Intrinsic quadrupole moment of the nucleon

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We address the question of the intrinsic quadrupole moment Q_0 of the nucleon in various models. All models give a positive intrinsic quadrupole moment for the proton. This corresponds to a prolate deformation. We also calculate the intrinsic quadrupole moment of the $\Delta(1232)$. All our models lead to a negative intrinsic quadrupole moment of the Δ^+ corresponding to an oblate deformation.

I. INTRODUCTION

Electron-proton scattering and atomic Lamb shift measurements have shown that the spatial extension of the proton charge distribution (charge radius) is about $r_p \approx 0.9$ fm [1]. In addition to the charge radius, the elastic electron scattering data provide precise information on the radial charge density $\rho(r)$ of the proton. However, they do not allow us to draw any definite conclusions concerning possible deviations of the proton's shape from spherical symmetry.

In order to learn something about the shape of a spatially extended particle one has to determine its *intrinsic* quadrupole moment [2],

$$Q_0 = \int d^3r \rho(\mathbf{r})(3z^2 - r^2), \tag{1}$$

which is defined with respect to the body-fixed frame. If the charge density is concentrated along the z direction (symmetry axis of the particle), the term proportional to $3z^2$ dominates, Q_0 is positive, and the particle is prolate (cigar shaped). If the charge density is concentrated in the equatorial plane perpendicular to z, the term proportional to r^2 prevails, Q_0 is negative, and the particle is oblate (pancake shaped). The intrinsic quadrupole moment Q_0 must be distinguished from the *spectroscopic* quadrupole moment Q measured in the laboratory frame. Due to angular momentum selection rules, a spin $J=1/2$ nucleus, such as the nucleon, does not have a spectroscopic quadrupole moment; however, it may have an *intrinsic* quadrupole moment as was realized more than 50 years ago [2]. Some information on the shape of the nucleon or any other member of the baryon octet can be obtained by electromagnetically exciting the baryon to spin $J=3/2$ or higher spin states.

With the Laser Electron Gamma Source at Brookhaven and various continuous electron beam accelerators, one can carry out high precision pion production experiments on the nucleon. In these experiments a photon (real or virtual) excites the $N(939)$ to, for example, the $\Delta(1232)$ resonance with spin $J=3/2$. Magnetic dipole ($M1$), electric quadrupole ($E2$), and/or charge quadrupole ($C2$) excitation modes are allowed by angular momentum conservation and invariance under the parity transformation. Once produced, the $\Delta(1232)$ hadronically decays to a nucleon and pion. By measuring the momenta of the final state nucleon and pion in coincidence, individual electromagnetic multipoles can be extracted.

The $E2$ and $C2$ multipoles carry the information about the intrinsic deformation of the nucleon. If the charge distribution of the initial and final three-quark states were spherically symmetric, the $E2$ and $C2$ amplitudes would be zero (Becchi-Morpurgo selection rule [3]). The experimental values for these quadrupole amplitudes are small compared to the dominant magnetic dipole transition, but they are clearly nonzero. Recent data [4–6] indicate that the ratio of the electric quadrupole amplitude to the magnetic dipole amplitude is at least $E2/M1 \approx -3\%$. A $C2/M1$ ratio of the same sign and comparable magnitude has been measured [7]. Recently, an experimental value for the $N \to \Delta$ quadrupole transition moment has been derived $Q_{exp}^{N \to \Delta} = -0.108 \pm 0.009 \pm 0.034$ fm^2 [5]. From these measurements one can conclude that the nucleon and the Δ are intrinsically deformed. However, the magnitude and sign of the intrinsic N and Δ deformation can only be calculated within a model.

There is considerable uncertainty in the literature concerning the implications of the experimental $C2/M1$ and $E2/M1$ ratios for the intrinsic deformation of the nucleon. Even with respect to the sign of the intrinsic nucleon deformation there is no consensus. For example, Refs. [8–10] conclude that the nucleon is oblate, while Refs. [11–13] find a prolate nucleon deformation. Several authors speak only about ''deformation'' without specifying the sign.

The purpose of this paper is to calculate the intrinsic quadrupole moment of the proton Q_0^p in various models. In particular, we want to predict its sign, i.e., we want to find out whether the proton is prolate or oblate. Before doing this, we discuss the possible origins of nucleon deformation in the quark model.

II. SOURCES OF QUADRUPOLE DEFORMATION

Two different sources contribute to a quadrupole deformation of baryons. First, tensor forces between quarks lead to D-state admixtures in the single-quark wave functions of a baryon, and consequently to a deviation of the *valence quark* distribution from spherical symmetry. The one-gluon exchange interaction was originally proposed to provide the

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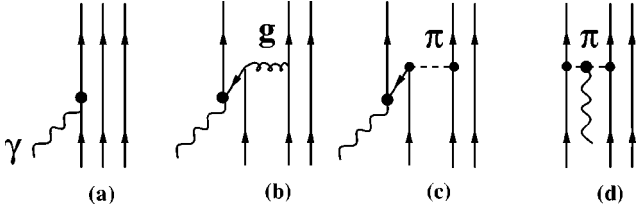†Electronic address: henley@phys.washington.edu

FIG. 1. Feynman diagrams of the four vector current $J^\mu = (\rho, \mathbf{J})$: (a) one-body current $\mathbf{J}^\mu_{[1]}$, and (b)–(d) two-body gluon and pion exchange currents $\mathbf{J}^\mu_{[2]}$. The one-body charge density leads to a quadrupole operator of the type given in Eq. (2). Both gluon and pion exchange charge densities produce a two-body quadrupole operator of the type given in Eq. (5).

required tensor force [14,15]. An external photon can induce a quadrupole transition, for example, by lifting an S state quark in the N into a D state in the Δ via the one-body current in Fig. 1(a).

