Relativistic contributions to the deuteron photodisintegration in the Bethe-Salpeter formalism

K. Yu. Kazakov* and S. Eh. Shirmovsky†

Laboratory of Theoretical Nuclear Physics, Far Eastern State University, Sukhanov Str. 8, Vladivostok 690600, Russia

(Received 12 June 2000; published 15 December 2000)

In the plane wave one-body approximation the reaction of deuteron photodisintegration is considered in the framework of the Bethe-Salpeter formalism for the two-nucleon system. Results are obtained for deuteron vertex function, which is the solution of the homogeneous Bethe-Salpeter equation with a multirank separable interaction kernel, with a given analytical form. A comparison is presented with predictions of nonrelativistic, quasipotential approaches and the equal time approximation. It is shown that important contributions come from the boost in the arguments of the initial state vertex function and the boost on the relative energy in the one-particle propagator due to recoil.

DOI: 10.1103/PhysRevC.63.014002 PACS number(s): 25.10.+s, 25.20.Dc, 21.45.+v, 11.10.St

I. INTRODUCTION

So far, experiments on elastic and quasielastic lepton-nucleus scattering have been considered as one of the fundamental and reliable resources of probing structure of nuclei over a wide range of energy-momentum transfer. Deuteron photodisintegration $\gamma+d\rightarrow p+n$ holds a most unique position among the reactions of this kind. It has been thoroughly investigated in the region of small and medium energies of the incoming γ quantum (an excellent review of experiments and theoretical frameworks applied to the study of the process can be found in Ref. [1]). The theoretical analysis of experimental data obtained in these kinematic regions has provided important information on the deuteron wave function, allowing us to discriminate various contributions to the differential cross section which stem from meson-exchange currents, isobar configurations, as well as spin-orbit and further relativistic corrections.

As of now, deuteron photodisintegration is rated among the leading trends at experimental facilities around the world. Here it seems worthwhile to point out an experiment performed with the polarized LADON γ ray beam [2] focusing on the existence of the narrow dibaryonic resonances in reaction on nuclei at a low excitation energies; an experimental program carried out on the linearly polarized photon beam at YEREVAN Synchrotron to study the cross section asymmetry of the deuteron photodisintegration process in the energy range 0.9–1.7 GeV for proton center-of-mass angle of 90° [3]. This is connected with the problem of the validity of the constituent quark counting rules for energies of a few GeV. Presently set experiments at SLAC with photons of energy 2.8 GeV [4] and the proposed experimental programs in RCNP Cyclotron with a photon beam at an energy up to 8 GeV [5] allow an investigation of hadronic systems at quark level to be made.

Prospective studies at the TJNAF are approved to extend measurements on the differential cross section to a wide range of reaction angles at high energies. The initial measurements of the cross section from $\gamma+d\rightarrow p+n$ up to 4.0 GeV correlate well with the previous low energy ones [6]. A comparison of the high precision Mainz data [7,8] (over the photon energy range 100–800 MeV) with the meson-exchange models incorporating relativistic effects [9–11] is supposed to show that one still lacks of the ultimate conclusion on the role of non-nucleonic degrees of freedom in nuclei. It is also understood that further development of deuteron photodisintegration theory is needed and consistent treatment of relativistic effects have to be applied. Everything mentioned above demands a construction of a genuine relativistic formalism for the description both the deuteron structure and the reaction mechanism.

The formulation of a completely relativistic formalism of hadronic bound states and reaction with them can be developed on the basis field-theoretical Bethe-Salpeter (BS) equation for the nucleon-nucleon (NN) scattering [12]. However, approximate methods evolved from the BS formalism due to substantial mathematical and computational problems. We bear in mind the quasipotential (QP) approach, which reduces the four-dimensional BS equation to a relativistic three-dimensional equation: the Logunov-Tavkhelidze [13], Blankenbeckler-Sugar (BbS) [14], Gross [15], and other approximations.

Actually, the relativistic description of the reaction with the deuteron has been extensively developed in the framework of the BS formalism, which is explicitly Lorentz covariant, provides two-body unitarity and includes symmetrically nucleon and antinucleon degrees of freedom in a hadronic state. So far applicability of the BS equation to the reactions with the deuteron has been bound to elastic electron-deuteron scattering [12], elastic pd-backward scattering [16], inclusive quasielastic electron-deuteron scattering [17,18], and the description of the static properties of the deuteron [19,20]. What all these processes have in common is that the reaction amplitude in the impulse approximation is proportional to the averaging of current operator between deuteron states. On the contrary, the reaction amplitude for deuteron photodisintegration is a nondiagonal matrix element between the incoming deuteron and outgoing $2N$ state.

An approach to a covariant description of the reaction $\gamma+d\rightarrow p+n$ where the basic degrees of freedom are taken as hadronic, is developed in the framework of the dispersion relation technique for laboratory photon energies $E_\gamma<400$ MeV [21] (this technique is appropriate for the analysis of

*Electronic address: kazakovk@ifit.phys.dvgu.ru

†Electronic address: suskovs@ifit.phys.dvgu.ru

partial wave amplitudes, as well as it takes final state rescatterings into account). Another approach describes deuteron photodisintegration as a simple parametrization of covariant deuteron in terms of a hard component and imposing gauge invariance on the cross section [22]. This approach looks into the onset of scaling in exclusive photodisintegration for high energies in the range 1–4 GeV, as it can be brought about by mechanisms which are different from those of PQCD.

The BS formalism comes up with an alternative idea. Formal application of the BS formalism to deuteron electrodisintegration and photodisintegration can be found in Ref. [23]. Here the rigorous derivation of the scattering amplitude in terms of BS amplitude of the initial and final $2N$ states and Mandelstam electromagnetic (EM) vertex comprising one-body and two-body parts, is proposed. The numerical analysis has not been performed. Reference [24] puts forward a framework based on the BS equation approach. This is applicable to both elastic and inelastic electron scattering. But more complete calculations are performed within a QP framework.

The aim of the present paper is to perform the fully relativistic analysis of deuteron photodisintegration in the framework BS formalism, and then to segregate and estimate contributions of various relativistic effects to the differential cross section. We resolve this issue by using the BS formalism, as well as by comparing different approximations to the BS equation within the context of the relativistic separable interaction kernel. The result, though it is of a comparative character, provides a consistent basis to account for the relativistic effects in the deuteron breakup.

This paper is structured as follows. In Sec. II we briefly discuss a connection between the BS formalism and the QP approach, as well as the equal time (ET) approximation. The above-mentioned relativistic formulations of the $2N$ dynamics, exploiting relativistic separable interactions, are applied to the study of the deuteron and its inelastic observables. Basic formulas for defining the Minkowski-space BS amplitude for bound and $2N$ continuum states are given. We also introduce the formulas for the relativistic separable interaction kernel. Solving the BS equation with the separable interaction, we find the vertex function used in computation of the unpolarized cross section.

In Sec. III the procedure of deriving deuteron photodisintegration cross section is covered. The EM interaction with $2N$ system in the framework of the BS formalism is determined by Mandelstam vertex generally dependent on the properties of the interaction kernel. The scheme incorporates two-body part of Mandelstam vertex in order to guarantee the gauge invariance.

Section IV deals with the derivation of the simplest contribution to the EM current matrix elements of deuteron breakup, namely, the plane wave one-body approximation (PWOA). We are concerned with the transformation properties of the deuteron vertex function between the laboratory and c.m. frames. Generally one-body part of Mandelstam vertex involves half off-mass-shell γNN form factors. However, as a consequence of gauge invariance the real photon scattering amplitude does not contain off-shell effects. In the PWOA the EM current matrix elements are proportional to the deuteron vertex function taken at a certain value of the relative energy (relative time) and three-momentum. These are directly related to the photon energy-momentum transfer. Finally, we have come up with the expression for the differential photodisintegration cross section.

The analysis of the relativistic effects required by the principles of relativity has become the objective of Sec. V. These are relativistic kinematics and dynamics, the relative energy dependence (or retardation), Lorentz contraction, and the spin precession. We compare results of our fully relativistic analysis with those of conventional nonrelativistic (NR) models, the QP approach, and the ET approximation. Our conclusions are drawn in Sec. VI.

II. RELATIVISTIC DESCRIPTION OF TWO-NUCLEON SYSTEM

The starting point for discussing relativistic scattering and bound state problems in the strongly interacting systems is formulation of integral equations for amplitudes. The off-shell T matrix for the elastic scattering of two nucleons with the relative four-momentum p, p', and the total momentum P satisfies to the inhomogeneous BS equation. In the momentum space this is the four-dimensional (4D) integral equation with respect to the relative momentum[1] $k=(k_0,\mathbf{k})$

$$T(p,p';P)=\mathcal{V}(p,p';P)+\frac{i}{4\pi^3}\int d^4k\,\mathcal{V}(p,k;P)G_0(k;P)T(k,p';P), \tag{1}$$

$k=(k_0\mathbf{k})$ where $\mathcal{V}$ is an interaction kernel obtained by summing of all irreducible $2N$ Feynman diagrams in a given field-theoretical model of the NN interaction, and $G_0(k;P)$ — the free two-nucleon propagator

$$G_0(k;P)=\frac{[\hat{P}/2+\hat{k}+m]^{(1)}}{\left(\frac{P}{2}+k\right)^2-m^2+i\epsilon}\,\frac{[\hat{P}/2-\hat{k}+m]^{(2)}}{\left(\frac{P}{2}-k\right)^2-m^2+i\epsilon}. \tag{2}$$

The BS amplitude for the $2N$ scattering state, $P^2=s>4m^2$, is expressed in terms of the half off-shell T matrix and the propagator function G_0 as follows:

$$\chi(k;\mathring{p}P)=[4\pi^3 i\,\delta^{(4)}(\mathring{p}-k)-G_0(k;P)T(k,\mathring{p};P)]\chi^{(0)}(\mathring{p};P),\quad \mathring{p}\cdot P=0, \tag{3}$$

where $\mathring{p}$ denotes the on-mass-shell relative four-momentum and $\chi^{(0)}(\mathring{p};P)$—the amplitude for the motion of free nucleons. The second term comprises rescattering contributions.

