

ρ - ω mixing in asymmetric nuclear matter via a QCD sum rule approach

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(Received 11 May 2000; published 21 December 2000)

We evaluate the operator product expansion (OPE) for a mixed correlator of the isovector and isoscalar vector currents in the background of the nucleon density with intrinsic isospin asymmetry (i.e., excess of neutrons over protons) and match it with its imaginary part, given by resonances and continuum, via the dispersion relation. The leading density-dependent contribution to ρ - ω mixing is due to the scattering term, which turns out to be larger than any density dependent piece in the OPE. We estimate that the asymmetric density of $n_n - n_p \sim 2.5 \times 10^{-2} \text{ fm}^{-3}$ induces the amplitude of ρ - ω mixing, equal in magnitude to the mixing amplitude in vacuum, with the constructive interference for positive and destructive for negative values of $n_n - n_p$. We revisit sum rules for vector meson masses at finite nucleon density to point out the numerical importance of the screening term in the isoscalar channel, which turns out to be one order of magnitude larger than any density-dependent condensates over the Borel window. This changes the conclusions about the density dependence of m_ω , indicating ~ 40 MeV increase at nuclear saturation density.

DOI: 10.1103/PhysRevC.63.015204

PACS number(s): 24.85.+p, 21.65.+f, 12.38.Lg

I. INTRODUCTION

Changes of hadronic properties in hot and dense nuclear medium are an intriguing issue which ties together modern particle and nuclear physics. The interest in these questions has intensified over the past decade due to the possibility of studying the transition from hadrons to the deconfining phase at heavy ion collisions. In particular, the modification of vector meson properties in nuclear medium has been a subject of a persistent theoretical activity [1]. This was initiated by the idea that in nuclear medium the vector meson masses should drop as a precursor to the chiral symmetry restoration [2]. Several experiments have also been proposed to study the changes of masses, widths, and coupling constants of vector resonances in dense (and/or hot) nuclear matter [3].

The properties of vector resonances in vacuum and the effects of isospin symmetry violation on the mixing of the ρ , ω resonances in vacuum have been investigated rather carefully by means of QCD sum rules in the past [4–6]. In the pioneering work of Ref. [6] it was found that the nonzero value for the ρ - ω mixing can be linked to the difference of light quark masses, and the possibility of $m_u = 0$ is seemingly excluded.

Later, the QCD sum rule method was extended to finite temperatures and densities [7]. A number of analyses [8–11] have found that the masses of ρ and ω resonances decrease in nuclear medium.¹ In Refs. [14,15] finite widths of the vector mesons have been taken into account by hand and by

calculation of the ρ -meson self-energy in a chiral model for the spectral function, respectively. For the ρ -meson channel it was found in Ref. [14] that at nuclear saturation density an increasing width of the ρ -resonance necessitates an increasing ρ -meson mass. However, for large values of the width the mass is blurred over a large window of possible values.

While appreciable efforts have been directed to estimate the density dependent modification of the masses and lifetime of the light vector mesons at finite density (and/or temperature), the question of ρ - ω mixing at finite densities (and/or temperature) has not received much attention. In fact finite nuclear densities can have a significant impact on this amplitude. The fact that nuclear matter can intrinsically be isospin asymmetric implies that the ρ - ω mixing in matter can potentially be larger than the vacuum part of the mixing which is induced by the difference in u and d quark masses, small in units of characteristic hadronic scales. This idea was suggested first in Ref. [16] where it has been pointed out that the presence of asymmetric nuclear matter has a profound effect on the mixing of the ρ and ω resonances. There the mixing angle was determined from the matter induced non-diagonal self-energy of the ρ^0 resonance by employing an $SU(2)_F$ symmetric hadronic model. Subsequently such a matter induced mixing has also been analyzed on a more elaborate footing in Ref. [17]. Along the same lines the author of Ref. [18] investigated the nucleonic density and temperature dependent ρ^0 - ω mixing at a fixed asymmetry. Thereby, an enhancement of the modulus of the vacuum mixing amplitude was found due to finite density. In all model descriptions the vacuum part of the mixing serves as an input parameter, to which all the results are normalized by hand in the limit of vanishing density. To this end, it is desirable to obtain an independent analysis of the mixing using finite density QCD sum rules, which hopefully would allow to treat the vacuum part and the density part from the first principles. At this point we already notice one principal

¹See, however, the work of Koike [12], where opposite behavior is claimed. A later analysis, based on the relation between the current-nucleon forward scattering amplitude and the scattering length of the vector meson off the nucleon in the static limit, again revealed negative mass shifts in the linear density approximation [13].

problem of finite density QCD sum rules. In the presence of nuclear matter there exist nonscalar condensates which can be related to the twists of different dimension. In general, going from mass dimension $2n$ to $2(n+1)$ the ratio of contributions R_{2k}^{2n} with a nonzero, fixed twist $2k$ is

$$R_{2k}^{2n} \propto \frac{A_{2(k+1)}}{A_{2k}} \left(\frac{m_N}{M} \right)^2. \quad (1)$$

This requires the external momenta to be much larger than m_N for the OPE to converge. However, the possibility to link properties of a ground state resonance to nonperturbative effects in the vacuum (the condensates) via the sum rule requires external momenta of $\sim m_N$. We will later show that for the contribution of twist operators there is a numerical suppression in the corresponding Wilson coefficients up to mass dimension six. Since at higher dimensions we have no parametrical smallness the above should limit the applicability of QCD sum rules at finite nucleonic density.