Second, quark-antiquark pairs and gluons are present in a physical baryon. These degrees of freedom also contribute to the observed quadrupole transition. We refer to the $q\bar{q}$ and gluon degrees of freedom generically as *nonvalence quark degrees of freedom*. The latter are effectively described as spin-dependent two-quark operators in the electromagnetic current [16]. The two-body terms in the charge and current operators shown in Figs. 1(b)–(d) arise as a result of eliminating the $q\bar{q}$ and gluon degrees of freedom from the wave function, very much in the same way as the two-body potentials in the Hamiltonian result from the elimination of the ''exchange particle'' degrees of freedom from Hilbert space.

Hence tensor force induced D waves in the single-quark wave function (one-quark quadrupole transition) *and* nonvalence quark degrees of freedom (two-quark quadrupole transition) contribute to the deformation of baryons. In principle, there can also be three-quark operators. We neglect them here. In a different paper [17] we argue that their contribution is suppressed by at least $1/N_c$ compared to the two-body terms.

Experimental amplitudes contain both mechanisms, so that one cannot readily distinguish between the two different excitation modes (one-quark vs two-quark currents). If, however, the single quark transition mode is strongly suppressed [3], and if one measures an $E2$ strength that is large compared to the single quark estimate, one may conclude that the deformation resides in the nonvalence quark degrees of freedom, effectively described by the two-body current operators.

A. Single quark operator: Deformed valence quark orbits

In a multipole expansion of the one-body charge operator, the term proportional to the spherical harmonic of rank two, $Y^2(\hat{\mathbf{r}}_i)$ provides the one-body quadrupole operator,

$$\hat{Q}_{[1]} = \sqrt{\frac{16\pi}{5}} \sum_{i=1}^{3} e_i r_i^2 Y_0^2(\mathbf{r}_i) = \sum_i e_i (3z_i^2 - r_i^2), \quad (2)$$

where the sum is over the three quarks in the baryon. Obviously, this one-body operator needs D waves in the nucleon

or Δ in order to make a nonvanishing contribution. In a two-state model (only S and D waves), the nucleon and Δ wave functions can be written as

$$|N\rangle = a_S |(S=1/2, L=0)J=1/2\rangle$$
$$+ a_D |(S=3/2, L=2)J=1/2\rangle,$$
$$\quad (3)$$
$$|\Delta\rangle = b_S |(S=3/2, L=0)J=3/2\rangle$$
$$+ b_D |(S=1/2, L=2)J=3/2\rangle,$$

where the quark spin S couples with the orbital angular momentum L to the total angular momentum J of the baryon. The D states in Eq. (3) are of mixed symmetry type with respect to the exchange of quarks 1 and 2. We have purposely omitted the symmetric D state in the Δ wave function $c_D |(S=3/2, L=2)J=3/2\rangle$, which is smaller in magnitude than the one listed here. The negative relative sign of the D wave amplitude $a_D = -0.04$ [14,15] with respect to the S wave amplitude indicates an oblate deformation of the valence quark distribution in the nucleon. Similarly, the positive sign of the D state amplitude $b_D = 0.07$ in the Δ corresponds to a prolate deformation of its valence quark distribution.

Applied to the $N \rightarrow \Delta$ quadrupole transition, the one-body quadrupole operator $\hat{Q}_{[1]}$ sandwiched between the N and Δ wave functions gives for the quadrupole transition moment [18]

$$Q_{p \rightarrow \Delta^+} = -b^2 \frac{4}{\sqrt{30}} (a_S b_D - a_D b_S), \quad (4)$$

where the small $a_D b_D$ term has been neglected. Here, the harmonic oscillator parameter b describes the spatial extension of the baryon wave function, and we refer to it as quark core (matter) radius. The two terms of this single quark current matrix element are schematically shown in Fig. 2.

Using standard D state admixtures [14,15] and an unphysically large quark core radius of $b=1$ fm in Eq. (4) one could describe the experimental transition quadrupole $Q_{p \rightarrow \Delta^+}^{exp} \approx -0.11$ fm^2 [5] by the one-body term alone. However, from the description of the baryon spectrum it is known that one needs an average single-quark excitation energy $\omega = 1/(m_q b^2) \approx 500$ MeV [18], which implies a quark core radius $b \approx 0.5$ fm. With this smaller value for the quark matter radius b one obtains only 20% of the experimental quadrupole strength.

