Electromagnetic properties of the $K=1$ band in the rotational limit of the neutron-proton interacting boson model

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Within the framework of the SU(3) limit of the neutron-proton interacting boson model the matrix elements of the angular momentum and the quadrupole operator between states belonging to the $K=1$ band are given in closed forms. To obtain the matrix elements analytically, we first derive the extended $U(6)\supset SU(3)$ isoscalar factors associated with low-lying bands by using the intrinsic SU(3) states. The extended $U(6)\supset SU(3)$ isoscalar factor is defined as the product of the ordinary $U(6)\supset SU(3)$ isoscalar factor and the U(6)-reduced matrix element of the one-body boson operator. Using these results, $M1$ and $E2$ intraband transition probabilities and the electromagnetic moments in the $K=1$ band are derived in closed forms.

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I. INTRODUCTION

Since the interacting boson model (IBM) was proposed by Arima and Iachello [1,2] in the 1970s, a wide variety of collective properties of low-lying energy levels in even-even nuclei have been explained successfully by the model. The basic assumptions of the original version of the interacting boson model (IBM-1) are that the low-lying quadrupole excitations of the even-even nucleus can be studied by considering a system of N interacting bosons with the angular momentum $L=0$ (s boson) and $L=2$ (d boson) and no distinction is made between the proton and the neutron bosons. The important extension of the IBM-1 is the neutron-proton interacting boson model (IBM-2), in which the proton and neutron degrees of freedom are explicitly taken into account [3]. In the IBM-2 the proton-neutron exchange symmetry of the wave function could be specified the quantum number F spin and the Majorana operator which separates states with different values of F spin [4–6]. Fully symmetric IBM-2 states with a maximal F spin $F=F_{\max}$, which are essentially symmetric respect to the proton-neutron interchange, are lowest in energy and identical with the states of IBM-1. A new class of collective states with $F\neq F_{\max}$ (mixed-symmetric states), for which the quadrupole degrees of the proton are excited in a different way than those of the neutron, is also predicted in the IBM-2. Such states are proved experimentally from the observation of low-lying collective 1^+ states of several deformed nuclei in the rare-earth region by Bohle *et al*. [7]. This 1^+ state corresponds to the bandhead of the $K=1$ mixed-symmetric rotational band in the SU(3) limit and $M1$ transition strengths to the 1^+ level have been extensively analyzed in the IBM-2 [8–10].

For the analysis of dynamic symmetries within the framework of the IBM-2, it is of some interest to obtain closed forms of observables. The dynamic symmetries in the SU(3) limit of the IBM-2 bring out algebraic closed forms in the energy level and some of the electromagnetic transitions in well-deformed nuclei [10–13]. To calculate the physical observables in the SU(3) basis of the IBM-2, knowledge of the matrix elements of the SU(3) generators, i.e., the angular momentum and the quadrupole operators, is needed.

In this paper, a method is introduced for the calculation of the matrix elements of the SU(3) generators based on group theory in the SU(3) limit of the IBM-2. In the process of reducing the matrix element within the group chain of the SU(3) limit, one obtains the $U(6)\supset SU(3)$ isoscalar factor and the U(6)-reduced matrix element of the one-body boson operator. These quantities necessary for the calculations of the matrix elements between states with full symmetry and between fully symmetric states and mixed-symmetric $K=1$ states are already given [14,15]. The electromagnetic properties of 1^+, 2^+, and 3^+ states in the $K=1$ band were studied previously by Van Isacker *et al*. [11], but the general expressions for the $U(6)\supset SU(3)$ isoscalar factors and the U(6)-reduced matrix elements which are necessary to calculate the $E2$ and $M1$ matrix elements between states to which belong the $K=1$ band have not been given. Since the $U(6)\supset SU(3)$ isoscalar factors and the U(6)-reduced matrix elements in the $K=1$ band cannot be determined separately, in the present work the product of these two quantities are derived by using the intrinsic states of the SU(3) limit of the IBM-2 [14]. From these results, closed expressions for the electromagnetic moments and intraband transitions in the $K=1$ band are calculated and a short discussion is also contained.

Although this work is only restricted to an exact SU(3) symmetry and follows the group theoretical approach of the IBM-2, the results in the present work provide first insight into the electromagnetic properties for mixed-symmetric states in deformed nuclei and may be applied to calculate analytically the realistic observables in the IBM-2 SU(3) limit, such as the proton-neutron interaction $Q_\pi\cdot Q_\nu$.

