Application of a generalized Nambu–Jona-Lasinio model to the calculation of the properties of scalar mesons and nuclear matter

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In this work we demonstrate the range of application of our generalized Nambu–Jona-Lasinio (NJL) model. We discuss the scalar-isoscalar mesons that are thought to have a $q\bar{q}$ structure and also consider some properties of nuclear matter. The spectrum of scalar-isoscalar mesons provides some insight into the sigma models that may be used to describe the dynamics of pseudoscalar and scalar mesons. Some authors have suggested that the lowest $q\bar{q}$ 0^{++} state is the $f_0(1370)$. [The $f_0(400-1200)$ that now appears in the data tables may be generated dynamically in π-π scattering and is not a ''preexisting'' state of the spectrum.] The $f_0(980)$ and $a_0(980)$ are often designated as K-$\bar{K}$ molecules. However, recent work does not support that assignment. Since there is no low-lying scalar state, the model of choice in describing pseudoscalar meson dynamics is the nonlinear sigma model. However, there are a number of applications of the Nambu–Jona-Lasinio model in the study of nuclear matter properties that can be described in terms of the linear sigma model. In the linear sigma model there is a low-mass σ meson, whose effects would be generated dynamically in the nonlinear model. While it is not necessary to ever introduce a low-mass σ meson, we describe some calculations which may be interpreted in terms of such a meson. That only represents a formal device, and no low-mass σ is to be found in experiment. However, the use of the low-mass σ as an order parameter for partial restoration of chiral symmetry at finite density is quite consistent with the picture obtained in the application of QCD sum rules to the calculation of the nucleon self-energy in nuclear matter. We discuss such sum rules to show that the linear sigma model has some limited application if we consider only spacelike values of the σ meson momentum. We also use our generalized NJL model to discuss the spectrum of scalar-isoscalar states under the assumption that the $f_0(1370)$ is the $1\,^3P_0$ $n\bar{n}$ state and the $f_0(1710)$ is the $1\,^3P_0$ $s\bar{s}$ state before consideration of quarkonium-glueball mixing. That analysis allows us to provide a microscopic model of the two $q\bar{q}$ states used by Close and Kirk in their recent description of quarkonium-glueball mixing in which they obtain wave functions of the $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ resonances. However, we also present an argument to show that the configuration assignments implied in the work of Close and Kirk are probably incorrect and that the standard scheme for the discussion of quarkonium-glueball mixing needs modification.

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I. INTRODUCTION

There is a good deal of discussion in the literature concerning the nature of the ''σ meson'' [1]. In this work we wish to see under what circumstances one may usefully introduce a low-mass σ. Various researchers have quite different interpretations of the nature of that meson. A scalar meson may be used to describe the strong attractive interaction seen in low-energy π-π scattering from threshold to about 1 GeV. That meson appears in the data tables as the $f_0(400-1200)$ resonance. [In some cases, the $f_0(400-1200)$ is identified as the scalar meson that appears in the linear sigma model.] The scalar attraction between two nucleons is generated by correlated two-pion exchange in a study of dispersion relations as they relate to the nucleon-nucleon interaction [2]. A low-mass scalar is often use to represent the effects of correlated pion exchange when describing the nucleon-nucleon interaction in the vacuum or in nuclear matter.

Since there are several phenomena associated with a ''σ meson,'' we need to clarify what we have in mind when we use that designation. For the purposes of this work, we will characterize the σ meson to be (predominately) a meson of $q\bar{q}$ character that is ''preexisting'' in the sense that it is not generated dynamically in π-π scattering.

It is useful to survey a few of the various opinions and results concerning the nature of scalar mesons. For example, Meissner suggests that the $f_0(400-1200)$ is not a ''preexisting'' resonance, but is a dynamic effect due to the strong pion-pion interaction in the $L=0$, $I=0$ wave [3]. He further states that the $f_0(400-1200)$ is certainly not the chiral partner of the pion that appears in the linear sigma model, for example. On the other hand, Pennington, while agreeing that the σ is a very short-lived correlation between pion pairs, suggests that such a sigma is expected to be the field whose nonzero vacuum value breaks chiral symmetry [1]. (That last observation is in contrast to Meissner's opinion.) Pennington also indicates that an analysis of the role of the $f_0(400-1200)$ in two-photon processes indicates that it has a $(u\bar{u}+d\bar{d})$ composition. Pennington also remarks that t- and u-channel ρ exchange can explain the strong π-π interaction in the $L=0$, $I=0$ state [1]. However, based upon duality arguments, he argues that the σ meson could also exist as an s-channel pole when studying π-π scattering [1].

Hannah remarks that the existence of the $f_0(400-1200)$

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may be considered controversial. In a study of the pion scalar form factor based upon dispersion relations and chiral perturbation theory, Hannah finds a resonance pole at $\sqrt{s}=445-i235\,\mathrm{MeV}$, which he claims provides further evidence for the existence of a σ meson [4]. A study of the nonstrange $(n\bar{n})$ scalar sector using QCD sum rule techniques argues that both the $f_0(400-1200)$ and the $a_0(980)$ are not $n\bar{n}$ states [5].

Another body of work concerning scalar mesons is that of Hatsuda and Kunihiro [6–9]. In that work the sigma that represents the strong π-π correlation at low energy is identified as the chiral partner of the pion in a theory with $\mathrm{SU}_L(2)\otimes\mathrm{SU}_R(2)$ symmetry. For example, a bosonization of the NJL model without confinement gives rise to a linear sigma model with a sigma meson of mass approximately equal to twice the constituent quark mass. If the up and down quark masses are 364 MeV, as in our model, one would expect $m_\sigma\simeq 728\,\mathrm{MeV}$. However, in our model, confinement moves this scalar state upward in energy [10]. In our earlier work we identified the $f_0(980)$ as the lowest $q\bar{q}\ 0^{++}$ state [10]. However, a recent study of quarkonium-glueball mixing supports the assignment of the $f_0(1370)$ as the lowest $0^{++}\ n\bar{n}$ state $[n\bar{n}=(u\bar{u}+d\bar{d})/\sqrt{2}]$ and the $f_0(1710)$ as the lowest $s\bar{s}$ state [11]. In Ref. [11] these states are mixed with the scalar glueball to become the experimentally determined $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ resonances.