That the single-quark excitation is not the dominant quadrupole excitation mechanism becomes particularly apparent when one considers the $E2/M1$ ratio in the electromagnetic excitation of the Δ^+. Calculations of the $E2/M1$ ratio based on spatial single quark currents give $E2/M1 = -0.1\%$ [20], which is an order of magnitude smaller than recent experiments. Thus, the experimental $E2/M1 \approx -3\%$ ratio [4,5] cannot be solely described by a single-quark transition. Other degrees of freedom must be taken into account. These points have recently been discussed in more detail [19].
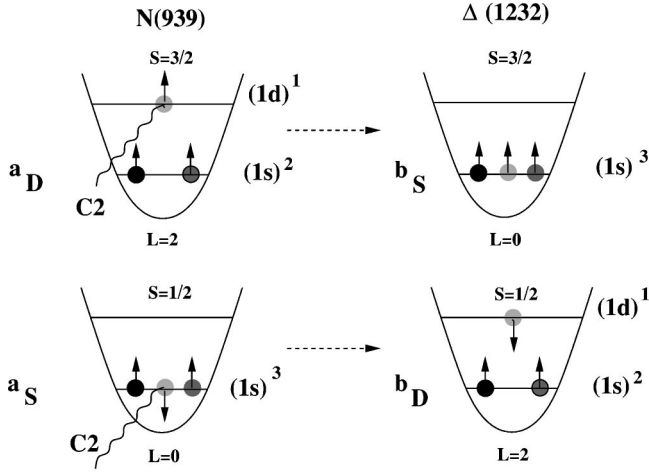
FIG. 2. $N \to \Delta$ quadrupole transition via the one-body quadrupole operator $\hat{Q}_{[1]}$ of Eq. (2) coming from the one-quark current in Fig. 1(a). In this single-quark transition, the absorption of a $C2$ photon is only possible if either the nucleon (left) or the Δ (right) contains a D-wave admixture (deformed valence quark orbit). Actually, the $N \to \Delta$ quadrupole transition due to $\hat{Q}_{[1]}$ is a coherent superposition of the two orbital angular momentum changing but intrinsic spin S conserving one-body transitions (upper and lower part of the figure). The resulting quadrupole transition matrix element of Eq. (4) is suppressed due to the small D-wave admixtures in the N und Δ wave functions.

B. Two-quark operator: Deformed $q\bar{q}$ cloud

Many people believe that the valence quarks must move in D waves in order to obtain a nonvanishing quadrupole moment. However, a two-body spin tensor in the charge operator also generates a quadrupole moment even when the valence quarks are in pure S states [19]. A similar observation was also made by Morpurgo [21]. The two-body quadrupole operator generated by the $q\bar{q}$ pair currents of Figs. 1(b) and 1(c),

$$\hat{Q}_{[2]} = B \sum_{i \neq j = 1}^{3} e_i (3 \sigma_{iz} \sigma_{jz} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), \tag{5}$$

acts in spin and isospin space, whereas the one-body operator in Eq. (2) acts in isospin and orbital space. The constant B with dimension fm^2 contains the orbital and color matrix elements. As a spin-tensor of rank 2, the operator $\hat{Q}_{[2]}$ may simultaneously flip the spin of two quarks (double spin flip) in such a way that the total spin changes from 1/2 to 3/2 (see Fig. 3).[1]

We emphasize that *although the operator $\hat{Q}_{[2]}$ formally acts on valence quark spin states, it does not describe the deformation of the valence quark core. Instead, it reflects*

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[1]The name double spin flip becomes clear if one rewrites one term in Eq. (5) in the spherical basis with spin raising (σ_+) and lowering (σ_-) operators, e.g., $B e_1 (2 \sigma_{1z} \sigma_{2z} + \sigma_{1-} \sigma_{2+} + \sigma_{1+} \sigma_{2-})$.
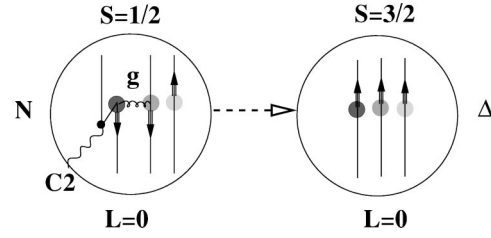

FIG. 3. $N \to \Delta$ quadrupole transition via the two-body quadrupole operator $\hat{Q}_{[2]}$ of Eq. (5) originating from, e.g., the two-body gluon exchange current in Fig. 1(b). The quadrupole transition proceeds by absorbing a $C2$ photon on a quark-antiquark pair with all valence quarks remaining in an S state. This double spin flip quadrupole amplitude describes the deformation of the $q\bar{q}$ cloud in the nucleon. It can be parameter-independently expressed in terms of the neutron charge radius $Q_{p \to \Delta^+} = r_n^2 / \sqrt{2} = -0.80$ fm^2 [19]. This prediction is in agreement with the recent extraction of $Q_{p \to \Delta}^{exp} = -0.108 \pm 0.009 \pm 0.034$ by the LEGS group [5] indicating that the major part of the quadrupole strength resides in the collective $q\bar{q}$ degrees of freedom.

that the physical nucleon contains $q\bar{q}$ sea quarks whose distribution deviates from spherical symmetry.

Using a quark model with two-body exchange currents the constant B has been calculated. It was found that the $N \to \Delta$ and Δ quadrupole moments can be expressed in terms of the neutron charge radius r_n^2 as follows [19]:

$$\sqrt{2} Q_{p \to \Delta^+} = Q_{\Delta^+} = 4B = r_n^2. \tag{6}$$

The reason for the existence of such a relation between Q_Δ and r_n^2 is that both observables are dominated by exchange currents. When expanding the gluon and pion exchange charge operators in Figs. 1(b) and (c) into Coulomb multipole operators one finds a fixed relative strength between the monopole term $C0 = -2B \Sigma e_i \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$ (giving rise to a nonvanishing neutron charge radius [16]) and the quadrupole term $C2 = B \Sigma e_i (3 \sigma_{iz} \sigma_{jz} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)$ (leading to a nonzero transition quadrupole moment [19]). As a result, one obtains the same analytic expression for r_n^2 and $\sqrt{2} Q_{p \to \Delta^+}$, suggesting that the deformation of the nucleon is closely connected to the nonvanishing neutron charge radius. The quadrupole moment calculated from the experimental neutron charge radius according to Eq. (6) is in good agreement with the transition quadrupole moments extracted from the $E2/M1$ and $C2/M1$ measurements [5].