II. MATRIX ELEMENTS OF THE SU(3) GENERATORS

Within the framework of the IBM, the rotational properties of even-even deformed nuclei are described by the SU(3) limit. We consider the SU(3) limit of the IBM-2, in which the proton and neutron degrees of freedom are joined at the level of U(6). The group chain in this limit is given as [12,16]

$$\mathrm{U}_{\pi}(6)\otimes\mathrm{U}_{\nu}(6)\supset\mathrm{U}_{\pi+\nu}(6)\supset\mathrm{SU}_{\nu+\pi}(3)\supset\mathrm{O}_{\pi+\nu}(3), \quad (1)$$

and the wave function is characterized by

$$|[N_\pi]\otimes[N_\nu];[N-f,f](\lambda,\mu)\kappa L\rangle, \quad (2)$$

where N_π (N_ν) is the number of proton (neutron) bosons and N is the total boson number ($N=N_\pi+N_\nu$). The additional quantum number κ is necessary to completely specify SU(3)⊃O(3) reduction. The irreducible representation (irrep) $[N-f,f]$ of $\mathrm{U}_{\pi+\nu}(6)$ is related to the F spin through $F=N/2-f$ [11,17]. Fully symmetric and the lowest mixed-symmetric states are characterized by the irreps $[N]$ and $[N-1,1]$ of $\mathrm{U}_{\pi+\nu}(6)$, respectively.

In the IBM-2, the generators of $\mathrm{SU}_{\pi+\nu}(3)$ consist of the angular momentum and the quadrupole operators:

$$L_q=L_{\pi,q}+L_{\nu,q} \quad (q=0,\pm1),$$
$$Q_q=Q_{\pi,q}+Q_{\nu,q} \quad (q=0,\pm1,\pm2), \quad (3)$$

with

$$L_{\rho,q}=\sqrt{10}(d^\dagger_\rho\tilde{d}_\rho)^{(1)}_q,$$
$$Q_{\rho,q}=(d^\dagger_\rho s_\rho+s^\dagger_\rho\tilde{d}_\rho)^{(2)}_q-\frac{\sqrt{7}}{2}(d^\dagger_\rho\tilde{d}_\rho)^{(2)}_q, \quad (4)$$

where ρ denotes π (proton) or ν (neutron) bosons. The matrix elements of the generator T_π and T_ν of the group $\mathrm{U}_\pi(6)\otimes\mathrm{U}_\nu(6)$ are simplified by the following relations [11,18]:

$$N_\pi\langle[N_\pi]\otimes[N_\nu];[N]\alpha|T_\nu|[N_\pi]\otimes[N_\nu];[N]\alpha'\rangle=N_\nu\langle[N_\pi]\otimes[N_\nu];[N]\alpha|T_\pi|[N_\pi]\otimes[N_\nu];[N]\alpha'\rangle, \quad (5a)$$

$$\langle[N_\pi]\otimes[N_\nu];[N-f,f]\alpha|T_\nu|[N_\pi]\otimes[N_\nu];[N-f',f']\alpha'\rangle=-\langle[N_\pi]\otimes[N_\nu];[N-f,f]\alpha|T_\pi|[N_\pi]\otimes[N_\nu];[N-f',f']\alpha'\rangle, \quad (5b)$$

if $f\neq f'$ and where α (α') is all additional quantum numbers to completely specify the state in Eq. (2). Then the matrix element of T_ρ ($\rho=\pi$ or ν) is related to that of the $\mathrm{U}_{\pi+\nu}(6)$ generators $T=T_\pi+T_\nu$ in the fully symmetric states as follows:

$$\langle[N_\pi]\otimes[N_\nu];[N]\alpha|T_\rho|[N_\pi]\otimes[N_\nu];[N]\alpha'\rangle=\frac{N_\rho}{N}\langle[N_\pi]\otimes[N_\nu];[N]\alpha|T|[N_\pi]\otimes[N_\nu];[N]\alpha'\rangle. \quad (6)$$

Therefore, the matrix elements of the ρ-boson angular momentum and quadrupole operators between interbands belonging to the irrep $[N]$ of $\mathrm{U}_{\pi+\nu}(6)$ vanish.

