Thermal equilibrium and nonuniform longitudinal flow in relativistic heavy ion collisions

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A model with nonuniform flow in the longitudinal direction is proposed for the relativistic heavy ion collisions and compared with the 14.6*A* GeV/*c* Si-Al and 10.8*A* GeV/*c* Au-Au collision data. The stronger influence of transparency on the distribution of heavier produced particles and the larger stopping in the heavier collision system are accounted for by using a new geometrical parametrization picture. The central dips in the proton and deuteron rapidity distributions for Si-Al collision are reproduced.

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I. INTRODUCTION

The experimental finding that colliding nuclei are not transparent but undergo a violent reaction in central collisions represents one of the major motivations for the study of ultrarelativistic heavy ion collisions at the CERN/SPS, BNL/AGS, and also at the future BNL/RHIC and CERN/LHC. Of central importance is the ability of understanding to what extent the nuclear matter has been compressed and heated.

The use of thermal models to interpret data on particle distribution from nuclear collisions is motivated by the hope and expectation that in collision between sufficiently large nuclei at sufficiently high energies a state of excited nuclear matter close to local thermodynamic equilibrium can be formed, allowing us to study the thermodynamics of QCD and the possible phase transition from a hadronic gas to a QGP. Since we do not yet reliably know how big the collision system and how large the beam energy have to be for this to occur (if at all), thermal models should be considered as an important phenomenological tool to test for such a behavior.

The study of collective flow in high energy nuclear collisions has attracted increasing attentions from both experimental [1] and theoretical [2] point of view. The rich physics of longitudinal and transverse flows is due to their sensitivity to the system evolution at early time. The expansion and cooling of the heated and highly compressed matter could lead to a considerable collectivity in the final state. Due to the high pressure, particles might be boosted in the transverse and longitudinal directions. The collective expansion of the system created during a heavy ion collision implies space-momentum correlation in particle distributions at freeze-out.

The experimental data on the rapidity distributions of produced particles in 14.6*A*GeV/*c* Si-Al collisions has been utilized to study the collective expansion using a cylindrical-symmetric longitudinal flow model (CSFM) [3,4]. In this model, the distribution of final-state particles comes from the superposition of a number of fireballs, distributed uniformly within the kinematical limit along the longitudinal (rapidity y) axis. The model-results fit well the experimental distribution of pion, but are too narrow in the case of heavier particles, such as proton and deuteron. In particular, the central dip, which can be clearly seen in the distribution of proton, is not reproduced.

More recently, E877 Collaboration [5] has published their data for 10.8*A* GeV/*c* Au-Au collisions, which provides a good chance to compare the stopping power in collision systems of different sizes. A possible central peak of the rapidity distribution of proton at around midrapidity, which was obtained through extrapolating the experimental data to midrapidity using RQMD model [6], has been taken as an evidence for the increasing of stopping power, but the reliability of this extrapolation is model-dependent.

Stopping and transparency describe the same physical aspect in relativistic heavy ion collision from two opposite sides. In order to settle a reliable model for relativistic heavy ion collision, this aspect must be taken into account carefully. The assumption of uniformly distributed fireballs in the CSFM model is a crude one. It does not account for the memory of the fireballs on the motion of the incident nuclei. In the present paper we propose a nonuniform longitudinal flow model (NUFM) using a new geometrical parametrization picture [7] to describe the nuclear stopping/transparency in a more proper way. The stronger influence of transparency on the distribution of heavier produced particles (proton and deuteron) as well as the larger stopping in the heavier collision system (Au-Au) are described by using this picture. The central dips in the proton and deuteron rapidity distributions for Si-Al collisions are reproduced, and at the same time the central peak in the pion rapidity distributions is maintained.

In Sec. II the nonuniform longitudinal flow model (NUFM) with a geometrical parametrization picture is formulated. The results of the model are given and compared with the experimental data in Sec. III. A short summary and conclusion is given in Sec. IV. In order to avoid the complexity associated with the production of strange particles and concentrate on the expansion of the system, we will discuss in this paper only normal nonstrange particles—pions, protons, and deuterons.

II. NONUNIFORM LONGITUDINAL FLOW

First, let us briefly recall the fireball scenario of relativistic heavy ion collisions.