Let us summarize. The quark model shows that *both* the $N(939)$ ground state and the excited $\Delta(1232)$ state are deformed. This conclusion is reached in the single quark transition model where tensor forces lead to D-state admixtures in *both* the N and Δ wave function. It is also obtained in the two-quark transition model, where exchange currents produce a nonzero quadrupole moment of the $S = 1$ diquark in the N and Δ. It is impossible to have a spherical nucleon and a deformed Δ or vice versa. If the quarks interact via vector or pseudoscalar type potentials, either both baryons are deformed or both are spherical. The latter possibility is ruled out by experiment. Furthermore, we understand that two-

body $q\bar{q}$ pair currents provide the major contribution to baryon quadrupole moment, suggesting that the deformation resides in the baryon's $q\bar{q}$ cloud.

III. INTRINSIC DEFORMATION OF THE NUCLEON

In this section we discuss the intrinsic quadrupole moment of the nucleon. Here, *intrinsic* quadrupole moment means the one obtained in a body-fixed coordinate system that rotates with the nucleon. Most work that deals with the problem of nucleon deformation in the quark model does not distinguish between intrinsic and spectroscopic (measured) quadrupole moment. This is all the more surprising since the shape of the nucleon is in the first place related to the intrinsic, and not to the spectroscopic quadrupole moment [2].

The spectroscopic quadrupole moment of the nucleon is zero. Nevertheless, the nucleon can have an *intrinsic* quadrupole moment. This is analogous to a deformed $J=0$ nucleus. All orientations of a deformed $J=0$ nucleus are equally probable, which results in a spherical charge distribution in the ground state and a vanishing quadrupole moment Q in the laboratory. The intrinsic quadrupole moment Q_0 can then only be obtained by measuring electromagnetic quadrupole transitions between the ground and excited states, or by measuring the quadrupole moment of an excited state with $J>1/2$ of that nucleus. If a sufficient number of quadrupole transitions to excited states are known, the intrinsic quadrupole moment could be extracted from the data in a model-independent way as suggested by Kumar [22]. Here, we follow a somewhat different approach, which is less general than Kumar's method.

Given only the experimental information of the Δ and the $N\rightarrow\Delta$ quadrupole moments what can we learn about the intrinsic quadrupole moment of the nucleon? The answer which we give is model dependent. However, with respect to the sign of the intrinsic quadrupole moment all models studied in this paper yield the same answer, namely that the nucleon is prolate shaped. In the following three sections, we will use three different models of the nucleon and Δ, (i) a quark model, (ii) a collective model, (iii) and a pion cloud model in order to calculate the intrinsic quadrupole moment of the nucleon.

A. Quark model

In standard notation the SU(4) spin-flavor part of the proton wave function is composed of a spin-singlet and a spin-triplet part:

$$|p\rangle=\frac{1}{\sqrt{2}}\left\{\frac{1}{\sqrt{6}}|(2uud-udu-duu)\rangle\right.$$

$$\times\frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)\rangle+\frac{1}{\sqrt{2}}|(udu-duu)\rangle$$

$$\left.\times|\frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)\rangle\right\}. \tag{7}$$

The angular momentum coupling factors 2, -1, -1 in front of the three terms in the spin symmetric proton wave function express (i) the coupling of the first two quarks to an $S=1$ diquark, and (ii) the coupling of the $S=1$ diquark with the third quark to total $J=1/2$.

Sandwiching the quadrupole operator $\hat{Q}_{[2]}$ between the proton's spin-flavor wave function yields a vanishing spectroscopic quadrupole moment. The reason is clear. The spin tensor $\hat{Q}_{[2]}$ applied to the spin singlet wave function gives zero, and when acting on the proton's spin triplet wave function it gives

$$(3\sigma_{1z}\sigma_{2z}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\frac{1}{\sqrt{6}}|(2\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)\rangle$$

$$=\frac{4}{\sqrt{6}}|(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow)\rangle, \tag{8}$$

where the right-hand side is a spin-3/2 wave function, which has zero overlap with the spin-1/2 wave function of the proton in the final state. Consequently, the spectroscopic quadrupole moment

$$Q_p=\langle p|\hat{Q}_{[2]}|p\rangle=B(2-1-1)=0 \tag{9}$$

vanishes due to the spin coupling coefficients in $|p\rangle$.