[1]For sake of simplicity we omit writing spinor, ρ-spin, and polarization quantum numbers of amplitudes below.

If the two-body system has a bound state of mass M_d, the T matrix has a pole at $P^2=M_d^2$

$$T(p,p';P)\propto\frac{\Gamma(p;P)\bar{\Gamma}(p';P)}{P^2-M_d^2}, \qquad (4)$$

where Γ and $\bar{\Gamma}$ are the vertex function of the $2N$ bound state and its conjugate, respectively. The vertex function satisfies the homogeneous BS equation with the same interaction kernel

$$\Gamma(p;P)=\frac{i}{4\pi^3}\int d^4k\,\mathcal{V}(p,k;P)\psi(k;P), \quad P^2=M_d^2, \qquad (5)$$

where the bound amplitude is defined as

$$\psi(k;P)=G_0(k;P)\Gamma(k;P). \qquad (6)$$

Equations (1) and (5) are manifestly Lorentz covariant and preserve the two-body elastic unitarity. Moreover, both equations do not discriminate between the positive and negative energy states, yielding transformation properties of the amplitudes consistent with the charge conjugation and time invariance.

Although the BS calculations are feasible, a rigorous treatment of Eqs. (1) and (5) is rather complicated due to the appearance of the relative energy in loop integrals and the presence of strong singularities in the interaction kernel. The great theoretical effort have been made to reduce the 4D bound-state equation to a 3D one. A productive method to obtain an approximate 3D equation has been developed by the QP approach, where the BS equation is reduced to the 3D one by using a new two-nucleon propagator with the internal relative energy variable restricted to a fixed value.

But there are several shortcomings in the relativistic equations obtained from the BS equation via the 3D reduction. First of all, one meets conceptual difficulties with the consistent treatment of both the $2N$ system and its EM interactions [25]. Secondly, by putting particles on the mass shell or using the positive energy projection operators, one results in a equation which violates the charge conjugation and CPT symmetries. Importance of the constraint put by discrete symmetries is discussed in Ref. [26].

An alternative choice, which has been made to study the EM interactions, is the ''instant'' or the equal time approximation [24,27]. The instant-ET approximation to EM current operator is the 3D reduction consistent with charge conjugation and unitarity. This approach has been applied to study of the elastic electron-deuteron scattering and the deuteron breakup in the electrodisintegration process.

We extend the ET approximation to describe the process of deuteron photodisintegration in a systematic way. In the plane wave one-body approximation that can be done by replacing the initial deuteron vertex function by the BbS one. The relative energy variable in the one-particle propagator is fixed by the condition that the final state describes the on-mass-shell particles [24]. The covariant BbS prescription, $P\cdot\hat{k}=0$, puts the relative energy equal to

$$\hat{k}_0=\frac{1}{2P_0}(E^2_{(1/2)\mathbf{P}+\mathbf{k}}-E^2_{(1/2)\mathbf{P}-\mathbf{k}}),$$

where $E_{\mathbf{k}}=\sqrt{m^2+\mathbf{k}^2}$ and $\hat{k}$ denotes the restricted four-vector k. The prescription leads to the relativistic equation of motion for the moving $2N$ system. By means of Lorentz boost it is transformed to the Schrödinger-type equation in the rest frame.[2] The BbS prescription is simple only in the $2N$ rest frame, $P_{(0)}=(\sqrt{s},\mathbf{0})$, where the BbS propagator $G_{\rm BbS}(\hat{k};P)$ is given by

$$G_{\rm BbS}(\hat{k};P_{(0)})=-2\pi i\,\delta(\hat{k}_0)\mathcal{G}(\hat{k};P_{(0)})\Lambda^{(1)}(\mathbf{k}_1)\Lambda^{(2)}(\mathbf{k}_2) \qquad (7)$$

with $\Lambda^{(i)}(\mathbf{k}_i)$ being the positive energy projection operator for the ith particle and $\mathcal{G}(\hat{k};P_{(0)})=4m^2/E_{\mathbf{k}}(s-4E^2_{\mathbf{k}}+i\varepsilon)$. The delta function in Eq. (7) sets both nucleons equally off the mass shell. It is an attractive feature in case of the deuteron because it treats both nucleons symmetrically and, as a consequence, it is consistent with the Pauli principle.

The QP wave function is defined in terms of the bound state vertex function $\hat{\Gamma}(\hat{k};P)$ defined in terms of the positive energy Dirac spinors [11]

$$\phi_{\rm QP}(\hat{k};P)=\sqrt{\frac{Q(\hat{k};P)}{m}}\mathcal{G}(\hat{k};P)\hat{\Gamma}(\hat{k};P), \qquad (8)$$

where $Q(\hat{k};P)=(P_0/M_d)\sqrt{m^2-\hat{k}^2}$. The equation of motion satisfied by the vertex function follows from the BbS reduction of the BS equation

$$\hat{\Gamma}(\hat{p};P)=\frac{1}{2\pi^2}\int d^3k\hat{\mathcal{V}}(\hat{p},\hat{k};P)\mathcal{G}(\hat{k};P)\hat{\Gamma}(\hat{k};P), \qquad (9)$$

where $\hat{\mathcal{V}}(\hat{p},\hat{k};P)$ is Lorentz invariant quasipotential with all relative four-momenta are restricted by the BbS condition. The QP wave function $\phi_{\rm QP}(\mathbf{k};P_{(0)})$ at the rest frame of the deuteron, $P_{(0)}=(M_d,\mathbf{0})$, is expressed in terms of Lorentz invariant $\sqrt{P_0/M_d}\phi_{\rm QP}(\hat{k};P)$. By means of the boost the relativistic equation of motion (9) for the moving deuteron transforms into the equation in the rest frame. This invariance property yields the NR equation for the wave function

$$\frac{M_d^2-4E^2_{\mathbf{p}}}{4m}\phi_{\rm QP}(\mathbf{p}_{(0)};P_{(0)}) =\frac{1}{2\pi^2}\int d^3kV(\mathbf{p}_{(0)},\mathbf{k}_{(0)};P_{(0)})\phi_{\rm QP}(\mathbf{k}_{(0)};P_{(0)}), \qquad (10)$$

[2] A detailed and systematic exposition of the covariant QP formalism for description of the electromagnetic (EM) properties and reactions involving the deuteron is given in Ref. [28].

where three-momenta **p** and **k** in the moving frame are mapped by the boost transformation to $\mathbf{p}_{(0)}$ and $\mathbf{k}_{(0)}$ at the rest frame, respectively, and V is the quasipotential modified by ‘‘minimal relativity.’’

Further, the discussion concerns partial wave decomposition of the BS vertex function of the deuteron $\Gamma(k;P)$. One has at the rest frame of the deuteron (we highlight dependence on the spin projection):

$$\Gamma_M(p;P_{(0)})=\sum_\alpha g_\alpha(p_0,|\mathbf{p}|;\sqrt{s})\Gamma^\alpha_M(-\mathbf{p}),$$
$$(M=\pm 1,0), \quad \sqrt{s}=M_d. \tag{11}$$

Here summation index α is determined by the following quantum numbers. $S=0,1$: spin, $L=0,1,2$: angular momentum, $J=1$: total angular momentum, and ρ spin: the projection of the total energy spin of the nucleon and antinucleon states; $\Gamma^\alpha_M(-\mathbf{p})$ is the spin-angular functions and g_α is the partial amplitudes. Eight partial states contribute to Eq. (11). Apart from two channels with the positive energy intermediate states, viz. ${}^3S_1^+ - {}^3D_1^+$, six ‘‘extra’’ states, which account for antinucleon degrees of freedom, come into play. In the spectroscopic notations ${}^{2S+1}L_J^\rho$ these are

$${}^3P_1^e, {}^3P_1^o, {}^1P_1^e, {}^1P_1^o, {}^3S_1^-, {}^3D_1^-,$$

where indices e and o stand for the even and odd parity relative to the ρ-spin functions. The partial amplitudes ${}^1P_1^e$, ${}^3P_1^o$ are even and ${}^1P_1^o$, ${}^3P_1^e$ are odd functions in the relative energy variable.

Influence of admixtures of P states and their contribution due to interference with the positive energy states to observables in the deuteron breakup and elastic proton-deuteron backward scattering are considered in Refs. [16,29,30]. It is shown that $N\bar{N}$ pair EM current term in the NR approach can be constructed from the P states in the deuteron BS amplitude. It is expected that the pair current term interferes with the one-body one to increase the deuteron photodisintegration cross section at $\Theta=90°$ and, then, to decrease it in the range of the laboratory photon energy 0.1–0.7 GeV [31].

