Is the polarized antiquark sea in the nucleon flavor symmetric?

R. S. Bhalerao

Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Colaba, Mumbai 400 005, India $(Received 2 May 2000; published 22 January 2001)$

We show that the model which naturally explains the $\overline{u} \neq \overline{d}$ asymmetry in the nucleon and is in quantitative agreement with the Gottfried sum rule data, also predicts that in the proton $\Delta \bar{u} > 0 > \Delta \bar{s} > \Delta \bar{d}$ and $\Delta \bar{u} - \Delta \bar{d}$ $\overline{\partial}$ \overline{d} \overline{u} \rightarrow \overline{d} \overline{u} . At the input scale, these results can be derived even analytically. Thus the violation of the flavor symmetry is more serious in the polarized case than in the unpolarized case. In contrast, many recent analyses of the polarized data have made a simplifying assumption that all the three $\Delta \bar{q}$'s have the same sign and magnitude. We point out the need to redo these analyses, allowing for the alternate scenario as described above. We present predictions of the model for the W^- asymmetry in polarized *pp* scattering, which can be tested at RHIC; these are quite different from those available in the literature.

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Several comprehensive analyses of the polarized deep inelastic scattering (DIS) data, based on next-to-leading-order quantum chromodynamics (QCD), have appeared recently $[1-11]$. In these analyses the polarized parton density functions (PDFs) are either written in terms of the well-known parametrizations of the *unpolarized* PDFs or parametrized independently, and the unknown parameters are determined by fitting the polarized DIS data. Additional simplifying assumptions are often made; the one that has been widely used in the literature $|1-8|$ is

$$
\Delta \overline{u} = \Delta \overline{d} = \lambda \Delta \overline{s}, \qquad (1)
$$

with a positive λ , which is usually set equal to unity. Recently the HERMES and SMC Collaborations $[12,13]$ too analyzed their inclusive and semi-inclusive DIS data assuming all $\Delta \bar{q}$'s to be of the same sign. The same assumption has also been used to make predictions for future accelerators; see, e.g., $[14]$. In some analyses $[11]$, the nonsinglet PDFs Δq_3 and Δq_8 are assumed to differ only by a constant multiplicative factor [see Eq. (21) below].

In this paper, we examine these simplifying assumptions made in the literature. This is important because a similar *ad hoc* assumption about the flavor decomposition of the unpolarized antiquark sea, $\overline{u} = \overline{d}$, turned out to be wrong when accurate data on muon DIS became available $[15]$, and the global analyses of the unpolarized data had to be redone. Here we derive a series of inequalities satisfied by the PDFs and point out the need to redo the global analyses of the polarized DIS data in the light of these inequalities, allowing, in particular, for the violation of the flavor symmetry in the polarized antiquark sea; see Eq. (20) below. We then present predictions of our model, which can be tested in polarized *pp* scattering at RHIC, BNL. Finally, we describe other recent works on flavor asymmetry of polarized sea distributions.

We use the framework of the statistical model for polarized and unpolarized structure functions and PDFs of the proton and the neutron, which was presented recently $\left[16,17\right]$. This model provided a natural explanation of the \overline{u} $\neq \overline{d}$ asymmetry in the nucleon and was in quantitative agreement with the Gottfried sum rule data. Additionally, it reproduced the data on $F_2^p(x, Q^2)$ for $0.00001 < x < 1$ and 2.5 $\langle Q^2 \langle 5000 \text{ GeV}^2, F_2^p(x) - F_2^n(x), F_2^n(x) / F_2^p(x), xg(x) \rangle$ $\overline{d}(x) - \overline{u}(x)$, $d(x)/u(x)$, the fractional momentum of charged partons and the polarized structure functions $g_1^{p,n}(x)$, at various Q^2 . Out of these, only the F_2^p and (F_2^p) $-F_2^n$ data, both at $Q^2=4$ GeV², were used as an input to fix the model parameters, and all other results served as model predictions. In particular, the $d(x)/u(x)$ ratio in the limit $x \rightarrow 1$ turned out to be 0.22 in good agreement with the QCD prediction 0.2 [18]. At the input scale $(Q^2 = Q_0^2 = M^2$, where *M* is the nucleon mass), all $xq(x)$ and $x\overline{q}(x)$ distributions were found to be valencelike, and $xg(x)$ was found to be constant in the limit $x \rightarrow 0$. *Thus the total number of gluons was logarithmically divergent providing a strong a posteriori justification for the statistical model ansatz* [17]. Contrary to common practice, the polarized and the unpolarized data were reproduced in a single framework and the simplifying assumption of charge symmetry was not made. Here we further explore the predictive power of the model.