In this paper we study the behavior of the isoscalar-isovector mixed correlator of the two vector currents in order to extract nuclear density effects. The asymptotic behavior of this correlator at large spacelike external momenta can be studied within the perturbative QCD framework, with the power corrections represented by quark, gluon, quark-gluon, etc., condensates. In the presence of finite nucleon density the power correction due to these condensates will change as compared to their vacuum values. Due to the presence of the preferred reference frame, in which the nucleons are at rest, new density-dependent power corrections will appear. In both cases we assume the small density regime and keep only the linear terms in the external nuclear density. As we shall see, this approximation is justified for densities not larger than the nuclear saturation density, which is small in proper ‘‘vacuum’’ units.

The asymptotics of the two-point correlation function, calculated this way, can be related to the ‘‘phenomenological part’’ which includes the contributions of vector resonances, continuum and the screening terms [7]. A success or a failure of the QCD sum rule analysis of vector meson properties would depend on how reliably the contribution of individual resonances (ρ, ω, \dots) can be separated from the rest of the contributions.

We carefully examine the density-dependent part of the operator product expansion (OPE) and find that the effects of matter-induced mixing due to nucleonic matrix elements of nonscalar and scalar QCD operators in asymmetric nuclear matter follow a certain hierarchy. The asymmetric density-induced effects start dominating vacuum contribution at asymmetries $\alpha_{pn} \equiv (n_p - n_n)/(n_p + n_n) \approx 0.2$ and an overall nucleonic density twice the nuclear saturation density $n_N^0 = 0.17 \text{ fm}^{-3} = (111 \text{ MeV})^3$. However, the analysis of the phenomenological part of the QCD sum rules shows that the scattering contribution, usually called screening term, turns out to be numerically by far more important. Brought to the OPE-side of the sum rule, the screening term can be regarded as a mass dimension two power correction. Already at intermediate asymmetries $\alpha_{pn} \approx 0.1$ and saturation density

vacuum and matter induced ρ - ω mixing are of the same sign and comparable in magnitude.

The smallness of the density-dependent pieces in the OPE as compared to the screening term indicates that any conclusion about the density-dependent piece in the ρ - ω mixing amplitude will mostly depend on the assumptions made about the spectral density, i.e., what is usually called the phenomenological part of the sum rules. This casts strong doubts on the applicability of the finite density QCD sum rules for the extraction of the isovector-isoscalar mixing since the density-dependent ‘‘QCD input’’ is negligibly small. This concern led us to reexamine the screening terms in the isovector-isovector and isoscalar-isoscalar correlators which were used in previous works [8–11] to investigate the modification of ρ and ω masses in nuclear matter. We have found that all previous analyses have used the same value for the screening terms in the isovector-isovector and isoscalar-isoscalar correlators. This is an unfortunate error because the screening term in the omega channel turns out to be 9 times larger than the value used in Refs. [8–11]. This changes dramatically all the conclusions about the behavior of m_ω in nuclear matter, and indicates that m_ω is a growing function of density in the linear density approximation.

II. ISOSINGLET-ISOTRIplet CORRELATOR OF THE TWO VECTOR CURRENTS AT FINITE DENSITIES

We start with the (causal) mixed correlator of isotriplet and isosinglet currents in asymmetric nuclear matter

$$\Pi_{\mu\nu} \equiv i \int d^4x e^{iqx} \langle T j_\mu^T(x) j_\nu^S(0) \rangle_{n_N}, \quad (2)$$

where

$$j_\mu^T \equiv \frac{1}{2} (\bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d), \quad j_\mu^S \equiv \frac{1}{2} (\bar{u} \gamma_\mu u + \bar{d} \gamma_\mu d). \quad (3)$$

We choose the same normalization of the two currents, which also means that their couplings to physical ρ and ω resonances are approximately equal. In Eq. (2) the Gibbs average $\langle \rangle_{n_N}$ (n_N indicating finite nucleon density) is approximated by a vacuum and one-particle nucleon states [8–11]. Due to the presence of a singled out rest frame with four-velocity u_μ there are, in general, two independent, current conserving tensor structures (longitudinal and isotropic) into which $\Pi_{\mu\nu}$ can be decomposed. However, in the limit $\vec{q} \rightarrow 0$ one of the corresponding invariants Π_l, Π_i becomes redundant [8], and we therefore concentrate on Π_l which satisfies the following dispersion relation [8]:

$$\Pi_l(Q_0^2 \equiv -q_0^2) \equiv \frac{\Pi_l^\mu}{3Q_0^2} = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} \Pi_l}{s + Q_0^2} + \text{subtractions}. \quad (4)$$