We focus on the positive energy states only. So we have two channels ${}^3S_1^+ - {}^3D_1^+$, and the corresponding vertex functions can be written in the matrix form [20]

$$\sqrt{8\pi}\,\Gamma_\lambda^{{}^3S_1^{++}}(p;P_{(0)})=\mathcal{N}_{\mathbf{p}}^2(m+\hat{p}_1)(1+\gamma_0)\hat{e}_\lambda$$
$$\times(m-\hat{p}_2)g_0(p_0,|\mathbf{p}|;s), \tag{12}$$

$$\sqrt{16\pi}\,\Gamma_\lambda^{{}^3D_1^{++}}(p;P_{(0)})$$
$$=-\mathcal{N}_{\mathbf{p}}^2(m+\hat{p}_1)\times(1+\gamma_0)\left(\hat{\varepsilon}_\lambda+\frac{3}{2}(\widehat{p_1}-\widehat{p_2})\frac{(p\cdot e_\lambda)}{\mathbf{p}^2}\right)$$
$$\times(m-\hat{p}_2)g_2(p_0,|\mathbf{p}|;s),$$

where $s=M_d^2$, $p_1=(E_{\mathbf{p}},\mathbf{p})$ ($p_2=(E_{\mathbf{p}},-\mathbf{p})$ are the on-mass-shell four-momenta, $\mathcal{N}_{\mathbf{p}}^{-1}=\sqrt{2E_{\mathbf{p}}(m+E_{\mathbf{p}})}$ is the normalization factor and $e_\lambda=(0,\mathbf{e}_\lambda)$ is four-polarization vector of the deuteron

$$\sum_{\lambda=-1}^{+1} e_\lambda^\mu e_\lambda^{\nu *}=-g^{\mu\nu}+\frac{P^\mu P^\nu}{M_d^2}, \quad e_\lambda P=0. \tag{13}$$

We are interested in calculating of matrix elements of the EM current operator between states containing free nucleons and deuteron. Thus we need to take into account change in the state amplitudes, when the BbS prescription is applied to the S-matrix element for a given process. To this end we can utilize the normalization condition for the bound state amplitude. Let us suppose that the interaction kernel $\mathcal{V}$ is independent on the total four-momentum P. Then we get

$$1=-\int\frac{d^4k}{2\pi^2 i}\bar{\Gamma}(p;P)\frac{\partial G(p;P)}{\partial P^2}\bigg|_{P^2=M_d^2}\Gamma(p;P). \tag{14}$$

In terms of the radial vertex functions g_L the normalization condition assumes the form

$$\frac{1}{2\pi^2 i M_d}\int_0^\infty dk_0\int_0^\infty d|\mathbf{k}|\,\mathbf{k}^2\frac{g_L(k_0,|\mathbf{k}|;s)^2\left(E_{\mathbf{k}}-\frac{M_d}{2}\right)}{\left[\left(\frac{M_d}{2}-E_{\mathbf{k}}+i\epsilon\right)^2-k_0^2\right]^2}$$
$$=P_L, \quad P_0+P_2=1. \tag{15}$$

We find for the QP vertex function $\hat{g}_L$ similarly

$$\frac{2m^2}{\pi^2 M_d}\int_0^\infty d|\mathbf{k}|\,\mathbf{k}^2\frac{\hat{g}_L(0,|\mathbf{k}|;s)^2}{E_{\mathbf{k}}(M_d^2-4E_{\mathbf{k}}^2)^2}=P_L. \tag{16}$$

One can deduce out of Eqs. (15) and (16) that the BS and QP vertex functions are approximately related to each other as

$$\frac{g_L(k_0,|\mathbf{k}|;s)}{E_{\mathbf{k}}-\frac{M_d}{2}}\Bigg|_{k_0=(M_d/2)-E_{\mathbf{k}}}\propto\sqrt{\frac{4m^2}{\pi E_{\mathbf{k}}}}\frac{\hat{g}_L(0,|\mathbf{k}|;s)}{4E_{\mathbf{k}}^2-M_d^2}. \tag{17}$$

In turn, the QP function (10) is related to the Schrödinger wave function by multiplying the latter by the minimal relativity factor

$$\phi_{\mathrm{QP}}(\mathbf{k};P_{(0)})\equiv\sqrt{\frac{m}{E_{\mathbf{k}}}}\phi_{\mathrm{NR}}(\mathbf{k}).$$

The presence of strong singularities in the interaction kernel can be avoided by special prescriptions referred to imply analytical properties of the amplitude. The ladder approximation, where the interaction kernel of the BS equation $\mathcal{V}$ is the sum of one-boson exchange diagrams, is widely used in solving BS equation for the NN scattering. In this case a solution of Eq. (5) (for example, we refer to Refs. [16– 20]), can be found in Euclidean space after Wick rotation. Actually, that presents a substantial obstacle in calculating observable in terms of the BS amplitude. The procedure of analytical continuation of the complex k_0 plane solution back to the real k_0 axis is extremely laborious and carries ambiguities. The physical solution can be also found via the method based on the perturbation theoretical integral representation of Nakanishi [33]. Here the BS equation for bound states is solved in terms of a generalized spectral representation directly in Minkowski space [34]. But the approach is developed for bound states in scalar theories only.

An alternative way to solve the BS equation is to use a nonlocal separable interaction kernel [35–37]:

$$\mathcal{V}(p,p';P)=\sum_{a,b=1}^{N} \lambda_{ab}v_a(p;P)v_b(p';P), \qquad (18)$$

where λ_{ab} is a symmetrical matrix of coupling constants. In this case both the bound state vertex function and the scattering T matrix are obtained directly in Minkowski space. The resultant amplitudes exhibit analytical properties determined by covariant form factors $v_a(p;P)$. Generally, the form factors include dependence on P^2, p^2, and $p\cdot P$. Leaving the dependence on p^2 means that a simple procedure for constructing relativistic separable interactions is chosen. These form factors are not expected to be genuine separable approximations to the realistic NN interaction. That has been already done in study of the elastic electron-deuteron scattering in the BS formalism (see Refs. [36,38,39]).

We employ the separable kernel of rank 3 [$N=3$ in Eq. (18)] for computation of the deuteron photodisintegration cross section. The kernel is a relativistically covariant generalization of the NR Graz-II potential for the description of the phase shifts of the NN scattering in the coupled $^3S_1$–$^3D_1$ waves (details can be found in Ref. [32]). The analytical properties of the radial vertex function are determined by poles in the relative energy

$$g_0(k_0,|\mathbf{k}|;s)=A(s)\frac{1-\gamma_1k^2}{(k^2-\beta_{11}^2)^2}+B(s)\frac{k^2}{(k^2-\beta_{12}^2)^2},$$

$$g_2(k_0,|\mathbf{k}|;s)=C(s)\frac{k^2(1-\gamma_2k^2)}{(k^2-\beta_{21}^2)^2(k^2-\beta_{22}^2)^2},\quad s=M_d^2, \qquad (19)$$

where $k^2=k_0^2-\mathbf{k}^2$, the coefficients A,B, and C are determined by the homogeneous set of three algebraic equations, the parameters β_{ab} and γ_a are chosen to reproduce $^3S_1$–$^3D_1$ NN scattering phase shifts up to a laboratory energy of $E_{\rm lab}=500$ MeV, the low-energy NN scattering parameters, as well as the static deuteron properties (such as the binding energy, the quadrupole, and magnetic moments). We adopted the coupling parameters of the interaction kernel corresponding to the value of $^3D_1^+$-state probability $P_2=4$ and 6%. The actual parameters β_{ab} and γ_a are chosen to have such values that the resulting T matrix $T_{LL'}(p_0,\mathbf{p},p_0',\mathbf{p}';s)$ satisfies the exact two-body unitarity relation at least up to a nucleon kinetic energies $E_{\rm lab}=(2/m)\beta_{11}(m+\beta_{11}/4)$ with $\beta_{11}=231$ MeV, which is about 500 MeV in the laboratory system.

The same operations are repeated in the QP approximation to the BS equation. Now the QP vertex function is the solution of the homogeneous BS equation with the BbS propagator function (7) and the Graz-II potential. The values of the parameters λ_{ab} are changed in order to reproduce the static deuteron properties, the low-energy scattering parameters and the phase shifts. The radial part of the deuteron vertex function $\tilde{g}_L$ depends on the relative three-momentum only

$$\hat{g}_0(0,|\mathbf{k}|;s)=\hat{A}(s)\frac{1+\gamma_1\mathbf{k}^2}{(\mathbf{k}^2+\beta_{11}^2)^2}+\hat{B}(s)\frac{\mathbf{k}^2}{(\mathbf{k}^2+\beta_{12}^2)^2},$$

$$\hat{g}_2(0,|\mathbf{k}|;s)=\hat{C}(s)\frac{\mathbf{k}^2(1+\gamma_2\mathbf{k}^2)}{(\mathbf{k}^2+\beta_{21}^2)^2(\mathbf{k}^2+\beta_{22}^2)^2},\quad s=M_d^2. \qquad (20)$$

The particular feature of the BbS reduction is that propagators are reduced to the static form. Speaking in the language of the meson-exchange model, the BbS reduction completely ignores the retardation or the dependence of the BS vertex function on the relative energy, which arises from noninstantaneous effects in the NN interaction.