Since the temperature at freeze-out exceeds 100 MeV, the Boltzmann approximation is used. Transformed into rapidity y and transverse momentum p_t this implies [4]

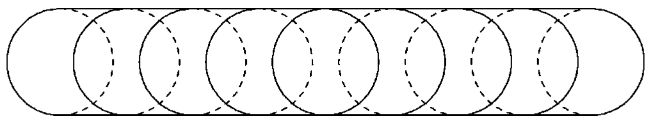

FIG. 1. Schematic sketch of the distribution of fireballs in the uniform flow model (CSFM).

$$E\frac{d^3n}{d^3p}\propto Ee^{(-E/T)}=m_t\cosh y e^{(-m_t\cosh y/T)}. \tag{1}$$

Here $m_t=\sqrt{m^2+p_t^2}$ is the transverse mass, m is the mass of the produced particles at freeze-out.

The rapidity is defined as $y=\tanh^{-1}(p_l/E)$, where p_l is the longitudinal momentum of the produced particle. Substituting into Eq. (1) and integrating over m_t, we get the rapidity distribution of the isotropic thermal source,

$$\frac{dn_{iso}}{dy}\propto\frac{m^2T}{(2\pi)^2}(1+2\xi_0+2\xi_0^2)e^{(-1/\xi_0)}, \tag{2}$$

where $\xi_0=T/m\cosh y$.

Equations (1) and (2) give the isotropic thermal distribution. As mentioned in Ref. [4], for pions with $m\approx T$, this distribution is close to that of massless particles, i.e., proportional to $1/\cosh^2 y$. For heavier particles isotropic emission implies a strongly narrow distribution, in contradiction to the experimental findings, see, e.g., the dashed lines in Fig. 6.

The measured momentum distribution of the final-state particles is certainly anisotropic. It is privileged in the direction of the incident nuclei. This is because the produced hadrons still carry their parent's kinematic information, making the longitudinal direction more populated than the transverse ones. The simplest way [3,4] to account for this anisotropy is to add up the contributions from a set of fireballs with centers located uniformly in the rapidity region $[-y_0,y_0]$, as sketched schematically in Fig. 1. The corresponding rapidity distribution is obtained through changing the ξ_0 in Eq. (2) into $\xi=T/m\cosh(y-y')$ and integrating over y' from $-y_0$ to y_0:

$$\frac{dn_{CSFM}}{dy}\propto\int_{-y_0}^{y_0}dy'\frac{m^2T}{(2\pi)^2}(1+2\xi+2\xi^2)e^{(-1/\xi)}, \tag{3}$$

where $\xi=T/m\cosh(y-y')$. Equivalently, we can also use the angular variable Θ defined by $\Theta=2\tan^{-1}\exp(-y')$, and change the integration variable in Eq. (3) to Θ,

$$\frac{dn_{CSFM}}{dy}=eKm^2T\int_{\Theta_{\min}}^{\Theta_{\max}}\frac{d\Theta}{\sin\Theta}\left(1+\frac{2T}{m\cosh(y-y')}+\frac{2T^2}{m^2\cosh^2(y-y')}\right)\exp(-m\cosh(y-y')/T), \tag{4}$$

cf. the solid circle and lines in Fig. 2.

This simple approach fits the rapidity distribution of pions well but fails to reproduce the central dip in heavier produced particles, which is clearly seen in the experimental distributions of protons and deuterons.

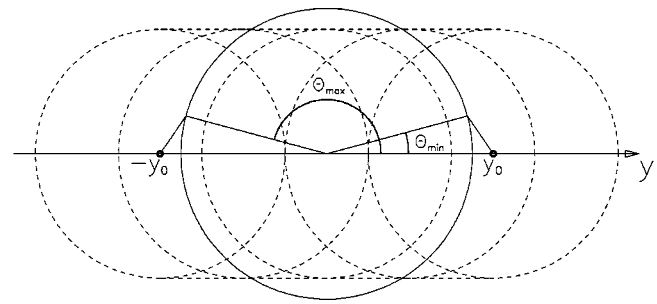

FIG. 2. Schematic sketch of the emission angle Θ (solid circle and lines) and the corresponding distribution of fireballs (dashed circles) in the uniform flow model (CSFM).

