

Dependence of the shell-model single-particle energies on different components of the nucleon-nucleon interaction

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The spin-orbit splittings in the spectra of nuclei with mass numbers 5, 15, and 17 are studied within the framework of shell-model configuration mixing calculations including $2\hbar\omega$ excitations. The contributions of the two-body spin-orbit and tensor components of the nucleon-nucleon interaction are studied in various model spaces. It is found that the effects of the two-body spin-orbit interaction are dominant and quite sensitive to the size of the model space considered. The effects of the tensor interaction are weaker. The correlations effects which are included in the larger $(0+2)\hbar\omega$ shell-model space reduce the spin-orbit splitting in the case of $A=5$ by 20%, and enhance it for $A=15$ by about the same 20%. However, it is found that the correlations have a very small effect on the $d_{3/2}-d_{5/2}$ splitting in $A=17$.

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I. INTRODUCTION, BACKGROUND, AND MOTIVATION

The problem of a microscopic understanding of the spin-orbit splitting in the spectra of nuclei and its relation to the nucleon-nucleon interaction has received a lot of attention over many years. As an example we quote from Bohr and Mottelson [1], “Finally, the tensor force contributes in second-order and higher order to the effective one-body spin-orbit potential.” They refer to the 1960 work of Terasawa *et al.* [2,3]. Indeed, using the tensor interaction, these authors obtained a large spin-orbit splitting with the correct sign and level ordering. However, Terasawa noted that other groups got very small effects, some even of the opposite sign. One of the main motivations for Terasawa’s work [2] was his feeling at that time (i.e., 1960) that it was not clear to what extent a two-body spin-orbit force would be required to explain the nucleon-nucleon (NN) data.

The contributions to the spin-orbit splitting in the single-particle energies for closed shell nuclei, which arise within the framework of a nonrelativistic solution of the nuclear many-body problem based on two-nucleon interactions, have been investigated by Scheerbaum [4,5]. His investigations utilized an effective interaction [6] which corresponds to a parametrization of the Brueckner G matrix derived from realistic interactions like the Reid soft-core potential [7]. Scheerbaum demonstrated that a large part of the spin-orbit splitting can be attributed to the effective NN spin-orbit interaction contained in the Brueckner G matrix. This contribution occurs already in the mean field or Brueckner-Hartree-Fock (BHF) approach.

In his work, Scheerbaum found that another important contribution to the spin-orbit splitting was related to the tensor component in the G matrix. This tensor force does not contribute to the spin-orbit splitting of spin-saturated nuclei within the mean-field approximation. A strong tensor force, however, leads to a sizable contribution of second order in this effective interaction. Since all particle-particle ladder diagrams are already included in the Brueckner G matrix,

Scheerbaum only considered terms of second order in G with intermediate hole-hole states. He observed that these terms of second order in G lead to a contribution to the spin-orbit splitting which is almost as large as the effect due to the spin-orbit component in the effective two-nucleon interaction [4].

However, most of the so-called realistic NN interactions, which were considered around 1970, for instance the Reid soft-core potential [7], contain a rather strong tensor component originating from the one-pion-exchange contribution. On the other hand, one-boson-exchange models for the NN interaction that have been developed more recently [8] take into account the fact that this tensor component, originating from the one-pion exchange, is compensated to some extent by the contribution of the ρ -meson exchange, which yields a tensor contribution with an opposite sign. Therefore modern NN interactions contain a weaker tensor component than did previous ones like the Reid potential. This is one motivation to reanalyze the contribution to the spin-orbit splitting of the various components of the NN interaction using a modern model of the NN interaction.

A second motivation for our studies in the present paper is to investigate in a consistent and nonperturbative way the effect on the spin-orbit splitting of long-range correlations induced by the tensor force and other components of the NN interaction. The short-range correlations leading to configurations with high-lying single-particle states are efficiently taken into account by means of the Brueckner G matrix. However, the effects of long-range correlations, involving shell-model configurations with lower excitation energy, may require an explicit treatment. Therefore one often splits the Hilbert space of all shell-model configurations into a model space (which includes, using the terminology of harmonic oscillator states, all configurations up to $n\hbar\omega$) and the rest of the Hilbert space. Correlations related to configurations outside the model space are treated by determining an effective Hamiltonian, which can be diagonalized within the model space.