The experimental data might be considered to argue against the $f_0(1370)$ being mainly the $1\ {}^3P_0\ s\bar{s}$ state, as found to be the case in Ref. [10], since the relative branching ratios for this state are [12]

$$\pi\pi:K\bar{K}:\eta\eta:4\pi=1:0.46\pm0.19:0.16\pm0.07:34.0^{+22}_{-9}.$$

[However, we can assume that the large 4π decay is driven by another component of the $f_0(1370)$, which might be a $q\bar{q}q\bar{q}$ state. We consider this an unresolved matter at this time.] The branching ratios for the $f_0(1500)$ and $f_0(1710)$ are also given in Ref. [12]. Close and Kirk have used these branching ratios and other data to determine the wave functions of the $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$, when quarkonium-glueball mixing is taken into account. Their model has eight parameters, four of which appear in the matrix that is diagonalized to obtain the wave functions [11]:

$$M=\begin{bmatrix} M_G & f & \sqrt{2}f \\ f & M_S & 0 \\ \sqrt{2}f & 0 & M_N \end{bmatrix}. \tag{1.1}$$

Here $f=\langle G|H|S\rangle=\langle G|H|N\rangle/\sqrt{2}$, with $|G\rangle$ denoting the bare glueball, $|N\rangle$ the $|n\bar{n}\rangle$ state, and $|S\rangle$ the $|s\bar{s}\rangle$ state. A typical parameter set has $M_G=1440\pm16\,\mathrm{MeV}$, $M_S=1672\pm9\,\mathrm{MeV}$, $M_N=1354\pm28\,\mathrm{MeV}$, and $f=91\pm11\,\mathrm{MeV}$, where the uncertainty is related to the data-fitting procedure [11]. The physical states are found to be

$$|f_0(1710)\rangle=0.39|G\rangle+0.91|S\rangle+0.14|N\rangle, \tag{1.2}$$

$$|f_0(1500)\rangle=-0.69|G\rangle+0.37|S\rangle-0.62|N\rangle, \tag{1.3}$$

and

$$|f_0(1370)\rangle=0.60|G\rangle-0.13|S\rangle-0.79|N\rangle. \tag{1.4}$$

In this work we show how the generalized NJL model can generate the ''bare'' $q\bar{q}$ states used above. That is, we would like to have the $1\ {}^3P_0\ s\bar{s}$ state at about 1670 MeV and the $1\ {}^3P_0\ n\bar{n}$ state at about 1350 MeV, corresponding to the parametrization of Ref. [11]. [These assignments are consistent with the fact that the 4π decay of the $f_0(1710)$ is very small [12].] While in this work we show that such an assignment of bare $q\bar{q}$ states, with the $1\ {}^3P_0\ s\bar{s}$ state at about 1670 MeV and the $1\ {}^3P_0$ state at 1350 MeV, is possible, we also stress that the formalism based upon Eq. (1.1) is unsatisfactory, given our most recent assignment of configurations of the scalar $q\bar{q}$ states.

The organization of our work is as follows. In Sec. II we determine a parameter of the confining field so that, when the short-range NJL interaction is considered, the states $|N\rangle$ and $|S\rangle$ have approximately the energies used in Ref. [11]. In that fashion we can provide wave functions for the state $|N\rangle$ and $|S\rangle$ introduced there. However, we provide evidence that the assignment of configurations implied in the work of Close and Kirk is quite unlikely to be correct. In Sec. III we describe the bosonization of the NJL model and remark upon the modification of such results in our generalized NJL model, which includes a covariant model of confinement. As we will see, bosonization of the standard NJL model (without confinement) gives rise to a linear sigma model, which may be used if the meson momentum is spacelike. We discuss the pion-nucleon sigma term and some properties of nuclear matter using the linear sigma model. We justify our application to the calculation of the mean scalar field in nuclear matter by using some results of QCD sum rules as applied in nuclear matter. Finally, Sec. IV contains some further discussion and conclusion.

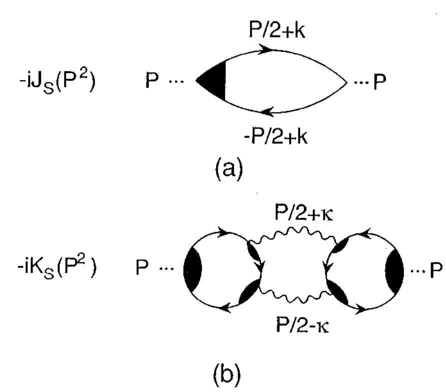

FIG. 1. (a) The polarization function $-iJ_s(P^2)$. Here the solid triangular region denotes our confinement vertex, which eliminates the spurious cut that would otherwise appear when the quark and antiquark go on mass shell [4]. (b) The polarization function $-iK_s(P^2)$. Here the wavy lines are pions, kaons, or eta mesons, corresponding to the decays $f_0\to\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, etc. The solid regions are confinement vertex functions.