Although the spin $S=1$ diquarks (uu and ud) in the proton have nonvanishing quadrupole moments, the angular momentum coupling of the diquark spin to the spin of the third quark prevents this quadrupole moment from being observed. Setting ''by hand'' all Clebsch-Gordan coefficients in the spin part of the proton wave function of Eq. (7) equal to 1, while preserving the normalization, one obtains a modified ''proton'' wave function $|\tilde{p}\rangle$:

$$|\tilde{p}\rangle=\frac{1}{\sqrt{2}}\left\{\left[\left|\frac{1}{\sqrt{6}}(2uud-udu-duu)\rangle\right.\right.\right.$$

$$\left.\left.+\frac{1}{\sqrt{2}}|(udu-duu)\rangle\right]\frac{1}{\sqrt{3}}|(\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow)\rangle\right\}. \tag{10}$$

The renormalization of the Clebsch-Gordan coefficients is undoing the averaging over all spin directions, which renders the intrinsic quadrupole moment unobservable. Note that we do not modify the flavor part of the wave function in order to ensure that we deal with a proton.[2]

We consider the expectation value of the two-body quadrupole operator $\hat{Q}_{[2]}$ in the state of the spin-renormalized proton wave function $|\tilde{p}\rangle$ as an estimate of the *intrinsic* quadrupole moment of the proton Q_0^p:

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[2]In the case of the neutron we must divide by the negative charge of the dd diquark. We then obtain $Q_0^n=Q_0^p$.

$$Q_0^p = \langle \tilde{p} | Q_{[2]} | \tilde{p} \rangle = 2B \left(\frac{2}{3} - \frac{8}{3} \right) = -4B = -r_n^2, \quad (11)$$

where the two contributions arise from the spin 1 diquark with projection $M=1$ and $M=0$. The latter dominates. Comparing with Eq. (6), we find that the *intrinsic* quadrupole moment of the proton is equal to the *negative* of the neutron charge radius r_n^2 and is therefore *positive*.

Similarly, with the Δ^+ wave function with maximal spin projection $M_J = 3/2$,

$$| \Delta^+ \rangle = \frac{1}{\sqrt{3}} | (uud + udu + duu) \rangle | \uparrow\uparrow\uparrow \rangle, \quad (12)$$

we find for the intrinsic quadrupole moment of the Δ^+

$$Q_0^{\Delta^+} = Q^{\Delta^+} = r_n^2. \quad (13)$$

In the case of the Δ, there are no Clebsch-Gordan coefficients that could be "renormalized," and there is no difference between the intrinsic $Q_0^{\Delta^+}$ and the spectroscopic quadrupole moment Q^{Δ^+}.

Summarizing, in the quark model, the intrinsic quadrupole moment of the proton and the Δ^+ are equal in magnitude but opposite in sign,

$$Q_0^p = -Q_0^{\Delta^+}. \quad (14)$$

We conclude that the proton is a prolate and the Δ^+ an oblate spheroid. The same conclusion is also obtained in a quite different approach to which we turn in the next section.

B. Collective model

Quadrupole moments of strongly deformed nuclei are not adequately described in a single-nucleon transition model. The measured quadrupole moments of strongly deformed nuclei exceed the quadrupole moment due to a single valence nucleon in a deformed orbit usually by a factor of 10 or more. The collective nuclear model, which involves the collective rotational motion of many nucleons of the nucleus, gives a more realistic description of the data.

In the collective nuclear model [2], the relation between the observable spectroscopic quadrupole moment Q and the intrinsic quadrupole moment Q_0 is

$$Q = \frac{3K^2 - J(J+1)}{(J+1)(2J+3)} Q_0, \quad (15)$$

where J is the total spin of the nucleus, and K is the projection of J onto the z axis in the body fixed frame (symmetry axis of the nucleus). The intrinsic quadrupole moment Q_0 characterizes the deformation of the charge distribution in the ground state. The ratio between Q_0 and Q is the expectation value of the Legendre polynomial $P_2(\cos \Theta)$ in the substate with maximal projection $M = J$. This factor represents the averaging of the nonspherical charge distribtion due to its rotational motion as seen in the laboratory frame.
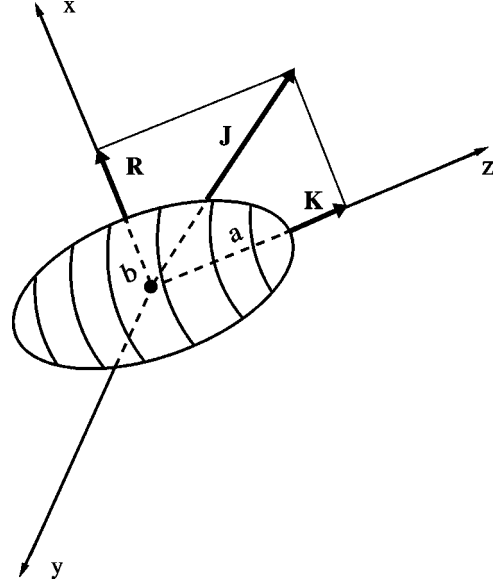

FIG. 4. Representation of the Δ isobar as a collective rotation of a prolate nucleon with intrinsic spin $K = 1/2$. The collective orbital angular momentum is denoted by R. As a result of the collective rotation of a cigar-shaped object (N) with intrinsic spin $K = 1/2$ one obtains a pancake-shaped object (Δ) with total angular momentum $J = 3/2$. The lengths of major half-axis a and minor half-axis b can be calculated in the model of a homogeneously charged spheroid (see text).