For the analytic calculation of the matrix elements of the SU(3) generators, it is necessary to know the tensor properties under the group chain given in Eq. (1). The generators of SU(3) are expressed as the tensor forms [13]

$$L_q=2T^{[2,1^4](1,1)}_{1q} \quad (q=0,\pm1),$$
$$Q_q=\sqrt{\frac{3}{2}}T^{[2,1^4](1,1)}_{2q} \quad (q=0,\pm1,\pm2). \quad (7)$$

The matrix element of a tensor operator can be calculated by applying the generalized Wigner-Eckart theorem. Formally, the matrix element of a tensor $T^{[2,1^4](1,1)}_{\rho,lq}$, where $\rho=\pi$ or ν, can be written as

$$\langle[N_\pi]\otimes[N_\nu];[N-f,f](\lambda,\mu)\kappa LM|T^{[2,1^4](1,1)}_{\rho,lq}|[N_\pi]\otimes[N_\nu];[N-f',f'](\lambda',\mu')\kappa'L'M'\rangle$$
$$=\langle[N_\pi]\otimes[N_\nu];[N-f,f]||||T^{[2,1^4]}_\rho||||[N_\pi]\otimes[N_\nu];[N-f',f']\rangle\langle LM',lq|LM\rangle$$
$$\times\langle[N-f',f'](\lambda',\mu'),[2,1^4](1,1)||[N-f,f](\lambda,\mu)\rangle\langle(\lambda',\mu')\kappa'L,(1,1)l||(\lambda,\mu)\kappa L\rangle. \quad (8)$$

In the process of reducing the matrix elements, one obtains the generalized coupling coefficient associated with the reduction U(6)⊃SU(3)⊃O(3)⊃O(2) and the U(6)-reduced matrix element denoted with four vertical bars (Wu *et al.* [14] called this the overlap amplitude). The generalized coupling coefficient is written as a product of two isoscalar factors [associated with the reductions U(6)⊃SU(3) and SU(3)⊃O(3), respectively] and the ordinary O(3)⊃O(2) Clebsch-Gordan coefficient according to Racah's factorization lemma [19]. The SU(3)⊃O(3) isoscalar factors necessary to our calculations have been given extensively by Vegados [20]. The U(6)⊃SU(3) isoscalar factor and U(6)-reduced matrix element can be calculated in the intrinsic SU(3) states of IBM-2 [14] and some available results have been given [15]. However, it is still insufficient applying to all cases. We revise and complement those which are necessary for the calculation of the matrix elements of the SU(3) generators in the IBM-2.

The U(6)-reduced matrix elements of the one-body boson operator between states with full symmetry and between the fully symmetric and the mixed-symmetric states could be easily determined by using the F spin [15]. However, for the calculation of the matrix elements between the mixed-symmetric states this method could not apply, because the values of the U(6)-reduced matrix element between the bands with mixed symmetry are not determined uniquely. Therefore, for consistency we calculate the $U(6)\supset SU(3)$ isoscalar factors and the U(6)-reduced matrix elements simultaneously rather than derive them separately. We define here the extended $U(6)\supset SU(3)$ isoscalar factor as the product of the $U(6)\supset SU(3)$ isoscalar factor and the U(6)-reduced matrix element of the one-body boson operator. We denote the extended $U(6)\supset SU(3)$ isoscalar factor for the ρ boson in the IBM-2 for simplicity as

$$\begin{aligned}&\langle[N-f',f'](\lambda',\mu'),\rho[2,1^4](1,1)||[N-f,f](\lambda,\mu)\rangle\\&=\langle[N_\pi]\otimes[N_\nu];[N-f,f]||||T_\rho^{[2,1^4]}||||[N_\pi]\otimes[N_\nu];\\&\times[N-f',f']\rangle\langle[N-f',f'](\lambda',\mu'),\\&\times[2,1^4](1,1)||[N-f,f](\lambda,\mu)\rangle.\end{aligned}\tag{9}$$

