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Reply to 'Comment on ''Radiative proton-deuteron capture in a gauge invariant relativistic model'' '

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In response to the preceding Comment we clarify the points raised and show that the arguments presented in the Comment are poorly substantiated.

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In the Comment [1] the author mentions several points which he regards as problems. We use this Reply to strengthen the arguments for the approach we followed in Ref. [2].

(i) It is indeed correct that the $pd\tau$ vertex (τ for ^{3}He), we have taken, does not obey an ''orthogonality'' condition (called gauge invariance in Ref. [3]). While for truly elementary (point) particles one may argue that this condition should be obeyed, it is not clear what the proper treatment is for composite particles which are not gauge particles. There does not exist a convincing argument why the coupling to off-shell composite particles may not have a component in the direction of the ''unphysical'' lower-spin component(s). For example, for spin-3/2 particle, in the conventional relativistic description of the Δ resonance [4,5] this orthogonality condition is not satisfied. As an aside, an example where imposing such a condition for a massive spin-1 particle leads to absurd results was pointed out in [4]. If one discarded spin-0 component in the propagator of the W boson, then the matrix element of the weak decay of the pion, $\pi\to l\overline{\nu}_l$ (l stands for electron or muon), would be identically zero.

The general structure of the $pd\tau$ vertex is given in Eqs. (9) and (10) of [2] for the case when at most one particle is off-shell. This form is more general than that in Eq. (1) of [1]. In the calculation we followed Ref. [6] and used the more restricted form in Eq. (11), which depends only on the relative pd momentum but is of course consistent with Lorentz and CPT invariance. This vertex has a more direct relation to a nonrelativistic wave function, written in terms of the S and D components.

The Ward-Takahashi identities (WTI)'s for the off-shell and on-shell electromagnetic vertices of the deuteron are not the same, contrary to what is stated in [1], Sec. (i). For a discussion of the half-off-shell case, which is of interest for pd capture, see Ref. [7] and Ref. [2] (Sec. II A and Appendix A). For the on-shell case the WTI reduces to current conservation. In our paper [2], as well as in the previous Ref. [8], the deuteron vertex is constructed in such a way that the WTI is exactly satisfied. Using a γdd vertex which does not obey [9] the proper WTI may cause problems with gauge invariance.

(ii) In the construction of the contact term (internal radiation amplitude) we followed Ref. [8] as was stated explicitly. This term resembles what is obtained if one follows the minimal substitution procedure proposed by Ohta [10], but differs in the kinematical point at which finite differences are calculated. Other methods also exist, for example, well-known [11] and less known [12] for the two nucleons in the Bethe-Salpeter formalism, [13] for pion photoproduction, new methods are also possible [14]. It was reassuring to know from [1] that the ''minimal photon insertion'' procedure [9], preferred by the author of the Comment, leads to a similar expression. Apparently the derivation has not been published yet.

However, since the claim in section (ii) of [1] is made, we attempted to reproduce Eq. (3) of the Comment. First, it is easy to see that the internal contribution in Eqs. (15) and (16) of [2], applied for real photon (and omitting G_-, H_-) gives the result

$$J^{\mu\nu}_{int}=2Q_3^{\mu}\left(\frac{M_r}{m_2}R_2^{\nu}-\frac{M_r}{m_1}R_1^{\nu}\right)+g^{\mu\nu}\left[\frac{M_r}{m_1}H_+(Q_1^2)-\frac{M_r}{m_2}H_+(Q_2^2)\right]\gamma_5, \qquad (1)$$

which differs from Eq. (3) of the Comment by an interchange of m_1 and m_2 in the first term.

In our notation m_1, m_2, and m_3 is the mass of the proton, deuteron, and ^{3}He, respectively, and M_r is the reduced mass of the pd system. p_1, p_2, and p_3 are the corresponding momenta, $Q_1^2=Q_3^2-2q\cdot Q_3(M_r/m_1)$, $Q_2^2=Q_3^2+2q\cdot Q_3(M_r/m_2)$, Q_3 is the relative pd four-momentum in the initial state, and q is the photon momentum. $R^{\nu}_{1,2}$ are expressed through components G_+, H_+ of the $pd\tau$ vertex (see [2], Sec. II B).

