Microscopic cluster study of the ^{5}H nucleus

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The ^{5}H nucleus is investigated in the generator coordinate method, using $^3\mathrm{H}+n+n$ three-cluster wave functions. The model is tested with the $^3\mathrm{H}+n$ and $^3\mathrm{He}+p$ properties which agree fairly well with the experimental data. The ^{5}H energy is found to be $E\approx 3$ MeV with respect to the $^3\mathrm{H}+n+n$ threshold, and the neutron width is $\Gamma_n\approx 1-2$ MeV or $\Gamma_n\approx 1-4$ MeV, according to the nucleon-nucleon interaction. We therefore suggest that the ^{5}H lifetime should be larger than the ^{4}H lifetime ($\Gamma_n\approx 5-6$ MeV).

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The search for heavy isotopes, and especially for ^{5}H, started more than 30 years ago [1]. In a ^{3}He(^{3}He,n) experiment, Adelberger *et al.* tried to observe the mirror system ^{5}Be, but did not find evidence for the ground state. They concluded that ^{5}Be is unstable by at least 4.2 MeV with respect to the $^3\mathrm{He}+p+p$ threshold and, from an R-matrix analysis, that the ^{5}H binding energy should be larger than 2.2 MeV. Subsequent attempts by Weisenmiller *et al.* [2] and by Belozyorov *et al.* [3] using stripping reactions on ^{8}B or ^{9}Be isotopes were also unsuccessful in the search for a narrow ^{5}H ground state, although the experiment of Belozyorov *et al.* gave evidence for the ^{4}H and ^{6}H ground states. In the theoretical point of view, shell-model calculations of Poppelier *et al.* [4] and of Bevelacqua [5] suggest that the ^{5}H ground state should be located between 3 and 4.5 MeV [4] or near 2.5 MeV [5] above the $^3\mathrm{H}+n+n$ threshold. Recently, Shul'gina *et al.* [6] investigated ^{5}H in a nonmicroscopic three-body model, using ^{3}H-n and n-n potentials; these authors find an energy of 2.5–3.0 MeV with a width of 3–4 MeV.

Recent experiments, using neutron-rich radioactive beams are partially achieved or in projects at Dubna [7]. Neutron-rich projectiles, such as ^{6}He or ^{8}He, open new perspectives for the production of heavy hydrogen isotopes with high statistics. At first sight, the ^{5}H nucleus should have properties rather similar to those of the well-known halo nucleus ^{6}He: a compact core, with no or high-energy excited states, surrounded by two neutrons. The main difference is of course that ^{6}He is particle bound whereas ^{5}H is unbound. Another interesting similarity between both systems concerns their ''Borromean'' nature. In ^{6}He and ^{5}H, none of the two-body subsystems is stable. The binding of ^{6}He (-0.98 MeV) is much larger than the corresponding $^4\mathrm{He}+n$ energy ($+0.89$ MeV with respect to the $^4\mathrm{He}+n$ threshold). The situation could be rather similar in ^{5}H since the $^3\mathrm{H}+n$ subsystem is known to be unstable by about 3 MeV [8]. A ^{5}H binding energy lower than 3 MeV could provide a ''quasi-Borromean'' system in the sense that ^{5}H, although unstable, would be more bound than the two-body subsystems.

In this Brief Report, we report on a microscopic calculation, using three-cluster generator coordinate method (GCM) wave functions [9,10]. In such a model, the five-body Hamiltonian H is given by

$$H=\sum_{i}^{5} T_i+\sum_{i<j}^{5} V_{ij}, \tag{1}$$

where T_i is the kinetic energy of nucleon i and V_{ij} the nucleon-nucleon interaction. The basis functions of the system are defined in the three-cluster approximation

$$\Phi_{\nu_1\nu_2\nu_3}(R_1,R_2,\alpha)=\mathcal{A}\phi_t^{\nu_1}\phi_n^{\nu_2}\phi_n^{\nu_3}, \tag{2}$$

where $\mathcal{A}$ is the five-nucleon antisymmetrizer, (ν_1,ν_2,ν_3) are the spin projections of the triton and of the external neutrons, respectively, and $(\phi_t^{\nu_1},\phi_n^{\nu_2},\phi_n^{\nu_3})$ are the corresponding wave functions, defined in the harmonic oscillator model and centered at locations depending on the generator coordinates R_1, R_2, and α (see Fig. 1). Such a model has been used successfully to investigate halo nuclei and especially the ^{6}He nucleus in an $\alpha+n+n$ model [11]. The good results obtained for this nucleus give some confidence in the ability of the model to describe the similar system $^3\mathrm{H}+n+n$.