Subtracting the terms attributed to the $\rho \rightarrow \gamma \rightarrow \omega$ electromagnetic mixing from the spectral representation [6] and the

OPE and appealing to the literature on density dependent OPE's of ρ and ω current correlators we arrive at the following sum rule:

$$\Pi'_l(Q_0^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} \tilde{\Pi}_l(s)}{s + Q_0^2} + \text{subtractions.} \quad (5)$$

The asymptotic behavior of the left-hand side of Eq. (5) can be calculated by means of the operator product expansion (OPE). The result is given in terms of the perturbative contribution and power corrections, proportional to the condensates, taken in the presence of the external nucleon density. Retaining terms up to the order Q_0^{-6} , we present the result in the following form:

$$\begin{aligned} \Pi'_l(Q_0^2) = & -\frac{\alpha}{16\pi^3} \frac{1}{4} \ln Q_0^2 + \frac{1}{Q_0^2} \frac{3}{2\pi^2} \frac{m_d^2 - m_u^2}{4} \\ & + \frac{1}{Q_0^4} \left[\frac{m_u}{2} \langle \bar{u}u \rangle_{n_N} - \frac{m_d}{2} \langle \bar{d}d \rangle_{n_N} + \frac{2}{3} \frac{Q_\mu Q_\nu}{Q^2} \langle S(\bar{u} \gamma_\mu \mathcal{D}_\nu u - \bar{d} \gamma_\mu \mathcal{D}_\nu d) \rangle_{n_N} \right] \\ & - \frac{1}{Q_0^6} \left[\frac{\pi \alpha_s}{2} \left\langle (\bar{u} \gamma_\mu \gamma_5 \lambda^a u)^2 - (\bar{d} \gamma_\mu \gamma_5 \lambda^a d)^2 + \frac{2}{9} [(\bar{u} \gamma_\mu \lambda^a u)^2 - (\bar{d} \gamma_\mu \lambda^a d)^2] \right\rangle_{n_N} \right. \\ & \left. + 2\pi\alpha \left\langle \frac{4}{9} (\bar{u} \gamma_\mu \gamma_5 u)^2 - \frac{1}{9} (\bar{d} \gamma_\mu \gamma_5 d)^2 + \frac{2}{9} \left[\frac{4}{9} (\bar{u} \gamma_\mu u)^2 - \frac{1}{9} (\bar{d} \gamma_\mu d)^2 \right] \right\rangle_{n_N} \right. \\ & \left. + \frac{8}{3} \frac{Q_\mu Q_\nu Q_\lambda Q_\sigma}{Q^4} \langle S(\bar{u} \gamma_\mu \mathcal{D}_\nu \mathcal{D}_\lambda \mathcal{D}_\sigma u - \bar{d} \gamma_\mu \mathcal{D}_\nu \mathcal{D}_\lambda \mathcal{D}_\sigma d) \rangle_{n_N} \right]. \quad (6) \end{aligned}$$

In Eq. (6), the symbol S denotes the operation of making tensors symmetric and traceless. As usual (for example, Refs. [8,14,12,15]) the averages over mixed operators and twist four contributions have been omitted in Eq. (6). The former can either be reduced to four quark operators by use of the equation of motion (these contributions are already included), or they are suppressed at $\mu^2 \approx 1 \text{ GeV}^2$ since there the gluon content of the nucleonic wave function is small [8]. The latter has been argued in Ref. [9] to have no substantial effect on the ρ and ω mass shifts, and we will therefore omit twist four operators. Further progress in calculating the OPE depends on how accurately we can predict the size of various contributions to Π' . We restrict ourselves to the case of low and medium densities, so that the linear (mean field) approximation is justified, and the density dependent part enters in the final expression multiplied by the matrix elements over the single nucleon states. We further make use of the vacuum saturation hypothesis [4], which becomes an exact relation in the limit of a large number of colors. This hypothesis is known to “work” reasonably well in vacuum. However, its application to the nucleon matrix elements is not fully justified. We use this hypothesis to estimate the order of magnitude of dim 6 contribution, noting that their numerical weight in the final result turns out to be small as the OPE is largely dominated by dim 4 contributions. With all these assumptions, Eq. (6) can be reduced to the following form:

$$\begin{aligned} \Pi'_l(Q_0^2) = & -\frac{\alpha}{16\pi^3} \frac{1}{12} \ln Q_0^2 + \frac{1}{Q_0^2} \left[\frac{3}{2\pi^2} \frac{m_d^2 - m_u^2}{12} - \right] \\ & + \frac{1}{Q_0^4} \left[\frac{m_u - m_d}{2} \left\{ \langle \bar{q}q \rangle_0(\mu^2) + \frac{\Sigma_{\pi N}(\mu^2)}{2\bar{m}} n_N \right\} - \bar{m} \langle p | \bar{u}u - \bar{d}d | p \rangle_{\vec{k}_p=0} \alpha_{pn} n_N + \frac{1}{2} m_p \alpha_{pn} n_N A_2^{u-d}(\mu^2) \right] \\ & + \frac{1}{Q_0^6} \left[-\frac{112}{81} \pi \left[\alpha_s \langle \bar{q}q \rangle_0^2(\mu^2) \right] \left\{ -\gamma + \frac{\alpha}{8\alpha_s(\mu^2)} \right\} + 2 \langle \bar{q}q \rangle_0 \langle p | \bar{u}u - \bar{d}d | p \rangle_{\vec{k}_p=0} \alpha_{pn} n_N - \frac{5}{12} m_p^3 \alpha_{pn} n_N A_4^{u-d}(\mu^2) \right]. \quad (7) \end{aligned}$$