To determine the extended $U(6)\supset SU(3)$ isoscalar factor, we use the intrinsic states for the SU(3) limit of the IBM-2 [14]. The intrinsic states span the irreducible space of the subgroup $SU(2)\otimes U(1)$ of SU(3) instead of O(3) in the group chain in Eq. (1). The three operators

$$\Lambda_0=\frac{1}{2}L_0,\quad \Lambda_{\pm1}=\mp\sqrt{\frac{2}{3}}Q_{\pm2}\tag{10}$$

generate the SU(2) group, and Q_0 is the sole generator of U(1). The intrinsic states are characterized by the eigenvalues of Q_0, Λ_0, and Λ^2 and these eigenvalues are specified by the quantum numbers ϵ, K, and Λ:

$$\begin{aligned}&\sqrt{8}Q_0|\epsilon\Lambda K\rangle=\epsilon|\epsilon\Lambda K\rangle,\\&\Lambda_0|\epsilon\Lambda K\rangle=\frac{K}{2}|\epsilon\Lambda K\rangle,\\&\Lambda^2|\epsilon\Lambda K\rangle=\Lambda(\Lambda+1)|\epsilon\Lambda K\rangle.\end{aligned}\tag{11}$$

Thus the intrinsic wave functions in the SU(3) limit of the IBM-2 are characterized by $|[N_\pi]\otimes[N_\nu];[N-f,f](\lambda,\mu)\epsilon\Lambda K\rangle$ and the SU(3) generators transform as the irreducible tensor operator $T^{[2,1^4](1,1)}_{\epsilon\Lambda K/2}$, where ϵ, Λ, and K are the $SU(2)\otimes U(1)$ weights of the SU(3) generator [14]. In the intrinsic space of the IBM-2 the tensor characters of the SU(3) generators are expressed as follows [15,21]:

$$\begin{aligned}&Q_0=-\sqrt{\frac{3}{2}}T^{[2,1^4](1,1)}_{000},\quad L_0=2T^{[2,1^4](1,1)}_{010},\\&Q_{\pm1}=-\frac{\sqrt{3}}{2}[T^{[2,1^4](1,1)}_{\mp3(1/2)\pm1/2}\pm T^{[2,1^4](1,1)}_{\pm3(1/2)\pm1/2}],\\&L_{\pm1}=\sqrt{2}[T^{[2,1^4](1,1)}_{\mp3(1/2)\pm1/2}\mp T^{[2,1^4](1,1)}_{\pm3(1/2)\pm1/2}],\\&Q_{\pm2}=\mp\sqrt{\frac{3}{2}}T^{[2,1^4](1,1)}_{01\pm1}.\end{aligned}\tag{12}$$

N-boson intrinsic wave functions for the low-lying bands have been derived explicitly by Wu *et al.* [14] and the matrix element of the one-body boson operator can be calculated from these states. On the other band, the matrix element of a tensor operator can be calculated by applying the generalized Wigner-Eckart theorem in the intrinsic SU(3) basis of the IBM-2. By comparing these two results one could obtain the extended $U(6)\supset SU(3)$ isoscalar factors. Since this procedure was summarized already [14,15], we do not repeat here. However, one has to keep in mind the multiplicity problem in the irreducible representation (λ,μ), because for $\mu\neq0$ the product $(\lambda,\mu)\otimes(1,1)$ contains the irreducible representation (λ,μ) twice. Therefore for $\mu\neq0$ the additional quantum number i is necessary in the extended $U(6)\supset SU(3)$ isoscalar factor such as $\langle[N-f,f](\lambda,\mu),\rho[2,1^4](1,1)||[N-f',f'](\lambda,\mu)\rangle_i$, where $i=1$ or 2. Hecht [21] had chosen the quantum number i such that the reduced matrix elements of the infinitesimal operators of SU(3) are nonzero only for the case $i=1$. The choice of i in this work is exactly the one adopted by Hecht. The extended $U(6)\supset SU(3)$ isoscalar factor $\langle[N-f,f](\lambda,\mu),\pi[2,1^4](1,1)||[N-f',f'](\lambda',\mu')\rangle$ for the proton boson are listed in Table I. For the neutron boson the corresponding expression $\langle[N-f,f](\lambda,\mu),\nu[2,1^4](1,1)||[N-f',f'](\lambda',\mu')\rangle$ is obtained by exchanging the subscript π and ν for $f=f'$ and adding minus sign for the $f\neq f'$ to the values given in Table I.

