Fermi-liquid theory of superfluid asymmetric nuclear matter

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Influence of asymmetry on superfluidity of nuclear matter with triplet-singlet pairing of nucleons (in spin and isospin spaces) is considered within the framework of a Fermi-liquid theory. Solutions of self-consistent equations for the critical temperature and the energy gap at T=0 are obtained with the use of Skyrme effective nucleon interaction. It is shown that if the chemical potentials of protons and neutrons are determined in the approximation of ideal Fermi-gas, then the energy gap for some values of density and asymmetry parameter of nuclear matter demonstrates double-valued behavior. However, accounting for the feedback of pairing correlations through the normal distribution functions of nucleons, two-valued behavior of the energy gap turns into universal one-valued behavior. At T=0 superfluidity arises and disappears as a result of a first order phase transition in density.

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It is well established, that neutron-proton (np) pairing plays an essential role in the description of superfluidity of finite nuclei with N=Z (see Refs. [1,2] and references therein) and symmetric nuclear matter [3-5]. In astrophysical context np pairing correlations can be important for the description of *r*-process [6,7] and cooling of neutron stars, which permit pion or kaon condensation [8,9]. In this Rapid Communication we shall investigate the influence of asymmetry on np superfluidity of nuclear matter. Previously this problem was treated with the use of various approaches and potentials of NN interaction. In particular, the cases of ${}^{3}S_{1} - {}^{3}D_{1}$ and ${}^{3}D_{2}$ pairing were considered in Refs. [3,10] on the basis of the Thouless criterion for the thermodynamic Tmatrix. As a potential of NN interaction, the Graz II and Paris potentials were chosen, respectively. Superfluidity in ${}^{3}S_{1} - {}^{3}D_{1}$ pairing channel was studied also in Ref. [11] within BCS theory of superconductivity with the use of the Paris potential in the separable form. Investigations, based on the Thouless criterion, deduce the suppression of np pairing correlations with increase of isospin asymmetry. However, the Thouless criterion can be exploited for finding the critical temperature only, but does not permit one to draw any conclusions about superfluidity with a finite gap. The studies in Ref. [11], based on the BCS theory, were carried out with the use of the bare interaction and the single particle spectrum of a free Fermi-gas and give, thus, overestimated values of the energy gap. The effect of ladder-renormalized single particle spectrum [12] on the magnitude of the energy gap in ${}^{3}S_{1} - {}^{3}D_{1}$ pairing channel was investigated in Ref. [13]. The Argonne V_{14} potential was explored as input for determination of the single particle energy and the bare interaction in the form of the Paris potential was used to evaluate the energy gap. The use of the bare interaction in the gap equation seems to be a very strong simplification, because medium polarization strongly reduces the magnitude of the gap (see Ref. [14] for the influence of the polarization effects on the pairing force in the ${}^{1}S_{0}$ channel). In principle, the effective pairing interaction should be obtained by means of Brueckner renormalization, which gives the correct interaction after modifying the bare interaction for the effect of nuclear medium. However, the issue of a microscopic manybody calculation of the effective pairing potential is a complex one and still is not solved. For this reason, it is quite a natural step to develop some kind of a phenomenological theory, where instead of microscopical calculation of the pairing interaction one exploits the phenomenological effective interaction. We shall investigate the influence of asymmetry on superfluid properties of nuclear matter, using Landau's semiphenomenological theory of a Fermi-liquid (FL). In the Fermi-liquid model the normal and anomalous FL interaction amplitudes are taken into account on an equal footing. This will allow us to consider consistently the influence of the FL amplitudes on superfluid properties of nuclear matter. Besides, as a potential of NN interaction we choose the Skyrme effective forces, describing the interaction of two nucleons in the presence of nucleon medium. The Skyrme forces are widely used in the description of nuclear system properties and, in particular, they were exploited for the description of superfluid properties of finite nuclei [15,16] as well as infinite symmetric nuclear matter [17-20].