Basis wave functions (2) are projected on total spin J and parity π of ^{5}H, using standard projection techniques [10]; this yields the projected basis functions $\Phi^{JM\pi}_{\nu_1\nu_2\nu_3}(R_1,R_2,\alpha)$. The total wave function of the system is then obtained from a linear combination of basis states, yielding

$$\Psi^{JM\pi}=\sum_{\nu_1\nu_2\nu_3}\int f^{J\pi}_{\nu_1\nu_2\nu_3}(R_1,R_2,\alpha)\times\Phi^{JM\pi}_{\nu_1\nu_2\nu_3}(R_1,R_2,\alpha)dR_1dR_2d\alpha, \tag{3}$$

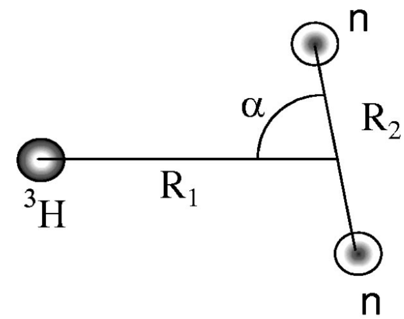

FIG. 1. Three-cluster structure of ^{5}H.

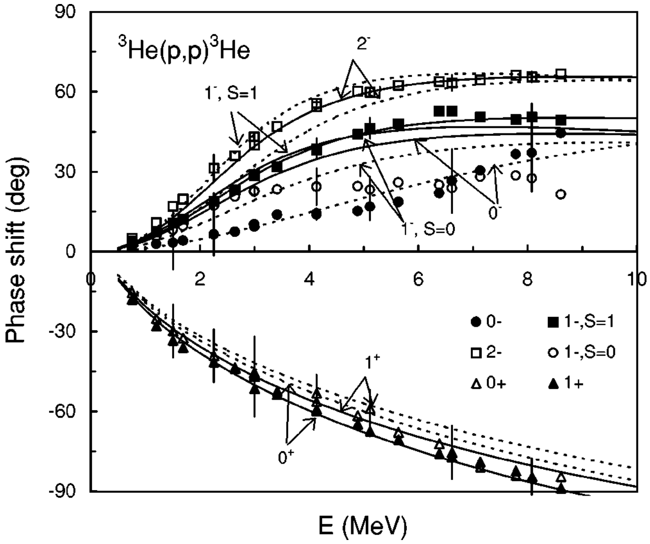

FIG. 2. ^{3}He$+p$ elastic phase shifts compared to the data of Tombrello [16]. The solid and dotted curves correspond to the MN and MH forces, respectively.

where the generator function $f^{J\pi}$ is determined from the Hill-Wheeler equation [12]

$$\sum_{\nu_1\nu_2\nu_3} \int dR_1 dR_2 d\alpha f^{J\pi}_{\nu_1\nu_2\nu_3}(R_1,R_2,\alpha) \times \langle \Phi^{JM\pi}_{\nu_1'\nu_2'\nu_3'}(R_1',R_2',\alpha')|H-E|\Phi^{JM\pi}_{\nu_1\nu_2\nu_3}(R_1,R_2,\alpha)\rangle = 0. \tag{4}$$

This equation involves the overlap and Hamiltonian kernels which are known to depend on three-dimensional integrals of unprojected matrix elements [10]. In practice the integrals over the generator coordinates are replaced by finite sums.

The main ingredient of a microscopic model is the nucleon-nucleon interaction. To evaluate the sensitivity of the final results with respect to this input, we use two different variants. A first calculation is performed with the Minnesota (MN) force [13] and a zero-range spin orbit force [14] with amplitude S_0. The MN interaction has been optimized on light nuclei and is very well adapted to five-nucleon systems. We complement the calculation with the force suggested by Mertelmeier and Hofmann (MH) [15], which is also adjusted on the properties of light nuclei, but contains a tensor term, absent in the MN interaction. Both calculations are achieved with the oscillator parameter which minimizes the triton binding energy ($b=1.58$ fm for MN and $b=1.40$ fm for MH).