Note that in this CSFM model the distribution of fireballs in the longitudinal direction of phase space is uniform which does not account properly for the interaction of the incident nuclei when they pass through each other. This is a crude approximation. A more reasonable assumption is that the fireballs keep some memory on the motion of the incident nuclei, and therefore the distribution of fireballs, instead of being uniform in the longitudinal direction, is more concentrated in the direction of motion of the incident nuclei, i.e., more dense at large absolute value of rapidity. A parametrization for such a distribution can be obtained by using an ellipselike picture on emission angle distribution, as shown in Fig. 3. In this scenario, the emission angle is

$$\theta=\tan^{-1}(e\tan\Theta), \tag{5}$$

where the parameter $e(0\leqslant e\leqslant 1)$ represents the ellipticity of the introduced ellipse which describes the nonuniformity of fireball distribution in the longitudinal direction, as sketched in Fig. 4.

Substituting Eq. (5) together with $y_e=-\ln\tan(\theta/2)$ into Eq. (3), a distribution function $Q(\theta)$ for the emission angle of nonuniform flow is introduced in the integration

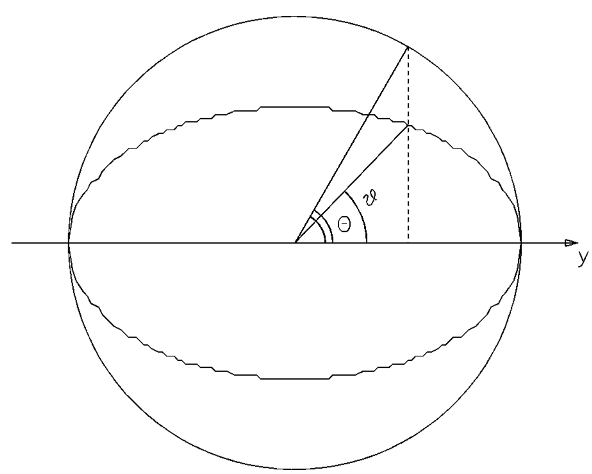

FIG. 3. Schematic sketch of the emission angle θ in the nonuniform flow model (NUFM).

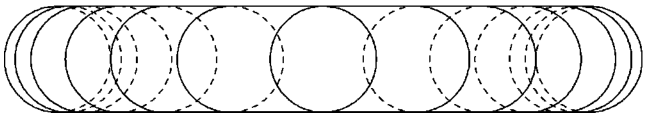

FIG. 4. Schematic sketch of the distribution of fireballs in the nonuniform flow model (NUFM).

$$\frac{dn_{\rm NUFM}}{dy}=eKm^2T\int_{\theta_{\rm min}}^{\theta_{\rm max}}\frac{Q(\theta)d\theta}{\sin\theta}\left(1+\frac{2T}{m\cosh(y-y_e)}+\frac{2T^2}{m^2\cosh^2(y-y_e)}\right)\exp(-m\cosh(y-y_e)/T), \tag{6}$$

$$y_e=-\ln\tan(\theta/2),\quad Q(\theta)=\frac{1}{\sqrt{e^2+\tan^2\theta}|\cos\theta|}. \tag{7}$$

Here $\theta_{\rm min}=2\tan^{-1}(e^{-y_{e0}})$, $\theta_{\rm max}=2\tan^{-1}(e^{-y_{e0}})$. y_{e0} is the rapidity limit which confines the rapidity interval of longitudinal flow.

Changing the integration variable in Eq. (6) back to y_e, the rapidity distribution can be rewritten as

$$\frac{dn_{\rm NUFM}}{dy}=eKm^2T\int_{-y_{e0}}^{y_{e0}}\left(1+\frac{2T}{m\cosh(y-y_e)}+\frac{2T^2}{m^2\cosh^2(y-y_e)}\right)\times\exp(-m\cosh(y-y_e)/T)\rho(y_e)dy_e, \tag{8}$$

$$\rho(y_e)=\sqrt{\frac{1+\sinh^2(y_e)}{1+e^2\sinh^2(y_e)}}. \tag{9}$$

Here $\rho(y_e)$ is the distribution function of nonuniform flow in the longitudinal direction. It can be seen from Fig. 5, that the larger is the parameter e, the flatter is the distribution $\rho(y_e)$ and the more uniform is the longitudinal-flow distribution. When $e\to 1$, the longitudinal-flow distribution is completely uniform $(\rho(y_e)\to 1)$, returning back to the CSFM model.