Since the electromagnetic transitions from the mixed-symmetric $K=1$ band to the fully symmetric states have been studied in detail elsewhere [11,13], the matrix elements of the quadrupole and the angular momentum operators between the states of $K=1$ band are only presented here. The reduced matrix element of the ρ-boson quadrupole operator Q_ρ between the mixed-symmetric states $|m;LM\rangle=|[N_\pi]\otimes[N_\nu];[N-1,1](2N-2,1)LM\rangle$ in the $K=1$ band is expressed as

$$\begin{aligned}&\langle m;L||Q_\rho||m;L'\rangle\\&=\sqrt{\frac{3}{2}}\langle m;L||T^{[2,1^4](1,1)}_{\rho,2}||m;L'\rangle\\&=\sqrt{\frac{3(2L+1)}{2}}\sum_{i=1,2}\langle[N-1,1](2N-2,1),\rho[2,1^4]\\&\times(1,1)||[N-1,1](2N-2,1)\rangle_i\langle(2N-2,1)\\&\times L',(1,1)2||(2N-2,1)L\rangle_i.\end{aligned}\tag{13}$$

TABLE I. Extended $U(6)\supset SU(3)$ isoscalar factors defined in Eq. (9) for the proton boson.

$[N-f,f](\lambda,\mu)$	$[N-f',f'](\lambda',\mu')$	$\langle[N-f,f](\lambda,\mu),\pi[2,1^4]\|\|[N-f',f'](\lambda',\mu')\rangle$
$[N](2N,0)$	$[N](2N,0)$	$\left[\frac{2(2N+3)}{3N}\right]^{1/2} N_\pi$
	$[N](2N-4,2)$	0
	$[N-1,1](2N-2,1)$	$-\left[\frac{2N+2}{N(2N-1)}\right]^{1/2}\sqrt{N_\pi N_\nu}$
	$[N-1,1](2N-4,2)$	0
$[N](2N-4,2)$	$[N](2N,0)$	0
	$[N](2N-4,2),\quad i=1$	$\left[\frac{4N^2-6N+6}{3N^2}\right]^{1/2} N_\pi$
	$[N](2N-4,2),\quad i=2$	0
	$[N-1,1](2N-2,1)$	$\left[\frac{3}{(N-1)(2N-1)}\right]^{1/2}\sqrt{N_\pi N_\nu}$
	$[N-1,1](2N-4,2),\quad i=1$	$-\left[\frac{3(N-2)(2N-1)}{N^2(2N^2-3N+3)}\right]^{1/2}\sqrt{N_\pi N_\nu}$
	$[N-1,1](2N-4,2),\quad i=2$	$\left[\frac{2(2N+1)}{(N-1)(2N^2-3N+3)}\right]^{1/2}\sqrt{N_\pi N_\nu}$
$[N-1,1](2N-2,1)$	$[N](2N,0)$	$\left[\frac{2}{N}\right]^{1/2}\sqrt{N_\pi N_\nu}$
	$[N](2N-4,2)$	$-\left[\frac{2N+1}{N(N-1)(2N-3)}\right]^{1/2}\sqrt{N_\pi N_\nu}$
	$[N-1,1](2N-1,1),\quad i=1$	$-\frac{3N-2N_\pi(2N+3)}{2\sqrt{3}N}$
	$[N-1,1](2N-1,1),\quad i=2$	$\left[\frac{3(N+1)}{4N^2(N-1)}\right]^{1/2}(N-2N_\pi)$
	$[N-1,1](2N-4,2)$	$-\left[\frac{(2N-1)(2N+1)}{2N(N-1)(N-2)(2N-3)}\right]^{1/2}(N-2N_\pi)$
$[N-1,1](2N-4,2)$	$[N](2N,0)$	0
	$[N](2N-4,2),\quad i=1$	$-\left[\frac{3(N-2)(2N-1)}{N^2(2N^2-3N+3)}\right]^{1/2}\sqrt{N_\pi N_\nu}$
	$[N](2N-4,2),\quad i=2$	$\left[\frac{2N+1}{(N-1)(2N^2-3N+3)}\right]^{1/2}\sqrt{N_\pi N_\nu}$
	$[N-1,1](2N-2,1)$	$\left[\frac{3}{2(N-1)(N-2)}\right]^{1/2}(N-2N_\pi)$