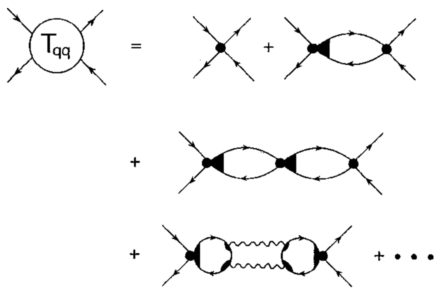

FIG. 2. The figure shows a perturbative expansion of $q\bar{q}$ T matrix.

II. CALCULATION OF QUARK-ANTIQUARK STATES IN A GENERALIZED NJL MODEL

The procedure for obtaining the spectrum of scalar states in our model was given in Ref. [10]. The important functions that appear in our analysis are shown in Fig. 1. Figure 1(a) shows the basic vacuum polarization function $J_S(P^2)$ of our generalized NJL model. Here the filled regions denote our covariant confinement vertex functions [10]. These vertex functions serve to eliminate spurious singularities that would otherwise arise when the quark and antiquark go on mass shell. In Fig. 1(b) we show the function $K_S(P^2)$ which describes the effects of coupling to two-meson decay channels. [Note that $J_S(P^2)$ is of order n_c and $K_S(P^2)$ is of order 1.] The T matrix for our model is shown in Fig. 2, where we have not shown the wave matrices that impose confinement for the initial and final $q\bar{q}$ pairs. (That feature is described in Ref. [13].)

Values obtained for $J_S(P^2)$, when the quarks have masses $m_u = m_d = 0.364$ GeV, are given in Fig. 3. The values for $J_S(P^2)$ when the quarks have mass $m_s = 0.565$ GeV are shown in Fig. 4. These functions have singularities at the energies of the bound states in the confining field, since the vertex functions are singular at those energies. (See the captions to Figs. 3 and 4.)

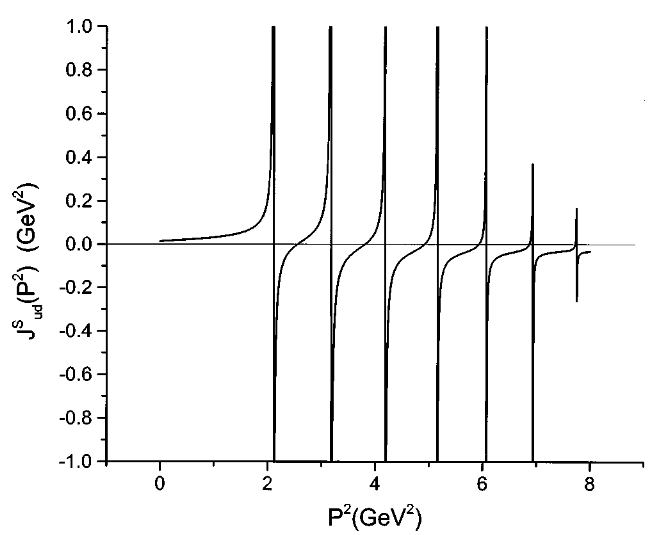

FIG. 3. The function $J^S_{ud}(P^2)$. Here $m_u = m_d = 0.364$ GeV. A Gaussian regulator of the form $\exp[-\vec{k}^2/\alpha^2]$ is used with $\alpha = 0.605$ GeV. The parameter of the confining field [10] is $\kappa = 0.065$ GeV^2.

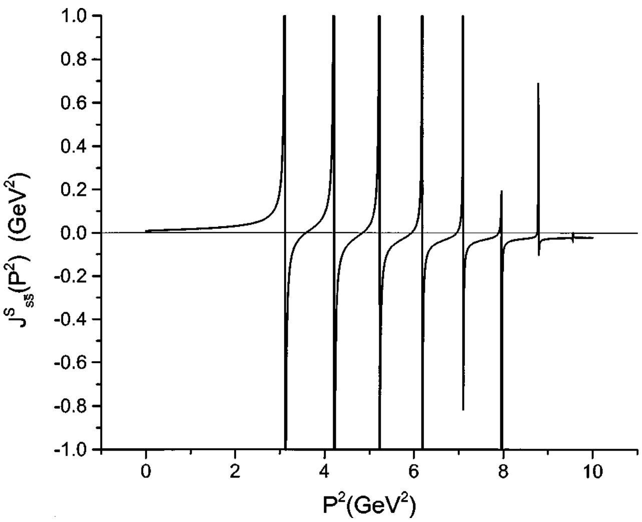

FIG. 4. The function $J^S_{s\bar{s}}(P^2)$. Here $m_s = 0.565$ GeV. (See the caption to Fig. 3.)

We now consider the short-range NJL interaction. The Lagrangian of our model is [10]

$$\mathcal{L} = \bar{q}(i\not\partial - m^0)q + \frac{G_S}{2}\sum_{i=0}^{8}[(\bar{q}\lambda^i q)^2 + (\bar{q}i\gamma_5\lambda^i q)^2]$$
$$-\frac{G_V}{2}\sum_{i=0}^{8}[(\bar{q}\gamma^\mu\lambda^i q)^2 + (\bar{q}\gamma^\mu\gamma_5\lambda^i q)^2]$$
$$+\frac{G_D}{2}\{\det[\bar{q}(1+\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]\}$$
$$+\mathcal{L}_{\rm tensor} + \mathcal{L}_{\rm conf}. \quad (2.1)$$

Here the fourth term is the 't Hooft interaction, and $\mathcal{L}_{\rm tensor}$ denotes interactions added to study tensor masons, while $\mathcal{L}_{\rm conf}$ denotes our model of confinement. In Eq. (2.1), m^0 is the current quark mass matrix $m^0 = \mathrm{diag}(m_u^0, m_d^0, m_s^0)$, the $\lambda_i (i = 1,...,8)$ are the Gell-Mann matrices, and $\lambda_0 = \sqrt{2/3}\mathbb{I}$, with $\mathbb{I}$ being the unit matrix in flavor space. In our previous work, we had $G_S = 11.83$ GeV^{-2} and $G_D = 86.39$ GeV^{-2} [10]. Thus the interaction in singlet states was $G_{00} = 10.46$ GeV^{-2} and the interaction in octet states was $G_{88} = 12.46$ GeV^{-2}. In the case of (approximate) ideal mixing, we put $G_{n\bar{n}} = 11.12$ GeV^{-2} and $G_{s\bar{s}} = 11.79$ GeV^{-2}. (The deviation from ideal mixing induced by the 't Hooft interaction is described in Ref. [10].)