If $n_{\alpha(\bar{\alpha})\uparrow(\downarrow)}$ denotes the number of quarks (antiquarks) of flavor α and spin parallel (antiparallel) to the proton spin, then any model of PDFs in the proton has to satisfy the following constraints:

$$
n_{u\uparrow} + n_{u\downarrow} - n_{u\uparrow} - n_{u\downarrow} = 2,\tag{2}
$$

$$
n_{d\uparrow} + n_{d\downarrow} - n_{\bar{d}\uparrow} - n_{\bar{d}\downarrow} = 1,\tag{3}
$$

$$
n_{s\uparrow} + n_{s\downarrow} - n_{s\uparrow} - n_{s\downarrow} = 0,\tag{4}
$$

$$
n_{u\uparrow} - n_{u\downarrow} + n_{u\uparrow} - n_{u\downarrow} = \Delta u + \Delta u, \tag{5}
$$

$$
n_{d\uparrow} - n_{d\downarrow} + n_{\bar{d}\uparrow} - n_{\bar{d}\downarrow} = \Delta d + \Delta \bar{d},\tag{6}
$$

$$
n_{s\uparrow} - n_{s\downarrow} + n_{s\uparrow} - n_{s\downarrow} = \Delta s + \Delta s. \tag{7}
$$

The right-hand sides $(RHSS)$ of Eqs. $(5)-(7)$ have been measured by several groups. We use $(\Delta u + \Delta \bar{u}) = 0.83$ $\pm 0.03, (\Delta d + \Delta \bar{d}) = -0.43 \pm 0.03, (\Delta s + \Delta \bar{s}) = -0.10 \pm 0.03;$

see [19]. The parton numbers $n_{\alpha(\bar{\alpha})\uparrow(\,\downarrow)}$ in Eqs. (2)–(7) are obtained by integrating the appropriate number density dn/dx over x. The various Δ 's are also x-integrated quantities.

The RHSs of Eqs. (2) – (4) are clearly Q^2 independent. The RHSs of Eqs. (5) – (7) are also Q^2 independent in the jet and Adler-Bardeen (AB) schemes: Recall that the nonsinglets $\Delta q_3 = (\Delta u + \Delta \bar{u}) - (\Delta d + \Delta \bar{d})$ and $\Delta q_8 = (\Delta u)$ $+\Delta \bar{u}$) + $(\Delta d + \Delta \bar{d}) - 2(\Delta s + \Delta \bar{s})$ are *Q*² independent in all renormalization schemes because of the conservation of the nonsinglet axial vector current, and the singlet $\Delta \Sigma = (\Delta u)^T$ $+\Delta \bar{u}$) + $(\Delta d + \Delta \bar{d}) + (\Delta s + \Delta \bar{s})$ is Q^2 independent in the jet and AB schemes because of the Adler-Bardeen theorem [20]. As a result, $(\Delta u + \Delta \bar{u})$, $(\Delta d + \Delta \bar{d})$, and $(\Delta s + \Delta \bar{s})$ which can be expressed as linear combinations of Δq_3 , Δq_8 , and $\Delta\Sigma$ are also Q^2 independent in these two schemes. In the MS scheme, on the other hand, $\Delta \Sigma$ is Q^2 independent at the leading order and only weakly *Q*² dependent at the next-toleading order. Empirically too $\Delta \Sigma$ is found to be almost Q^2 independent; see e.g., Fig. 5 of $[21]$. Hence in the MS scheme the RHSs of Eqs. (5) – (7) are expected to be nearly Q^2 independent.