A first approximation for this effective Hamiltonian is to

consider the Brueckner G matrix calculated from a solution of the Bethe-Goldstone equation with a Pauli operator adjusted to the model space. Such effective Hamiltonians, based on the Bonn A and Bonn C [8] potentials have been considered by Zamick *et al.* [9]. They investigated the single-particle energies using the BHF approximation supplemented by an explicit treatment of two-particle – one-hole and three-particle–two-hole diagrams for configurations inside the model space. Inspecting the spin-orbit splitting, those authors found a strong cancellation between the two-hole diagrams and the two-particle diagrams. The effect of the hole-hole terms, which in agreement with the findings of Scheerbaum [4] enhanced the spin-orbit splitting, was essentially compensated for by the corresponding particle-particle terms.

The question is, does this cancellation hold beyond second-order perturbation theory? To answer this question we are going to diagonalize the effective Hamiltonian in model spaces including configurations beyond one major shell. Furthermore, we want to study the relative importance of the various terms in the effective NN interaction, the central, tensor, and spin-orbit parts of the NN interaction, as they contribute to the spin-orbit splitting. For that purpose we shall use a parametrization of the model-space G matrix derived by Zheng and Zamick [10], which has the form

$$V(r) = V_c(r) + x \cdot V_{s.o.} + y \cdot V_t, \quad (1)$$

where $s.o.$ stands for the two-body spin-orbit interaction, t for the tensor interaction, and $V_c(r)$ is everything else, especially the (spin-dependent) central interaction. The interaction terms V_c , $V_{s.o.}$, and V_t have been adjusted so as to obtain a good fit to the G -matrix elements for the Bonn A potential with $x=1$, $y=1$. We can study the effects of the spin-orbit and tensor interactions by varying x and y . More details about the interaction are given in Ref. [10].

The diagonalization of the effective Hamiltonian in large model spaces is achieved by employing the OXBASH program [11], taking care of spurious states by using the Gloeckner-Lawson technique [12].

In our present paper, we investigate the spin-orbit splitting within the nonrelativistic many-body approach using realistic two-body NN interactions. It has been demonstrated by Pieper *et al.* [13], that three-nucleon forces may enhance the spin-orbit splitting, bringing it close to the experimental value in the case of $p_{1/2}$ and $p_{3/2}$ hole states in $A=15$. In fact they obtained a contribution of 2.84 MeV to this spin-orbit splitting from the Urbana VII model for the two-pion exchange three nucleon force [14].

Another mechanism, which may be very important to the spin-orbit splitting, is the change of the Dirac spinors for the nucleons in the nuclear medium as predicted by the Dirac-Brueckner-Hartree-Fock approach [15,16]. The strong and attractive scalar component in the relativistic self-energy for the nucleon leads to an enhancement of the small components for the Dirac spinor in the nuclear medium, which may be characterized in terms of a reduced Dirac mass for the nucleons. This leads to an enhancement by about 2 MeV of the spin-orbit splitting of the $p_{1/2}$ and $p_{3/2}$ hole states in $A=15$ [9].

TABLE I. The effective spin-orbit splitting $ESO = E(1/2^-) - E(3/2^-)$ for $A=5$ varying y (strength of tensor force) and x (strength of the two-body spin-orbit interaction).

x	y	ESO [MeV]		
		$0 \hbar \omega$	$(0+2) \hbar \omega$	$(0+2+4) \hbar \omega$
1	1	3.375	2.959	2.659
0	0	0	0	0
0	0.5	0	0.046	0.050
0	1	0	0.216	0.230
0	1.5	0	0.542	0.572
0	2	0	1.034	1.092
0	3	0	2.457	2.640
0.5	0	1.688	1.375	1.238
1	0	3.375	2.716	2.431
1.5	0	5.063	4.012	3.584