We consider the Δ with spin $J = 3/2$ as a collective rotation of the entire nucleon with an intrinsic angular momentum $K = 1/2$ (see Fig. 4). This is how the $\Delta(1232)$ is viewed in the Skyrme model.[3] In this model, the (rotational) energy of the Δ is inversely proportional to its moment of inertia [23]. Therefore it is energetically favorable to increase its moment of interia by assuming a deformed shape. Inserting the quark model relation for the spectroscopic quadrupole moment $Q_\Delta = r_n^2$ on the left-hand side[4] one finds for the intrinsic quadrupole moment of the proton

$$Q_0^p = -5 r_n^2. \quad (16)$$

The large value for Q_0^p is certainly due to the crudeness of the model. The rigid rotor model for the nucleon which underlies Eq. (15) is most certainly an oversimplification. A more realistic description would treat nucleon rotation as being partly irrotational, e.g., only the peripheral parts of the nucleon participate in the collective rotation. This results in smaller intrinsic quadrupole moments [2]. However, we speculate that the sign of the intrinsic quadrupole moment given by Eq. (16) is correct. If so, the nucleon is a prolate spheroid.

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[3]With hindsight the Skyrme model is seen as an effective field theory of QCD corresponding to the limit of infinitely many colors (and quarks) [23].

[4]This may be justified because in the quark model Q_Δ is dominated by the collective $q\bar{q}$ degrees of freedom.

We can also use the collective model to estimate Q_0^Δ. For this purpose one regards the Δ^+ as the $K=J=3/2$ ground state of a rotational band. We then obtain from Eq. (15) a negative intrinsic quadrupole moment for the Δ^+,

$$Q_0^{\Delta^+}=5r_n^2=-Q_0^p. \tag{17}$$

Obviously, the intrinsic quadrupole moments of the proton and the Δ^+ have the same magnitude but different sign, a result that was also obtained in the quark model with two-quark operators. The sign change between Q_0^p and $Q_0^{\Delta^+}$ can be explained by imagining a cigar-shaped ellipsoid (N) collectively rotating around the x axis. This leads to a pancake-shaped ellipsoid (Δ).

In classical electrodynamics the simplest model for a non-spherical homogeneous charge distribution is a rotational ellipsoid with charge Z, major axis a along, and minor axis b perpendicular to the symmetry axis (see Fig. 4). Its quadrupole moment is given by

$$Q_0=\frac{2Z}{5}(a^2-b^2)=\frac{4}{5}ZR^2\delta, \tag{18}$$

with the deformation parameter $\delta=2(a-b)/(a+b)$ and the mean radius $R=(a+b)/2$. We use this model to estimate the degree of baryon deformation.

From the collective model we get $Q_0^p=0.565$ fm^2, and with the recent value for the proton charge radius [1] we obtain for the ''equivalent'' radius $R_p=\sqrt{5/3}r_p=1.15$ fm. Eq. (18) then leads to a deformation parameter $\delta_N\approx0.53$, and a ratio of major to minor semi-axes $a/b\approx1.73$. Similarly, for the Δ one gets with the help of the charge radius relation $r_{\Delta^+}^2=r_p^2-r_n^2$ [25,26], $\delta_\Delta=-0.48$, and $a/b=-0.62$. On the other hand, if we insert the quark model result $Q_0^p=-r_n^2=0.113$ fm^2 on the left-hand side of Eq. (18), we obtain a deformation parameter $\delta_N=0.11$. This corresponds to a ratio of major to minor semi-axes $a/b=1.11$. For the deformation parameter of the Δ we find $\delta_\Delta=-0.09$ and a half-axis ratio $a/b=-0.91$.

Summarizing, the collective model leads in combination with the experimental information to a positive intrinsic quadrupole moment of the nucleon and a negative intrinsic quadrupole moment for the Δ^+. Although the magnitude of the deformation is uncertain, we are confident that our assignment of a prolate deformation for the nucleon and an oblate deformation for the Δ is correct.

C. Pion cloud model

Finally, we consider the physical proton with spin up, denoted by $|p\uparrow\rangle$, as a coherent superposition of three different terms: (i) a spherical quark core contribution with spin 1/2, called a bare proton p'; (ii) a bare neutron n' surrounded by a positively charged pion cloud; and (iii) a bare p' surrounded by a neutral pion cloud [24]. In the last two terms the spin(isospin) of the bare proton and of the pion cloud are coupled to total spin and isospin of the physical proton. Similarly, the physical Δ^+ is considered as superposition of a spherical quark core term with spin 3/2, called a

bare $\Delta^{+'}$, a bare n' surrounded by a π^+ cloud, and a bare p' surrounded by a π^0 cloud. In each term, the spin/isospin of the quark core and pion cloud are coupled to the total spin and isospin of the physical Δ^+. We then write

$$|p\uparrow\rangle=\alpha|p'\uparrow\rangle+\beta\frac{1}{3}(|p'\uparrow\pi^0Y_0^1\rangle-\sqrt{2}|p'\downarrow\pi^0Y_1^1\rangle$$
$$-\sqrt{2}|n'\uparrow\pi^+Y_0^1\rangle+2|n'\downarrow\pi^+Y_1^1\rangle),$$

$$|n\uparrow\rangle=\alpha|n'\uparrow\rangle+\beta\frac{1}{3}(-|n'\uparrow\pi^0Y_0^1\rangle+\sqrt{2}|n'\downarrow\pi^0Y_1^1\rangle$$
$$+\sqrt{2}|p'\uparrow\pi^-Y_0^1\rangle-2|p'\downarrow\pi^-Y_1^1\rangle),$$