The basic formalism is laid out in more detail in Ref. [19], where superfluidity of symmetric nuclear matter was studied. As shown there, superfluidity with triplet-singlet (TS) pairing of nucleons (total spin *S* and isospin *T* of a pair are equal S=1, T=0) is realized near the saturation density in symmetric nuclear matter with the Skyrme interaction. Therefore, we shall study further the influence of asymmetry on superfluid properties of TS phase of nuclear matter. For the states with the projections of total spin and isospin $S_z=T_z=0$ the normal distribution function *f* and the anomalous distribution function *g* have the form

$$f(\mathbf{p}) = f_{00}(\mathbf{p}) \sigma_0 \tau_0 + f_{03}(\mathbf{p}) \sigma_0 \tau_3, \quad g(\mathbf{p}) = g_{30}(\mathbf{p}) \sigma_3 \sigma_2 \tau_2,$$
(1)

where σ_i, τ_k are the Pauli matrices in spin and isospin spaces. The operator of quasiparticle energy ε and the matrix order parameter Δ of the system for the energy functional, being invariant with respect to rotations in spin and isospin spaces, have the analogous structure

$$\varepsilon(\mathbf{p}) = \varepsilon_{00}(\mathbf{p})\sigma_0\tau_0 + \varepsilon_{03}(\mathbf{p})\sigma_0\tau_3, \quad \Delta(\mathbf{p}) = \Delta_{30}(\mathbf{p})\sigma_3\sigma_2\tau_2.$$
(2)

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Using the minimum principle of the thermodynamic potential and procedure of block diagonalization [21], one can express evidently the distribution functions f_{00} , f_{03} , and g_{30} in terms of the quantities ε and Δ :

$$f_{00} = \frac{1}{2} - \frac{\xi_{00}}{4E} \left(\tanh \frac{E + \xi_{03}}{2T} + \tanh \frac{E - \xi_{03}}{2T} \right), \quad (3)$$

$$f_{03} = -\frac{1}{4} \left(\tanh \frac{E + \xi_{03}}{2T} - \tanh \frac{E - \xi_{03}}{2T} \right), \tag{4}$$

$$g_{30} = -\frac{\Delta_{30}}{4E} \left(\tanh \frac{E + \xi_{03}}{2T} + \tanh \frac{E - \xi_{03}}{2T} \right).$$
(5)

Here

$$E = \sqrt{\xi_{00}^2 + \Delta_{30}^2}, \quad \xi_{00} = \varepsilon_{00} - \mu_{00}^0, \quad \xi_{03} = \varepsilon_{03} - \mu_{03}^0$$
$$\mu_{00}^0 = \frac{\mu_p^0 + \mu_n^0}{2}, \quad \mu_{03}^0 = \frac{\mu_p^0 - \mu_n^0}{2},$$

T is temperature, μ_p^0 and μ_n^0 are chemical potentials of protons and neutrons. To obtain the closed system of equations for the quasiparticle energy ε and the energy gap Δ , it is necessary to express the quantities ε, Δ through the distribution functions *f* and *g*. For this purpose one has to set the energy functional $\mathcal{E}(f,g)$ of the system. In the case of asymmetric nuclear matter with TS pairing of nucleons the energy functional is characterized by two normal U_0, U_2 and one anomalous V_1 FL amplitude [19]. Differentiating the functional $\mathcal{E}(f,g)$ with respect to *g* [21] and using Eq. (5), one can obtain the gap equation in the form

$$\Delta_{30}(\mathbf{p}) = -\frac{1}{4\mathcal{V}} \sum_{\mathbf{q}} V_1(\mathbf{p}, \mathbf{q}) \frac{\Delta_{30}(\mathbf{q})}{E(\mathbf{q})} \times \left\{ \tanh \frac{E(\mathbf{q}) + \xi_{03}(\mathbf{q})}{2T} + \tanh \frac{E(\mathbf{q}) - \xi_{03}(\mathbf{q})}{2T} \right\}.$$
(6)

The anomalous interaction amplitude V_1 in the Skyrme model reads [19]

$$V_{1}(\mathbf{p},\mathbf{q}) = t_{0}(1+x_{0}) + \frac{1}{6}t_{3}\varrho^{\beta}(1+x_{3}) + \frac{1}{2\hbar^{2}}t_{1}(1+x_{1})(\mathbf{p}^{2}+\mathbf{q}^{2}),$$
(7)

where ρ is density of nuclear matter, t_i, x_i, β are some phenomenological parameters, which differ for various versions of the Skyrme forces (later we shall use the SkP potential [15]). Equation (6) should be solved jointly with equations

