of the system in this study, the plots of Fig. 2 convey the notion of thermal scaling.

The evolution of the parameter m for various bins of E^*/A is given in Table I. Also shown in Table I are the values of the primary source charged Z_{src}, the sum of the observed charge Z_{obs}, and the average Q_{value} of all channels at a given excitation energy. Over the excitation energy range shown in the table, m varies by a factor of 2.3, from $m=3.36$ at $E^*/A=2$ MeV to ~ 7.79 at $E^*/A=9$ MeV, while Z_{src} decreases by a factor of 1.3. Clearly the parameter m is not a constraint related to source size of the multifragmenting system as suggested by Moretto *et al.* [14]. While Z_{obs} also increases with E^*/A, it changes twice as fast as m. The only variable that appears to track rather well m is the mass-energy balance or Q_{value}. This would seem to point to an interpretation of m as representing an energy constraint, allowing only certain (IMF) partitions. Therefore, the number of throws m is limited by the cost of producing these partitions and not necessarily the number of charges available to emit IMF.

The behavior of the parameter p and m for fixed E^*/A bins has also been explored to create event samples that can be sorted as a function of other variables. The goal of the exercise is to verify the properties of the binomial parameters if the intensive variable, here E^*/A, is kept constant. Two variables were chosen, the transverse energy of all the charged particles, E_t, and Z_{src}. The bottom panels of Fig. 3 show the parameter p for $E^*/A=3.0$, 5.0, and 7.5 MeV with a bin width of 0.5 MeV. Here p remains essentially constant over the full range for both variables with values comparable with those in Fig. 2. In fact, the limit in value that p can take is nicely predicted by the width of the E^*/A bins, i.e., p at 2.75A MeV is different than at 3.25 MeV.

However, the parameter m varies strongly as a function of E_t and Z_{src} (top panels), increasing with Z_{src} while decreasing with E_t (for our data set, E_t is known to be anticorrelated to Z_{src}). Thus for fixed excitation energies, it can be argued that m fulfils its intended role in the binomial distribution very well; m accounts for the increasing available charge that can produce IMFs, according to a probability p. Therefore, contrary to the report by Bauer and Pratt [23], the changes in the sum of the observed charge are mostly accounted for by the binomial parameter m and are not the primary factor influencing a change in the binomial probability p. The parameter p does behave as an elementary probability in the sense that p remains constant when the excitation energy is constant. On the other hand, our results seem to validate part of their approach, i.e., the increase in the elementary probability p is in fact related to a change in the slope of the charge distribution *with* E^*/A, as previously shown by power-law fits for this data set [30], again consistent with the thermal-like behavior of p.

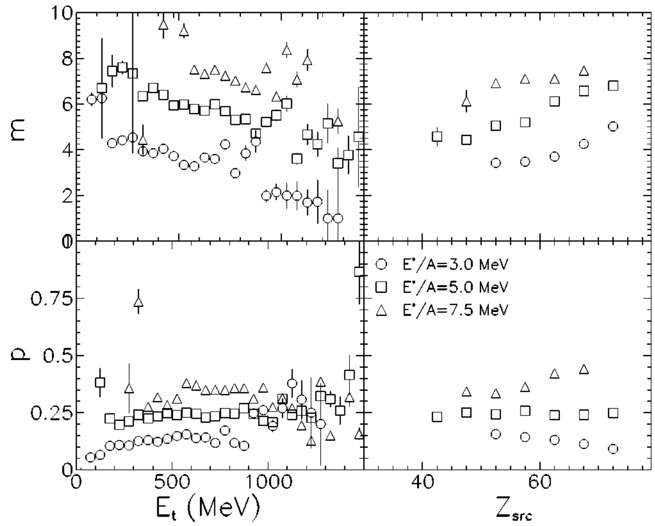

FIG. 3. Upper left panel: The extracted binomial parameter m (top panels) and p (bottom panels) as a function of the transverse energy E_t (left panels) and Z_{src} (right panels) for three bins of E^*/A: 3.0 MeV represented by the open circle, 5.0 MeV by the open square, and 7.5 MeV by the open triangle.

It is however important to point out that in the analysis made here, the E^*/A bin width was kept small. For an event sample where the width of the correlation between E^*/A and E_t or Z_{src} would be large, p could take a wide range of values just because of that width. It would therefore be very difficult to disentangle the effect of varying source size and that of the temperature.