$$|\Delta^+\uparrow\rangle=\alpha'|\Delta^{+'}\uparrow\rangle+\beta'\frac{1}{3}(2|p'\uparrow\pi^0Y_0^1\rangle+\sqrt{2}|p'\downarrow\pi^0Y_1^1\rangle$$
$$+\sqrt{2}|n'\uparrow\pi^+Y_0^1\rangle+|n'\downarrow\pi^+Y_1^1\rangle), \tag{19}$$

where β and β' describe the amount of pion admixture in the N and Δ wave functions. These amplitudes satisfy the normalization conditions $\alpha^2+\beta^2=\alpha'^2+\beta'^2=1$, so that we have only two unknows β and β'. The p and Δ^+ wave functions are normalized and orthogonal. Here, Y_0^1 and Y_1^1 are spherical harmonics of rank 1 describing the orbital wave functions of the pion. Because the pion moves predominantly in a p wave, the charge distributions of the proton and Δ^+ deviate from spherical symmetry, even if the bare proton and bare neutron wave functions are spherical.

The quadrupole operator to be used in connection with these states is

$$\hat{Q}_\pi=e_\pi\sqrt{\frac{16\pi}{5}}r_\pi^2Y_0^2(\hat{\mathbf{r}}_\pi), \tag{20}$$

where e_π is the pion charge operator divided by the charge unit e, and r_π is the distance between the center of the quark core and the pion. Our choice of $\hat{Q}_\pi$ implies that the quark core is spherical and the entire quadrupole moment comes from the pion p-wave orbital motion.[5]

The π^0 terms do not contribute when evaluating the operator $\hat{Q}_\pi$ between the wave functions of Eq. (19). We then obtain for the spectroscopic Δ and $N\rightarrow\Delta$ quadrupole moments

$$Q_{\Delta^+}=-\frac{2}{15}\beta'^2r_\pi^2,\quad Q_{p\rightarrow\Delta^+}=\frac{4}{15}\beta'\beta r_\pi^2. \tag{21}$$

Only the Y_1^1 part of the pion wave function (pion cloud aligned in x-y plane) contributes to Q_{Δ^+}. This leads to an oblate intrinsic deformation of the Δ^+.

We have to determine three parameters β, β', and r_π. From the experimental $N\rightarrow\Delta$ quadrupole transition moment,

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[5]A possible intrinsic deformation of the pion is neglected.

$Q_{p \to \Delta}^{exp} \approx -0.11 = r_n^2$ [5], we can fix only one of them. Therefore we also calculate the nucleon and Δ charge radii in the pion cloud model and obtain

$$r_p^2 = (1 - \beta^2) r_{p'}^2 + \beta^2 \left(\frac{1}{3} r_{p'}^2 + \frac{2}{3} r_\pi^2 \right),$$

$$r_n^2 = \beta^2 \left(\frac{2}{3} r_{p'}^2 - \frac{2}{3} r_\pi^2 \right),$$

$$r_{\Delta^+}^2 = (1 - \beta'^2) r_{p'}^2 + \beta'^2 \left(\frac{2}{3} r_{p'}^2 + \frac{1}{3} r_\pi^2 \right). \tag{22}$$

Here, $r_{p'}^2$ is the charge radius of the bare proton. We assume that the charge radii of the bare proton and of the bare Δ^+ are equal. Adding the first two equations gives $r_{p'}^2 = r_p^2 + r_n^2$, which expresses the bare proton charge radius in terms of the experimental isoscalar nucleon charge radius. Subtracting the first and third equations one gets

$$r_p^2 - r_{\Delta^+}^2 = r_{p'}^2 \left(\frac{1}{3} \beta'^2 - \frac{2}{3} \beta^2 \right) + r_\pi^2 \left(\frac{2}{3} \beta^2 - \frac{1}{3} \beta'^2 \right) = r_n^2$$

$$= \beta^2 \left(\frac{2}{3} r_{p'}^2 - \frac{2}{3} r_\pi^2 \right). \tag{23}$$

Because the correction to $r_p^2 - r_{\Delta^+}^2 = r_n^2$ is of order $\mathcal{O}(1/N_c^2)$ [25,26] and therefore small, we obtain $\beta' = -2\beta$. When the latter condition is used in Eq. (21), we get

$$Q_{\Delta^+} = Q_{p \to \Delta^+} = r_n^2. \tag{24}$$

This is in the same ballpark as the quark model prediction of Eq. (6).

We can now eliminate the model parameters and express them through the experimental charge radii: $\beta^2 = -(3/8) r_n^2 / (r_p^2 + r_n^2)$ and $r_\pi^2 = 5(r_p^2 + r_n^2)$. The resulting numerical values $\beta = 0.26$, $\beta' = -0.52$, $r_\pi = 1.77$ fm correspond to a pion probability of 7% in the nucleon. The spatial extension of the pion cloud r_π is close to the Compton wave length of the pion. Due to the larger lever arm of r_π compared to $r_{p'}$, the major part of the neutron charge radius and the nucleon's intrinsic quadrupole moment comes from the pion cloud.