We now show how the statistical model naturally leads to a violation of the flavor symmetry in the unpolarized and polarized seas in the nucleon. Consider the following six equations:

$$
2n_{u\uparrow} - 2n_{u\downarrow} = 2.83,\tag{8}
$$

$$
2n_{u\downarrow} - 2n_{u\uparrow} = 1.17, \tag{9}
$$

$$
2n_{d\uparrow} - 2n_{\bar{d}\downarrow} = 0.57, \tag{10}
$$

$$
2n_{d\downarrow} - 2n_{\bar{d}\uparrow} = 1.43, \tag{11}
$$

$$
2n_{s\uparrow} - 2n_{s\downarrow} = -0.10, \tag{12}
$$

$$
2n_{s\downarrow} - 2n_{s\uparrow} = 0.10. \tag{13}
$$

These are obtained from Eqs. (2) – (7) by linearly combining the latter set of equations in pairs. For example, Eqs. (8) and (9) are obtained by adding or subtracting Eqs. (2) and (5) .

It was shown in $[16]$ that the parton number density dn/dx in the infinite-momentum frame, at the input scale, is given by

$$
\frac{dn}{dx} = \frac{M^2x}{2} \int_{xM/2}^{M/2} \frac{dE}{E^2} \frac{dn}{dE},
$$
\n(14a)

where

$$
dn/dE = g f(E)(VE^2/2\pi^2 + aR^2E + bR), \qquad (14b)
$$

is the density in the nucleon rest frame. Here *M* is the nucleon mass, *E* is the parton energy in the nucleon rest frame, *g* is the spin-color degeneracy factor, $f(E)$ is the usual Fermi or Bose distribution function $f(E) = \{ \exp[(E)$ $-\mu$ /*T*] \pm 1}⁻¹, *V* is the nucleon volume, and *R* is the radius of a sphere with volume V . The three terms in Eq. $(14b)$ are the volume, surface, and curvature terms, respectively; in the thermodynamic limit only the first survives. The two free parameters a and b in Eq. $(14b)$ were determined in $[17]$ by fitting the structure function $F_2(x, Q^2)$ data at Q^2 $=4$ GeV². Their values as well as the values of the temperature (T) and chemical potential (μ) , which get determined due to Eqs. (2) – (7) , were given in [17].

At the input scale, with the help of Eq. (14) , Eq. (8) can be written in full form as

$$
\int_0^1 dx \frac{M^2 x}{2} \int_{xM/2}^{M/2} \frac{dE}{E^2} g(VE^2/2\pi^2 + aR^2E + bR)
$$

$$
\times \left[\frac{2}{e^{\beta(E - \mu_{u\uparrow})} + 1} - \frac{2}{e^{\beta(E - \mu_{u\downarrow})} + 1} \right] = 2.83. \quad (15)
$$

It is straightforward to show that the chemical potentials for quarks and antiquarks satisfy the relations

$$
\mu_{q\uparrow} = -\mu_{q\downarrow},\qquad(16a)
$$

$$
\mu_{q\downarrow} = -\mu_{q\uparrow} \,. \tag{16b}
$$

So it follows from Eqs. (15) and (16b) that $\mu_{u\uparrow} > 0$. Similar arguments show that $\mu_{u\downarrow}$, $\mu_{d\uparrow}$, $\mu_{d\downarrow}$, and $\mu_{s\downarrow}$ are positive and $\mu_{s\uparrow}$ is negative. Moreover, since the RHSs of Eqs. (12) and (13) differ only in sign, we have $\mu_{s\uparrow} = -\mu_{s\downarrow}$. Since RHSs of Eqs. (8) – (13) can be arranged as 2.83 >1.43 $>1.17>0.57>0.10>-0.10$, the corresponding chemical potentials satisfy

$$
\mu_{u\uparrow} > \mu_{d\downarrow} > \mu_{u\downarrow} > \mu_{d\uparrow} > (\mu_{s\downarrow} = \mu_{s\downarrow}) > 0 > (\mu_{s\uparrow} = \mu_{s\uparrow})
$$

>
$$
\mu_{\bar{d}\downarrow} > \mu_{\bar{u}\uparrow} > \mu_{\bar{d}\uparrow} > \mu_{\bar{u}\downarrow}.
$$
 (17)

It will be useful to recall the actual values of the μ 's given in [17]. They are (in MeV) μ_{μ} ⁼210, $\mu_{d\downarrow}$ =106, $\mu_{\mu\downarrow}$ $=86, \mu_{d\uparrow}=42, \mu_{s\downarrow}=7, \mu_{s\uparrow}=-7, \mu$'s for the antiquarks follow from Eq. (16) . The RHSs of Eqs. (8) – (13) are sufficiently different from each other so that the experimental errors in $(\Delta q + \Delta \bar{q})$, quoted above, will not alter the ordering in Eq. (17) . Equation (17) , together with Eq. (14) , yields, *at the input scale* Q_0^2 (= M^2 = 0.88 GeV²),

$$
n_{u\uparrow} > n_{d\downarrow} > n_{u\downarrow} > n_{d\uparrow} > (n_{s\downarrow} = n_{s\downarrow})
$$

