

Nonexistence of a ηNN quasibound state

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We have solved the Faddeev equations for ηd elastic scattering using realistic separable interactions for the NN and coupled ηN - πN subsystems. We found that including explicitly the pion channel in the integral equations drastically reduces the attraction that is present in the system. As a consequence, the existence of a ηNN quasibound state is excluded by the modern ηN amplitude analysis.

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The possible existence of a ηNN quasibound state in the ηd system was first suggested by Ueda [1]. He solved the Faddeev equations of elastic ηd scattering using separable interactions for the NN and coupled ηN - πN subsystems. At the time of Ueda's prediction, however, very little was known about the ηN channel, so that he fitted his ηN and πN interactions basically to the πN data only. He found that his model predicted the existence of a ηNN quasibound state very near threshold with a mass of 2430 MeV and a width of 10–20 MeV.

More recent calculations [2–5] have used in the case of the coupled ηN - πN subsystem only the ηN subsector by means of a Yamaguchi separable potential with a complex energy-dependent strength. They found that the existence of the quasibound state depended strongly on the value of the real part of the ηN scattering length, such that $\text{Re } a_{\eta N} \approx 1$ fm is required in order for the quasibound state to exist. This value of $\text{Re } a_{\eta N}$ is within the range of values given by modern ηN amplitude analysis [6–8].

However, in a recent paper [9] we have pointed out that a true measure of the attraction or repulsion present in a three-body system can only be obtained by assuming two-body interactions which are real and energy independent. Therefore, in Ref. [9] we constructed separable potential models of the coupled ηN - πN subsystem in which the strength of the potentials is real and energy independent, so that the imaginary part of the ηN scattering length is generated by the coupling to the πN channel. These models were required to fit not only the ηN scattering length but also the ηN amplitude in the vicinity of the S_{11} resonance as obtained by the recent ηN data analysis [6–8]. In Ref. [9] we used the diagonal $\eta N \rightarrow \eta N$ part of the full ηN - πN amplitude to calculate ηd elastic scattering in a truncated approximation where the pion was not included explicitly in the integral equations but only implicitly through his contribution in the propagator of the ηN interacting pair. We used for the NN interaction in the 3S_1 channel the so-called PEST separable potential [10] which takes into account the NN repulsion at short distances. We found in Ref. [9] that the truncated model does not give rise to a ηNN quasibound state for any of the models based on modern ηN amplitude analyses. However, two questions that immediately arise are (a) how important is the explicit contribution of the pion? and (b) is it attractive or repulsive? We will answer these two questions in this paper.

In Ref. [9] we constructed six different phenomenological models of the coupled ηN - πN subsystem which were fitted to the amplitudes of recent data analyses [6–8]. All six potentials have separable form

$$\langle p | V_{\eta\eta} | p' \rangle = -g_{\eta}(p)g_{\eta}(p'), \quad (1)$$

$$\langle p | V_{\pi\pi} | p' \rangle = -g_{\pi}(p)g_{\pi}(p'), \quad (2)$$

$$\langle p | V_{\eta\pi} | p' \rangle = \pm g_{\eta}(p)g_{\pi}(p'), \quad (3)$$

with

$$g_{\eta}(p) = \sqrt{\lambda} \frac{A + p^2}{\eta(\alpha_2^2 + p^2)^2}, \quad (4)$$

$$g_{\pi}(p) = \sqrt{\lambda} \frac{1}{\pi(\alpha_{\pi}^2 + p^2)}. \quad (5)$$

The parameters of the six models are given in Table III of Ref. [9]. If one substitutes the potentials (1)–(3) into the Lippmann-Schwinger equation of the coupled ηN - πN subsystem one obtains that the T matrices are of the form

$$\langle p | t_{\eta\eta}(E) | p' \rangle = g_{\eta}(p)\tau_2(E)g_{\eta}(p'), \quad (6)$$

$$\langle p | t_{\pi\pi}(E) | p' \rangle = g_{\pi}(p)\tau_2(E)g_{\pi}(p'), \quad (7)$$

$$\langle p | t_{\eta\pi}(E) | p' \rangle = \pm g_{\eta}(p)\tau_2(E)g_{\pi}(p'), \quad (8)$$