In Eq. (7) n_N (n_p, n_n) denotes the total nucleonic (proton, neutron) density; α_{pn} the p - n asymmetry, defined as $\alpha_{pn} \equiv (n_p - n_n)/n_N$; m_p the proton mass; $\langle \bar{q}q \rangle_0$ the value of the u -quark condensate; γ the asymmetry of u -quark and d -quark condensate defined as $\gamma \equiv \langle \bar{d}d \rangle_0 / \langle \bar{u}u \rangle_0 - 1$; $\Sigma_{\pi N} = (45 \pm 7) \text{ MeV}$ [9] the nucleon sigma term; and \bar{m} is defined as $\bar{m} \equiv 1/2(m_u + m_d)$. The electromagnetic coupling is α , the strong coupling at scale μ is $\alpha_s(\mu^2)$. In Eq. (7) we have already used

the numerical smallness of isospin and chiral symmetry violating parameters as compared to the normal hadronic scale and thus neglected terms proportional to $m_{u(d)}\gamma$, γ^2 and so on. For similar reasons it is justified to neglect the effects of isospin violation in nucleon matrix elements and take $\langle p|\bar{u}u - \bar{d}d|p\rangle = -\langle n|\bar{u}u - \bar{d}d|n\rangle$, which leads to the dependence on the asymmetry factor α_{pn} . The scalar matrix element $\langle p|\bar{u}u - \bar{d}d|p\rangle$ is related to the baryon octet mass splitting, $(m_\Xi - m_\Sigma)/m_s$, and it is numerically close to 0.7 [19].

As for the contribution of symmetric and traceless twist two quark bilinears [9] of dimension 4 and 6, their nucleonic matrix elements are determined by the quark parton distributions $A_2^{u-d}(\mu^2)$ and $A_4^{u-d}(\mu^2)$ as [8]

$$\begin{aligned} & \langle S\bar{q}\gamma_{\mu_1}D_{\mu_2}q\rangle_{N(k)}(\mu^2) \\ &= -iA_2^{u-d}(\mu^2)\left(k_{\mu_1}k_{\mu_2} - \frac{1}{4}g_{\mu_1\mu_2}k^2\right), \\ & \langle S\bar{q}\gamma_{\mu_1}D_{\mu_2}D_{\mu_3}D_{\mu_4}q\rangle_{N(k)}(\mu^2) \\ &= iA_4^{u-d}(\mu^2)(k_{\mu_1}k_{\mu_2}k_{\mu_3}k_{\mu_4} - \text{traces}), \end{aligned} \quad (8)$$

where k_μ denotes the nucleon momentum, D_μ is the gauge covariant derivative. In general, the factor $A_k^q(\mu^2)$ can be obtained from the parton distributions $Q(x, \mu^2)$ and $\bar{Q}(x, \mu^2)$ in the proton as

$$A_k^q(\mu^2) = 2 \int_0^1 dx x^{k-1} (Q(x, \mu^2) + (-)^k \bar{Q}(x, \mu^2)). \quad (9)$$

In Ref. [21] the parton distributions in the nucleon have been fitted to experiment at a resolution scale $\mu^2 = 0.26 \text{ GeV}^2$. Using these distributions and performing the integrations of Eq. (9), we obtain

$$\begin{aligned} A_2^{u-d}(\mu^2 = 0.26, \text{ GeV}^2) &= 0.429, \\ A_4^{u-d}(\mu^2 = 0.26, \text{ GeV}^2) &= 0.097. \end{aligned} \quad (10)$$

To generate the respective values at the scale $\mu^2 \approx 1 \text{ GeV}^2$ relevant for the sum rule we simply use the conversion factors $f_2(\mu_2^2, \mu_1^2) \equiv A_2^{u+d}(\mu_2^2)/A_2^{u+d}(\mu_1^2)$ and $f_4(\mu_2^2, \mu_1^2) \equiv A_4^{u+d}(\mu_2^2)/A_4^{u+d}(\mu_1^2)$. Using A_2^{u+d} and A_4^{u+d} at 1 GeV from Ref. [9], we arrive at the following values of matrix elements of interest:

$$A_2^{u-d}(1 \text{ GeV}^2) = 0.32, \quad A_4^{u-d}(1 \text{ GeV}^2) = 0.062. \quad (11)$$