>
$$
(n_{s\uparrow} = n_{s\uparrow}) > n_{\bar{d}\downarrow} > n_{\bar{u}\uparrow} > n_{\bar{d}\uparrow} > n_{u\downarrow} > 0. \quad (18)
$$

As a check, it is easy to verify that Eq. (18) reproduces the correct signs of the RHSs of Eqs. (2) – (7) . Notice the symmetric arrangement of the μ 's in Eq. (17) and the consequent arrangement of the n 's in Eq. (18) .

To recapitulate, the statistical model provides a quantitative method to incorporate the effects of the Pauli exclusion principle into the PDFs: the RHSs of the number constraints (2)–*(7) or equivalently Eqs. (8)*–*(13), force the various chemical potentials and hence the parton distributions to be arranged as in Eqs. (17) and (18), respectively, at the input scale*.

Further consequences of Eq. (18) are easy to derive (note $n_q = n_{q\uparrow} + n_{q\downarrow}$ and $\Delta q = n_{q\uparrow} - n_{q\downarrow}$.

(a) The general positivity constraints on the polarized and unpolarized PDFs: $|\Delta q| \leq n_q$ are satisfied trivially.

(b) $\Delta u > 0$, $\Delta d < 0$, $\Delta s < 0$.

(c) $\Delta \bar{u} > 0$, $\Delta \bar{d} < 0$, $\Delta \bar{s} < 0$. This is in contrast to the assumption (1) made in the literature $[1-8,12-14]$ that all the three $\Delta \bar{q}$'s have the same sign.

(d) $\Delta u_v = \Delta u - \Delta \bar{u} = n_{u\uparrow} - n_{u\downarrow} - n_{\bar{u}\uparrow} + n_{\bar{u}\downarrow} > 0$, because the two n_{μ}^- terms are too small compared to the two n_{μ} terms [see Eq. (18)] to change the sign of the RHS.

(e) $\Delta d_v = \Delta d - \Delta \overline{d} = n_{d\uparrow} - n_{d\downarrow} - n_{\overline{d}\uparrow} + n_{\overline{d}\downarrow} < 0$, because the two $n_{\bar{d}}$ terms are too small compared to the two n_d terms [see Eq. (18)] to change the sign of the RHS.

(f) $\Delta s_v = \Delta s - \Delta s = 0$.

 $(g) \Delta q_3 = (\Delta u + \Delta \bar{u}) - (\Delta d + \Delta \bar{d}) > 0$; see (b),(c).

(h) $n_{\bar{d}} > n_{\bar{u}}$ which leads to the Gottfried sum rule violation. *Thus the statistical model naturally leads to the* $\overline{u} \neq \overline{d}$ *asymmetry in the unpolarized sea* [16]. Moreover, it was shown in [17] that the model is in *quantitative* agreement with the data on $(F_2^p - F_2^n)$ vs *x* and the Gottfried sum S_G .

(i) $\Delta \bar{u} - \Delta \bar{d} > n_{\bar{d}} - n_{\bar{u}} > 0$. *Thus the violation of the flavor symmetry is more serious in the polarized case than in the unpolarized case.*

(j) $\Delta d - \Delta s = n_{d\uparrow} - n_{d\downarrow} - n_{s\uparrow} + n_{s\downarrow} < 0$, because $n_{s\uparrow}$ and

 $n_{s\perp}$ tend to cancel each other, unlike $n_{d\uparrow}$ and $n_{d\perp}$. Combining this result with (b) above, one gets $|\Delta d| > |\Delta s|$, and

$$
\Delta u > 0 > \Delta s > \Delta d. \tag{19}
$$

 (k) $\Delta \overline{d} - \Delta \overline{s} = n_{\overline{d}\uparrow} - n_{\overline{d}\downarrow} - n_{\overline{s}\uparrow} + n_{\overline{s}\downarrow} < 0$, because $n_{\overline{s}\uparrow}$ and $n_{\bar{s}\downarrow}$ tend to cancel each other, unlike $n_{\bar{d}\uparrow}$ and $n_{\bar{d}\downarrow}$. Combining this result with (c) above, one gets $|\Delta \bar{d}| > |\Delta \bar{s}|$, and

$$
\Delta \bar{u} > 0 > \Delta \bar{s} > \Delta \bar{d}.
$$
 (20)