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$$\frac{1}{\mathcal{V}} \sum_{\mathbf{p}} \left\{ 2 - \frac{\xi_{00}(\mathbf{p})}{E(\mathbf{p})} \left(\tanh \frac{E(\mathbf{p}) + \xi_{03}(\mathbf{p})}{2T} + \tanh \frac{E(\mathbf{p}) - \xi_{03}(\mathbf{p})}{2T} \right) \right\} = \varrho, \qquad (8)$$

$$\frac{1}{\mathcal{V}}\sum_{\mathbf{p}}\left\{\tanh\frac{E(\mathbf{p})+\xi_{03}(\mathbf{p})}{2T}-\tanh\frac{E(\mathbf{p})-\xi_{03}(\mathbf{p})}{2T}\right\}=\alpha\varrho,$$
(9)

being the normalization conditions for the normal distribution functions f_{00}, f_{03} . In Eq. (9) the quantity $\alpha = (\varrho_n - \varrho_p)/\varrho$ is the asymmetry parameter of nuclear matter, ϱ_p, ϱ_n are the partial number densities of protons and neutrons. Note that the account of the normal FL amplitudes in the case of the effective Skyrme interaction, being quadratic in momenta, is reduced to the renormalization of free nucleon masses and chemical potentials. Expressions for the quantities ξ_{00}, ξ_{03} , which enter in Eqs. (6),(8),(9), with regard for the explicit form of the amplitudes U_0, U_2 [19] read

$$\xi_{00} = \frac{p^2}{2m_{00}} - \mu_{00}, \quad \xi_{03} = \frac{p^2}{2m_{03}} - \mu_{03},$$

where the effective nucleon mass m_{00} and effective isovector mass m_{03} are defined by the formulas

$$\frac{\hbar^2}{2m_{00}} = \frac{\hbar^2}{2m_{00}^0} + \frac{\varrho}{16} [3t_1 + t_2(5 + 4x_2)],$$
$$\frac{\hbar^2}{2m_{03}} = \frac{\alpha \varrho}{16} [t_1(1 + 2x_1) - t_2(1 + 2x_2)], \quad (10)$$

 m_{00}^0 being the bare mass of a nucleon. The renormalized chemical potentials μ_{00}, μ_{03} should be determined from Eqs. (8), (9) and in the leading approximation on the ratios $T/\varepsilon_F, \Delta/\varepsilon_F$ have the form

$$\mu_{00} = \frac{1}{2} (\mu_p + \mu_n), \quad \mu_{03} = \frac{1}{2} (\mu_p - \mu_n); \quad \mu_{p,n} = \frac{\hbar^2 k_{F_{p,n}}^2}{2m_{p,n}},$$
(11)

where $k_{F_{p,n}} = (3 \pi^2 \rho_{p,n})^{1/3}$, m_p and m_n are the proton and neutron effective masses, defined as

$$\frac{2}{m_{00}} = \frac{1}{m_p} + \frac{1}{m_n}, \quad \frac{2}{m_{03}} = \frac{1}{m_p} - \frac{1}{m_n}$$

The critical temperature of transition to TS superfluid phase is found from Eq. (6), determining the energy gap, in the linear on Δ approximation. Considering, that the interaction amplitude V_1 is not equal to zero only in a narrow layer near the Fermi surface, $|\xi_{00}| \leq \theta$ (we shall set $\theta = 0.1\mu_{00}$) and entering new variables $x = \xi_{00}/\mu_{00}$, $\theta_0 = \theta/\mu_{00}$, T_{μ} $= T/\mu_{00}$, we present this equation in the form

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FIG. 1. Critical temperature as a function of density.

$$1 = \frac{g}{4} \int_{-\theta_0}^{\theta_0} \frac{dx}{x} \left\{ \tanh \frac{x \left[1 + \frac{m_{00}}{m_{03}} \right] + \psi}{2T_{\mu}} + \tanh \frac{x \left[1 - \frac{m_{00}}{m_{03}} \right] - \psi}{2T_{\mu}} \right\},$$
 (12)

$$g = -\nu_F V_1(p = p_F, q = p_F), \quad \psi = \frac{m_{00}}{m_{03}} - \frac{\mu_{03}}{\mu_{00}},$$

$$\nu_F = \frac{m_{00} p_F}{2 \, \pi^2 \hbar^3}, \quad p_F = \sqrt{2 m_{00} \mu_{00}}$$