By inserting appropriate values of the extended $U(6)\supset SU(3)$ isoscalar factors in Table I and the $SU(3)\supset O(3)$ isoscalar factors [20], the matrix elements of the ρ-boson quadrupole operator Q_ρ between states within the $K=1$ band can be obtained and the results are listed in Table II. The matrix elements of L_ρ are also calculated with the help of Eq. (8). For $L=$ odd the results are

$$\langle m;L\|L_\rho\|m;L\rangle = \sqrt{\frac{2L+1}{L(L+1)}}\left[\frac{N-2N_\rho}{N-1}+\frac{L(L+1)(2N_\rho-1)}{2(N-1)}\right], \tag{14a}$$

$$\langle m;L\|L_\rho\|m;L+1\rangle = -\sqrt{\frac{L(L+2)(2N-L-1)}{2(L+1)N}}\,\frac{N-2N_\rho}{N-1}, \tag{14b}$$

$$\langle m;L\|L_\rho\|m;L-1\rangle = \sqrt{\frac{(L-1)(L+1)(2N+L)}{2LN}}\,\frac{N-2N_\rho}{N-1}, \tag{14c}$$

and for $L=$even

$$\langle m;L\|L_\rho\|m;L\rangle = \sqrt{\frac{2L+1}{L(L+1)}}\left[\frac{N-2N_\rho}{N-1}+\frac{L(L+1)[N_\rho(N+1)-N]}{N(N-1)}\right], \tag{15a}$$

TABLE II. Reduced matrix elements of the ρ-boson quadrupole operator Q_ρ between states in the $K=1$ band.

L'	$\langle m;L\|\|Q_\rho\|\|m;L'\rangle$, $L=$odd
L	$\sqrt{\frac{2L+1}{2L(L+1)(2L-1)(2L+3)}}\,\frac{3[(4N-1)L(L+1)-6N]+2N_\rho[12N+6-(4N+5)L(L+1)]}{4(N-1)}$
$L+1$	$\sqrt{\frac{3(2N-L-1)}{N(L+1)}}\,\frac{N(L+4)-2N_\rho(2N+L+2)}{4(N-1)}$
$L+2$	$\sqrt{\frac{3L(L+3)(2N+L+2)(2N-L-1)}{(2L+3)}}\,\frac{(2N_\rho-1)}{4(N-1)}$
$L-1$	$\sqrt{\frac{3(2N+L)}{NL}}\,\frac{N(L-3)+2N_\rho(2N-L+1)}{4(N-1)}$
$L-2$	$\sqrt{\frac{3(L-2)(L+1)(2N-L+1)(2N+L)}{2L-1}}\,\frac{(2N_\rho-1)}{4(N-1)}$
L'	$\langle m;L\|\|Q_\rho\|\|m;L'\rangle$, $L=$even
L	$\sqrt{\frac{2L+1}{2L(L+1)(2L-1)(2L+3)}}\,\frac{6N[L(L+1)-3N]-2N_\rho[L(L+1)(4N^2-N+3)-6N(2N+1)]}{4N(N-1)}$
$L+1$	$-\sqrt{\frac{3(2N+L+1)}{N(L+1)}}\,\frac{N(L-2)+2N_\rho(2N-L)}{4(N-1)}$
$L+2$	$\sqrt{\frac{3L(L+3)(2N+L+1)(2N-L-2)}{2L+3}}\,\frac{N_\rho(N+1)-N}{2N(N-1)}$
$L-1$	$\sqrt{\frac{3(2N-L)}{NL}}\,\frac{N(L+3)+2N_\rho(2N+L+1)}{4(N-1)}$
$L-2$	$\sqrt{\frac{3(L-2)(L+1)(2N-L)(2N+L-1)}{2L-1}}\,\frac{N_\rho(N+1)-N}{2N(N-1)}$

$$\langle m;L||L_\rho||m;L+1\rangle=-\sqrt{\frac{L(L+2)(2N+L+1)}{2(L+1)N}}\,\frac{N-2N_\rho}{N-1}, \tag{15b}$$

$$\langle m;L||L_\rho||m;L-1\rangle=\sqrt{\frac{(L-1)(L+1)(2N-L)}{2LN}}\,\frac{N-2N_\rho}{N-1}. \tag{15c}$$