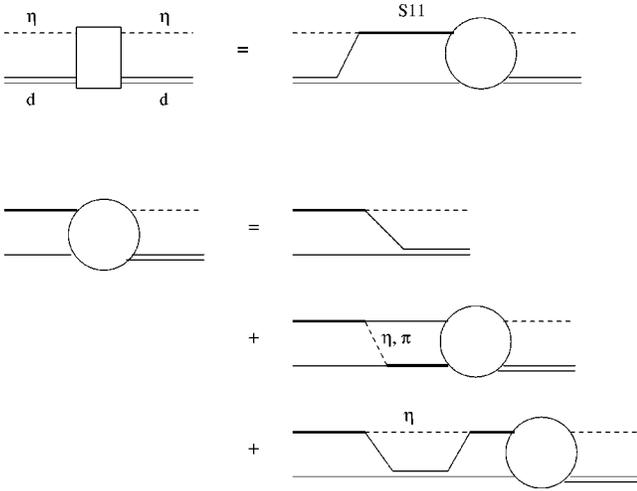
where

$$\tau_2(E) = [-1 - G_{\eta}(E) - G_{\pi}(E)]^{-1}, \quad (9)$$

$$G_{\eta}(E) = \int_0^{\infty} p^2 dp \frac{g_{\eta}^2(p)}{E - p^2/2\mu_2 + i\epsilon}, \quad (10)$$

$$G_{\pi}(E) = \int_0^{\infty} p^2 dp \frac{g_{\pi}^2(p)}{E + p_0^2/2\mu_{\pi} - p^2/2\mu_{\pi} + i\epsilon}. \quad (11)$$

μ_2 and μ_{π} are the ηN and πN reduced masses, respectively, while p_0 is the πN relative momentum at the ηN threshold, i.e.,


 FIG. 1. Faddeev equations for ηd elastic scattering.

$$p_0^2 = \frac{[s_0 - (m_\pi + m_N)^2][s_0 - (m_\pi - m_N)^2]}{4s_0}, \quad (12)$$

with

$$s_0 = (m_\eta + m_N)^2. \quad (13)$$

Thus, the Faddeev equations for ηd elastic scattering take the form diagrammatically depicted in Fig. 1. In the second equation of this figure, there is only a term with a nucleon-nucleon interaction proceeding while a meson (the η) is a spectator, since the term where the spectator meson is a pion involves an intermediate state (formed by a pion and a NN state in the 3S_1 channel) of isospin 1, while the ηd system has isospin 0. Similarly, the intermediate state where a pion is the spectator and the NN state is in the 1S_0 channel can not proceed either due to the fact that this state has total spin 0, while the ηd system has total spin 1.

The integral equation depicted in Fig. 1 has the analytical form

$$T_2(q_2; E) = 2K_{21}(q_2, q_{10}; E) + \int_0^\infty q_2'^2 dq_2' M(q_2, q_2'; E) \times \tau_2(E - q_2'^2/2\nu_2) T_2(q_2'; E), \quad (14)$$

where the kernel $M(q_2, q_2'; E)$ is given by

$$M(q_2, q_2'; E) = K_{23}(q_2, q_2'; E) - K_{23}^\pi(q_2, q_2'; E) + 2 \int_0^\infty q_1^2 dq_1 K_{21}(q_2, q_1; E) \times \tau_1(E - q_1^2/2\nu_1) K_{12}(q_1, q_2'; E). \quad (15)$$

If one drops the term K_{23}^π , Eqs. (14) and (15) are identical to Eqs. (2) and (3) of Ref. [9]. The kernels K_{ij} have been defined in [9] and the new term K_{23}^π is equal to K_{23} except that particle 1 is now a π instead of a η .

We note at this point that the $\eta N \rightarrow \pi N$ transition amplitude, describing a pion exchange followed by an η exchange,

TABLE I. ηd scattering length (in fm) for the six models of the $\eta N - \pi N$ subsystem. The first column indicates the reference of the $\eta N - \pi N$ amplitude analysis on which the model is based, the second column indicates the ηN scattering length (in fm) of that model, the third column gives $A_{\eta d}$ from the impulse approximation, the fourth column gives $A_{\eta d}$ from the full model without pion contribution, and the fifth column gives $A_{\eta d}$ from the full model with pion contribution.