II. RESULTS

A. The $A=5$ system

We present in Table I the results of our shell-model calculations for the spin-orbit splitting $\Delta E = E(1/2^-) - E(3/2^-)$ in mass $A=5$, considering three different model spaces: using a harmonic oscillator notation these model spaces are characterized by the excitation energies of the shell-model configurations that are included and denoted as $0 \hbar \omega$, $(0+2) \hbar \omega$, and $(0+2+4) \hbar \omega$, respectively. Thus $0 \hbar \omega$ corresponds to the case where we have a closed $0s$ shell and the one valence particle is in $0p_{3/2}$ or $0p_{1/2}$. For $(0+2) \hbar \omega$ we have the above valence configuration plus all $2 \hbar \omega$ excitations, etc. In our case, due to computational limitations, the $(0+2+4) \hbar \omega$ space includes only the $0s$, $0p$, $0s-1d$, and $0f-1p$ shells, and is thus not quite complete. This is also true for the $(0+2) \hbar \omega$ space in ^{17}O .

For the parametrization of the realistic G matrix [$x=1$, $y=1$ in Eq. (1)] we find that the values for the spin-orbit splitting ΔE decrease with increasing sizes of the model spaces. The values listed in Table I are 3.375, 2.959, and 2.659 MeV in the 0 , $(0+2)$, and $(0+2+4) \hbar \omega$ spaces, respectively. Thus, in higher order, we get a noticeable *reduction* of the effective spin-orbit splitting for $A=5$. What is the cause of this reduction? Is it the two-body tensor interaction in play or the two-body spin-orbit interaction? To answer this question we performed shell-model calculations varying the strength of the two-body spin orbit and the tensor interaction in terms of the variables x and y as defined in Eq. (1) and again show our results in Table I.

For $x=0$, $y=0$, there is *no* effective ‘‘spin-orbit’’ splitting (ESO) in any of the model spaces. This reflects the fact that a central interaction, indeed even a *spin-dependent* central interaction, cannot induce any ESO even if correlations in large model spaces are considered.

We also note that, in the $0 \hbar \omega$ space, the ESO is zero when $x=0$. The tensor interaction does not contribute to the ESO for a spin saturated system (i.e., for a closed LS shell plus or minus one particle) if the mean-field approximation is

considered (which corresponds to the calculation in the $0\hbar\omega$ space). As we vary y (keeping $x=0$), we see an approximate quadratic rise in the effective spin-orbit splitting in each of the larger model spaces. In fact, the rise is a bit faster than quadratic in y . This shows that the tensor force contributes to the ESO only in second order and higher order of a perturbation expansion, with the dominant terms being of the form $V_t \times V_t$ in the notation of Eq. (1). The contribution of the tensor force to the ESO has the correct sign. However, that contribution is rather small for all reasonable values of y (i.e., for $y \leq 1$), and increases only by 5–9% for any one of our values of y in Table I when we include configurations of $4\hbar\omega$.

In Table I we also study in the $A=5$ system the effects of varying the two-body spin-orbit strength x in the absence of the tensor interaction (i.e. for $y=0$). In the $0\hbar\omega$ space, the ESO varies linearly with x . We see the linear relation between the ESO and the two-body spin-orbit interaction in the mean-field approach. Interestingly, also in the larger spaces the ESO varies almost linearly with x . This indicates that a perturbative inclusion of these correlations in the larger model space would be dominated by terms of the form $V_c \times V_{s.o.}$ using the nomenclature of Eq. (1).

Perhaps the most important result of Table I is that for $A=5$, there is a systematic *decrease* in the spin-orbit splitting as one goes to larger model spaces. For example, in the 0 , $(0+2)$, and $(0+2+4)\hbar\omega$ spaces the values of the ESO for $x=1$ ($y=0$) are 3.375, 2.716, and 2.431 MeV, respectively. In each of the three cases that we studied in Table I: $(x,y) = (0.5,0)$, $(1,0)$, $(1.5,0)$, we find that going from $0\hbar\omega$ to $(0+2)\hbar\omega$ decreases the ESO by about 20%, and going from $(0+2)\hbar\omega$ to $(0+2+4)\hbar\omega$ in each case further decreases the ESO by about 10%. The percentages of change are the same in all three cases due to the linearity of the ESO's with x .