III. ELECTROMAGNETIC PROPERTIES OF THE MIXED-SYMMETRIC STATES IN THE $K=1$ BAND

In the SU(3) limit of the IBM-2 the one-body $E2$ and $M1$ transition operators are given by

$$T(E2)=e_\pi Q_\pi+e_\nu Q_\nu, \tag{16a}$$

$$T(M1)=\sqrt{\frac{3}{4\pi}}(g_\pi L_\pi+g_\nu L_\nu), \tag{16b}$$

where e_ρ and g_ρ ($\rho=\pi,\nu$) are the ρ-boson effective charge and g factor, given in units of e b and μ_N, respectively. The electromagnetic moments of the mixed-symmetric $K=1$ states and $E2$ and $M1$ transition probabilities between states within the $K=1$ band can now be easily derived by using the results in Eqs. (14), (15), and Table II. The magnetic dipole moment of the states of the $K=1$ band in the SU(3) limit is given as

$$\mu_L=\frac{1}{(L+1)(N-1)}\begin{cases} g_S[2N-L(L+1)]+g_A N(L+2)(L-1), & L=\text{odd},\\ g_S[2N-2L(L+1)]+g_A[(N+1)L(L+1)-2N], & L=\text{even},\end{cases} \tag{17}$$

where $g_S=(g_\pi+g_\nu)/2$ and $g_A=(g_\pi N_\pi+g_\nu N_\nu)/N$. The magnetic dipole moment of the 1_m^+ state is simply given as $\mu(1_m^+)=\frac{1}{2}(g_\pi+g_\nu)=g_S$ and independent of the proton and neutron boson number in this model. In the condition of F-spin symmetry for the IBM Hamiltonian, the g factor of the mixed-symmetric states strongly depends on the angular momentum, whereas for all levels with full symmetry the g factor is independent of the angular momentum L and has the constant value $g_{FS}=g_A$, when the g factor is defined as $\mu=gL$ [9]. However, the dependence of the angular momentum for the magnetic dipole moment of the $K=1$ band is not proved from the experiment for lack of experimental data.

The electric quadrupole moment of the mixed-symmetric $K=1$ state is given as follows:

$$Q_L=-\sqrt{\frac{2\pi}{5}}\frac{1}{(L+1)(2L+3)(N-1)}\begin{cases}3e_S[6N-(4N-1)L(L+1)]-e_A N[12N+6-(4N+5)L(L+1)], & L=\text{odd},\\ 3e_S[6N-2L(L+1)]-e_A[6N(2N+1)-(4N^2-N+3)L(L+1)], & L=\text{even},\end{cases} \quad (18)$$

where $e_S=(e_\pi+e_\nu)/2$ and $e_A=(e_\pi N_\pi+e_\nu N_\nu)/N$. The electromagnetic moments of the 1_m^+ and 2_m^+ states have already been obtained by Van Isacker *et al.* [11] and their results are identical with those of the present work.

The reduced $M1$ transition probability between states of the $K=1$ band is obtained as

$$B(M1;mL+1\to mL)=\frac{3}{4\pi}(g_\pi-g_\nu)^2\frac{L(L+2)(2N\mp L\mp 1)}{2(L+1)(2L+3)N(N-1)^2}\times(N_\pi-N_\nu)^2, \quad (19)$$

where the value with the negative (positive) sign in the term $(2N\mp L\mp 1)$ corresponds to that for $L=$odd (even). For $M1$ intraband transitions between states within the $K=1$ band, the interesting result is obtained; that is, the $M1$ transition between mixed-symmetric states within $K=1$ band is allowed, whereas the $M1$ transitions are forbidden between fully symmetric states in the SU(3) limit of the IBM-2. The $B(M1)$ strengths within the $K=1$ band are proportional to $(g_\pi-g_\nu)^2$ as like transitions from the $K=1$ band to the fully symmetric states [11,13], and strongly depend on the difference of the proton and neutron boson number, $N_\pi-N_\nu$. Therefore for the nucleus with $N_\pi=N_\nu$ the $M1$ transition between adjacent states belonging to the $K=1$ band is forbidden.

The reduced $E2$ transition probabilities for $L=$odd are obtained as

$$B(E2;mL+1\to mL)=\frac{3N(2N-L-1)[e_A(2N+L+2)-e_S(L+4)]^2}{4(L+1)(2L+3)(N-1)^2}, \quad (20a)$$

$$B(E2;mL+2\to mL)=\frac{3L(L+3)(2N+L+2)(2N-L-1)[e_A N-e_S]^2}{4(2L+3)(2L+5)(N-1)^2}, \quad (20b)$$

and for $L=$even as

$$B(E2;mL+1\to mL)=\frac{3N(2N+L+1)[e_A(2N-L)+e_S(L-2)]^2}{4(L+1)(2L+3)(N-1)^2}, \quad (21a)$$

$$B(E2;mL+2\to mL)=\frac{3L(L+3)(2N+L+1)(2N-L-2)[e_A(N+1)-2e_S]^2}{4(2L+3)(2L+5)(N-1)^2}. \quad (21b)$$