For the Minkowski-space calculations reported here and in our earlier work, we found it useful to neglect energy transfer by the confining field in the meson rest frame. For example, if we start with the potential $V^C(r) = \kappa r \exp[-\mu r]$ and form the Fourier transform, we have the form of the interaction used in the meson rest frame:

$$V^C(\vec{k}-\vec{k}')=-8\pi\kappa\left[\frac{1}{[(\vec{k}-\vec{k}')^2+\mu^2]^2}-\frac{4\mu^2}{[(\vec{k}-\vec{k}')^2+\mu^2]^3}\right]. \tag{2.2}$$

Here μ is a small parameter used to soften the infrared singularities of V^C. If μ is small enough, the potential approximates a linear potential with ''string tension'' κ over the range of r relevant for our problem. (Since we use Lorentz-vector confinement, our value of κ differs from that usually quoted for a Lorentz-scalar model of confinement.) The potential of Eq. (2.2) may put in a covariant form if we use the four-vectors $\hat{k}^\mu$ and $\hat{k}'^\mu$,

$$\hat{k}^\mu=k^\mu-\frac{(k\cdot P)P^\mu}{P^2} \tag{2.3}$$

and

$$\hat{k}'^\mu=k'^\mu-\frac{(k'\cdot P)P^\mu}{P^2}, \tag{2.4}$$

since, when $\vec{P}=0$, we have $\hat{k}^\mu=[0,\vec{k}]$ and $\hat{k}'^\mu=[0,\vec{k}']$. The calculation of bound states and vertex functions using the potential of Eq. (2.2) is described in detail in Ref. [10].

To find the bound states in the presence of both confinement and the short-range NJL interaction, we consider

$$G_{n\bar{n}}^{-1}-J_{ud}^S(P^2)=0 \tag{2.5}$$

and

$$G_{s\bar{s}}^{-1}-J_{s\bar{s}}^S(P^2)=0. \tag{2.6}$$

In our previous studies, we found $\kappa=0.055\,\mathrm{GeV}^2$ from a fit to a very large number of states of light mesons [10]. In order to obtain bare $q\bar{q}$ $1\ {}^3P_0$ $s\bar{s}$ and $n\bar{n}$ masses compatible with the Kirk-Close assignments, it is necessary to use $\kappa=0.065\,\mathrm{GeV}^2$. [See Figs. 3 and 4 where we present $J_{u\bar{d}}^S(P^2)$ and $J_{s\bar{s}}^S(P^2)$ for $\kappa=0.065\,\mathrm{GeV}^2$.]

In Figs. 5(a) and 5(c) we show the energies of the $n\bar{n}$ and $s\bar{s}$ bound states in the confining field. In Fig. 5(b) we show the results of including the short-range NJL interaction. In that manner we generate states that correspond to the states $|N\rangle$ and $|S\rangle$ of Ref. [11]. [See Fig. 5(b).] The figure also shows the energies of the states with nodes in their wave functions. We find that when $\kappa=0.065\,\mathrm{GeV}^2$, the $1\ {}^3P_0$ $n\bar{n}$ state is at 1445 MeV and the $1\ {}^3P_0$ $s\bar{s}$ state is at 1758 MeV. Inclusion of the short-range interaction moves the $n\bar{n}$ state down to 1334 MeV, where it can be identified with the state $|N\rangle$ of Ref. [11]. At the same time, the $1\ {}^3P_0$ $s\bar{s}$ state moves down to 1673 MeV where it can be identified with the state $|S\rangle$ of Ref. [11]. Our analysis, therefore, provides a microscopic model for the starting point of Ref. [11], since we now have both the wave functions and energies of the states $|N\rangle$ and $|S\rangle$ used there.

We now wish to argue that, while the spectrum of scalar states could be fit using our generalized NJL model, other information precludes the use of Eq. (1.1). We note that the

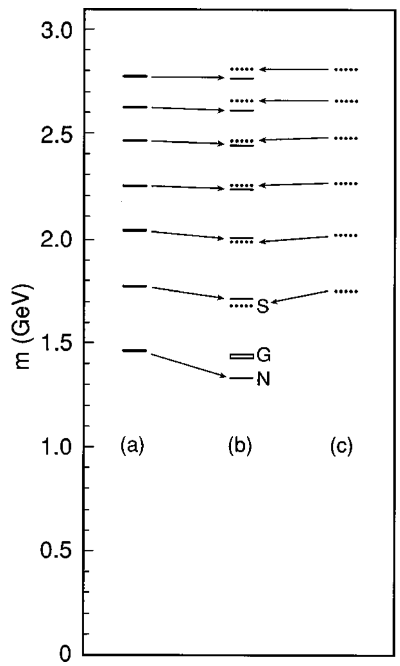

FIG. 5. The energies of bound $q\bar{q}$ states of our generalized NJL model. (a) The energies of the bound $n\bar{n}$ states in the confining field. There are bound states at 1458, 1783, 2045, 2274, 2467, 2633, and 2783 MeV. (b) The energies obtained when the short-range NJL interaction is included. There are bound $n\bar{n}$ states at 1337, 1717, 2013, 2251, 2454, and 2629 MeV and bound $s\bar{s}$ states at 1680, 2004, 2265, 2471, 2660, 2822, and 2965 MeV. (c) The energies of the bound $s\bar{s}$ states in the confining field. There are bound states at 1767, 2048, 2284, 2491, 2664, 2820, and 2964 MeV.