While there has been some discussion of the enhancement of the spin-orbit interaction for $A=5$ due to second-order tensor effects [2], we are not aware of any discussion of the spin-orbit interaction in higher order.

We see the combined effects of the spin-orbit and tensor interactions by comparing the $x=1$, $y=1$ case in Table I with the $x=1$, $y=0$ case. The small effects of the tensor force can essentially be added to the results obtained with the central plus spin-orbit interaction. This is true both in the $(0+2)\hbar\omega$ and in the $(0+2+4)\hbar\omega$ spaces. In each case, the contribution of the tensor interaction is less than 10% of that of the spin-orbit interaction.

The observed values for the $1/2^- - 3/2^-$ level separation in the $A=5$ system have large experimental uncertainties, being 7.5 ± 2.5 MeV for ${}^5\text{Li}$ and 4.0 ± 1.0 MeV for ${}^5\text{He}$ [17]. The calculated results in Table I agree with the observed data better for $x=1.5$ than for $x=1.0$. Such an enhancement of the strength of the two-body spin-orbit interaction in actual nuclei was also suggested for nuclei in the beginning of the $1s-0d$ shell by the work of Fayache *et al.* [18].

B. The $A=15$ System

Next we consider the $E(3/2_1^-) - E(1/2_1^-)$ splitting in mass 15. In lowest order ($0\hbar\omega$) these states are described as hole

TABLE II. The $3/2^- - 1/2^-$ splitting in $A=15$ with various x and y combinations.

x	y	ESO [MeV]	
		$0\hbar\omega$	$(0+2)\hbar\omega$
1	1	5.063	5.698
0	0	0	0
0	0.5	0	-0.002
0	1	0	-0.009
0	1.5	0	-0.019
0	2	0	-0.036
0	2.5	0	-0.059
0	3	0	-0.088
0.5	0	2.531	3.026
1	0	5.063	6.008
1.5	0	7.593	8.934

states relative to the closed shell nucleus ${}^{16}\text{O}$. In that picture, the ground state of the $A=15$ system is a $p_{1/2}$ hole, and the first excited state is a $p_{3/2}$ hole. The results for the calculations in the $0\hbar\omega$ and $(0+2)\hbar\omega$ spaces are presented in Table II.

In the $A=15$ system, and for $y=0$ (no tensor interaction), the ESO is linear in x in the $0\hbar\omega$ space and very nearly linear in x in the $(0+2)\hbar\omega$ space. For the $x \neq 0$, $y=0$ cases, for *each* x value ($x=0.5$, 1.0 , or 1.5), the ESO for $A=15$ *increases* by about 20% as we go from $0\hbar\omega$ to $(0+2)\hbar\omega$. We recall that under these circumstances, the ESO for the $A=5$ system *decreased* by about 20%. We again understand in the $A=15$ system that the percentages of change for the three x values ($x \neq 0$, $y=0$) are the same because of the linearity of the ESO's with x (for $y=0$) in both of the spaces considered. However, it is interesting to note that (but harder to explain why) there is an increase in $A=15$ but a decrease in $A=5$, and also why the percent change in both systems is the same (about 20%). The linearity with x (for $y=0$) indicates again that the corrections to the ESO's in second order are dominated by terms of the form $V_c \times V_{s.o.}$.

For the $A=15$ system, when we vary the tensor interaction with the spin-orbit interaction turned off ($x=0$), we get again a nearly quadratic dependence of the ESO on y . Once more, the magnitude of the ESO rises slightly faster than quadratically with y . This shows that, for the $A=15$ system as well, the tensor force contributes to the ESO only in second and higher orders of a perturbation expansion with the dominant terms being of the form $V_t \times V_t$. The contribution of the tensor term by itself to the ESO (i.e., when $x=0$) is very small (an order of magnitude smaller than its already small contribution in the $A=5$ case), and has the wrong sign.