III. DEUTERON PHOTODISINTEGRATION CROSS SECTION

Let us consider disintegration of a deuteron with total four-momentum K by a photon with four-momentum q^μ, $q^2=0$, into a free neutron-proton (np) pair, characterized with the total and relative four-momenta P and p, respectively. In the rest frame of the np pair, i.e., $P^\mu_{(0)}=(\sqrt{s},\mathbf{0})$, $p^\mu=(0,\mathbf{p})$, where $\sqrt{s}$ is the total energy of the pair, the differential absorption cross section of a photon with energy ω can be written as

$$\frac{d\sigma}{d\Omega_p}=\frac{\alpha}{16\pi s}\frac{|\mathbf{p}|}{\omega}\overline{|\varepsilon_\lambda\cdot\mathcal{M}_{\rm fi}|^2}, \qquad (21)$$

where $\alpha=e^2/(4\pi)$ is the fine structure constant, $\mathcal{M}^\mu_{fi}$ is the transition matrix element $\mathcal{M}^\mu_{fi}=\langle f|\widehat{J^\mu}|i\rangle$ of the EM current operator $\widehat{J^\mu}$ between the deuteron bound state and the $2N$ continuum; ε^μ_λ is a photon polarization four-vector with $\lambda=\pm1$. The four-momentum conservation at the photon-deuteron vertex gives $K+q=P$.

Since polarizations of the particles involved in the process will not be considered here, averaging and summing over the photon and nuclear polarizations in the initial and final states, respectively, are assumed. We may choose such a coordinate system, where the photon three-momentum is directed along the z axis, $q^\mu=(\omega,0,0,\omega)$. In the laboratory system, being the rest frame of the deuteron, the deuteron has the four-momentum $K_{(0)}=(M_d,\mathbf{0})$ and the photon energy is denoted as E_γ.

In experiments on two-body photodisintegration of the deuteron the differential cross section (21) is viewed as a function of the laboratory photon energy E_γ and angle Θ_p between incoming-photon and outgoing-proton three-momenta in the c.m. system of the np pair. One can obtain the following kinematic relations between the laboratory photon energy and variables in the c.m. frame:

$$|\mathbf{p}|=\sqrt{\frac{s}{4}-m^2},\quad s=M_d^2+2E_\gamma M_d,\quad \omega=\frac{M_d}{\sqrt{s}}E_\gamma. \tag{22}$$

The invariant amplitude $\mathcal{M}^\mu_{fi}$, following Ref. [23], can be written in terms of the BS amplitude of the initial (6) and final (3) states

$$\mathcal{M}^\mu_{\rm fi}=\frac{1}{4\pi^3}\int d^4k d^4l\overline{\chi_{Sm_s}}(l;pP)\Lambda^\mu(l,k;P,K)\psi_M(k;K), \tag{23}$$

where $S=0,1$ is the total spin of the np pair and m_s is its projection on to the z axis, M is the projection the total angular momentum of the deuteron; Λ_μ denotes Mandelstam vertex, which determines the EM interaction with $2N$ system in the framework of the BS formalism.

Let us make some remarks on the current conservation. Mandelstam vertex consists of one-body and two-body parts $\Lambda_\mu(p,k;P,K)=\Lambda^{[1]}_\mu(p,k;P,K)+\Lambda^{[2]}_\mu(p,k;P,K)$. The second part of Mandelstam vertex determines the so-called two-body contributions. The specific form of $\Lambda^{[2]}_\mu(p,k;P,K)$ depends on a given model for the interaction kernel in the BS equation and it cannot be associated with the pair and meson-exchange currents in the NR approach. The gauge independence of EM current transition matrix element, $q\cdot\mathcal{M}_{fi}=0$, will be fulfilled if Mandelstam current meets the following relations:

$$iq\cdot\Lambda^{[1]}(p,k;P,K)=\Bigg\{\pi_p(1)\,\delta\left(p-k-\frac{q}{2}\right)\left[S^{(1)}\left(\frac{K}{2}+k\right)^{-1}-S^{(1)}\left(\frac{P}{2}+p\right)^{-1}\right]S^{(2)}\left(\frac{K}{2}-k\right)^{-1} \tag{24}$$

$$+\pi_p(2)\,\delta\left(p-k+\frac{q}{2}\right)\left[S^{(2)}\left(\frac{K}{2}-k\right)^{-1}-S^{(2)}\left(\frac{P}{2}-p\right)^{-1}\right]S^{(1)}\left(\frac{K}{2}+k\right)^{-1}\Bigg\} \tag{25}$$

and

$$iq\cdot\Lambda^{[2]}(p,k;P,K)=\sum_{l=1,2}\left[\pi_p(l)\mathcal{V}\left(p+(-1)^l\frac{q}{2},k;K\right)-\mathcal{V}\left(p,k-(-1)^l\frac{q}{2};K\pi_p(l)\right)\right], \tag{26}$$

where $S^{(l)}(p)$ is the fermion propagator and $\pi_p(l)=1/2[1+\tau_z(l)]$ is the projector on to proton state. Moreover, the BS amplitudes for the initial and final states have to satisfy to the BS equations with the same interaction kernel.

It is shown in Ref. [36] that the gauge independence condition for the elastic electron-deuteron scattering amplitude is fulfilled even in the impulse approximation for the many-rank separable kernels, implying that $q\cdot\mathcal{M}^{[2]}_{fi}=0$. In the case of the deuteron breakup the isospin $I=1$ state is present in the final $2N$ state, yielding a nonvanishing isovector contribution, $q\cdot\mathcal{M}^{[2]}_{fi}\neq 0$. Consequently, the two-body Mandelstam current operator should be added to guarantee the gauge independence of the scattering amplitude.

IV. PLANE WAVE ONE-BODY APPROXIMATION

The invariant amplitude in Eq. (23) contains the contributions due to the final state interaction. This means that the half-off-mass shell NN scattering T matrix is needed, see Eq. (3). Rescattering contributions and the two-body processes are not taken into account, which constitutes a shortcoming of our present work. At present, this is bound up with computational difficulties. In a forthcoming paper we will consider the FSI interactions for $J=0$, ${}^1S_0$, and $J=1$, ${}^3S_1$–${}^3D_1$, channels and work on the pair processes is in progress. We stress that the full and detailed analysis of the FSI cannot clearly be avoided without reconsidering an entirely different form of the interaction kernel. Thus, in the present investigation we confine ourselves to the simplest contribution to the EM current operator, where the photon couples to either of the two nucleons in the deuteron and the FSI between the outgoing nucleons are omitted—the plane-wave one-body approximation.

A. The BS amplitude for the continuous spectrum

According to the approximations we made above, the BS amplitude of the final state in Eq. (3) is the antisymmetrical combination of the two free Dirac positive energy spinors[3]

$$\chi_{Sm_s}(k_0,\mathbf{k};\sqrt{s}\mathbf{p})=4\pi^3\delta(k_0)\times[\chi_{Sm_s}(\mathbf{p})(\eta_0+\eta_1)\delta^{(3)}(\mathbf{k}-\mathbf{p})+(-1)^{S+1}\chi_{Sm_s}(-\mathbf{p})(\eta_0-\eta_1)\times\delta^{(3)}(\mathbf{k}+\mathbf{p})], \tag{27}$$

where

$$\chi_{Sm_s}(\mathbf{p})=\sum_{\lambda_p,\lambda_n=\pm\frac{1}{2}}C^{Sm_s}_{\frac{1}{2}\lambda_p\frac{1}{2}\lambda_n}u_{\lambda_p}(\mathbf{p})u_{\lambda_n}(-\mathbf{p});$$

η_0 and η_1 stand for isopin singlet and triplet functions, respectively. Since the outgoing nucleons are on mass-shell,

[3]We use the covariant normalization of the Dirac spinors $u^+u=2E$.

we have the constraint $P\cdot\hat{p}=0$, which sets the relative energy p_0 to be equal to zero in the rest frame of the np pair.