We have derived the results (a) – (k) analytically, at the input scale. They are borne out by actual numerical calculations; see Fig. 1 which shows our polarized PDFs at the input scale $Q_0^2 = M^2 = 0.88$ GeV². We have evolved our polarized PDFs in the next-to-leading-order QCD, in the MS scheme, in the range $Q_0^2 < Q^2 < 6500$ GeV². We find that the results (a) – (k) are valid throughout this range. Figure 2 shows that the violation of the flavor symmetry is more serious in the polarized case than in the unpolarized case, throughout this range.

Incidentally, we have examined another simplifying assumption made e.g., in $|11|$, namely

$$
\Delta q_3(x, Q^2) = C \Delta q_8(x, Q^2),\tag{21}
$$

FIG. 2. Solid curves, $x(\Delta \overline{u} - \Delta \overline{d})$; dashed curves, $x(\overline{d} - \overline{u})$. Curves are labeled by Q^2 in GeV².

where *C* is a constant independent of *x* and Q^2 . The present model predicts that Eq. (21) is not justified (Fig. 1).

The statistical model makes concrete predictions for various asymmetries in polarized *pp* scattering, which can be tested at RHIC. For example, parity-violating single- and double-spin asymmetries for *W* production in the reactions $\vec{p}p \rightarrow W^{\pm}X$ and $\vec{p}p \rightarrow W^{\pm}X$, respectively, are given by $[22, 23]$

$$
A_L^{PV}(W^+) = \frac{\Delta u(x_a, M_W^2) \overline{d}(x_b, M_W^2) - \Delta \overline{d}(x_a, M_W^2) u(x_b, M_W^2)}{u(x_a, M_W^2) \overline{d}(x_b, M_W^2) + \overline{d}(x_a, M_W^2) u(x_b, M_W^2)},
$$
\n(22)

$$
A_L^{PV}(W^-) = \frac{-\Delta \bar{u} \, d + \Delta d \, \bar{u}}{\bar{u} \, d + d \, \bar{u}},\tag{23}
$$

$$
A_{LL}^{PV}(W^+) = \frac{\Delta u \, \bar{d} - u \, \Delta \bar{d} - \Delta \bar{d} \, u + \bar{d} \, \Delta u}{u \, \bar{d} - \Delta u \, \Delta \bar{d} + \bar{d} \, u - \Delta \bar{d} \, \Delta u},\tag{24}
$$

$$
A_{LL}^{PV}(W^{-}) = \frac{\bar{u}\ \Delta d - \Delta \bar{u}\ d - d\ \Delta \bar{u} + \Delta d\ \bar{u}}{\bar{u}\ d - \Delta \bar{u}\ \Delta d + d\ \bar{u} - \Delta d\ \Delta \bar{u}},\tag{25}
$$

where $x_a = \sqrt{\tau}e^y$, $x_b = \sqrt{\tau}e^{-y}$, $\tau = M_W^2/s$, y is the rapidity of *W* and \sqrt{s} is the *pp* center-of-mass energy. The arguments x_a , x_b and M_W^2 are suppressed in Eqs. (23)–(25) for brevity of notation.

FIG. 3. Predictions for asymmetries in W^- production in polarized *pp* scattering at \sqrt{s} =500 GeV vs rapidity *y*. Solid curves, present model. Other curves: Long-dashed [24], short-dashed [2], dotted, and dot-dashed [3] for two separate parametrizations—as reported in [23].