For the $A=15$ system, and in the $0\hbar\omega$ space where the tensor interaction *cannot* contribute, the ESO is 5.063 MeV for both the $x=1$, $y=0$ and the $x=1$, $y=1$ cases. In the $(0+2)\hbar\omega$ space the ESO is 6.008 MeV for $x=1$, $y=0$ and 5.698 MeV for $x=1$, $y=1$. We thus see from Table II that the effect of the tensor interaction (with $y=1$) is more sig-

TABLE III. The $3/2^+ - 5/2^+$ splitting in $A=17$, as well as the $1s_{1/2}$ energy (relative to $0d_{5/2}$ for various x and y combinations).^a

x	y	$0\hbar\omega$	$(0+2)\hbar\omega$	$0\hbar\omega$	$(0+2)\hbar\omega$
x	y	ESO [MeV]		$E(1s_{1/2}) - E(0d_{5/2})$ [MeV]	
1	1	5.562	5.662	-0.119	-1.430
0	0	0	0	-2.343	-3.853
0	0.5	0	-0.005	-2.343	-3.806
0	1	0	-0.010	-2.343	-3.661
0	1.5	0	-0.004	-2.343	-3.419
0	2	0	0.024	-2.343	-3.085
0	2.5	0	0.078	-2.343	-2.671
0	3	0	0.160	-2.343	-2.195
0.5	0	2.782	2.849	-1.231	-2.723
1	0	5.562	5.689	-0.119	-1.618
1.5	0	8.344	8.522	0.994	-0.534

^aIn all cases, at the $0\hbar\omega$ level, $E_{c.m.} - E(1s_{1/2}) = 2.343$ MeV. For the $(0+2)\hbar\omega$ case, for finite x but with $y=0$ there is at most a 3% deviation from the $x=0, y=0$ value of 3.853 MeV.

nificant (-0.32 MeV) when the spin-orbit interaction is present ($x=1$) than when the spin-orbit interaction is absent (-0.009 MeV for $x=0$). For $A=15$, and unlike the $A=5$ case, the tensor and the spin-orbit effects are not additive, indicating the presence in $A=15$ (but not in $A=5$) of a larger second-order term of the form $V_{s.o.} \times V_t$.

When for $A=15$ we consider the realistic interaction $x=y=1$, we see again that in both spaces the ESO is very largely due to the spin-orbit interaction, while the effect of the tensor interaction is small and of the wrong sign. In contrast to the $A=5$ case, the ESO in the $A=15$ system for $x=1, y=1$ is larger in the $(0+2)\hbar\omega$ space than in the $0\hbar\omega$ space, with most of the enhancement again due to the spin-orbit interaction.

For $A=15$, the observed $E(3/2_1^-) - E(1/2_1^-)$ splitting is 6.324 MeV for ^{15}N and 6.176 MeV for ^{15}O [17]. The results of Table II suggest that for $A=15$, including the $2\hbar\omega$ excitations and taking $x=1$ lead to results in closer agreement with the observed level separations.

C. The $A=17$ System

The results of calculations of the ESO for $A=17$, considering the spin-orbit partners are $0d_{5/2}$ and $0d_{3/2}$, are given in Table III. For the realistic $x=1, y=1$ interaction, there is hardly any change (a mere increase of 0.1 MeV) in the ESO in going from $0\hbar\omega$ to $(0+2)\hbar\omega$. Again, for $y=0$ the ESO's are proportional to x in the $0\hbar\omega$ space and very nearly so in the $(0+2)\hbar\omega$ space. For all the $y=0$ cases, the effect on the ESO's of going from $0\hbar\omega$ to $(0+2)\hbar\omega$ is an increase, but by less than 3%. The small enhancement of the ESO for $x=1, y=1$ as we go to the larger space is again largely due to the two-body spin-orbit interaction. The effect of the tensor force is again very weak; for $x=0$ and a variable y , the ESO is again zero in the $0\hbar\omega$ space and has a magnitude of 0.02 MeV or less for $y \leq 2$ in the $(0+2)\hbar\omega$ space. For $x=0$ in the $(0+2)\hbar\omega$ space, the behavior of the ESO as a