B. The BS amplitude for the bound state

The BS equation for the deuteron is solved in the c.m. frame. Since the vertex function in Eq. (23) is referred to a moving frame, it has to be boosted to the c.m. frame. Lorentz transformation between the laboratory and c.m. frames is defined by $K^{\mu}=\mathcal{L}^{\mu}_{\nu}K^{\nu}_{(0)}$, $P^{\mu}=\mathcal{L}^{\mu}_{\nu}P^{\nu}_{(0)}$, where $K^{\nu}_{(0)}$ and $P^{\nu}_{(0)}$ are four-momenta of the deuteron and np pair in their rest frame, respectively. As the boost along the z axis is needed, the explicit expression for the matrix $\mathcal{L}^{\nu}_{\mu}$ is given by

$$\mathcal{L}^{\nu}_{\mu}=\begin{pmatrix} \sqrt{1+\eta} & 0 & 0 & -\sqrt{\eta} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sqrt{\eta} & 0 & 0 & \sqrt{1+\eta} \end{pmatrix}, \tag{28}$$

where Lorentz factor $\gamma\equiv\sqrt{1+\eta}$ and $v\gamma\equiv\sqrt{\eta}$ with v being the velocity of c.m. frame in the laboratory frame are defined in terms of the dimensionless boost parameter η:

$$\sqrt{\eta}=\frac{E_\gamma}{\sqrt{s}},\quad \sqrt{1+\eta}=\frac{E_\gamma+M_d}{\sqrt{s}}.$$

Under the boost the vertex function is transformed according to the general rule of the transformation of spinor amplitudes

$$\Gamma_M(k;K)=\Lambda(\mathcal{L})\Gamma_M(\mathcal{L}^{-1}k;K_{(0)}), \tag{29}$$

where $\Lambda(\mathcal{L})=\Lambda^{(1)}(\mathcal{L})\Lambda^{(2)}(\mathcal{L})$ and $\Lambda^{(l)}(\mathcal{L})$ is the boost operator in the spinor space of lth nucleon

$$\Lambda^{(l)}(\mathcal{L})=\left(\frac{1+\sqrt{1+\eta}}{2}\right)^{\frac{1}{2}}\left(1+\frac{\gamma_0\gamma_3\sqrt{\eta}}{1+\sqrt{1+\eta}}\right)^{(l)}. \tag{30}$$

As $\eta\to 0$ the matrix (28) and operator (30) turn to unity, i.e., $\mathcal{L}\to I$ and $\Lambda\to I$. It corresponds to the static limit for the BS amplitude and takes place on the threshold of the deuteron photodisintegration, a case of $E_\gamma/m\ll 1$ [see Eq. (22)]. The $\gamma_0\gamma_3$ term in Eq. (30) affects the spin degrees of freedom of the BS amplitude.

C. The electromagnetic vertex

The one-body part of Mandelstam vertex has the form

$$\Lambda^{[1]}_{\mu}(p,k;P,K)=i\delta^{(4)}\left(p-k-\frac{q}{2}\right)\Gamma^{(1)}_{\mu}\left(\frac{P}{2}+p,\frac{K}{2}+k\right)\times S^{(2)}\left(\frac{K}{2}-k\right)^{-1}+i\delta^{(4)}\left(p-k+\frac{q}{2}\right)\times\Gamma^{(2)}_{\mu}\left(\frac{P}{2}-p,\frac{K}{2}-k\right)S^{(1)}\left(\frac{K}{2}+k\right)^{-1}, \tag{31}$$

where $\Gamma^{(l)}_{\mu}(p,k)$ is off-mass-shell γNN vertex for the lth nucleon. Here we meet a common problem of the direct application of the BS formalism to the NN interaction, since we have to work with the off-mass-shell amplitudes. The consequence of the gauge constraints for off-shellness in the EM vertices has been recently considered in Ref. [40] (see also references therein). In our case we deal with half off-mass-shell EM vertex. This type of vertices occur in the $(e,e'N)$ reaction, e.g., nucleon knockout and inclusive electron scattering, where the initial nucleon is taken to be bound (off-mass-shell) and the knocked-out nucleon is assumed to be in physical state. It is shown in Ref. [40] that the off-shell behavior of the nucleon EM vertex in a real Compton scattering does not play a role as consequence of gauge invariance.

Thus we deal with the well-known on-mass-shell version of the EM vertex

$$\Gamma^{(l)}_{\mu}(q)=\gamma_\mu[F^{(s)}_1(q)+\tau^{(l)}_3F^{(v)}(q)]+\frac{i}{2m}\sigma_{\mu\nu}q^{\nu}[F^{(s)}_2(q)+\tau^{(l)}_3F^{(v)}_2(q)], \tag{32}$$

where $F^{(s,v)}_{1,2}(q)$ are the isoscalar and isovector Pauli-Dirac form factors of the lth nucleon, normalized as $F^{(s)}_1(0)=\frac{1}{2}$, $F^{(s)}_2(0)=(\kappa_p+\kappa_n)/2$, $F^{(v)}_1(0)=\frac{1}{2}$, $F^{(v)}_2(0)=(\kappa_p-\kappa_n)/2$, with the anomalous part of the proton (neutron) magnetic moments denoted as $\kappa_{p(n)}$.

Employing the above written transformation laws, substituting Eqs. (27), (29), (31) into Eq. (23) and finally integrating over the intermediate four-momentum, we derive the expression for the transition matrix element in terms of the c.m. vertex function and the single nucleon propagators

$$\mathcal{M}^{\mu}_{\rm fi}=\sum_{l=1,2}\overline{\chi}^{(0)}_{Sm_s}(0,\mathbf{p};\sqrt{s}\mathbf{p})\Gamma^{(l)}_{\mu}\times(q^2=0)\Lambda(\mathcal{L})S^{(l)}(k;K_0)\Gamma_M(k_{0l},\mathbf{k}_l;K_0), \tag{33}$$

where $k_l=\mathcal{L}^{-1}[p+(-1)^l(q/2)]$. All possible contributions to the matrix element in the PWOA are depicted in Fig. 1. Note that the four diagrams are identical, since both the initial and final $2N$ states are antisymmetrical.

It is seen that the matrix element is proportional directly to the deuteron vertex function evaluated at a specific point of the nucleon relative energy-momentum. As the relative four-momentum k_l is restricted by the energy-momentum conservation in the photon-nucleon vertex, the relative energy k_{0l} and three-momentum $\mathbf{k}_l$ variables depend on the photon energy in the lab frame and the boost parameter η

$$k_{0l}=\sqrt{\eta}|\mathbf{p}_{||}|+(-1)^l\frac{E_\gamma}{2},$$

$$\mathbf{k}_{\perp l}=\mathbf{p}_{\perp},\quad \omega=M_d\sqrt{\eta}, \tag{34}$$

$$\mathbf{k}_{||l}=\sqrt{1+\eta}\,\mathbf{p}_{||}+(-1)^l\frac{\mathbf{q}}{2},$$

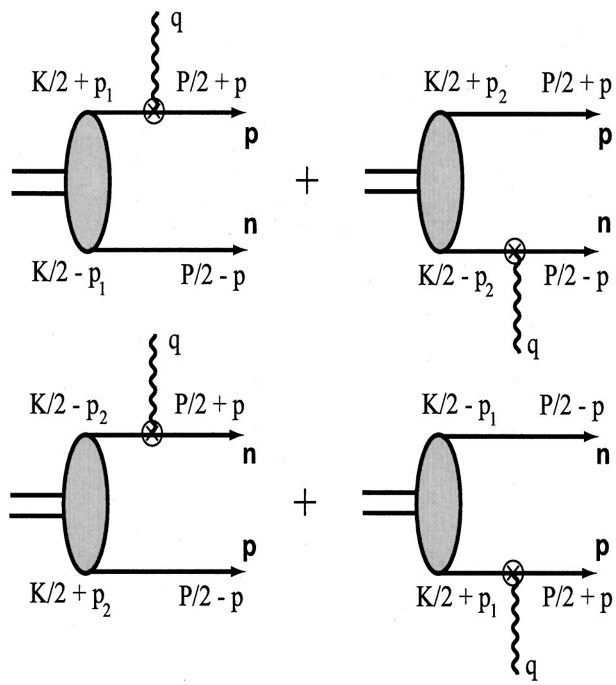

FIG. 1. Diagrams of the deuteron photodisintegration in the plane wave one-body approximation. Outgoing particles are on mass shell.

where indices $||$ and $\perp$ denote the longitudinal and transverse components of vector $\mathbf{k}$ with respect to the direction of the incoming photon three-momentum $\mathbf{q}$, here $|\mathbf{q}|=E_\gamma$. The situation is illustrated on Fig. 2. For photon energies $E_\gamma \leqslant 0.2$ GeV one is probing the energy-momentum distribution of the bound nucleons in the deuteron, where the high-momentum ''tails'' of the nucleon density contribution in the deuteron are especially relevant. For given c.m. angles Θ_p it is found that both the relative energy (it accounts for the retardation in the vertex function of the deuteron) and the modulus of the three-momentum of the proton (neutron) grow strongly in increasing order of E_γ. They become smaller at forward scattering angles, covering wide bands from 100 MeV to 2 GeV and from 500 MeV to 2.5 GeV for the rest scattering angles.

Introducing the matrix representation for the deuteron vertex function and the $2N$ continuum amplitude [see Eqs. (12) and (27)], the transition matrix element can be transformed into traces of γ-matrix expressions

$$\mathcal{M}_{\text{fi}}^{\mu}=-\text{Sp}\left[\overline{\chi}_{Sm_s}(\mathbf{p})\Gamma_{p,\mu}\Lambda(\mathcal{L})S\left(\frac{K_0}{2}+k_1;K_0\right)\times\Gamma_M(k_1;K_0)\Lambda(\mathcal{L}^{-1})\right]-\text{Sp}\left[\overline{\chi}_{Sm_s}(\mathbf{p})\Lambda(\mathcal{L})\Gamma_M(k_2;K_0)\widetilde{S}\times\left(\frac{K_0}{2}-k_2;K_0\right)\Lambda(\mathcal{L}^{-1})\Gamma_{n,\mu}\right], \tag{35}$$