Ref.	$a_{\eta N}$	Impulse	Full (η)	Full ($\eta + \pi$)
[6]	$0.72 + i0.26$	$1.33 + i0.36$	$2.46 + i1.62$	$1.55 + i0.49$
[7]	$0.75 + i0.27$	$1.37 + i0.36$	$2.61 + i1.72$	$1.65 + i0.53$
[8](D)	$0.83 + i0.27$	$1.48 + i0.34$	$3.10 + i2.03$	$1.96 + i0.62$
[8](A)	$0.87 + i0.27$	$1.52 + i0.34$	$3.36 + i2.19$	$2.12 + i0.67$
[8](B)	$1.05 + i0.27$	$1.74 + i0.30$	$4.81 + i3.19$	$3.03 + i0.96$
[8](C)	$1.07 + i0.26$	$1.76 + i0.29$	$5.02 + i3.14$	$3.17 + i0.98$

enters an even number of times at every order of iteration of the integral equation in Fig. 1 [i.e., Eq. (15)]. Therefore, the ambiguity in sign in the $\eta N \rightarrow \pi N$ transition amplitude, explicit in Eqs. (3) and (8), is immaterial for this calculation.

The most important point in Eq. (15) is that K_{23} and K_{23}^π appear with opposite signs. These signs come from the reduction of the Faddeev equations when one has two identical fermions [11,12]. Since we are assuming that the meson is particle 1 so that 2 and 3 are the two fermions and all orbital angular momenta are equal to zero, then following the reduction procedure of Refs. [11,12] leads to the result that the kernel K_{23} must be multiplied by a factor of F_{23} , where

$$F_{23} = F_{23}^{identical} F_{23}^{spin} F_{23}^{isospin}, \quad (16)$$

and

$$F_{23}^{identical} = -(-)^{s_1 + s_3 - S_2 + i_1 + i_3 - I_2}, \quad (17)$$

$$F_{23}^{spin} = (-)^{S_3 + s_3 - S} \sqrt{(2S_2 + 1)(2S_3 + 1)} W(s_3 s_1 S_2; S_2 S_3), \quad (18)$$

$$F_{23}^{isospin} = (-)^{I_3 + i_3 - I} \sqrt{(2I_2 + 1)(2I_3 + 1)} W(i_3 i_1 I_2; I_2 I_3), \quad (19)$$

with W the Racah coefficient, and s_i , S_i , and S (i , I_i , and I) are the spins (isospins) of particle i , of the pair jk , and the three-body system. It is straightforward to see that the factor of F_{23} is equal to 1 when particle 1 is a η but it is equal to -1 when particle 1 is a π . All other spin-isospin recoupling coefficients that would appear in Eqs. (14) and (15) are equal to 1.

In Eq. (14) the propagator $\tau_2(E - q_2'^2/2\nu_2)$ is the one appropriate for a ηN interacting pair since ν_2 is the reduced mass of a nucleon and a ηN pair. In principle, one should have two amplitudes $T_2(q_2; E)$ corresponding to the two possibilities of decay for the S_{11} isobar, into a ηN or a πN pair. However, if one assumes that

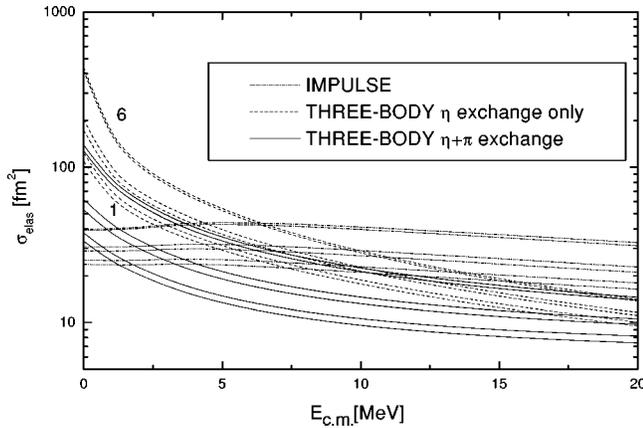


FIG. 2. Integrated ηd elastic cross sections of the three-body model with the pion contribution (solid lines), of the three-body model without the pion contribution (dashed lines), and of the impulse approximation (dot-dashed lines) for the six models of the ηN - πN subsystem, as a function of the c.m. ηd kinetic energy.