The results of numerical integration of Eq. (12) are shown in Fig. 1. For small values of asymmetry α there exist such regions of large and low densities of nuclear matter, for which we have two critical temperatures. When α increases, these regions begin to approach and at some value α $= \alpha_c \ (\alpha_c \approx 0.071)$ it takes place contiguous to the regions, so that we have always only two critical temperatures (for the regions, where solutions exist). For $\alpha > \alpha_c$ the phase curves are separated from the density axis and turn into the closed oval curves. Under further increase of α the dimension of the oval curves is reduced and at some $\alpha = \alpha_m$ the oval curves contract to a point ($\alpha_m \approx 0.092$). For the values $\alpha > \alpha_m$ the triplet-singlet superfluidity fails. Note, that our results concerning two-valued behavior of the critical temperature qualitatively agree with the results of Ref. [10], where ${}^{3}D_{2}$ pairing of nucleons with the Paris NN potential was considered.

As follows from Eq. (6), an equation determining the energy gap at T=0 has the form

$$\Delta_{30}(\mathbf{p}) = -\frac{1}{4\mathcal{V}} \sum_{\mathbf{q}} V_1(\mathbf{p}, \mathbf{q}) \frac{\Delta_{30}(\mathbf{q})}{E(\mathbf{q})} \{ \operatorname{sgn}(E(\mathbf{q}) + \xi_{03}(\mathbf{q})) + \operatorname{sgn}(E(\mathbf{q}) - \xi_{03}(\mathbf{q})) \}.$$
(13)



FIG. 2. Energy gap as a function of density in uncoupled calculations.

Passing in Eq. (13) to integration on a layer, we arrive at the equation for determining the dimensionless gap $y = \Delta_{30}/\mu_{00}$:

$$1 = \frac{g}{4} \int_{-\theta_0}^{\theta_0} \frac{dx}{\sqrt{x^2 + y^2}} \left\{ 1 + \operatorname{sgn}\left(\sqrt{x^2 + y^2} - \frac{m_{00}}{m_{03}}x - \psi\right) \right\}$$
(14)

(for $\alpha > 0$ it holds true $m_{03} > 0$, $\mu_{03} < 0$). The contribution to the integral gives the domain on *x*, for which the function, standing as an argument of the function sgn, is positive. In particular, such values of the gap, density, and asymmetry parameter of nuclear matter are possible, that this function has no roots with respect to *x*. In this case the whole domain from $-\theta_0$ to θ_0 gives the contribution to the integral in Eq. (14) and we arrive at the equation of the BCS type at T=0with the solution $y = \theta_0/(\sinh(1/g))$.

Let us first find the solutions of Eq. (14) in the case when the chemical potentials μ_{00}, μ_{03} are given in the main approximation on Δ/ε_F . The results of numerical integration of Eq. (14) are presented in Fig. 2. In the case of symmetric nuclear matter ($\alpha = 0$) we obtain the phase curve with onevalued behavior of the gap. For small values of asymmetry α there exist such regions of large and low densities of nuclear matter (excluding some vicinity of the point $\rho = 0$), for which we have two values of the energy gap, where one of these values is the solution of the BCS type and practically coincides with the corresponding value of the gap for the case $\alpha = 0$. For α less than some α_c , the energy gap demonstrates double-valued behavior in the intervals $(\varrho_{min}, \varrho'_{min})$ and $(\varrho'_{max}, \varrho_{max})$, while in the interval $(\varrho'_{min}, \varrho'_{max})$ it has one-valued behavior (see Table I for the values of various boundary points ϱ). When α increases, the regions with double-valued behavior of the gap begin to approach and at $\alpha = \alpha_c$ [the same α_c as for the phase curves $T_c(\varrho)$ it takes place contiguity of the regions with two solutions. For $\alpha > \alpha_c$ two branches of the phase curves are separated from the density axis and combine to one curve, beginning and ending in some points of the phase curve with