III. COMPARISON WITH EXPERIMENTS

We now proceed to compare the model results with the AGS data.

The rapidity distributions of pion, proton and deuteron for 14.6*A* GeV/*c* Si-Al collisions [9,10] are given in Figs. 6(a)–6(c). The dashed, dotted, and solid lines (band) correspond to the results from isotropical thermal model, uniform longitudinal flow model (CSFM), and the nonuniform longitudinal flow model (NUFM), respectively. The rapidity limit y_{e0} and the ellipticity e used in the calculation are listed in Table I and illustrated in Fig. 7. The rapidity limit y_0 used in the CSFM model of Ref. [4] is also listed in Table I for comparison. The parameter T is chosen to be 0.12 GeV following Ref. [4].

Note that the distribution of the light particle (pion) is insensitive to the ellipticity e. Changing e from 0.35 to 0.7 results in a narrow band shown in Fig. 6(a). Since pions are produced through the interaction of colliding nuclei, they have less memory on the motion of the incident nuclei before interaction. Therefore, physically the value of ellipticity e for pion should be bigger than that for proton and deuteron, but the exact value cannot be fixed through fitting the model results to the experimental data.

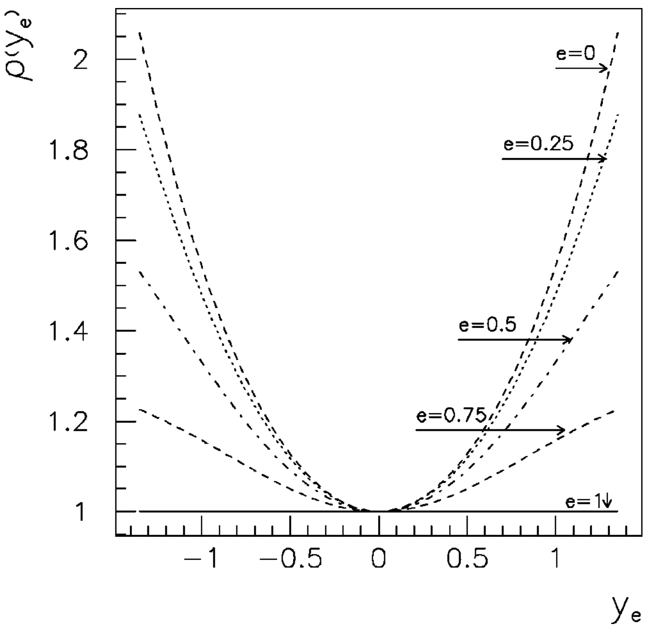

FIG. 5. The distributions $\rho(y_e)$ of the center of fireballs as a function of y_e. When $e\to 1$, the nonuniform distribution turns to the uniform distribution $\rho(y_e)\to 1$.

It can be seen from the figures that the NUFM model reproduces the central dip of the rapidity distribution of heavier particles (proton and deuteron) in agreement with the experimental findings, while for light particles (pions) there is a peak instead of dip at central rapidity. Note that the appearance or disappearance of central dip is insensitive to the rapidity limit y_{e0} but depends strongly on the magnitude of the ellipticity e and the mass m of the produced particles. For the heavier particles (proton and deuteron) a central dip appears for $e<0.8$, but for light particles (pions) there is no dip even when e is as small as 0.35, cf. Table I and Fig. 6.

On the other hand, the width of the rapidity distributions is mainly controlled by the amplitude y_{e0} of the longitudinal flow. A single value (1.35) of y_{e0} can account for the wide distribution of heavier particles (protons and deuterons) and at the same time fit the pion-distribution well. The difference in the width of the dN/dy distribution originates mainly from the difference in the mass of the particles [3,4].