Keeping this in mind, we will now proceed to the numerical evaluation of OPE. Performing a Borel transformation of the OPE of Eq. (7) and omitting the numerically strongly suppressed dimension two power correction, we obtain

$$\begin{aligned} \Pi'_l(M^2) &= 3.7 \times 10^{-6} + \frac{|\langle \bar{q}q \rangle|}{M^4} \left(2 \text{ MeV} - 1.5 \text{ MeV} \frac{\alpha_{np}}{0.2} \frac{n_N}{n_N^0} \right) \\ &\quad - \frac{|\langle \bar{q}q \rangle|}{M^4} \frac{0.1 \text{ GeV}^2}{M^2} \\ &\quad \times \left(1.4 \text{ MeV} + \frac{\alpha_{np}}{0.2} \frac{n_N}{n_N^0} [3.8 \text{ MeV} - 2.4 \text{ MeV}] \right). \end{aligned} \quad (12)$$

In this expression we have used the following set of values: $m_u = 5 \text{ MeV}$; $m_d = 9 \text{ MeV}$; $\bar{m} = 7 \text{ MeV}$; $m_p = 940 \text{ MeV}$; $n_N = n_N^0 = (111 \text{ MeV})^3$ (the nuclear matter saturation density); $\Sigma_{\pi N} = 45 \text{ MeV}$; $\langle \bar{q}q \rangle_0 = -(225 \text{ MeV})^3$; $\gamma = -10^{-2}$ [20]; $\alpha = 1/137$; and $\alpha_s(1 \text{ GeV}^2) = 0.5$ [9]. The quark condensate has been factored out numerically for the sigma-term and the twist contributions.

Several important observations should be made at this point. For $M \sim 1 \text{ GeV}$, a pure perturbative contribution is negligibly small as compared to power corrections. The latter are dominated by dimension 4, with the constant term originating from the m_u and m_d mass difference and the $\alpha_{np}n_N$ -dependent piece given by the A_2^{u-d} contribution. At the level of dimension 6 we observe three different terms [second line of Eq. (12)]: vacuum part, density-dependent scalar condensate and twist contributions. At this dimension the density dependent contributions from scalar condensate and twist tend to cancel each other. This cancellation can be an artifact of the chosen parameters and/or of the crude nature of the approximations made in estimating the size of the four-quark matrix elements over the nucleon. Nevertheless, $M \sim 1 \text{ GeV}$, and the OPE is dominated by dim 4 terms, where at $(\alpha_{np}/0.2)(n_N/n_N^0) \sim 1$ the suppression of dim 6 is about 50%. For higher values of asymmetric density $\Pi'(M^2 = 1 \text{ GeV})$ changes sign.

What does this behavior of $\Pi'(M^2 = 1 \text{ GeV})$ mean in terms of the ω - ρ resonance mixing amplitude? To answer this question we should parametrize the spectral function in terms of the resonance contributions and analyze the resulting sum rule (5).

Following Refs. [6,7], we approximate the imaginary part of the correlator by contributions of ρ , ω , ρ' , ω' resonances and the QCD-continuum:

$$\begin{aligned} \frac{1}{\pi} \text{Im} \tilde{\Pi}_l(s, \alpha_{pn}, n_N) &= \frac{1}{4} \left[f_\rho \delta(s - m_\rho^2) - f_\omega \delta(s - m_\omega^2) \right. \\ &\quad \left. + f_{\rho'} \delta(s - m_{\rho'}^2) - f_{\omega'} \delta(s - m_{\omega'}^2) \right. \\ &\quad \left. + \frac{\rho_{sc}^{ST}}{8\pi^2} \delta(s) + \frac{\alpha}{16\pi^3} \theta(s - s_0) \right]. \end{aligned} \quad (13)$$

The contribution to the mixing due to the electromagnetic continuum is small [4]. Therefore, we will neglect it in the subsequent consideration. In Eq. (13) f_ρ and f_ω refer to the ρ

and ω residues of the ρ - ω current propagator, and ρ' and ω' symbolize the cumulative effect of higher resonances² introduced in the original analysis [6] in order to have consistent asymptotic behavior of $\Pi'(M^2)$. In addition to the ‘‘usual’’ annihilation continuum above a certain threshold s_0 , Eq. (13) exhibits a scattering term which behaves as a pole at $s=0$ (Landau pole) [7]. The corresponding coefficient can be calculated explicitly, and the result in the leading order in Fermi momentum (p_f/m_N expansion) is given by

$$\rho_{sc}^{ST} = \frac{2\pi^2}{m_N} [F_p^S F_p^T n_p + F_n^S F_n^T n_n] = \frac{6\pi^2}{m_N} \alpha_{pn} n_N. \quad (14)$$

Here the coefficients $F_{p(n)}^{S(T)}$ are defined via nucleon matrix elements of quark vector currents at vanishing momentum transfer

$$F_p^S = F_n^S = \langle p | \bar{u} \gamma_0 u + \bar{d} \gamma_0 d | p \rangle = 3, \\ F_p^T = -F_n^T = \langle p | \bar{u} \gamma_0 u - \bar{d} \gamma_0 d | p \rangle = 1. \quad (15)$$