Next, we calculate the spectroscopic quadrupole moment of the proton in the pion cloud model. We find

$$Q^p = \frac{4}{3} \beta^2 r_\pi^2 \left[\frac{1}{3} \left(\frac{2}{5} \right) + \frac{2}{3} \left(-\frac{1}{5} \right) \right]. \tag{25}$$

The factors $1/3$ and $2/3$ are the squares of the Clebsch-Gordan coefficients that describe the angular momentum coupling of the bare neutron spin $1/2$ with the pion orbital angular momentum $l = 1$ to total spin $J = 1/2$ of the proton. They ensure that the spectroscopic quadrupole moment of the proton is zero. The factors $2/5$ and $-1/5$ are the expectation values of the Legendre polynomial $P_2(\cos \Theta)$ evaluated between the pion wave function $Y_0^1(\hat{\mathbf{r}}_\pi)$ (pion cloud
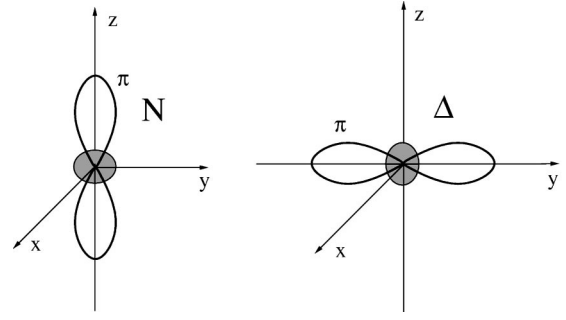

FIG. 5. Intrinsic quadrupole deformation of the nucleon (left) and Δ (right) in the pion cloud model. In the N, the p-wave pion cloud is concentrated along the polar (symmetry) axis, with maximum probability of finding the pion at the poles. This leads to a prolate deformation. In the Δ, the pion cloud is concentrated in the equatorial plane producing an oblate intrinsic deformation.

aligned along z axis) and $Y_1^1(\hat{\mathbf{r}}_\pi)$ (pion cloud aligned along an axis in the x-y plane). If we set *by hand* each of the coupling coefficients in front of $\langle Y_0^1 | P_2 | Y_0^1 \rangle$ and $\langle Y_1^1 | P_2 | Y_1^1 \rangle$ equal to $1/2$, the cancellation between the two orientations of the cloud disappears. The normalization of the sum of coupling coefficients is thereby preserved. We note that the first term in Eq. (25), which comes from the Y_0^1 part of the pion wave function, dominates. Therefore the probability for finding the pion in the nucleon is largest at the poles. This term is just the negative of the spectroscopic Δ^+ quadrupole moment.

By this procedure we are undoing the geometric averaging over all angles, which prevents the nonsphericity of the pion cloud from being observed in the laboratory. One then finds for the intrinsic quadrupole moment of the proton and the Δ^+

$$Q_0^p = \frac{4}{3} \beta^2 r_\pi^2 \left[\frac{1}{2} \left(\frac{2}{5} \right) + \frac{1}{2} \left(\frac{2}{5} \right) \right] = \frac{8}{15} \beta^2 r_\pi^2 = -r_n^2, \quad Q_0^{\Delta^+} = r_n^2. \tag{26}$$

Again, the intrinsic quadrupole moment of the p is positive and that of the Δ^+ negative. They are identical in magnitude but opposite in sign.

The positive sign of the intrinsic proton[6] quadrupole moment has a simple geometrical interpretation in this model. It arises because the pion is preferably emitted along the spin (z axis) of the nucleon (see Fig. 5). Thus the proton assumes a prolate shape. Here, we neglect the deformation of the bare nucleon quark bag due to the pressure of the surrounding pion cloud. We emphasize that in this model all of the deformation comes from the pion cloud itself, none from the valence quark core. Previous investigations in a quark model with pion exchange [9] concluded that the nucleon assumes an oblate shape under the pressure of the surrounding pion cloud, which is strongest along the polar axis. However, in these studies the deformed shape of the pion cloud itself was

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[6]After dividing by the negative sign of the π^- cloud, the neutron's intrinsic quadrupole moment is also positive, i.e., $Q_0^n = Q_0^p$.

ignored. Inclusion of the latter leads to a prolate deformation that exceeds the small oblate quark bag deformation by a large factor.

IV. SUMMARY

The experimental evidence for a nonvanishing $N \to \Delta$ transition and Δ quadrupole moments can be seen as an indication for an intrinsic nucleon deformation. In the present paper, the intrinsic quadrupole moment of the nucleon has been estimated in (i) a quark model, (ii) a collective model, and (iii) a pion cloud model, using the empirical information on the $p \to \Delta^+$ quadrupole moment and the nucleon charge radii.

The quark model with the two-body quadrupole operator, and the pion cloud model predict a negative sign for the *spectroscopic* Δ^+ quadrupole moment, and that $Q_{\Delta^+} \approx Q_{p \to \Delta^+} \approx r_n^2$. In all three models we find that the *intrinsic* quadrupole moment of the nucleon is *positive*. This indicates a prolate shape of the proton charge distribution. On the other hand, the intrinsic quadrupole moment of the Δ^+ is found to be *negative*, indicative of an oblate Δ^+ charge distribution. As to the magnitude of the deformation, the models vary within a wide range $Q_0^p = 0.11 - 0.55$ fm^2.

Despite their differences, all models emphasize collective over single-particle degrees of freedom and lead to an appreciable spectroscopic $p \to \Delta^+$ transition quadrupole moment, in agreement with recent experiments. In our opinion this reflects that the intrinsic nucleon deformation resides mainly in the $q\bar{q}$ cloud surrounding an almost spherical valence quark core. It would be interesting to calculate the intrinsic quadrupole moment of the nucleon in other models, in order to check whether our finding of a prolate nucleon shape can be confirmed.

ACKNOWLEDGMENTS

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