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TABLE I. The values of the boundary points (in fm^{-3}), determining the intervals of double- and one-valued behavior of the energy gap in uncoupled calculations.

| α | ϱ_{min} | ϱ'_{min} | ϱ'_{max} | ϱ_{max} |
|-------|-----------------|------------------|------------------|-----------------|
| 0.06 | 0.0004 | 0.0045 | 0.070 | 0.121 |
| 0.07 | 0.0008 | 0.016 | 0.035 | 0.116 |
| 0.072 | 0.0017 | _ | _ | 0.113 |
| 0.09 | 0.0018 | _ | _ | 0.102 |
| 0.14 | 0.0053 | - | - | 0.068 |

 $\alpha = 0$. In this case the energy gap differs from zero only in the interval $(\varrho_{min}, \varrho_{max})$, where it has double-valued behavior. When α increases further, the boundary points of the phase curves move towards and at some $\alpha = \alpha'_m$ [not equal to α_m for the phase curves $T_c(\varrho)$] the branches of solutions contract to a point. The value α'_m determines the maximum value of the asymmetry parameter, when TS superfluidity exists at T=0 ($\alpha'_m \approx 0.179$).

Let us now find the solutions of Eq. (14) while accounting for the influence of the finite size of the gap on the chemical potentials μ_{00}, μ_{03} . The results of integration of Eq. (14) in this case are presented in Fig. 3. Here the solutions of Eqs. (14), (8), and (9), obtained for different α , correspond to the different parts of the dome-shaped curve, contained by the dashed lines of the same type for a given α . One can see that taking into account the feedback of the finite size of the gap through the normal distribution functions f_{00} , f_{03} in Eqs. (3), (4) leads to the qualitative change: instead of two-valued behavior of the gap we have universal one-valued behavior. The first solution of the BCS type, obtained in uncoupled calculation, remains practically unchanged in self-consistent treatment of the gap equation (14) and with sufficiently high accuracy equals to its value at $\alpha = 0$ in the self-consistent determination. The second solution in uncoupled scheme, to which corresponds the smaller gap width, under simultaneous iterations of Eqs. (8), (9), (14) tends to the first solution of the BCS type. Taking into account the finiteness of the gap results in the reduction of the threshold asymmetry, at which superfluidity disappears, to the value $\alpha'_m \approx 0.029$. Thus, in spite of the smallness of the ratio Δ/ε_F (Δ/ε_F ≤ 0.12 for all densities ρ), the backward influence of pairing correlations is significant. This is explained by the fact that if the quantity Δ in Eqs. (8), (9) differs from zero, then the absolute value of the chemical potential μ_{03} increases a few times as against its value at $\Delta = 0$. The increase of $|\mu_{03}|$ is equivalent to the increase of the effective shift between neutron and proton Fermi surfaces, that leads to significant reduction of the threshold asymmetry. As at $\alpha \neq 0$ the gap is finite everywhere, superfluidity arises and disappears under changing density by means of a first order phase transition. In principle, this phase transition can be observed in laboratory conditions under the study of intermediate-energy heavy ion reactions. If we assume that the final stage of the reaction can be described as an expansion of a compound nucleon system [4], formed in a heavy ion collision, then under lowering density this disassembling phase can undergo a first

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FIG. 3. Energy gap as a function of density in a self-consistent scheme.

order phase transition in density from the normal to superfluid state.

The temperature dependence of the energy gap was studied in Refs. [11,13] with the use of the bare interaction in the gap equation in the form of the Paris and Argonne V_{14} potentials, respectively. Our results agree qualitatively with theirs at T=0. However, in our calculations with the effective density-dependent *NN* interaction we obtain the gap as a function of density (not at fixed density) and this allows us to find new important features in behavior of the energy gap.