In Fig. 8 are shown the rapidity distributions of pions and protons for Au+Au collisions at 10.8*A* GeV/*c* [5]. The dotted and solid lines (band) correspond to the results of CSFM and NUFM models (with parameters listed in Table I), respectively. In the calculation the CSFM results we have also used the NUFM model with the same rapidity limit y_{e0} listed in Table I but with ellipticity $e=1$. The histogram is the result from the RQMD model.

It can be seen from Fig. 8 that in the NUFM model there

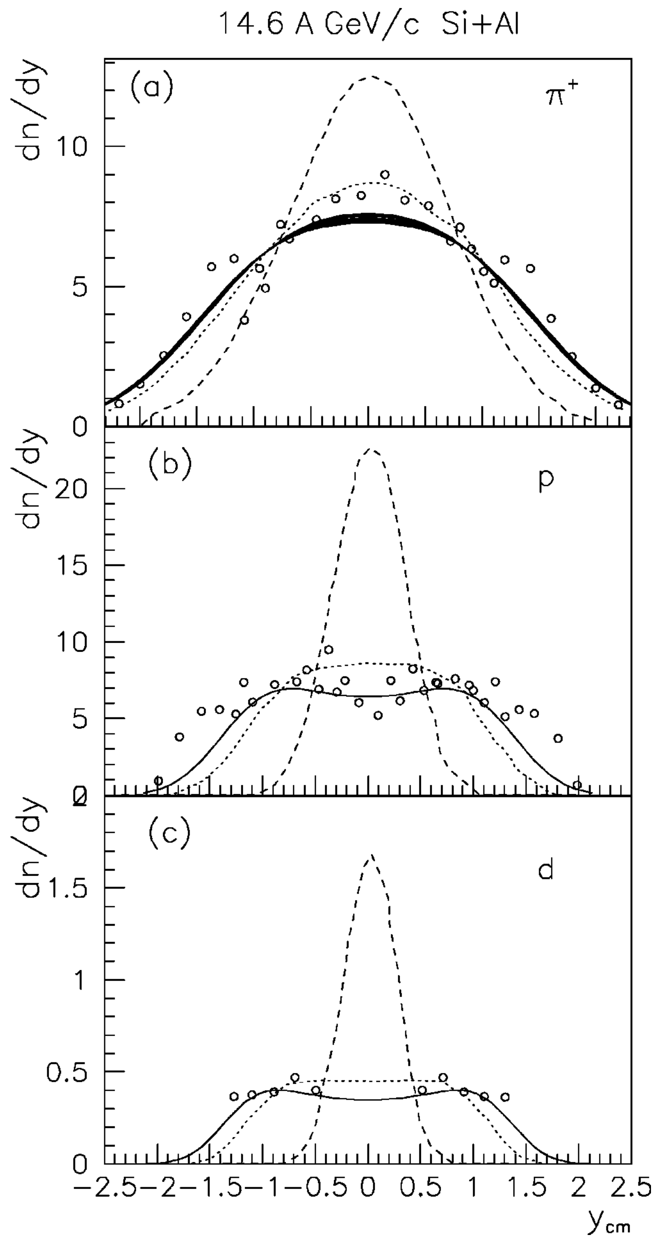

FIG. 6. Rapidity distributions for central 14.6*A* GeV/*c* Si+Al collisions. Open circles are the experimental data for Si+Al collisions taken from Refs. [9,10]. Dashed, dotted, and solid lines are the distributions from the isotropical thermal model, cylindrical-symmetric longitudinal flow model (CSFM), and nonuniform longitudinal flow model (NUFM), respectively. (a), (b), and (c) are the pion, proton, and deuteron distributions, respectively. The ellipticity *e* for pion is within a range [0.35,0.7] and therefore the NUFM results are present as a band. The temperature $T=0.12$ GeV.