use of the relations $f=\langle G|H|S\rangle=\langle G|H|N\rangle/\sqrt{2}$ implies that the states $|N\rangle$ and $|S\rangle$ have the same number of nodes. In the model of Close and Kirk, these states would both be $1\ {}^3P_0$ states, with the $f_0(980)$ eliminated by considering that resonance to be a $K\bar{K}$ molecule. However, recent work of Maltman, who uses QCD sum rules and other techniques, shows that the $a_0(980)$ has a much larger decay constant than it would have if it were a $K\bar{K}$ molecule [14,15]. In Table I we show the values obtained by Maltman and compare them to values we have calculated using the standard (unmodified)

TABLE I. Meson decay constants [16].

Meson	Configuration	$m_i^2 f_i$ (GeV^3) TDA[a]	RPA[b]	Maltman [14,15]
$a_0(980)$	$1\ {}^3P_0$	0.0649	0.0868	0.0447 ± 0.0085
$a_0(1450)$	$2\ {}^3P_0$	0.0357	0.0420	0.0647 ± 0.0123
$a_0(1834)$[c]	$3\ {}^3P_0$	0.0377	0.0424	
$K_0^*(1430)$	$1\ {}^3P_0$	0.0614	0.0508	0.0842 ± 0.0045
$K_0^*(1730)$[c]	$2\ {}^3P_0$	0.0425	0.0375	
$K_0^*(1950)$	$3\ {}^3P_0$	0.0414	0.0378	

[a]Tamm-Dancoff approximation.
[b]Random phase approximation.
[c]These states are predicted to exist in our formalism.

values of the parameters of our generalized NJL model [16] with $\kappa=0.055\,\mathrm{GeV}^2$. [The fact that our value for the $a_0(980)$ coupling constant is about a factor of 2 larger than Maltman's value suggests that the $a_0(980)$ has an important $K\overline{K}$ component.]

We also note that our most recent analysis [17] of the scalar spectrum using $\kappa=0.055\,\mathrm{GeV}^2$ places the $f_0(980)$ at 980 MeV and the $a_0(980)$ at 950 MeV in what is essentially a parameter-free calculation, since the parameters have been fixed in our studies of the pseudoscalar and vector mesons. In our model the $f_0(980)$ and $a_0(980)$ are $1\,{}^3P_0$ states.

Using this body of information, we infer that the two states $|N\rangle$ and $|S\rangle$ used by Close and Kirk have a different number of nodes. For example, in our model the $f_0(1370)$ is a $s\overline{s}$ state before quarkonium-glueball mixing is taken into account and the $f_0(1500)$ is the $2\,{}^3P_0$ state in our model. We have sufficient confidence in these assignments that we can conclude that Eq. (1.1) is not appropriate for the study of quarkonium-glueball mixing. An analysis of quarkonium-glueball mixing based upon our NJL model was presented in Refs. [18,19] and is more appropriate for this problem.

III. BOSONIZATION OF THE NJL MODEL

We may consider the equation analogous to Eq. (2.5) in the case we deal with scalar states in the SU(2)-flavor NJL model without confinement:

$$G_S^{-1}-\hat{J}_S(m_\sigma^2)=0. \tag{3.1}$$

It is well known that $m_\sigma^2=(2m_q)^2+m_\pi^2$, where m_q is the constituent quark mass of the up (or down) quark. Thus the low-mass σ meson found in this manner is just above the threshold for the $q\overline{q}$ states that starts at $2m_q$. However, if we include our model of confinement, we have

$$G_S^{-1}-J_S(P^2)=0, \tag{3.2}$$

where $J_S(P^2)$ is calculated with the confinement vertex (see Fig. 1). Since $J_S(P^2)<\hat{J}_S(P^2)$, the mass obtained from Eq. (3.2) is a good deal larger than $m_\sigma\simeq 2m_q$. Indeed, a solution of Eq. (3.2) is obtained for a mass of about 1.3 GeV when $\kappa=0.065\,\mathrm{GeV}^2$ (see Fig. 5) and at about 1.0 GeV when $\kappa=0.055\,\mathrm{GeV}^2$. Thus we see that confinement eliminates a low-mass σ meson from our results. However, for *small* $P^2>0$ or $P^2<0$, confinement is a quite small effect and may be neglected. Since that restriction on P^2 is appropriate for most nuclear physics studies, we will neglect confinement in the following discussion. [Again, we stress that there is no low-mass sigma to be found in experiment if we exclude from consideration the strong π-π interaction that leads to the introduction of the $f_0(400–1200)$. That meson is unrelated to the low-mass σ meson obtained upon bosonization of the NJL model.]

We recall the Lagrangian of the NJL model with SU(2) flavor symmetry

$$\mathcal{L}(x)=\overline{q}(i\not\partial-m^0)q+\frac{G_S}{2}[(\overline{q}q)^2+(\overline{q}i\gamma_5\vec{\tau}q)^2]. \tag{3.3}$$

Here $m^0=\mathrm{diag}(m_u^0,m_d^0)$ is the current quark mass matrix, which we will neglect in part of the following discussion. We have found that if $G_S=11.83\,\mathrm{GeV}^{-2}$ and a cutoff of $\Lambda_3=0.622\,\mathrm{GeV}$ is used for the quark momentum, a pion of mass 138 MeV is obtained when $m_u^0=m_d^0=5.5\,\mathrm{MeV}$ [20]. The Goldberger-Trieman relation is satisfied with $m_q=0.364\,\mathrm{GeV}$, $f_\pi=0.093\,\mathrm{GeV}$, and $g_{\pi qq}=3.91$. The relation between the constituent mass and the condensate is

$$m=m^0-G_S\langle 0|u\overline{u}+d\overline{d}|0\rangle, \tag{3.4}$$

which leads to $\langle 0|u\overline{u}|0\rangle=\langle 0|d\overline{d}|0\rangle=-(0.247\,\mathrm{GeV})^3$. We also see that the Gell-Mann–Renner–Oaks relation $f_\pi^2m_\pi^2=m^0\langle 0|u\overline{u}+d\overline{d}|0\rangle$ is well satisfied.