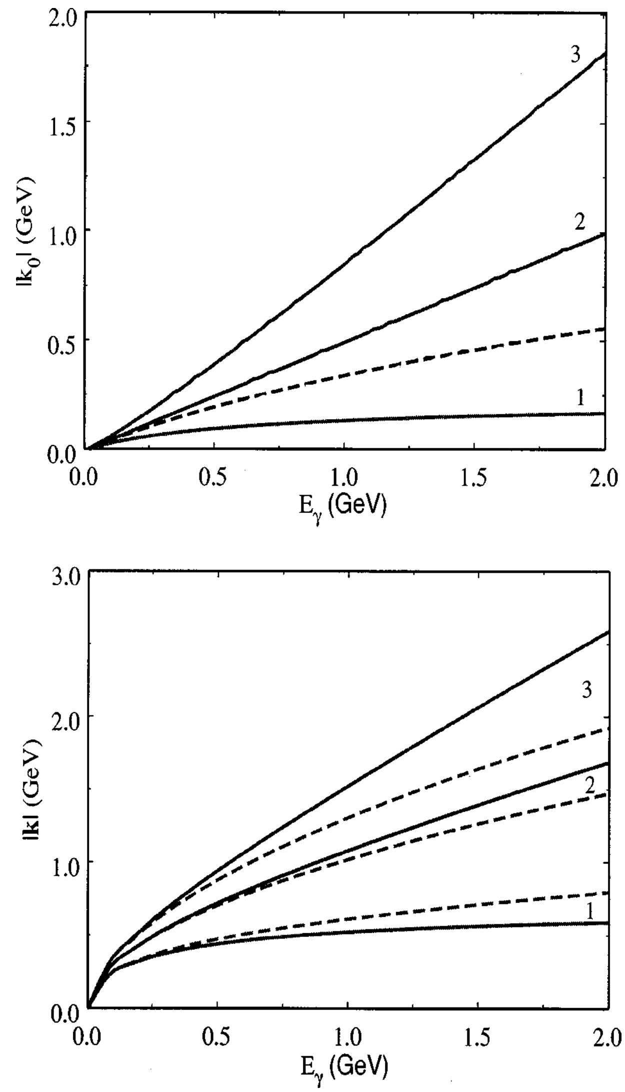

FIG. 2. Modulus of the relative energy $|k_0|$ and the three-momentum $|\mathbf{k}|$ of a nucleon in the deuteron versus the photon energy E_γ (solid lines). Dashed lines correspond to solid ones when neglecting the boost parameter, i.e., $\sqrt{\eta}=0$ (the static approximation). The set of fixed proton scattering angles $\Theta_p=0°$, 90°, and 180° in the c.m. frame correspond to the curves labeled as 1, 2, and 3, respectively. The same curves are related to the energy and the three-momentum of a spectator nucleon at c.m. frame angles $\Theta_p=180°$, 90°, and 0°.

with

$$\overline{\chi}_{1m_s}(\mathbf{p})=\frac{\mathcal{N}_{\mathbf{p}}^2}{2\sqrt{2}}(m-\hat{p}_2)\hat{\xi}_{m_s}^{*}(1+\gamma_0)(m+\hat{p}_1), \tag{36}$$

$$\overline{\chi}_{00}(\mathbf{p})=\frac{\mathcal{N}_{\mathbf{p}}^2}{2\sqrt{2}}(m-\hat{p}_2)\gamma_5(1+\gamma_0)(m+\hat{p}_1),$$

where ξ_{m_s} is the np pair polarization four-vector with the following completeness and orthogonality relations:

$$\sum_{m_s=-1}^{+1} \xi^{\mu}_{m_s}\xi^{\nu *}_{m_s} = -g^{\mu\nu} + \frac{P^{\mu}P^{\nu}}{s}, \quad \xi \cdot P = 0. \tag{37}$$

Normalization constants $\mathcal{N}_{\mathbf{p}}$ and vectors $p_{1,2}$ are defined in Eq. (12).

Since the EM nucleon form factors can be taken in the on-shell form, we have for the charge-current operator

$$\Gamma_{p,\mu} = \gamma_{\mu} + \frac{i\kappa_p}{2m}\sigma_{\mu\nu}q^{\nu}, \quad \Gamma_{n,\mu} = \frac{i\kappa_n}{2m}\sigma_{\mu\nu}q^{\nu}. \tag{38}$$

The fermion propagator in Eq. (35)

$$\tilde{S}(k) = \frac{\hat{k} - m}{k^2 - m^2 + i\epsilon} \tag{39}$$

is related to the propagator $\tilde{S}$ by $\tilde{S} = -CS^TC$, where $C = i\gamma^2\gamma^0$.

A little manipulation with the expression (35) yields that the differential photoabsorption cross section can be written as

$$\frac{d\sigma}{d\Omega_p} = \frac{d\sigma_0}{d\Omega_p} + \frac{d\sigma_{\rm SP}}{d\Omega_p}, \tag{40}$$

where the σ_0 is the part of the cross section which makes up the shape of the angular distributions

$$\frac{d\sigma_0}{d\Omega_p} = \frac{\alpha}{4\pi s}\left(\frac{1+\sqrt{1+\eta}}{2}\right)^2 \sum_{S=0,1} |X_0^S|^2 \tag{41}$$

and the $d\sigma_{\rm SP}$ accounts for the effect of the boost on the spin degrees of freedom (the spin precession) of the nucleons

$$\frac{d\sigma_{\rm SP}}{d\Omega_p} = \frac{\alpha}{4\pi s}\left(\frac{1+\sqrt{1+\eta}}{2}\right)^2 \sum_{S=0,1} |\mathcal{M}^S_{\rm SP}|^2. \tag{42}$$

The square modulus of the amplitude $\mathcal{M}^S_{\rm SP}$ is given by

$$|\mathcal{M}^S_{\rm SP}|^2 = 2\beta\,{\rm Re}(X_0^S X_1^{S*}) + \beta^2[|X_1^S|^2 - 2\,{\rm Re}(X_0^S X_2^{S*})] - 2\beta^3\,{\rm Re}(X_1^S X_2^{S*}) + \beta^4|X_2^S|^2 \tag{43}$$

with $\beta = \sqrt{\eta}/(1+\sqrt{1+\eta})$ and the amplitudes X_i^S $(i=0,1,2)$ are expressed as

$$X_0^S = {\rm Sp}[\overline{\chi}_{Sm_s}(\mathbf{p})\,\Gamma_{p,\lambda}\,S^{(1)}(s_1;K_0)\Gamma_M(k_1;K_0)] + p \leftrightarrow n,$$

$$X_1^S = {\rm Sp}[\overline{\chi}_{Sm_s}(\mathbf{p})\,\Gamma_{p,\lambda}\,\gamma_0\hat{n}_3 S^{(1)}(s_1;K_0)\Gamma_M(k_1;K_0)] - {\rm Sp}[\overline{\chi}_{Sm_s}(\mathbf{p})\,\Gamma_{p,\lambda}\,S^{(1)}(s_1;K_0)\Gamma_M(k_1;K_0)\,\gamma_0\hat{n}_3] + p \leftrightarrow n, \tag{44}$$

$$X_2^S = {\rm Sp}[\overline{\chi}_{Sm_s}(\mathbf{p})\,\Gamma_{p,\lambda}\,\gamma_0\hat{n}_3 S^{(1)}(s_1;K_0)\Gamma_M(k_1;K_0)\,\gamma_0\hat{n}_3] + p \leftrightarrow n,$$

where $n_3 = (0,0,0,1)$ stands for the unit spacelike vector and $s_1 = K_0/2 - k_1$.

These are the general expressions for the deuteron photodisintegration cross section (21) in the PWOA. Since we omit the two-body contribution to the transition matrix element we do not preserve the gauge independence of the amplitude. When averaging over the photon state polarizations, we make use of Coulomb gauge $\varepsilon^0 = 0$, $\varepsilon \cdot \mathbf{q} = 0$ with the completeness relation of the form

$$\sum_{\lambda=\pm 1} (\varepsilon_\lambda)_i^* (\varepsilon_\lambda)_j = \delta_{ij} - \frac{q_i q_j}{\mathbf{q}^2}, \quad i,j = x,y. \tag{45}$$

All the γ-matrix expressions in the matrix elements (44) and the square modulus of the amplitude (43) are evaluated with the computer algebraic program REDUCE. When summing over nuclear polarizations, we make use of the corresponding completeness relations, Eqs. (13) and (37).

V. ANALYSIS OF RELATIVISTIC EFFECTS

Now we are in a position to do final computations. The results for the angular distributions for the deuteron photodisintegration at four different photon energies E_γ are depicted in Fig. 3. The invariant matrix element of the differential cross section is calculated in the framework of the BS formalism. The deuteron vertex function is the solution of the homogeneous BS equation with the separable interaction kernel (with sets of the coupling parameters corresponding to two different strength of the D state, $P_2 = 4\%$ and $P_2 = 6\%$). The interaction kernel is similar to that employed in the calculations of the EM elastic form factors of the deuteron in Ref. [36]. For the sake of simplicity we disregard contributions to the matrix element stemming from the negative-energy partial states of the deuteron vertex function. Subsequently, their role in the cross section will be investigated in the BS formalism with the one-boson exchange interaction kernel.