In summary, we studied TS superfluidity of asymmetric nuclear matter in the FL model with density-dependent Skyrme effective interaction (SkP force). In the FL approach the normal and anomalous FL amplitudes are taken into account on an equal footing and this allows us to consider consistently within the framework of a phenomenological theory the influence of medium effects on superfluid properties of nuclear matter. It is shown that for some values of density and asymmetry parameter of nuclear matter the critical temperature of a second order phase transition demonstrates double-valued behavior, that agrees with the results of previous studies. In the case when the chemical potentials μ_{00}, μ_{03} (half of a sum and half of a difference of the proton and neutron chemical potentials, respectively) are determined in the approximation of ideal Fermi-gas, the energy gap demonstrates for some values of density and asymmetry parameter the double-valued behavior. If we consider the feedback of pairing correlations through the dependence of the normal distribution functions of nucleons from the energy gap, then the energy gap drastically changes its behavior from two-valued to universal one-valued character. In spite of relative smallness of the ratio Δ/ε_F , taking into account the finite size of the gap in chemical potentials leads to the significant increase of absolute value of μ_{03} and, hence, to considerable reduction of the threshold asymmetry, at which superfluidity at T=0 disappears. In self-consistent determination the energy gap at T=0 as a function of density has a finite width and normal-to-superfluid and

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superfluid-to-normal phase transitions should appear as a first order phase transition in density. Among the other problems we note here the study of multigap superfluidity [20] in asymmetric nuclear matter.

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- [1] A. L. Goodman, Phys. Rev. C 60, 014311 (1999).
- [2] G. Röpke, A. Schnell, P. Schuck, and U. Lombardo, Phys. Rev. C 61, 024306 (2000).
- [3] Th. Alm, B. L. Friman, G. Röpke, and H. Schulz, Nucl. Phys. A551, 45 (1993).
- [4] M. Baldo, U. Lombardo, and P. Schuck, Phys. Rev. C 52, 975 (1995).
- [5] E. Garrido, P. Sarriguren, E. Moya de Guerra, and P. Schuck, Phys. Rev. C 60, 064312 (1999).
- [6] B. K.-L. Kratz, J.-P. Bitouzet, F.-K. Thielmann, and B. Pfeiffer, Astrophys. J. 403, 216 (1993).
- [7] B. Chen, J. Dobaczewski, K.-L. Kratz, K. Langanke, B. Pfeiffer, F.-K. Thielmann, and P. Vogel, Phys. Lett. B 355, 37 (1995).
- [8] B. E. Brown and H. A. Bethe, Astrophys. J. 423, 659 (1994).
- [9] S. Tsuruta, Phys. Rep. 292, 1 (1998).
- [10] Th. Alm, G. Röpke, A. Sedrakian, and F. Weber, Nucl. Phys. A604, 491 (1996).
- [11] A. Sedrakian, T. Alm, and U. Lombardo, Phys. Rev. C 55, R582 (1997).
- [12] W. Zuo, I. Bombaci, and U. Lombardo, Phys. Rev. C 60,

024605 (1999).

- [13] A. Sedrakian and U. Lombardo, Phys. Rev. Lett. 84, 602 (2000).
- [14] H.-J. Schulze, J. Cugnon, A. Lejeune, M. Baldo, and U. Lombardo, Phys. Lett. B 375, 1 (1996).
- [15] J. Dobaczewski, H. Flocard, and J. Treiner, Nucl. Phys. A422, 103 (1984).
- [16] P.-G. Reinhard, D. J. Dean, W. Nazarewicz, J. Dobaczewski, J. A. Maruhn, and M. R. Strayer, Phys. Rev. C 60, 014316 (1999).
- [17] R. K. Su, S. D. Yang, and T. T. S. Kuo, Phys. Rev. C 35, 1539 (1987).
- [18] M. F. Jiang and T. T. S. Kuo, Nucl. Phys. A481, 294 (1988).
- [19] A. I. Akhiezer, A. A. Isayev, S. V. Peletminsky, A. P. Rekalo, and A. A. Yatsenko, Zh. Éksp. Teor. Fiz. **112**, 3 (1997) [Sov. Phys. JETP **85**, 1 (1997)].
- [20] A. I. Akhiezer, A. A. Isayev, S. V. Peletminsky, and A. A. Yatsenko, Phys. Lett. B 451, 430 (1999); J. Low Temp. Phys. 119, 299 (2000).
- [21] A. I. Akhiezer, V. V. Krasil'nikov, S. V. Peletminsky, and A. A. Yatsenko, Phys. Rep. 245, 1 (1994).