function of y is rather complicated and even changes sign. It starts from $y=0$ by being very slightly negative and reaches a minimum of about -0.01 MeV for $y \approx 1$. The ESO then increases as y increases further, becoming positive for $y \geq 1.63$. Indeed, for $x=0$ and $y \geq 1.63$, ΔESO shows a rapid but less than quadratic increase with Δy . These last observations can be taken as a possible indication that there is a cancellation between two effects. This would support the results of the calculations of Ref. [9] in which Zheng *et al.* observed that in perturbation theory there is a similar cancellation between the contributions from two-particle-one-hole states and those from three-particle-two-hole states.

It is interesting to study the variation with x and y of two other quantities in the $A=17$ system. One is $E(1s_{1/2})$, the energy of the $1s_{1/2}$ level, and the other is $E_{c.m.}$, the energy of the center of mass of the $0d_{5/2} - 0d_{3/2}$ spin-orbit doublet, where

$$E_{c.m.} \equiv 0.6E(0d_{5/2}) + 0.4E(0d_{3/2}). \quad (2)$$

In the $0\hbar\omega$ space, both the $E(1s_{1/2})$ and the $E_{c.m.}$ are strictly independent of both x and y . In that small space (with one particle being outside a closed-shell core and no particle-hole excitations), the two-body tensor force has no effect on the $E(1s_{1/2})$, $E(0d_{5/2})$, or $E(0d_{3/2})$, and hence no effect on the $E_{c.m.}$. This explains why in Table III both the ESO and $E(1s_{1/2}) - E(0d_{5/2})$ are independent of y in the $0\hbar\omega$ space.

Furthermore, in the $0\hbar\omega$ space the two-body spin-orbit interaction acts like a one-body spin-orbit force, and thus it has no effect either on the $E_{c.m.}$ or on the energy of the $1s_{1/2}$ level which has $l=0$. Hence, in the $0\hbar\omega$ space, both the $E_{c.m.}$ and the $E(1s_{1/2})$ are unaffected also by changes in x . As x increases from $x=0$, $E(0d_{5/2})$ decreases by an amount δ , proportional to x , while $E(0d_{3/2})$ increases by 1.5δ , so that indeed $E_{c.m.}$ is left unchanged.

In the $0\hbar\omega$ space we calculate that for all x, y values

$$E_{c.m.} - E(1s_{1/2}) = 2.343 \text{ MeV.} \quad (3)$$

This energy difference is due to the attractive central force component $V_c(r)$ in the effective NN interaction of Eq. (1). The experimental data [17] for ^{17}O shows a $5/2^+$ ground state with excitation energies (in MeV) of 0.871 for $1/2^+$ and 5.084 for $3/2^+$. With this observed data, $E_{c.m.} - E(1s_{1/2})$ is equal to 1.465 MeV. From Eqs. (2) and (3) and the definition of the ESO, we obtain in the $0\hbar\omega$ space the following relationship:

$$[E(1s_{1/2}) - E(0d_{5/2})] - 0.4 \text{ ESO} = -2.343 \text{ MeV.} \quad (4)$$