After Borel transformation the contribution of the Landau screening term is usually carried to the lhs of the sum rule to effectively become a power correction of dimension 2 in the expansion of $\Pi'_l(M^2)$. Defining $f_{\rho\omega} \equiv 1/2(f_\rho + f_\omega)$, $\bar{m}_r^2 \equiv 1/2(m_\rho^2 + m_\omega^2)$, $\Delta m_r^2 \equiv m_\omega^2 - m_\rho^2$, and $\beta = (f_\omega - f_\rho) \bar{m}_r^2 / (f_{\rho\omega} \Delta m_r^2)$ (the primed quantities are defined analogously), we quote the result of Ref. [6] relating $f_{\rho\omega}$ to the measurable quantities \bar{m}_r^2 , Δm_r^2 , g_ρ , and g_ω

$$f_{\rho\omega} \approx - \frac{12\bar{m}_r^2}{g_\rho g_\omega} \frac{\delta_{\rho\omega}}{\Delta m_r^2} \equiv \frac{m_r^4}{\Delta m_r^2} \xi, \quad (16)$$

where g_ρ , g_ω are the respective decay constants, and $\delta_{\rho\omega}$ enters the measurable mixing parameter ε as follows:

$$\varepsilon = \frac{\delta_{\rho\omega}}{(m_\omega - 1/2i\Gamma_\omega)^2 - (m_\rho - 1/2i\Gamma_\rho)^2}. \quad (17)$$

Thereby, ε is defined as

$$\omega = \omega_0 + \varepsilon \rho_0, \quad \rho = \rho_0 - \varepsilon \omega_0, \quad (18)$$

and Γ denotes the width of the respective resonance. It is fair to remark at this point that the observable combination, $\varepsilon \approx \delta_{\rho\omega} \Gamma_\rho^{-1} m_\rho^{-1}$ will have an additional dependence on density due to a substantial increase of Γ_ρ with n_N [23]. Thus, finding the decrease of ξ with density would certainly allow to conclude that ε is decreasing. The opposite behavior, a rising ξ , would complicate the prediction of $\varepsilon(n_N)$.

The final sum rule is given by the following expression:

²In Ref. [4] the cumulative values m_ρ^2, m_ω^2 were chosen to be about 1.5 GeV², which is well below the physical masses (~ 1.7 GeV) of the resonances ρ', ω' .

$$\frac{1}{4} \xi \frac{\bar{m}_r^2}{M^2} \left(\frac{\bar{m}_r^2}{M^2} - \beta \right) e^{-\bar{m}_r^2/M^2} + (\rho \rightarrow \rho', \omega \rightarrow \omega') \\ = 1.1 \times 10^{-2} \text{ GeV}^{-1} \left\{ \frac{18 \text{ MeV}}{M^2} \frac{\alpha_{np}}{0.2} \frac{n_N}{n_N^0} + \frac{1}{M^4} \right. \\ \times \left(2 \text{ MeV} - 1.5 \text{ MeV} \frac{\alpha_{np}}{0.2} \frac{n_N}{n_N^0} \right) - \frac{0.1}{M^6} \\ \left. \times \left(1.4 \text{ MeV} + \frac{\alpha_{np}}{0.2} \frac{n_N}{n_N^0} [3.8 \text{ MeV} - 2.4 \text{ MeV}] \right) \right\}, \quad (19)$$

where all masses and the Borel parameter are taken in units of GeV. It is remarkable that the screening term, brought to the OPE side of this sum rule, completely dominates other density dependent contributions. This shows that in the asymmetric nuclear matter background the influence of the screening term on the ω - ρ mixing is by far more important than any changes of the QCD condensates. Moreover, for any realistic M^2 the screening term becomes comparable to the vacuum contribution to the mixing at $n_N \approx n_N^0$ and asymmetries as low as $\alpha_{np} \sim 0.05$.

In the limit of vanishing density, relation (19) reduces to the known sum rule for ρ - ω mixing. A naive evaluation of this sum rule at ρ -meson mass $M^2 = (0.77)^2$, and at $n_N = 0$, gives a reasonable agreement with experimentally measured value $\xi = 1.1 \times 10^{-3}$ [22] with $\beta \approx 0.5$, advocated in Ref. [6]. Next, we parametrize the linear dependence of ξ and β on the density as follows:

$$\xi = \xi^{(0)} + \xi^{(1)} \tilde{n}; \quad \beta = \beta^{(0)} + \beta^{(1)} \tilde{n}, \quad (20)$$

where \tilde{n} denotes $\alpha_{np} n_N$ in units of $0.2 n_N^0$.