Since very little is known about ^{4}H and ^{5}H at low energies, we first investigate the ^{3}He$+p$ phase shifts (see Fig. 2) which are measured in a wide energy range [16]. For the MN force, the mixing parameter u and the spin-orbit strength have been determined on the 2^- and 1^- ($S=1$) experimental phase shifts, yielding $u=1.12$ and $S_0=25$ MeV fm^5. When these parameters are fixed, the model does not contain any degree of freedom. The MH force does not involve any free parameter. In positive parity, both interactions yield similar results, although the MN interaction is closer to experiment. The MH force slightly overestimates the 1^- ($S=1$) phase shift but the presence of a tensor component significantly improves the 0^- phase shift, with respect to the MN interaction.

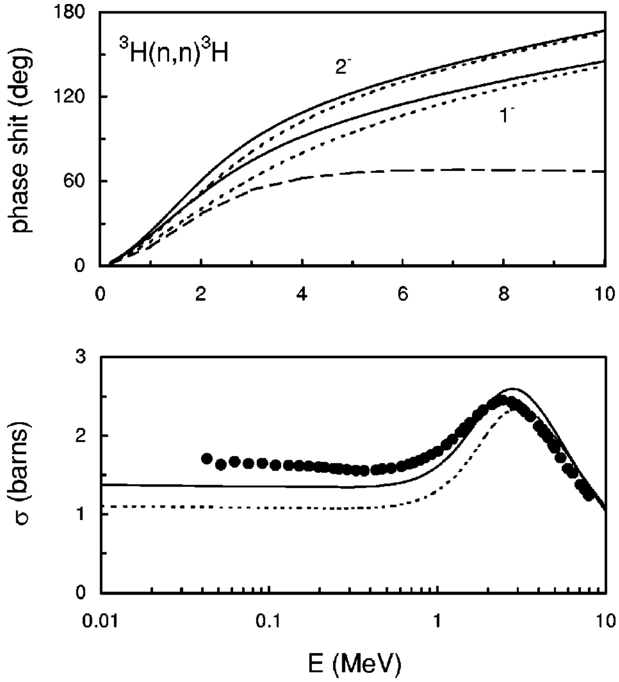

FIG. 3. Upper panel: ^{3}H$+n$ phase shifts after removal of the hard-sphere phase shift (with a radius of 5 fm); the dashed curve is the 2^- phase shift (MN force) including the hard-sphere component. Lower panel: elastic cross section compared to the data of Ref. [17]. The solid and dotted curves correspond to the MN and MH forces, respectively.

Before investigating ^{5}H, we start with the ^{3}H$+n$ system. According to the recent compilation of Tilley *et al.* [8], the ^{4}H spectrum presents two low-lying states: the 2^- ground state ($E=3.19$ MeV) and the 1^- excited state ($E=3.50$ MeV). The neutron widths are 5.4 and 6.7 MeV, respectively. For such broad resonances, the energy and width are, at least partly, model dependent. In Fig. 3, we show the GCM phase shifts, after removal of the hard-sphere phase shift (the channel radius is $a=5$ fm). As shown in Fig. 3, the hard-sphere phase shift is quite important beyond 3 MeV. We define the resonance energy as the energy where the phase shift δ is 90°; the width Γ_n is obtained from the energy derivative $d\delta/dE=2/\Gamma_n$. The theoretical values are given in Table I, and compared to experiment. Notice that the experimental values are obtained from *R*-matrix fits of ^{3}He$+p$ data which affects the accuracy of the quoted values for ^{4}H. From Table I, we conclude that, without a fitting procedure, the model gives a realistic description of p waves in ^{4}H and that the sensitivity with respect to the nucleon-nucleon interaction is fairly weak. Reproducing p waves is quite important since they are expected to dominate the ^{5}H structure. The lower panel of Fig. 3 shows the ^{3}H$+n$ elastic cross section, where the maximum near 3 MeV arises from the superposition of the 2^- and 1^- resonances. Below 1

TABLE I. Energies (with respect to the ^{3}H$+n$ or ^{3}H$+n+n$ threshold) and neutron width of ^{4}H and ^{5}H (in MeV).

	MN	MH	Expt. [a]
^{4}H			
$E(2^-)$	3.05	3.39	3.19
$\Gamma(2^-)$	5.1	5.1	5.42
$E(1^-)$	3.89	4.65	3.50
$\Gamma(1^-)$	7.6	8.2	6.73
^{5}H			
$E(1/2^+)$	2.8–3.0	3.0–3.2	
$\Gamma(1/2^+)$	1–2	1–4	

[a]Reference [8].