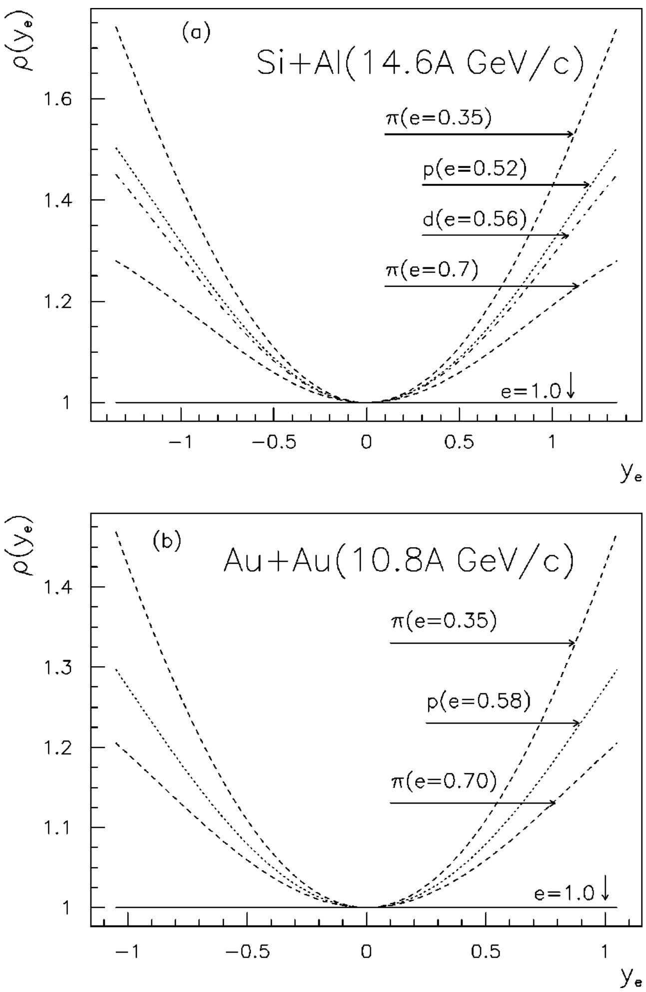

FIG. 7. The fireball distribution functions $\rho(y_e)$ versus rapidity y_e in the nonuniform flow of different particles for the collisions of Si+Al at 14.6*A* GeV/*c* (a) and Au+Au at 10.8*A* GeV/*c* (b), respectively. For pion the distribution functions $\rho(y_e)$ are only plotted for the two boundaries of the used region $0.35 \leqslant e \leqslant 0.7$.

is a shallow dip in the central rapidity of the distribution of proton, instead of a central peak as predicted by the CSFM model. However, the presently available experimental data are restricted to the large rapidity. The peak at central rapidity is the extrapolation of data using RQMD and is model dependent. It is interesting to see whether the prediction of a central dip (plateau) or a central peak will be observed in future experiments.

Comparing the parameter values for Si-Al (smaller colliding nuclei) and Au-Au (larger colliding nuclei) collisions listed in Table I, it can be seen that the rapidity limit y_{e0} is

TABLE I. The value of model parameters.

	Si-Al collisions			Au-Au collisons	
Parameter	π	p	d	π	p
e	0.35–0.7	0.52	0.56	0.35–0.7	0.58
y_{e0}	1.35	1.35	1.35	1.05	1.05
y_0	1.15	1.15	1.15		