Following Moretto and co-workers [12,14], it was further assumed that if p represents a ''true'' elementary binary probability in the sense of time sequential emission, then p can be expressed in terms of a partial decay width of that particular binary channel by

$$p=\frac{\Gamma}{\hbar\omega_0}=e^{-B/T}, \tag{4}$$

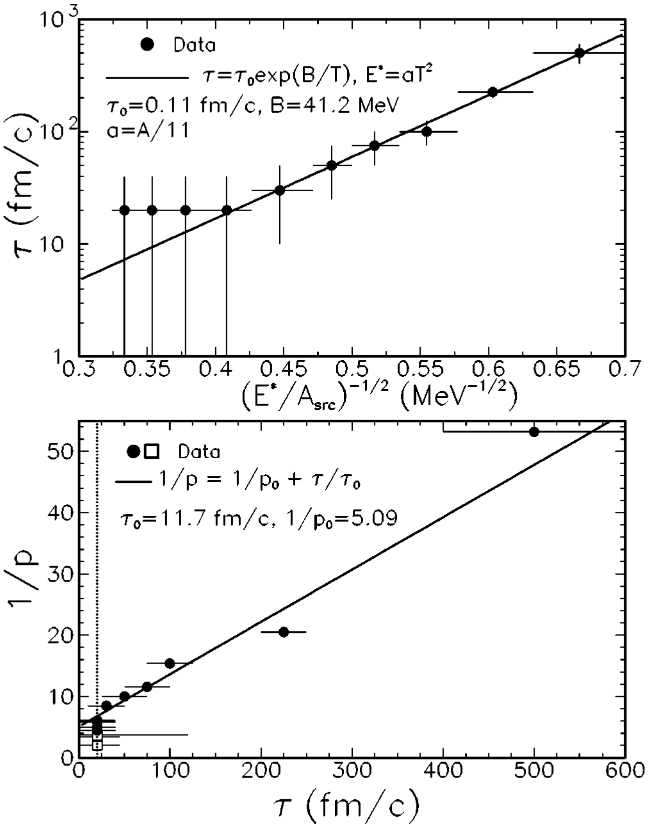

FIG. 4. Top panel: IMF emission time τ as a function of E^*/A, from Ref. [33]. The line corresponds to a fit using Eq. (5). Bottom panel: Graph of $1/p$ vs τ for the same bins in E^*/A as in the upper panel. The solid line is a linear fit to the data. The dotted line indicates the ''apparent'' saturation in emission time.

where ω_0 can be interpreted as the frequency of assault on the barrier B and T is the temperature of the system. Defining the intrinsic τ_0 as $1/\omega_0$, the corresponding emission time is given by

$$\tau=\tau_0 e^{B/T} \quad \text{therefore} \quad p=\tau_0/\tau. \tag{5}$$

The IMF emission time for this reaction has been extracted by Beaulieu *et al.* [33] using excitation-energy-gated two-IMF correlation functions and an IMF range defined as $4\leqslant Z_{\rm IMF}\leqslant 9$. The result of that analysis is summarized in the upper panel of Fig. 4. The emission time τ decreases exponentially with E^*/A up to about $6A$ MeV and saturates afterward at $\tau\sim 20$ fm/c. The solid line is a fit to the data using Eq. (5). The obtained barrier B is found to be around 41 MeV and is larger than the one extracted from the $1/p$ vs $1/\sqrt{E^*/A}$ plot by a factor of 2 for the same $Z_{\rm IMF}$ range, with $B'=21$ MeV and the same level-density parameter. From Eqs. (4) and (5) one would expect the barrier to be the same. However, it is known experimentally that the values of p follow the minimum charge of the IMF range, here $Z=4$, and as such in the above interpretation each IMF charge would have different emission time [13,14]. In contrast, for the emission time scale extracted from IMF-IMF correlation, it is assumed that each IMF species have the same emission time. Therefore the relation between τ and p might not be straightforward.

Still using the above definition for IMFs, the binomial analysis was redone for the same excitation energy bins as in the time scale analysis. The reciprocal of the extracted parameter p is shown in the bottom panel of Fig. 4 as a function of the emission time τ. As predicted by Moretto *et al.* [12,14], $1/p$ is well represented by a simple linear relation to the emission time down to about 20 fm/c, corresponding to $E^*/A\sim 6A$ MeV. At higher excitation energy, the extracted time saturates while $1/p$ still decreases. For completeness, values of $1/p$ at higher excitation energy are shown as open squares. Because of the limited statistics, it was not possible to extract emission time at these excitation energies. However, since τ is nearly constant with a value of 20 fm/c above $E^*/A=6$ MeV, a value for τ of 20 fm/c was assigned to each value of $1/p$. The evolution of $1/p$ at that point seems independent of time, which would argue for a spacelike interpretation of p, rather than sequential, at high excitation energy [14]. Therefore the break in this linear behavior of $1/p$ vs τ suggests the possibility of a change in the emission mechanism from a sequential process (surface dominated emission) to a simultaneous process (bulk emission). This conclusion was reached recently for this data set [33] by looking at the global behavior of the time, thermally driven expansion energy and IMF emission probability.

Finally, the parameter τ_0, extracted in the upper panel, corresponds to the infinite temperature limit, and the extrapolation of the exponential fit as such has little meaning. In the bottom panel, τ_0 is represented by the slope, and its value of 11.7 fm/c is more in line with what could be expected for a characteristic emission time (fluctuation time) and is very close to the experimentally measured saturation in the emission time.