The primary reason for the introduction of the $\rho' - \omega'$ contribution in Ref. [6] was the absence of the $1/M^2$ term in the OPE side of the sum rule, so that ρ and ω contribution alone would not be consistent with the asymptotic behavior of Π' . Thus, the role of $\rho' - \omega'$ is to imitate the cancellation of $1/M^2$ terms in contributions of various resonances at large M^2 . For a semiquantitative determination of the linear density dependence of ξ and β we proceed as in Ref. [4]. There the vacuum values of ξ and β were estimated by choosing $M = m_\rho$, which strongly suppresses the higher resonances. It should then be legitimate to compare powers of M^{-2} in the OPE and the lowest resonance contribution. The result is given by

$$\frac{\xi^{(1)}}{\xi^{(0)}} + \frac{\beta^{(1)}}{\beta^{(0)}} = -2.0 \times 10^{-4} \frac{4}{\xi^{(0)} \beta^{(0)} \bar{m}_r^2}. \quad (21)$$

Using this relation, we can find $\beta^{(1)}$ and $\xi^{(1)}$ separately, evaluating (19) at $M^2 = 0.59$. The final estimate of $\xi^{(1)}$ reads

$$\xi^{(1)} \approx [2.3 - 0.8] \times 10^{-3} = 1.5 \times 10^{-3}, \quad (22)$$

where 2.3 originates from the screening term and -0.8 comes from the OPE. A similar number can be obtained from the combination of Eq. (19) and its first derivative in M^2 . This value of $\xi^{(1)}$ leads to the doubling of mixing amplitude and complete screening at $n_n - n_p \sim \pm 2.5 \times 10^{-2} \text{ fm}^{-3}$, respectively.

III. IMPORTANCE OF THE SCREENING TERM FOR THE ISOSCALAR-ISOSCALAR CORRELATOR

Having found such an important role of the screening term in the isoscalar-isovector mixed correlator, we would like to return to previous analyses of diagonal correlators (isovector-isovector and isoscalar isoscalar) which were used to extract the behavior of m_ρ and m_ω at finite nucleon density [8–11]. In all these papers it was found that masses and coupling constants of ρ and ω resonances behave similarly in nuclear matter, simply because the OPE sides of the sum rules in both cases are the same after the application of the vacuum saturation hypotheses.

We use the same symmetric normalization of the two currents, Eq. (3). From now on we neglect the asymmetry of the nuclear matter and other isospin breaking effects. Then the sum rules for isovector-isovector and isoscalar-isoscalar correlators in medium take the following symbolic form:

$$\frac{1}{M^2} F_\rho^* e^{-m_\rho^{*2}/M^2} = \frac{1}{8\pi^2} (1 - e^{-S_\rho^*/M^2}) - \frac{1}{4} \frac{n_N}{m_N M^2} + \frac{c_4}{M^4} + \frac{c_6}{2M^6}, \quad (23)$$

$$\frac{1}{M^2} F_\omega^* e^{-m_\omega^{*2}/M^2} = \frac{1}{8\pi^2} (1 - e^{-S_\omega^*/M^2}) - \frac{9}{4} \frac{n_N}{m_N M^2} + \frac{c_4}{M^4} + \frac{c_6}{2M^6}, \quad (24)$$

where c_4 and c_6 are the same for both expressions. Obviously, at vanishing nucleon density $F_\omega \simeq F_\rho$ and $S_\omega \simeq S_\rho$.

It is remarkable that the screening terms in Eqs. (23), (24) are different by a factor of 9. The enhancement of the screening term in the isoscalar-isoscalar channel is due to

$$\frac{\rho_{Sc}^{SS}}{\rho_{Sc}^{TT}} = \frac{F_p^S F_p^S}{F_p^T F_p^T} = 9. \quad (25)$$

This difference was overlooked in Refs. [8,9,11].³

The coefficients c_2 and c_4 can be computed along the same standard technique (again with considerable degree of

³In Ref. [11] ρ_{sc} was taken as a free search parameter and determined from the sum rules at the level consistent with ρ_{Sc}^{TT} for both correlators. It casts a strong doubt on the validity of the whole approach, since the actual value of the screening term for ω should be 9 times larger.

uncertainty for c_6). When plugging these values into the sum rules (23), (24), we obtain the following numerical relations:

$$\frac{1}{M^2} F_\rho^* e^{-m_\rho^{*2}/M^2} = \frac{1}{8\pi^2} (1 - e^{-S_\rho^*/M^2}) - \frac{3.4 \times 10^{-4}}{M^2} \frac{n_N}{n_N^0} + \frac{10^{-4}}{M^4} \left[4.1 + 3.8 \frac{n_N}{n_N^0} \right] + \frac{10^{-4}}{2M^6} \left[-2.8 + 1.2 \frac{n_N}{n_N^0} \right], \quad (26)$$

$$\frac{1}{M^2} F_\omega^* e^{-m_\omega^{*2}/M^2} = \frac{1}{8\pi^2} (1 - e^{-S_\omega^*/M^2}) - \frac{31 \times 10^{-4}}{M^2} \frac{n_N}{n_N^0} + \frac{10^{-4}}{M^4} \left[4.1 + 3.8 \frac{n_N}{n_N^0} \right] + \frac{10^{-4}}{2M^6} \left[-2.8 + 1.2 \frac{n_N}{n_N^0} \right], \quad (27)$$

where again all masses and dimensional coupling constants are taken in GeV units. It is remarkable that at $M \sim 1$ GeV the screening term in the ω sum rule is larger by an order of magnitude than any other density-dependent term from the OPE.