All the $0\hbar\omega$ data in Table III is fitted perfectly by this relationship. In the $(0+2)\hbar\omega$ space, and for $x=0$ $y=0$, we have $E(0d_{5/2}) = E(0d_{3/2}) \equiv E_{c.m.}$, and due to the central force term in Eq. (1) we calculate that

$$E_{c.m.} - E(1s_{1/2}) = 3.853 \text{ MeV.} \quad (5)$$

In the $(0+2)\hbar\omega$ space, and keeping $y=0$, we note that as x increases a relationship similar to Eq. (4) but with -2.343 replaced by -3.853 holds to within 3% or better (see Table III). In this large space, however, and keeping $x=0$, we note that an increase in y also renormalizes the central force term. All three energies $E(0d_{5/2})$, $E(0d_{3/2})$, and $E(1s_{1/2})$ decrease in the large space with increasing y . The $E(0d_{5/2})$ and the $E(0d_{3/2})$ (and hence the $E_{c.m.}$) all decrease at the same rate. Hence, in Table III, and for $(0+2)\hbar\omega$, the ESO's are very small for $x=0$ and any y . But the above rate of decrease is about 15% larger than the corresponding rate of decrease for the $E(1s_{1/2})$. Hence, for $x=0$, as y increases in Table III, the separation $E(1s_{1/2}) - E(0d_{5/2})$, which is calculated to be negative for $y=0$ in the $(0+2)\hbar\omega$ space, becomes less negative with increasing y .

III. ADDITIONAL REMARKS AND SUMMARY

To summarize our paper, we have shown that the contribution to the effective spin-orbit splitting of the two-body tensor term in the effective interaction is generally much smaller than the contribution from the two-body spin-orbit term. This is consistent with modern NN interactions which have weaker tensor components. An earlier investigation [4,5] used older models of realistic interactions with stronger tensor components and obtained a much larger contribution from the tensor term to the effective spin-orbit splitting.

We see in our nonperturbative calculations that the effects of higher-shell admixtures on the ESO's cannot be ignored, being generally in about the 10–20% range for $A=5$ and $A=15$, but less than 3% for $A=17$.

Additional insights into some of our results are provided by the perturbative work of Ref. [9]. A comparison with their

results suggests similarities between the behavior of the second-order two-body spin-orbit interaction term and some Hartree-Fock-type diagrams. For both the $A=5$ and $A=15$ systems, there do not seem to be nearly complete cancellations of the effects of particle-particle admixtures and hole-hole admixtures, which respectively tend to reduce and enhance the ESO [9] (see also Ref. [4]). In the $0p$ -shell nuclei, the reduction effects are more important for $A=5$ (one particle beyond a closed shell), while the enhancement effects prevail for $A=15$ (one hole away from a closed shell). However, for $A=17$ the cancellation is nearly complete, and going from $0\hbar\omega$ to $(0+2)\hbar\omega$ increases the ESO by less than 3%.

We found that even in the larger $(0+2)\hbar\omega$ space there is an almost linear relationship between the ESO and the strength of the two-body spin-orbit component of the effective interaction. On the other hand, in this larger space, the ESO's dependence on the strength of the two-body tensor component of the effective interaction is close to being quadratic.

Recalling the situation for $A=15$ and $A=17$, we find that the effect of $2\hbar\omega$ configurations yields a larger enhancement for $A=15$ than for $A=17$. The spin-orbit splittings for the particle states tend to be reduced as compared to those for hole states, as noted in Ref. [9]. This is supported from experiment. The splitting $E(3/2_1^-) - E(1/2_1^-) = 6.0$ MeV is larger than the corresponding $A=17$ splitting $E(3/2_1^+) - E(5/2_1^+) = 5.1$ MeV (although the orbital angular momentum is larger in the latter case) and the corresponding $E(1/2_1^-) - E(3/2_1^-) = 4.0$ MeV splitting in $A=5$. Thus large space calculations are essential in this context to properly account for the differences in spin-orbit splittings of single particle states above the Fermi energy and of single hole states below the Fermi energy.

Finally, the correlation effects which are included in the $(0+2)\hbar\omega$ shell-model space *reduce* the contribution to the ESO of the dominant two-body spin-orbit interaction term by about 20% in the $A=5$ system, but *increase* the contribution by about the same 20% in the $A=15$ system. The fact that both magnitudes are essentially the same requires further investigation.

Note added in proof. We are now considering adding a corrective monopole-monopole interaction to the one in this paper in order to investigate how important the Hartree-Fock contributions are.

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