It is useful to proceed with a bosonization of the NJL Lagrangian via the introduction of a sigma field and a pion field. Thus we introduce

$$\sigma=-\frac{G_S}{g_{\pi qq}}\overline{q}q \tag{3.5}$$

and

$$\vec{\pi}=-\frac{G_S}{g_{\pi qq}}\overline{q}i\gamma_5\vec{\tau}q. \tag{3.6}$$

If we neglect m^0 and write $m=-G_S\langle 0|\overline{q}q|0\rangle$, we see that the vacuum value of σ is f_π, as is well known.

As a formal device, we may introduce a sigma dominance model by relating a chain of NJL polarization diagrams to the propagator of a "σ meson." For example, we may write, with $g_{\pi qq}=g_{\sigma qq}$,

$$\frac{G_S}{1-G_SJ_S(P^2)}\simeq-\frac{g_{\sigma qq}^2}{P^2-m_\sigma^2} \tag{3.7}$$

for P^2 small or $P^2<0$. Thus

$$-\left[\frac{G_S}{g_{\pi qq}}\right]\frac{1}{1-G_SJ_S(P^2)}=\frac{g_{\sigma qq}}{P^2-m_\sigma^2}. \tag{3.8}$$

On the left-hand side of Eq. (3.8), we see the factor $(-G_S/g_{\pi qq})$ that appears in Eq. (3.5), while the "σ propagator" is on the right-hand side of the equation. The point to keep in mind is that Eq. (3.8) with $m_\sigma\simeq 2m_q$ is only valid for $P^2<0$ or for small $P^2>0$.

We take this occasion to update our calculation of the pion-nucleon sigma term made in Ref. [21], where we used a sigma dominance model with a quark mass of $m_q=260\,\mathrm{MeV}$. We found $\sigma_N=51.8\,\mathrm{MeV}$ in that work. Since we are now using $m_u=m_d=0.364\,\mathrm{GeV}$ in our most recent work, we can repeat the calculation given there for the new mass value.

We had defined a scalar form factor of the nucleon [21],

$$F_s(P^2)\overline{u}(\vec{k}+\vec{P},s')u(\vec{k},s)\,\delta_{\tau\tau'}=\langle N,\vec{k}+\vec{P},s',\tau'|\vec{q}(0)q(0)|N,\vec{k},s,\tau\rangle, \tag{3.9}$$

and wrote

$$F_s(0)=\langle N|\bar{q}q|N\rangle_{\rm val}+\langle N|\bar{q}q|N\rangle_c\,, \tag{3.10}$$

where the second term is due to the ‘‘meson cloud.’’ In the σ dominance model, we found that

$$\langle N|\bar{q}q|N\rangle=\frac{1}{1-G_S[J_S(0)+K_S(0)]}\langle N|\bar{q}q|N\rangle_{\rm val}. \tag{3.11}$$

With $m_u=m_d=0.364\,\text{GeV}$, we have $J_s(0)=0.0485\,\text{GeV}^2$ and $K_s(0)=0.0110\,\text{GeV}^2$. Since we now neglect confinement, we can use the value of $G_s=11.83\,\text{GeV}^{-2}$ mentioned above. Thus

$$\langle N|\bar{q}q|N\rangle=3.39\langle N|\bar{q}q|N\rangle_{\rm val}. \tag{3.12}$$

The value of $\langle N|\bar{q}q|N\rangle_{\rm val}=3.02\pm0.09$ has been obtained in a lattice QCD calculation [22]. Therefore, with the definition

$$\sigma_N=\left[\frac{m_u^0+m_d^0}{2}\right]\langle N|\bar{q}q|N\rangle, \tag{3.13}$$

we have

$$\sigma_N=56.1\pm1.7\ \ \text{MeV}, \tag{3.14}$$

which is a satisfactory result for σ_N. Thus we see that near $P^2=0$, where this calculation is made, the linear sigma model with a sigma mass of $m_\sigma^2=4m_q^2+m_\pi^2$ may be used and confinement may be neglected. [The inclusion of $K_S(0)$ in the calculation will reduce the sigma mass somewhat from the value obtained in the simplest bosonization scheme. Recall that $J_S(P^2)$ is of order n_c and $K_S(P^2)$ is of order 1.]

Another well-known relation is the density dependence of the condensate [23–25]:

$$\frac{\langle\bar{q}q\rangle_\rho}{\langle\bar{q}q\rangle_{\rm vac}}=1-\frac{\sigma_N\rho}{f_\pi^2m_\pi^2}, \tag{3.15}$$

where σ_N is the pion-nucleon sigma term. At nuclear matter density $\rho=(0.111\,\text{GeV})^3$, so with $\sigma_N\simeq45\,\text{MeV}$, we have a 37.4% reduction of the condensate and a corresponding reduction of the σ field so that, in matter, $\sigma=f_\pi-\sigma'$, with $\sigma'=34.8\,\text{MeV}$.