It is seen in Fig. 3 that the resultant curves reproduce shapes of the angular distributions [1]. They exhibit an almost perfect $\sin^2\Theta_p$ behavior above the threshold, which corresponds to $E1$ transition to the ${}^3P_0$ continuum state. At higher photon energies the overall magnitude of the cross section rapidly falls off and the maximum is shifted from 90° to 70° and, further, to 60°. The distributions, which are dominated by magnetic transitions, become flat, the ratio of the forward cross section to the maximum decreases. The role of the D state becomes more pronounced. Notwithstanding these similarities, the theory appears systematically less than the experimental distributions. At $E_\gamma = 20$ MeV it is less by a factor of 1.4, at $E_\gamma = 100$ MeV by a factor of 3, in the Δ-resonance region it is expectedly lower by almost a factor of 10, and at $E_\gamma = 500$ MeV by a factor of 3 with respect to D-state weight $P_2 = 6\%$. In the NR approach, incorporating meson-nucleon degrees of freedom, a good deal of discrepancy between the experiment and theory is diminished by contributions from pion-exchange currents and the isobar configurations.

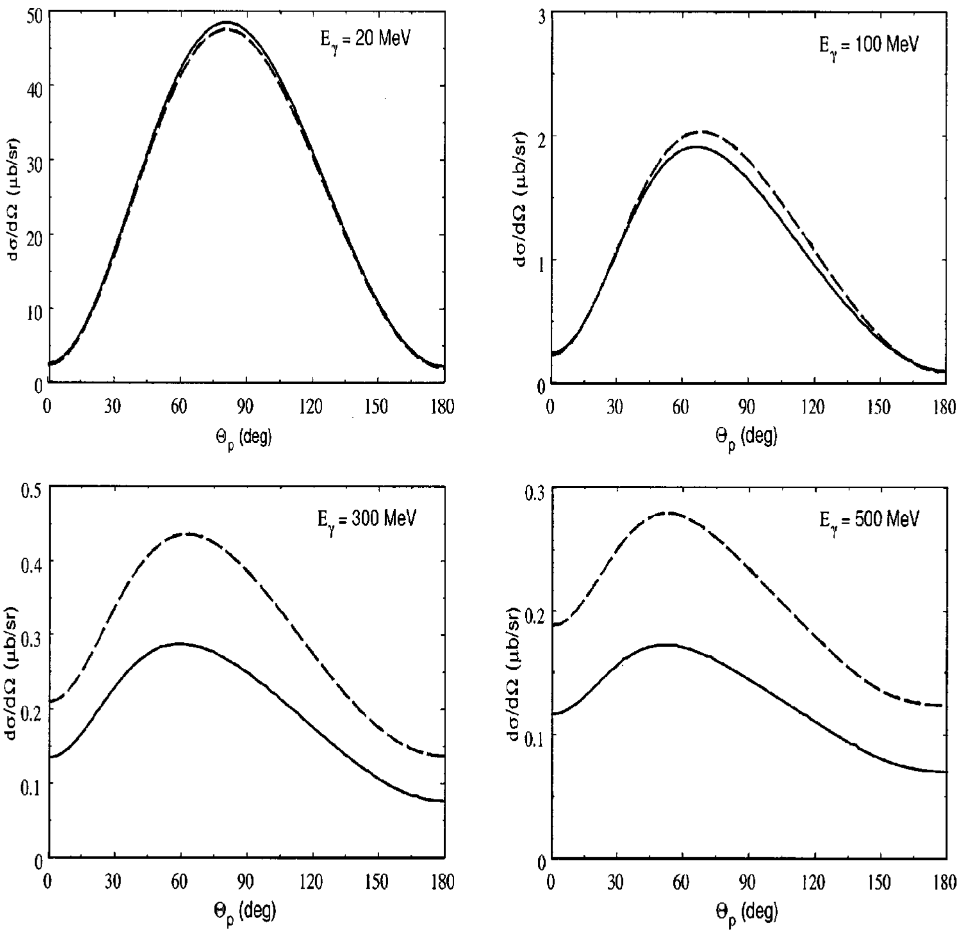

FIG. 3. The differential cross section in the plane wave one-body approximation at different photon energies E_γ. Curves present two different probabilities of ${}^3D_1^+$ partial state. Solid line, $P_2=4\%$, and dashed line, $P_2=6\%$.

A. Approximate calculations

Despite the fact that our theoretical results do not describe the experimental data, we are able to make definitive statements concerning the relative importance of the various relativistic effects. We distinguish the following classes of the relativistic effects in our consistent treatment: (1) the role of the relativistic kinematics and the covariance of the EM current operator; (2) the role of the relativistic *NN* dynamics, which forbids instantaneous interactions and leads to the relative energy dependence of the deuteron vertex function (the effect of retardation); (3) the boost transformation affecting the internal variables of the BS amplitude of the deuteron (the effect of Lorentz contraction) and its spin degrees of freedom (the spin precession). The corresponding elements of the cross section are denoted as $d\sigma_0/d\Omega$ and $d\sigma_{\rm SP}/d\Omega$, respectively.

We start our discussion with estimation of the size of contributions due to the retardation, Lorentz contraction and the spin precession. Let us consider a number of approximations with respect to the exact positive-energy state BS calculations.

(1) First, the static approximation (BS-SA) to the BS cross section is carried out. That amounts to neglecting the boost on the arguments of the deuteron vertex function and the one-particle propagator, see Eq. (33). It is achieved by setting the boost parameter $\eta=0$ in the deuteron vertex function $\Gamma_L(k_{0l},\mathbf{k}_l)|_{\eta=0}=\Gamma_L(p_{0l},\mathbf{p}_l)$, where $p_{0l}=(-)^l(E_\gamma/2)$ and $\mathbf{p}_l=\mathbf{p}+(-)^l\mathbf{q}/2$ $(l=1,2)$. This effect has a direct bearing on the dynamical variables, as is shown in Fig. 2. The booster in Eq. (30) is approximated by $\Lambda(\mathcal{L})|_{\eta=0}=I$. Note that the static approximation excludes contributions due to Lorentz contraction and the spin precession.

(2) Special attention is paid to investigation of the influence of the boost on the nucleon relative energy in the one-particle propagator. As is shown in Ref. [35] that is the most important relativistic contribution to the deuteron EM form factors. Here we consider the case (BS-BR), which is the same as BS-SA, but includes the boost on the one-particle propagator $S^{(l)}(k_{0l},|\mathbf{p}_l|)$ due to recoil, $k_{0l}=\sqrt{\eta}|\mathbf{p}|+(-1)^l(E_\gamma/2)$.

(3) Finally, we find out the relativistic correction associated with the relative energy dependence of the matrix elements. We consider the zeroth order approximation (BS-ZO) for the vertex function, i.e., in the BS-BR approximation we compute the radial parts of the vertex function with k_{0l} equal to zero.

In Fig. 4 we present the results for the angular distributions at the previous set of lab photon energies. The solid curve corresponds to the exact positive-energy state calculation. We would like to stress the importance of the effect concerning the booster. In Fig. 4 it is clearly seen the pronounced role of the $d\sigma_{\rm SP}$ cross section (dot-dashed line) with rise of the photon energy. It becomes more noticeable at the scattering in the forward semisphere, where it is almost

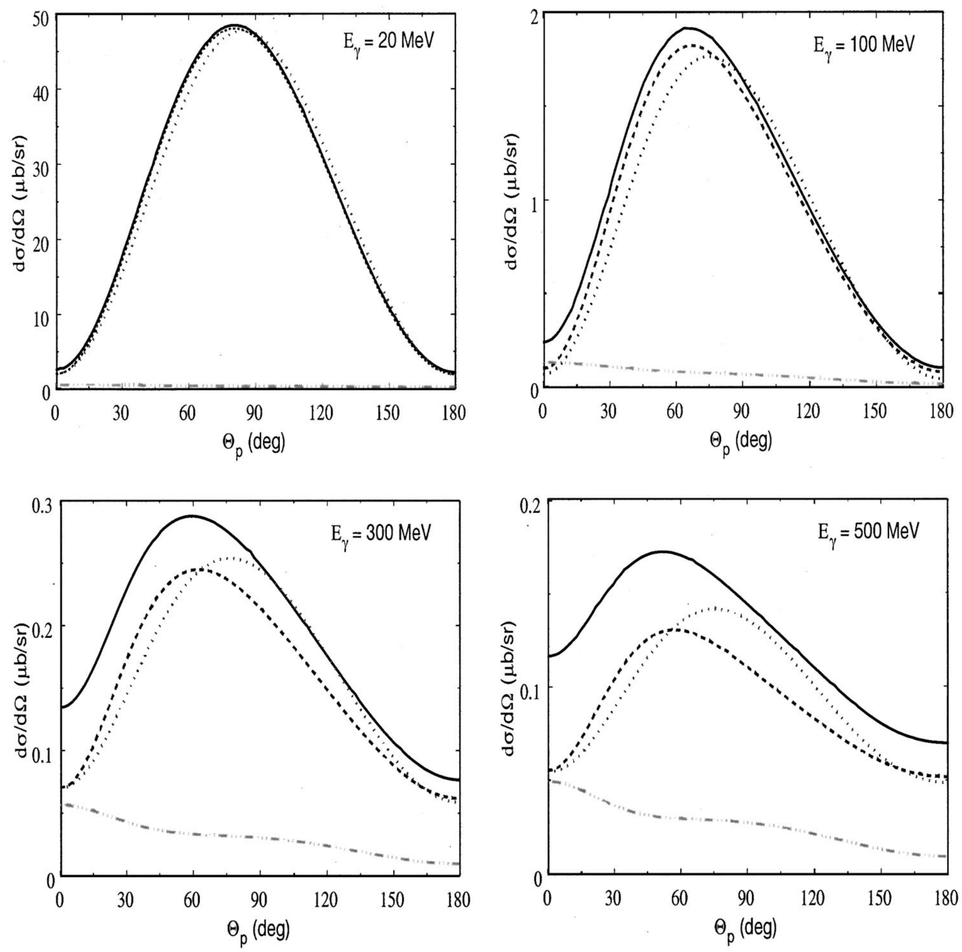

FIG. 4. The differential cross section in the plane wave one-body approximation at different photon energies E_γ. Curves: solid line, the exact positive-energy BS calculation; dotted line, the static approximation (with exclusion of Lorentz contraction); dashed line, the static approximation taking into account the boost on the one-particle propagator due to recoil; dot-dashed line, the contribution due to the spin precession, Eq. (42). Probability of ${}^3D_1^+$ partial state, $P_2=4\%$.

half the size of the cross section. One should allow for such a contribution starting at the medium photon energies. When added to the BS-BR approximation (dashed line), the latter becomes a plausible one for the region of the photon energies in question. The BS-BR approximation, including the boost on the one-particle propagator, gives the shape of the angular distributions close to the exact ones. We can conclude that the BS-BR approximation supplemented with $d\sigma_{\mathrm{SP}}$ contribution accounts for the major relativistic effects in the differential cross section. On the other hand, the SA approximation (dotted line) ceases to be reasonable for E_γ above 100 MeV, as it has a wrong position of the cross section maximum.