Second, let us consider the vertex in Eq. (11) of our paper [2] $A^{\nu}(p_1,p_2,p_3)=A^{\nu}(Q)=[\gamma^{\nu}G_+(Q^2)-Q^{\nu}H_+(Q^2)]\gamma_5$ which depends on the relative pd momentum $Q=(M_r/m_1)p_1-(M_r/m_2)p_2$ (discarding G_- and H_-). Inserting this vertex in Eq. (2) of [1] and carrying out integration (for $q^2=0$) we obtain the internal amplitude [labeled by a tilde to contrast it with [1] and Eq. (1)]

$$\tilde{J}^{\mu\nu}_{\rm int}=2Q_3^\mu\left(\frac{M_r}{m_2}R_2^\nu-\frac{M_r}{m_1}R_1^\nu\right)+\frac{Q_3^\mu q^\nu}{qQ_3}\left[\frac{M_r}{m_1}H_+(Q_1^2)-\frac{M_r}{m_2}H_+(Q_2^2)\right]\gamma_5+\left(\frac{Q_3^\mu q^\nu}{qQ_3}-g^{\mu\nu}\right)\frac{1}{2qQ_3}\times\left[\int_{Q_3^2}^{Q_1^2}H_+(x)dx+\int_{Q_3^2}^{Q_2^2}H_+(x)dx\right]\gamma_5. \qquad (2)$$

Further simplification is not possible without specifying the explicit form of the function $H_+(Q^2)$. Comparison of this equation with our original real-photon amplitude in Eq. (1) [and with Eq. (3) of the Comment] shows that the claim in section (ii) of [1] is wrong.

We would like to mention that the procedure suggested in [8] applies for virtual photons as well. It was used to calculate electron scattering on ^{3}He [2] and production of e^-e^+ pairs in pd capture [8]. Also for virtual photons the internal amplitude of [8,2] is different from that obtained from Eq. (2) of [1].

The amplitudes in Eq. (1) and Eq. (2) have equal four-divergences, i.e., $q_\mu J^{\mu\nu}_{\rm int}=q_\mu \tilde{J}^{\mu\nu}_{\rm int}$, and thus differ by a term which is gauge invariant by itself. However $J^{\mu\nu}_{\rm int}$ and $\tilde{J}^{\mu\nu}_{\rm int}$ give different contributions to observables. The situation is an example of an ambiguity inherent in methods of deriving the internal radiation amplitude for composite particles.

(iii) The term in the electromagnetic vertex of ^{3}He, proportional to $\kappa_{\rm eff}-\kappa$, is indeed introduced in [2] (Sec. III A) on an *ad hoc* basis (as was stated) with the interesting result that with it a longstanding discrepancy between data and calculation for the cross section could be removed. It is a matter of simple algebra to rewrite this term as a four-point (contact) term which is current conserving by itself. There is thus no question about double counting. Moreover, there is no modification of the anomalous magnetic moment κ of ^{3}He, because, by construction, $\kappa_{\rm eff}$ coincides with κ if the excitation energy is zero.

The caveat in the argument presented in (iii) of [1] is that it is assumed that the relativistic $pd\tau$ vertex, extracted (following a certain procedure) from a nonrelativistic ground-state wave function of ^{3}He, corresponds to a reducible vertex. The latter would include predictions for the structure of the vertex also for off-shell ^{3}He momenta, i.e., for invariant mass W not equal to the bound-state mass m_3. Among other effects, this would be determined by mesonic and isobar components in the off-shell τ leg. It is hard to believe that these processes are completely included in the nonrelativistic wave function. To account for possible physics occurring at finite off-shellness of the ^{3}He we introduced the self-energy (which is equivalent to a contact term) in the approach [2].

(iv) Contrary to what is stated in the Comment our treatment generates an imaginary component to the $pd\tau$ vertex only for an invariant mass of the τ leg above particle breakup ($W\geqslant m_1+m_2$), fully consistent with unitarity conditions. Since a complete inclusion of the initial-state interaction can only be done based on (relativistic) Faddeev equations, we used the approximate method of including a complex phase in the vertex which is similar to what is known as the Watson theorem in pion photoproduction. The phase is directly related to the pd-scattering phase shift, and the inelasticity accounts for the loss of flux to the ppn channel. For energies W below the breakup of ^{3}He the imaginary part vanishes and the vertex is expressed in terms of the real-valued functions. There is no violation of the time-reversal invariance.

(v) As argued in sections (i) and (iii) the $pd\tau$ vertex is not unique. In our paper we have written how the components $G_-(Q^2)$, $H_-(Q^2)$ of the vertex, which are present in general for off-shell proton or ^{3}He, were included. We followed Ref. [6], where arguments were presented for this particular construction. In short, only the positive-energy part of the fermion propagator should remain in the low-energy limit. This natural assumption was thoroughly investigated in [15]. Though the solution suggested in [6,15] is not unique we found it reasonable and used it in the calculation. Note that putting $G_-(Q^2), H_-(Q^2)$ equal to zero (another solution which seems to be favored by the author of [1]) cannot be shown to be theoretically ''better'' in any sense. Actually the effect on observables turns out to be very small anyway, as was demonstrated explicitly in Fig. 2 of [2].

In summary, we have found no reasons to change any conclusions of our previous paper [2]. We also hope that the discussion presented in this Reply may draw attention to some issues in effective (''elementary-particle'') relativistic approaches for electromagnetic processes in the three-nucleon systems, as at present, a fully consistent microscopic description is absent.

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