In the linear sigma model, Eq. (3.15) may also be written as

$$\frac{\sigma}{f_\pi}=1-\frac{\sigma_N\rho}{f_\pi^2m_\pi^2}. \tag{3.16}$$

In order to see that the use of the σ field as an order parameter for the scalar quark condensate yields a consistent picture, we may make reference to the use of QCD sum rules in the calculation of the nucleon self-energy in matter [24,25].

IV. USE OF THE LINEAR SIGMA MODEL IN THE DESCRIPTION OF MEAN-FIELD ASPECTS OF NUCLEAR MATTER

In this section we wish to show that the linear σ model finds some limited application in the study of properties of nuclear matter, even though there is no low-mass σ meson of $q\bar{q}$ character to be found in experiment. That is, the partial restoration of chiral symmetry described by a reduced value for the σ field in matter is consistent with the calculation of the self-energy of a nucleon in matter made using QCD sum rules [24,25]. For example, the scalar potential experienced by a nucleon in nuclear matter is $V_S=-G_{\sigma NN}\sigma'$, where $G_{\sigma NN}$ is the constant describing the coupling of the sigma field to the nucleon. We can provide a rough estimate of $G_{\sigma NN}$ by writing $G_{\sigma NN}=3g_{\sigma qq}=3g_{\pi qq}\simeq11.7$, so that $V_S\simeq-407\,\text{MeV}$. That is close to the value used in Dirac phenomenology [26] or in the Walecka model [27] to describe the scalar potential felt by a nucleon in nuclear matter. To provide some justification of this viewpoint, we consider the correlator of two Ioffe currents,

$$\Pi(x)=\langle T[\eta_N(x)\,\bar{\eta}_N(0)]\rangle_\rho\,, \tag{4.1}$$

which is related to the propagator of a nucleon in nuclear matter [25]. After transformation to momentum space, one has

$$\Pi(q)=\Pi_1(q^2,q\cdot u)+\not{q}\,\Pi_q(q^2,q\cdot u)+\not{u}\,\Pi_u(q^2,q\cdot u), \tag{4.2}$$

where u^μ describes the flow of nuclear matter. (In the rest frame, we have $u^\mu=[1,0,0,0]$.) It is found that, for a nucleon in matter,

$$\Pi_1(q^2,q\cdot u)=\frac{1}{4\pi^2}q^2\ln(-q^2)\frac{\langle\bar{q}q\rangle_\rho}{2}+\cdots, \tag{4.3}$$

$$\Pi_q(q^2,q\cdot u)=-\frac{1}{64\pi^2}(q^2)^2\ln(-q^2)+\frac{1}{3\pi^2}q\cdot u\ln(-q^2)\frac{\langle q^\dagger q\rangle_\rho}{2}+\cdots, \tag{4.4}$$

and

$$\Pi_u(q^2,q\cdot u)=\frac{2}{3\pi^2}q^2\ln(-q^2)\frac{\langle q^\dagger q\rangle_\rho}{2}+\cdots, \tag{4.5}$$

where

$$\langle\bar{q}q\rangle_\rho=[\langle\bar{u}u\rangle_\rho+\langle\bar{d}d\rangle_\rho] \tag{4.6}$$

and

$$\langle q^\dagger q\rangle_\rho=[\langle u^\dagger u\rangle_\rho+\langle d^\dagger d\rangle_\rho]. \tag{4.7}$$

Recall that $\sigma=f_\pi-\sigma'$ and

$$\sigma=-\frac{G_s}{g_{\sigma qq}}\langle\bar{q}q\rangle_\rho, \tag{4.8}$$

so that

$$\langle\bar{q}q\rangle_\rho=\langle\bar{q}q\rangle_0-\frac{g_{\sigma qq}}{G_S}\sigma'. \tag{4.9}$$

We will concentrate on the scalar field and rewrite Π_1 as

$$\Pi_1(q^2,q\cdot u)=\frac{1}{4\pi^2}q^2\ln(-q^2)\frac{\langle\bar{q}q\rangle_0}{2}+\frac{1}{4\pi^2}q^2\ln(-q^2)\times\left[-\frac{g_{\sigma qq}}{2G_s}\right]\sigma'+\cdots. \tag{4.10}$$

The nucleon mass in matter m_N^* is related to the scalar part of the self-energy, $m_N^*=m_N+\Sigma_S$, with

$$\Sigma_s=\frac{-8\pi^2}{M^2}\left[\frac{g_{\sigma qq}}{2G_s}\right]\sigma', \tag{4.11}$$

where M is the Borel mass [25]. If we put $g_{\sigma qq}=3.91$, $G_S=11.83\,\mathrm{GeV}^{-2}$, $M=1.0\,\mathrm{GeV}$, and $\sigma'=34.8\,\mathrm{MeV}$ (from the last section), we find $\Sigma_S\simeq-454\,\mathrm{MeV}$. We expect about a 10% reduction in this value, when short-range correlations are included, so that $\Sigma_S\sim-409\,\mathrm{MeV}$ [26]. That value is close to the value used in Dirac phenomenology and in the Walecka model, as noted earlier.

We remark that the Goldberger-Treiman relation

$$f_\pi^2 m_\pi^2=-m_q^0\langle\bar{q}q\rangle_0 \tag{4.12}$$

generalizes to

$$\sigma f_\pi m_\pi^2=-m_q^0\langle\bar{q}q\rangle_\rho \tag{4.13}$$

in matter. That this is correct may be checked by using a relation obtained in the bosonization procedure,

$$\frac{G_S}{g_{\sigma qq}}=\frac{m_q^0}{m_\pi^2 f_\pi}, \tag{4.14}$$

and recalling that

$$f_\pi-\sigma'=-\frac{G_S}{g_{\sigma qq}}\langle\bar{q}q\rangle_\rho. \tag{4.15}$$

If we assume that m_π^2 is unchanged in matter, we see that we reproduce Eq. (3.16) upon taking the ratio of Eqs. (4.13) and (4.12).