MeV, the cross section is essentially given by the $l=0$ component, and the model underestimates the data by 15% (MN force) and 30% (MH force).

Let us now come to the ^{5}H system. The generator coordinates R_1, R_2, and α are chosen as $R_1=2.5$–8.5 fm (step 1 fm), $R_2=0.5$–4.5 fm (step 1 fm), and $\alpha=0°$–$90°$ (step 30°). Other sets have been used with minor influence on the physical results. In Fig. 4, we present the ^{3}H$+n+n$ energy surfaces, which correspond to the energy of the system for given values of R_1 and R_2; all α values are taken into account. Since the MN and MH forces provide similar energy surfaces, we show the MN results only. For both interactions, the energy surface presents a shallow minimum located at large values of R_1 and R_2, larger than for ^{6}He. This minimum corresponds to the ^{5}H ground state, which is expected to be fairly broad. In the bound-state approximation, which neglects the asymptotic behavior of the wave function, the rms radius of ^{5}H is about 3.6 fm. Even if this value is qualitative only, it corresponds to a radius larger by 50% than the ^{6}He radius.

The study of broad resonances in a multicluster model raises several problems to derive energies and widths. Correct asymptotic boundary conditions are difficult to take exactly into account. An interesting method, called ‘‘analytic continuation in the coupling constant’’ (ACCC), has been recently developed by Tanaka *et al.* [18,19] and has been shown to be well adapted to cluster models. In this method, one linear parameter involved in the Hamiltonian is used to define a Padé approximation of the square root of the energy. The coefficients of the Padé approximant are obtained from diagonalization of the Hamiltonian for several values of the linear parameters. Then the approximant is continued in the complex plan yielding the energy and width of a resonance. We refer the reader to Refs. [18,19] for more detail.

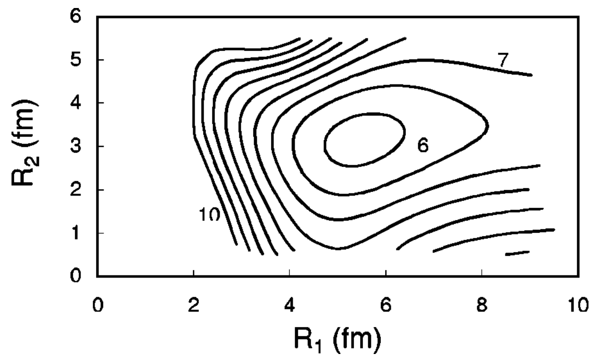

FIG. 4. Energy surface of the ^{3}H$+n+n$ system with the MN interaction. The curves are plotted by steps of 0.5 MeV.

In the present calculation, the coupling constant is chosen as the mixing parameter u for the MN interaction and the strength of the tensor force in the MH force. Energies and widths obtained for ^{5}H are gathered in Table I. The ^{5}H ground state is found close to 3 MeV for both interactions. This value is slightly smaller that the ^{4}H ground state energy. For these rather high energies, the parameters provided by the ACCC method depend on numerical conditions, such as the degree of the Padé approximant. The sensitivity is larger for the width than for the energy. Several calculations have been performed, and yield a neutron width between 1 and 2 MeV for the MN force, and between 1 and 4 MeV for the MH force. These values are significantly lower than the ^{4}H width, indicating that the ^{5}H lifetime should be larger than the ^{4}H lifetime. The ^{5}H properties do not significantly depend on the nucleon-nucleon interaction. The energy and width found here are consistent with previous theoretical estimates [4–6], although the models are rather different.

In conclusion, we have investigated the ^{5}H system in the microscopic three-cluster model, using $t+n+n$ wave functions. This approach has been tested on the ^{3}H$+n$ and ^{3}He$+p$ properties, which agree reasonably well with the available experimental data. For ^{5}H, we find an energy $E \approx 3$ MeV with respect to the ^{3}H$+n+n$ threshold and a neutron width $\Gamma_n \approx 1$–4 MeV. Except for the width, those results are nearly insensitive to the nucleon-nucleon force and to the choice of the bases functions. We conclude that the ^{5}H lifetime should be larger than the ^{4}H lifetime.

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