In conclusion, we have shown the applicability of the binomial reducibility analysis and thermal scaling on a reference data set obtained in hadron-induced thermal multifragmentation. For such well-characterized systems, the n-fold IMF probability distributions can be described by the binomial equation. As long as the first-stage cascade particles are removed, linear plots of $\ln 1/p$ vs $1/\sqrt{E^*/A}$ are found. We have shown that the parameter m acts as an energy constraint that is related to and tracks with the changes in the average Q_{value}. Finally, this analysis is consistent with a picture in which p reflects the elementary probability of the system, the source size has little influence on p, and p has the expected correlation with the emission time in a time sequential interpretation up to $\sim 6A$ MeV. The sudden change of the $1/p$ versus τ plot at very short emission times might suggest a transition in the mechanism from sequential emission to simultaneous multifragmentation at high excitation energies. At the very least, it can be concluded in this picture that the emission times have reached values close to the characteristic time τ_0. Further investigation is needed to assess the validity of this interpretation. In particular, exploring the difference in the extracted barriers by comparison of the emission time for various Z_{IMF} pairs would be useful.

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[1] D.H.E. Gross, Rep. Prog. Phys. **53**, 605 (1990).
[2] J.P. Bondorf, A.S. Botvina, A.S. Iljinov, I.N. Mishustin, and K. Sneppen, Phys. Rep. **257**, 133 (1995).
[3] J. Aichelin, Phys. Rep. **202**, 233 (1991).
[4] W.A. Friedman, Phys. Rev. C **42**, 667 (1990).
[5] *Multifragmentation*, Proceedings of the XXVII International Workshop on Gross Properties of Nuclei and Nuclear Excitations, edited by H. Feldmeyer, J. Knoll, W. Nörenberg, and J. Wambach (GSI, Darmstadt, Germany, 1999), ISSN No. 0720-8715.
[6] J. Pochodzalla, Prog. Part. Nucl. Phys. **39**, 443 (1997).
[7] L.G. Moretto and G.J. Wozniak, Annu. Rev. Nucl. Part. Sci. **43**, 379 (1993).
[8] B. Borderie, Ann. de Phys. **17**, 349 (1992).
[9] D. Guerreau, *Formation and Decay of Hot Nuclei: The Experimental Situation* (Plenum, New York, 1989).
[10] W.G. Lynch, Annu. Rev. Nucl. Part. Sci. **37**, 493 (1987).
[11] J. Toke and U. Schroder, Phys. Rev. Lett. **82**, 5008 (1999).
[12] L.G. Moretto *et al.*, Phys. Rev. Lett. **74**, 1530 (1995).
[13] K. Tso *et al.*, Phys. Lett. B **361**, 25 (1995).
[14] L.G. Moretto *et al.*, Phys. Rep. **287**, 249 (1997).
[15] L. Phair *et al.*, Phys. Rev. Lett. **75**, 213 (1995).
[16] L.G. Moretto *et al.*, Phys. Rev. Lett. **76**, 372 (1996).
[17] L. Phair *et al.*, Phys. Rev. Lett. **77**, 822 (1996).
[18] L. Beaulieu *et al.*, Phys. Rev. Lett. **81**, 770 (1998).
[19] J.B. Elliot *et al.*, Phys. Rev. Lett. **85**, 1194 (2000).
[20] A. Wieloch and D. Durand, Z. Phys. A **359**, 345 (1997).
[21] A. Wieloch *et al.*, Phys. Lett. B **432**, 29 (1998).
[22] M.B. Tsang and P. Danielewicz, Phys. Rev. Lett. **80**, 1178 (1998).
[23] W. Bauer and S. Pratt, Phys. Rev. C **59**, 2695 (1999).
[24] J. Toke *et al.*, Phys. Rev. C **56**, R1683 (1997).
[25] L.G. Moretto, L. Phair, and G.J. Wozniak, Phys. Rev. C **60**, 031601(R) (1999).
[26] L.G. Moretto *et al.*, nucl-ex/9709011.
[27] A.S. Botvina and D.H.E. Gross, Phys. Lett. B **344**, 6 (1995).
[28] R. Donangelo and S.R. Souza, Phys. Rev. C **56**, 1504 (1997).
[29] T. Lefort *et al.*, Phys. Rev. Lett. **83**, 4033 (1999).
[30] L. Beaulieu *et al.*, Phys. Lett. B **463**, 159 (1999).
[31] K. Kwiatkowski *et al.*, Nucl. Instrum. Methods Phys. Res. A **360**, 571 (1995).
[32] W.-c. Hsi *et al.*, Phys. Rev. Lett. **79**, 817 (1997).
[33] L. Beaulieu *et al.*, Phys. Rev. Lett. **84**, 5971 (2000).