V. DISCUSSION

A source of confusion in the study of scalar mesons is the attempt to understand the spectrum without introducing a model of confinement. For example, the result of the standard bosonization procedure for the scalar mass, $m_\sigma^2=(2m_q)^2+m_\pi^2$, puts the σ mass in the quark-antiquark continuum, which starts at $P^0=2m_q$. Above that energy, the function $J_S(P^2)$ becomes complex. If confinement is included in the analysis, $J_S(P^2)$ is a real function whose value is reduced relative to the function calculated without confinement. With $\kappa=0.055\,\mathrm{GeV}^2$, the lowest-energy $q\bar{q}$ scalar state is at about 1.0 GeV. That is about 300 MeV greater than the value obtained from the bosonization scheme. On the other hand, studies of the nucleon-nucleon interaction or the partial restoration of chiral symmetry at finite density require that we analyze the theory for $P^2\leqslant 0$. Confinement is a rather small effect for spacelike P^2, so that $J_S(P^2)$, with P^2 small or $P^2<0$, is largely unchanged when confinement is introduced. That has the consequence that the usual bosonization procedure may be used to obtain the effective σ mass for *spacelike* P^2.

The small mass $m_\sigma\simeq 600-700\,\mathrm{MeV}$, obtained from bosonization, leads to the belief that the $f_0(400-1200)$ might be the chiral partner of the pion. (Here we agree with Meissner that that is an incorrect assumption [3].) The two-pion decay width of the $f_0(400-1200)$ is very large, since that resonance may be described as a state with two pions loosely coupled by the interaction arising from ρ exchange. In our earlier work [10], we made the assumption that the $f_0(980)$ was a $1\,^3P_0$ $n\bar{n}$ state. That led to the identification of the $f_0(1370)$ as the $1\,^3P_0$ $s\bar{s}$ state, which at first appears incompatible with the very large 4π decay of that state. [However, one may always make the assumption that a $q\bar{q}\bar{q}\bar{q}$ state that decays strongly to the 4π continuum is admixed in the $f_0(1370)$ wave function.] In this work we have considered the suggestion that the $a_0(980)$ and $f_0(980)$ are not $n\bar{n}$ states, but may be K-$\bar{K}$ molecules [28] or $q\bar{q}\bar{q}\bar{q}$ states. With this new approach we were able to make contact with the phenomenological study of quarkonium-glueball mixing carried out by Close and Kirk [11]. However, we have stressed that that assumption is not tenable, given the recent analysis of the scalar decay constants [14,15].

Recently we have seen some quite excellent fits to the scattering of light pseudoscalar mesons made using chiral perturbation theory in a model that respects unitarity [29]. The pole position of the σ meson is found at $\sqrt{s}=442-i227\,\mathrm{MeV}$. (The effective mass is approximately 600 MeV and the effective width is very large.) If one is interested in calculating phase shifts, the work of Ref. [29] is quite appropriate. However, in our work we have been interested in the quark configurations to be assigned to the scalar mesons and to the use of a low-mass scalar meson as an order parameter. In that regard, our generalized NJL model provides a suitable formalism to discuss the issues of interest to us. Oller, Oset, and Ramos [30] come to the conclusion that the singlet contribution to the $f_0(980)$ and a scalar octet around 1.35 GeV are the lightest preexisting scalar states, while the $\sigma,\kappa,a_0(980)$, and some part of the $f_0(980)$ are generated dynamically by multiple scattering using the lowest-order chiral Lagrangian. [Here the σ refers to what is usually designated as the $f_0(400-1200)$.] Their remarks seem to be in accord with the conclusion of Ref. [6] where it is also stated that the $f_0(400-1200)$ and $a_0(980)$ are not standard $n\bar{n}$ states. If the $a_0(980)$ is generated dynamically and is not "preexisting," one would have to put the $a_0(1450)$ in the same octet as the $K_0^*(1430)$. If we do that, we do not see the upward shift in the mass that is expected when you change an up or down quark to a strange quark. For example, the $\rho(770)$ is in the same octet as the $K^*(892)$. That represents an upward shift in mass of about 120 MeV. [If one replaces both the up and down quarks by strange quarks, one finds the $\phi(1020)$, so that the upward shift in mass from the ρ is about 250 MeV.] This problem, related to the energies of the states in the scalar nonet, has recently been discussed in Ref. [31], where it is suggested that mixing with $q\bar{q}q\bar{q}$ states may be important in understanding the spectrum of isospinor and isovector scalar mesons. We hope to investigate the role of $q\bar{q}q\bar{q}$ states in the case of the isoscalar states in the future. That is a more difficult problem than the one addressed in Ref. [31], as the authors of that reference point out.

Although we have described the $f_0(980)$ as the $1\,{}^3P_0\ n\bar{n}$ state in Ref. [10], the physical situation is more complicated. For example, if we introduce the propagator of the $f_0(980)$, the self-energy of that resonance is strongly P^2 dependent due to cusp behavior at the opening of the $K\bar{K}$ channel. Therefore, the strength at the propagator pole is reduced from unity, indicating a significant admixture of the $K\bar{K}$ continuum in the $f_0(980)$ resonance. We hope to discuss this matter more fully in a future work.

The bosonized NJL model has also been used to describe the nucleon-nucleon interaction [32–34]; however, we do not take up that matter here. A recent discussion of scalar exchange in the *NN* interaction, made within the chiral unitary approach [30], may be found in Ref. [35], where references to related work may be found. An alternate method for the study of radial excitations of light mesons, which makes use of a bosonization scheme, may be found in Refs. [36–38]. However, the assignment of the quark configurations to the various scalar states in those works differs from ours.

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