As in the previous case, it is convenient to parametrize the density dependence of masses and coupling constants as follows:

$$m = m^{(0)} \left(1 + \frac{m^{(1)}}{m^{(0)}} \frac{n_N}{n_N^0} \right); \quad F = F^{(0)} \left(1 + \frac{F^{(1)}}{F^{(0)}} \frac{n_N}{n_N^0} \right); \quad S_0 = S_0^{(0)} \left(1 + \frac{S_0^{(1)}}{S_0^{(0)}} \frac{n_N}{n_N^0} \right). \quad (28)$$

Using the sum rules (26) and (27), and the first derivatives of these expressions, we solve for m^{*2} as a function of S_0^* , and Borel parameter M . The dependence of the threshold on the density is obtained by requiring the Borel curves $m^*(M^2, S^*, n)$ be parallel over the Borel window which we take from 0.6 to 1.2 GeV for different values of densities. The slope of the Borel curve $m(M^2)$ in the Borel window at zero density represents a ‘‘systematic uncertainty’’ introduced by sum rules and the requirement of the Borel curves to be parallel at different densities is equivalent to the requirement that this uncertainty does not change while going to finite but small densities. The resulting dependence of S_0 on the density, $S_0^{(1)}/S_0^{(0)} = -0.2$ for ρ and -0.1 for ω , allows us to deduce the following estimates for the linear dependence of masses on the density:

$$\frac{m_\rho^{(1)}}{m_\rho^{(0)}} \sim -0.15, \quad \frac{m_\omega^{(1)}}{m_\omega^{(0)}} \sim 0.05. \quad (29)$$

Our estimate for $m_\rho^{(1)}$ agrees with the results of previous analyses [8,9,11]. The result for $m_\omega^{(1)}$ has the opposite sign and correspond to an 40 MeV increase of m_ω at the nuclear saturation density. This difference could be easily explained by the error in the screening term for ω sum rule in Refs. [8,9,11]. The disagreement with the results of Ref. [15], where the correct form of the screening terms is used, is harder to explain, and we hypothesize that it could be an artifact of different numerical methods used to extract the density dependence of the resonance masses and thresholds.

IV. DISCUSSION

Apart from the question of (non)convergence of the OPE we would like to point out some concerns about usefulness and validity of the sum rules at finite densities.

(1) *Vacuum factorization at dim 6*. It is unclear what the status of factorization procedure is, especially in the presence of nuclear matter. In principle, one could try to relate four-fermion matrix elements over the nucleon states, which appear in the calculation of the OPE, to some measured processes induced by weak interactions. Indeed, nonleptonic hyperon decays and parity violating pion-nucleon coupling constants could be reduced to similar matrix elements from the four-quark operators. It is unclear, though, whether such an analysis is feasible.

(2) *The importance of a particular choice of the spectral function*. In linear density approximation the analysis of the examples of the ρ - ω and ω - ω sum rules suggest that there are large contributions from the respective screening terms. In fact, these contributions dominate all density dependent pieces in the OPE. It means that the ‘‘QCD input’’ in these channels is not important in comparison with the choice of the spectral function at finite density.

(3) *Is the linear density approximation valid up to n_0 and beyond?* The use of the dilute Fermi gas to model the behavior of the scattering terms and the QCD condensates has its limitations, and a more realistic description may greatly affect the resulting sum rule. However, it seems unfeasible to

calculate QCD operator averages over interacting multi-nucleon states which one would have to consider when going beyond the dilute gas approximation. An inclusion of Fermi motion of noninteracting nucleons is practically doable but does not alter the zero momentum result significantly.

In conclusion, we have considered the correlator of isovector and isosinglet vector currents in the presence of the asymmetric nuclear matter in linear density approximation. We see a significant dependence of the OPE on $n_n - n_p$, which becomes comparable to vacuum contributions at $n_n - n_p \sim 0.05n_N^0$. An attempt to extract the ρ - ω mixing, using the dispersion relation has shown that this mixing is more affected by the presence of the scattering term than by density-dependent part of the OPE. A similar tendency exists in the isosinglet-isosinglet channel, which is normally used to deduce the dependence of m_ω on density. Hence, in linear density approximation the explicitly density dependent part of the *spectral functions* (scattering terms) in the ρ - ω and ω - ω channels dominantly drive the density dependence of hadronic parameters. The density dependence of the width of the resonances, which has been neglected here, does not alter this finding. However, it may drastically change the conclusions about the direction of resonance mixing at finite, asymmetric density.

ACKNOWLEDGMENTS

R.H. and M.P. would like to thank V. Eletsky and A. Vainshtein for sharing their skepticism about the relevance of QCD sum rules at finite density. M.P. is grateful to the McGill University nuclear theory group, where this work was started, for the hospitality extended to him during his visit. The work of R.H. was funded by Deutscher Akademischer Austauschdienst (DAAD). The work of A.K.D.M. has been supported by the Natural Sciences and Engineering Research Council of Canada and the Fonds FCAR of the Québec Government. The work of M.P. was supported in part by the Department of Energy under Grant No. DE-FG02-94ER40823.

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