$$\tau_2(E - q_2'^2/2\nu_2) = \tau_2(E - q_2'^2/2\nu_\pi), \quad (20)$$

where ν_π is the reduced mass of a nucleon and a πN pair, one obtains a single equation. We have checked numerically that the effect of separating Eq. (14) into two equations, that is of considering

$$\tau_2(E - q_2'^2/2\nu_2) \neq \tau_2(E - q_2'^2/2\nu_\pi), \quad (21)$$

is to produce changes in the ηd scattering length of less than 1%. We should point out that in a relativistic Faddeev theory [13] the energy of the isobar is independent of the mode into which it decays so that the equivalent of Eq. (20) always holds.

We solved the integral equation (14) using the method of contour rotation [14]. We give in Table I the results for the ηd scattering length for the case of the impulse approxima-

tion and for the full calculation with and without the pion contribution. As one sees, the effect of including the pion channel explicitly is quite large and it reduces the ηd scattering length. This reduction is a direct consequence of the minus sign in front of the kernel K_{23}^π representing the pion contribution. The equations for ηd elastic scattering without the pion contribution were not attractive enough to produce a ηNN quasibound state (the signal that a quasibound state exists for a given model is that the real part of $A_{\eta d}$ becomes negative while the imaginary part becomes large), but it turns out that the inclusion of the pion reduces even further the attraction, completely ruling out the existence of a quasibound state in this system. It is worth pointing out that the minus sign for the second term of the right-hand side of equation (15) is critical: if one takes the pion contribution with a plus sign instead of the correct minus sign, the six models of the coupled ηN - πN subsystem will give rise to a quasibound state in the ηd system.

We show in Fig. 2 the results for the cross section of ηd elastic scattering in the region near threshold again for the cases of the impulse approximation and the full calculation with and without the pion contribution. As one sees, the strong enhancement of the cross section near threshold is greatly reduced when the pion contribution is included. Unexpectedly, one reencounters here the pattern of cancellation between the π and η rescattering processes found in Ref. [15], in a one-loop calculation for the $\pi d \rightarrow \eta NN$ reaction.

To conclude, we have shown that the explicit contribution of the pion drastically reduces the amount of attraction that is present in the ηd system, such that there is no possibility for a ηNN quasibound state to exist in this system.

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- [1] T. Ueda, Phys. Rev. Lett. **66**, 297 (1991).
 - [2] S. A. Rakityansky, S. A. Sofianos, M. Braun, V. B. Belyaev, and W. Sandhas, Phys. Rev. C **53**, 2043 (1996).
 - [3] A. M. Green, J. A. Niskanen, and S. Wycech, Phys. Rev. C **54**, 1970 (1996).
 - [4] N. V. Shevchenko, S. A. Rakityansky, S. A. Sofianos, V. B. Belyaev, and W. Sandhas, Phys. Rev. C **58**, R3055 (1998).
 - [5] A. Deloff, Phys. Rev. C **61**, 024004 (2000).
 - [6] M. Batinić, I. Dadić, I. Šlaus, A. Švarc, B. M. K. Nefkens, and T.-S. H. Lee, Phys. Scr. **58**, 15 (1998).
 - [7] A. M. Green and S. Wycech, Phys. Rev. C **55**, R2167 (1997).
 - [8] A. M. Green and S. Wycech, Phys. Rev. C **60**, 035208 (1999).
 - [9] H. Garcilazo and M. T. Peña, Phys. Rev. C **61**, 064010 (2000).
 - [10] H. Zankel, W. Plessas, and J. Haidenbauer, Phys. Rev. C **28**, 538 (1983).
 - [11] I. R. Afnan and A. W. Thomas, Phys. Rev. C **10**, 109 (1974).
 - [12] H. Garcilazo and T. Mizutani, πNN Systems (World Scientific, Singapore, 1990).
 - [13] H. Garcilazo, Phys. Rev. C **35**, 1804 (1987).
 - [14] J. H. Hetherington and L. H. Schick, Phys. Rev. **137**, B935 (1965).
 - [15] H. Garcilazo and M. T. Peña, Phys. Rev. C **62**, 011002(R) (2000).