In the present model, Δu and $\Delta \overline{u}$ are positive and Δd and $\Delta \bar{d}$ are negative [see (b), (c), and Fig. 1]. Also note that $\Delta u \le u$ and $|\Delta \overline{d}| \le \overline{d}$ [see (a) above]. Hence it is straightforward to show that $0 < A_L^{PV}(W^+) < 1$. Similarly, -1 $\langle A_L^{PV}(W^-) \langle 0, A_{LL}^{PV}(W^+) \rangle = 0, A_{LL}^{PV}(W^-) \langle 0, \text{It is some-}$ what tedious but again straightforward to show, using Eqs. $A_L^{PV}(W^+) > A_L^{PV}(W^+)$ and $|A_{LL}^{PV}(W^-)|$ $> |A_L^{PV}(W^-)|$. A quick and crude way to convince oneself that $A_{LL}^{PV}(W^+) > A_L^{PV}(W^+)$ is to ignore the (small) " $\Delta \Delta$ " terms in the denominator of Eq. (24) , which makes the denominators of Eqs. (22) and (24) identical, and then to compare their numerators. In fact, at $y=0$ (or $x_a=x_b$), $A_{LL}^{PV}(W^+)$ is seen to be almost twice as big as $A_L^{PV}(W^+)$.

Figure 3 shows our predictions for A_L^{PV} and A_{LL}^{PV} for $W^$ production in polarized *pp* scattering at \sqrt{s} = 500 GeV as a function of the rapidity *y*. The above inequalities for $A_L^{PV}(W^-)$ and $A_{LL}^{PV}(W^-)$, which we derived analytically here are borne out by the actual numerical results in Fig. 3. Also shown for comparison are results reported in $[23]$. These are based on the parametrizations of polarized PDFs given in [2,3,24]. Asymmetries for W^- production are sensitive to the sign of $\Delta \overline{u}$, which is positive in the present model, negative in $[2,24]$, and *x* dependent in $[3]$. The recent work of de

Florian and Sassot $|25|$ has yielded a clear preference for a positive $\Delta \overline{u}$ distribution.

As stated earlier, the HERMES and SMC Collaborations [12,13] analyzed their inclusive and semi-inclusive DIS data assuming all $\Delta \bar{q}$'s to be of the same sign. Recently, Morii and Yamanishi [26] have reanalyzed these data and have estimated $\Delta \bar{d}(x) - \Delta \bar{u}(x)$ at $Q^2 = 4$ GeV². It is evident from their Fig. 1 that $\Delta \bar{u}(x) - \Delta \bar{d}(x)$ is positive and has a peak at $x \approx 0.06$ where $x\{\Delta \bar{u}(x) - \Delta \bar{d}(x)\}$ is ≈ 0.05 . All these observations are consistent with our Fig. 2.

Another model which is able to generate flavor asymmetric polarized antiquark sea is the chiral quark soliton model $(CQSM)$ [27–32]. Results in our Fig. 2 are strikingly similar to those in $[27,28,30]$. This is remarkable because the physics inputs of the two models are quite different. It is also noteworthy that the origin of the $\overline{u} \neq \overline{d}$ and $\Delta \overline{u} \neq \Delta \overline{d}$ asymmetries is quite simple in the statistical model. While the role of gluons is yet to be understood in CQSM, the statistical model predicts a positive $\Delta g(x, Q^2)$. The pion cloud model also gives rise to the $\overline{u} \neq \overline{d}$ asymmetry (for a recent review, see [33]), and there have been some attempts to generate polarization by including spin-1 resonances in that model. These attempts have been commented upon in $[30,31]$. Recently, Glück and Reya [34] have discussed the issue of flavor asymmetry, in a phenomenological way making use of the Pauli exclusion principle. We recall that the statistical model $\left[16,17\right]$ provides a quantitative method to incorporate the effects of the Pauli exclusion principle into the PDFs.

We have treated all partons as massless: $m_u = m_d = m_s$ $=0$. If m_s is taken to be nonzero, then Eq. (14) will have to be generalized, but the parton densities still have to satisfy Eqs. (2) – (7) and equivalently Eqs. (8) – (13) . So it is not obvious how this will affect the symmetric arrangement of the μ 's in Eq. (17) and the consequent arrangement of the n 's in Eq. (18) , at the input scale. This is a nontrivial problem which needs to be investigated further.

In conclusion, we have derived, on rather general grounds, a series of inequalities for the polarized PDFs; see (a) – (k) above. This points to the need to redo the analyses $[1-8,12,13]$ of polarized data, allowing for the alternate scenario as in Eqs. (19) and (20) . Some of the inequalities can be tested in the forthcoming spin-physics program at RHIC, BNL. To illustrate, we have given our predictions for the $W^$ asymmetries; these are quite different from those available in the literature.

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