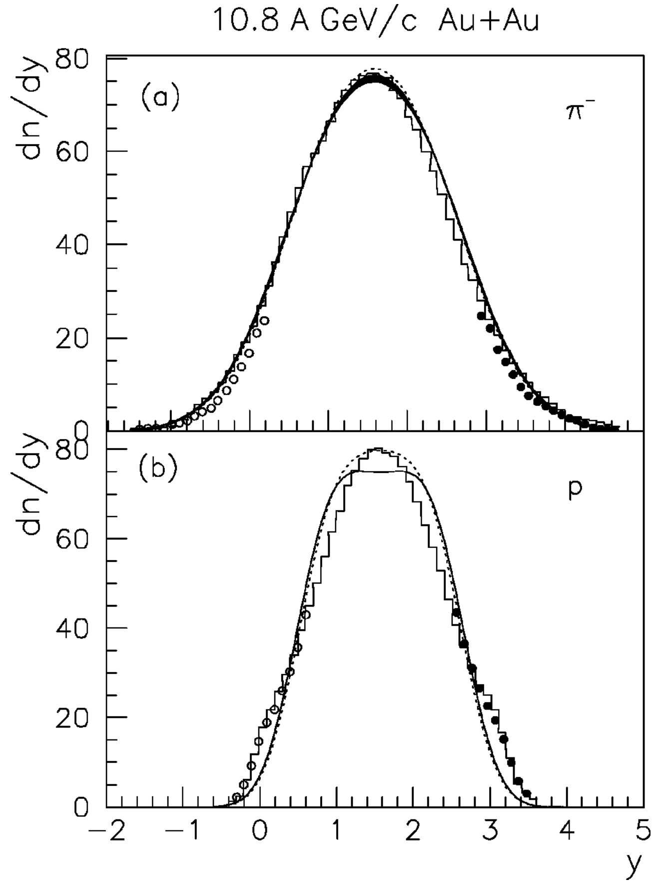

FIG. 8. Rapidity distributions for pions (a) and protons (b) in central Au+Au collisions at 10.8*A* GeV/*c*. Full circles represent the experimental data taken from Ref. [5]. Open circles are the reflection of the data. The solid line is our calculation using the NUFM model. The ellipticity e for pion is within a range [0.35,0.7] and therefore the results for pion are present as a band. The histogram shows the results from RQMD and the dotted lines are the results from the prediction of CSFM model. The temperature $T=0.14$ GeV.

smaller and the ellipticity e for proton is bigger for the larger colliding nuclei than for the smaller ones. Both of these two show that the hadronic system formed from the larger colliding nuclei is less spread out in rapidity, i.e., there is stronger nuclear stopping in the collision of larger nuclei.

IV. SUMMARY AND CONCLUSIONS

In high energy heavy ion collisions, due to the transparency of nucleus the participants will not lose the historical memory and the produced hadrons will carry some of their parent's memory of motion, leading to the unequivalence in longitudinal and transverse directions. So it is reasonable to assume that the flow of produced particles is privileged in the longitudinal direction. This picture has been used by lots of models [3,8]. Here we would mention two thermal and hydrodynamic models, one is the boost-invariant longitudinal expansion model postulated by Bjorken [8] which can explain such an anisotropy already at the level of particle production in hadron-hadron collisions. This model has been formulated for asymptotically high energies, where the rapidity distribution of produced particles establishes a plateau at midrapidity. The second model is the CSFM postulated first by Schnedermann, Sollfrank, and Heinz [3] which accounts for the anisotropy of longitudinal and transverse direction by adding the contribution from a set of fireballs with centers located uniformly in the rapidity region $[-y_0,y_0]$ in the longitudinal direction, sketched schematically in Figs. 1 and 2 as dashed circles. It can account for the wider rapidity distribution when comparing to the prediction of the pure thermal isotropic model but fails to reproduce the central dip in the proton and deuteron rapidity distributions.

In this paper, we argue that the transparency/stopping of relativistic heavy ion collisions should be taken into account more carefully. It will not only lead to the anisotropy in longitudinal-transverse directions, but also render the fireballs (especially for those of proton and deuteron) concentrate more in the direction of motion of the incident nuclei. A nonuniform longitudinal flow model is proposed, which assumes that the centers of fireballs are distributed nonuniformly in the longitudinal phase space. An ellipticity parameter e is introduced through a geometrical parametrization which can express the nonuniformity of flow in the longitudinal direction, i.e., the centers of fireballs of proton and deuteron prefer to accumulate in the two extreme rapidity regions ($y_e \approx \pm y_{e0}$) in the c.m.s. frame of relativistic heavy ion collisions, and accordingly the distribution is diluted in the central rapidity region ($y_e \approx 0$).

It is found that the depth of the central dip depends on the magnitude of the parameter e and the mass of produced particles, i.e., the nonuniformity of longitudinal flow which is described by the parameter e determines the depth of the central dip for heavier particles. On the other hand, the central peak in the pion distribution turns out to be insensitive to the value of e and can be well reproduced from this model simultaneously with the central dip in the proton and deuteron rapidity distributions.

Through comparing the feature of collision systems of different size, it is found that the longitudinal flow amplitude are smaller for the heavier collision system than for the lighter ones, which suggests, together with the larger e, a larger stopping in the bigger collision system.

ACKNOWLEDGMENTS

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