For the intraband $E2$ transition, the difference of the effective proton and neutron boson charges does not appear in the IBM-2 calculations. When the boson effective charges e_ν and e_π are taken equal, the reduced $E2$ transition probabilities within the $K=1$ band are expressed simply as follows:

$$B(E2;mL+1\to mL)=\frac{3N(2N\mp L\mp 1)}{(L+1)(2L+3)}e^2,$$

$$B(E2;mL+2\to mL)=\frac{3L(L+3)(2N\mp L\mp 1)(2N\pm L\pm 2)}{4(2L+3)(2L+5)}e^2, \quad (22)$$

where $e_\pi=e_\nu=e$ and the value with the upper (lower) sign in the numerator indicates the value for $L=$odd (even). In the limit of large N, the $B(E2)$ strength between states of the $K=1$ band is proportional to N^2 except the geometrical factor and has same order with $B(E2)$ values for the intraband transitions in the fully symmetric states.

The reduced $M1$ and $E2$ transition probabilities within the $K=1$ band can be expressed in the following form:

$$B(R\lambda;mL'\to mL)=\langle L'1,\lambda 0|L1\rangle^2 M_{mm}^2(R\lambda;L,L'), \quad (23)$$

where $R\lambda=E2$ or $M1$. In the geometrical model, $M_{mm}(R\lambda)$ is interpreted as the electromagnetic multipole intrinsic matrix element that is independent of the angular momentum. In the classical limit of the IBM, i.e., for $N\to\infty$, M_{mm} is independent of L and so it is called the intrinsic matrix element. In the classical limit of the present model the ratio of two reduced electromagnetic transition probabilities between states within the $K=1$ band depends only on the ratio of two Clebsch-Gordan coefficients. This result is identical with that from Alaga rule. For large N the $M1$ intrinsic matrix element becomes

$$M_{mm}(M1)=\sqrt{\frac{3}{4\pi}}\,\frac{(g_\pi-g_\nu)(N_\pi-N_\nu)}{N}. \tag{24}$$

The $E2$ intrinsic matrix element within the $K=1$ band is given as $M_{mm}(E2)=\sqrt{2}(e_\pi N_\pi+e_\nu N_\nu)=\sqrt{2}Ne_{\mathrm{A}}$, which is the same with the value of the $E2$ intrinsic matrix element within the ground state band, when the $E2$ operator is defined as Eq. (16a). Therefore as described above the $E2$ intraband transition strengths between states within the ground state band and within the $K=1$ band have the same order in the pure SU(3) limit of the IBM-2.

IV. SUMMARY

In this paper, a method for the calculations of the matrix elements of the angular momentum and the quadrupole operators has been introduced within the framework of the IBM-2 SU(3) limit based on the group theory. In the process of calculating the matrix elements with the group theoretical method, one obtains the generalized Clebsch-Gordan coefficient corresponding to the given group chain. Since the U(6)-reduced one-body matrix elements and $U(6)\supset SU(3)$ isoscalar factors, which are contained in the matrix elements between states belonging to the $K=1$ band, cannot be determined separately, we have derived the extended $U(6)\supset SU(3)$ isoscalar factor, which is defined as the product of the $U(6)\supset SU(3)$ isoscalar factor and the U(6)-reduced matrix element of one-body operator, from the SU(3) intrinsic states of the IBM-2. The matrix elements of the generators of $SU_\rho(3)(\rho=\pi,\ \nu)$, i.e., the ρ-boson angular momentum and quadrupole operator between states of the $K=1$ band, are presented. We have applied these results to the electromagnetic properties of the $K=1$ band; especially the $M1$ and $E2$ intraband transition rates in the $K=1$ band are derived in closed forms and some of the properties are discussed. It has been checked that the $M1$ transitions between states of the $K=1$ band are allowed, even if these states have the identical F spin with $F=F_{\max}-1$ in the pure SU(3) limit.

Although the present results were not compared with the experimental data directly, it is considered that the method and results obtained in this work are useful for studying the electromagnetic properties of mixed-symmetric states within the framework of the IBM-2 SU(3) limit.

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