We also demonstrate the influence of the boost transformation on the arguments of the initial-state vertex function. It is worthwhile to mention that the boost leaves the arguments of the radial part of the vertex function unchanged [since the radial function $g_L(p;K)$ depends on Lorentz invariants K^2, p^2, and $p\cdot K$]. It only has a direct bearing on its spin-orbital part. In Fig. 5 the relative deviation of the BS-BR (dashed line) and the BS-ZO (dot-dashed line) approximations with respect to the BS cross section $d\sigma_0$ are displayed. The deviation of the BS-BR cross section is due to the boost effect on the orbital part of the deuteron vertex function. The contribution is quite great, especially at forward and backward proton c.m. angles. The effects of the retardation are responsible for the discrepancy between the two curves in Fig. 5. One can see that it is practically uniform difference reaching up to 5%. If one ignores the dependence on the relative energy in the deuteron vertex function, one comes up with the cross section which is slightly smaller but keeps the same shape.

B. Comparison with other approaches

The BS formalism consistently accounts for the relativistic effects associated with manifest Lorentz covariance of the scattering amplitudes. As far as the matter of relativity is concerned, we compare the exact results of the BS framework with the following approaches.

(1) In the PWOA the ET approximation can be obtained immediately from the exact expressions by replacing the deuteron partial states for the ''++'' channel with the QP vertex function. The latter is a solution of the 3D QP equation with the BbS propagator (7) and with the refitted version of the separable interaction kernel Graz-II [36]. Essentially, the ET approximation makes use of instantaneous interactions, i.e., with the relative time (the relative energy) set to zero in the deuteron vertex function g_L and the one-particle propagator.

(2) A minimally relativistic approach employs the same QP vertex function of the deuteron corresponding to the solution of the equation the BbS propagator. That represents the static limit for the ET approximation (ET-SA). The ap-

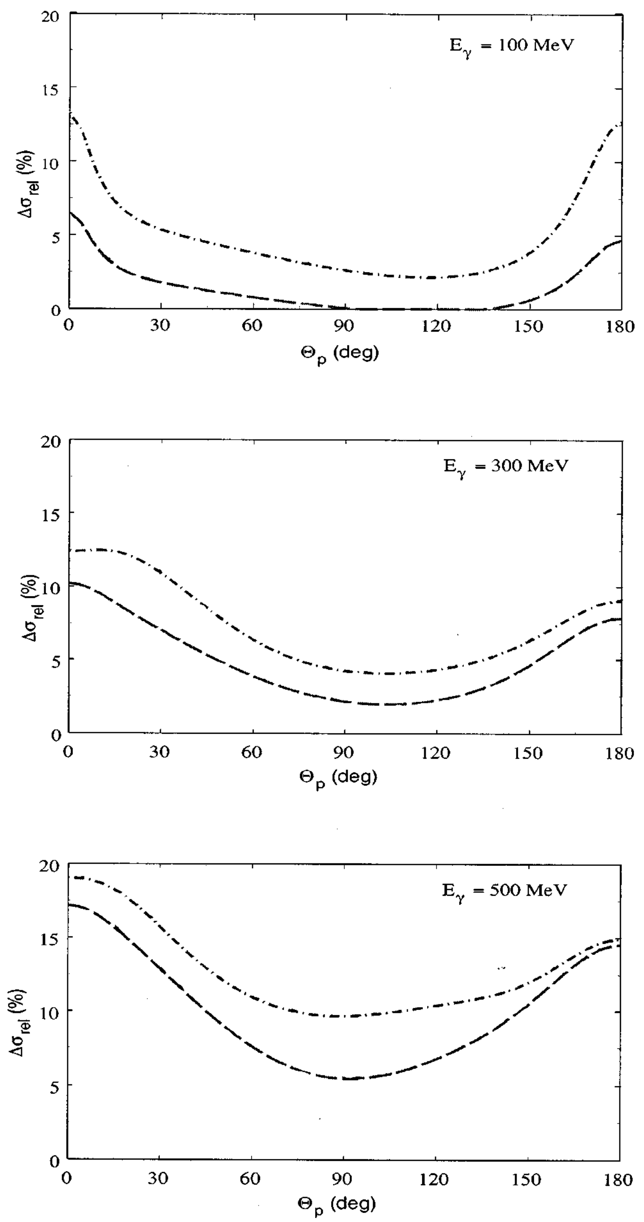

FIG. 5. The relative deviation of the deuteron photodisintegration cross section for the photon energies $E_\gamma = 100$, 300, and 500 MeV for the different approximations with respect to the exact positive-energy BS result BS-ZO (dashed line) and BS-BR (dot-dashed line). On the y axis it is plotted $(\sigma_0 - \sigma_{\text{BS-}\alpha)/\sigma_0} \times 100\%$, where $\alpha = \text{BR, ZO}$. Probability of $^3D_1^+$ partial state, $P_2 = 4\%$.

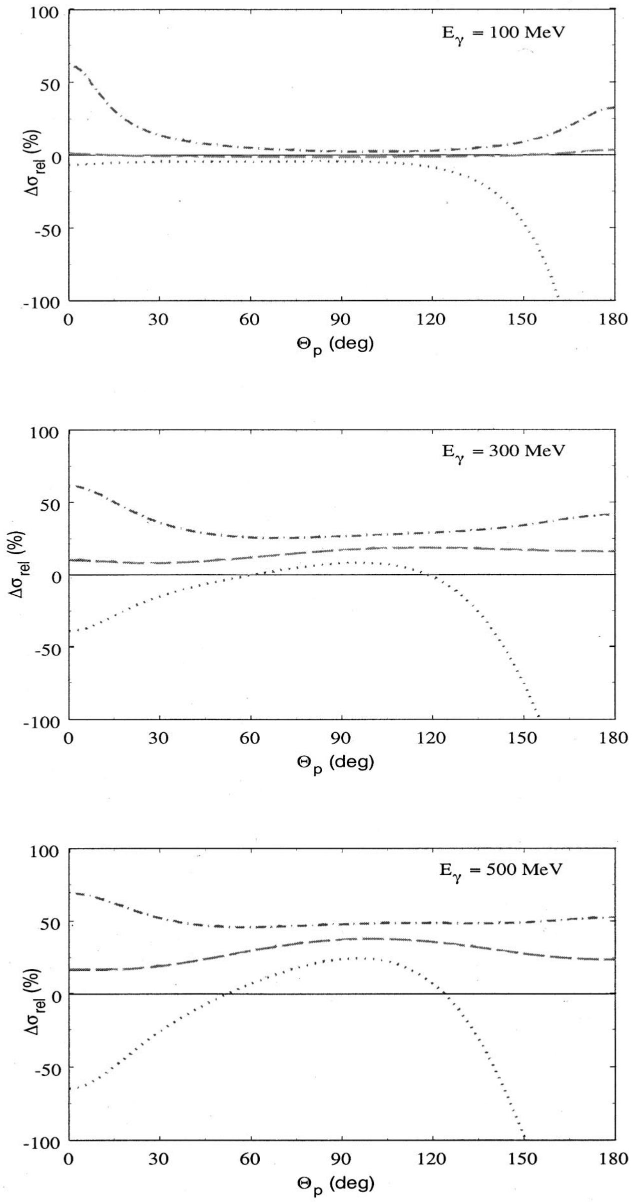

FIG. 6. The relative difference of the deuteron photodisintegration cross sections for the photon energies $E_\gamma = 100$, 300, and 500 MeV. The following cases are considered with respect to the exact positive-energy BS result: the equal time approximation (dashed line), the minimally relativistic approach (dot-dashed line), and the nonrelativistic approach (dotted line). On the y axis it is plotted $(\sigma - \sigma_\alpha)/\sigma \times 100\%$, where $\alpha = \text{ET, ET-SA, and NR}$. Probability of $^3D_1^+$ partial state $P_2 = 4\%$.

proach incorporates the relativistic kinematics and the covariant form of the EM current operator.

(3) A purely nonrelativistic approach makes use of the wave function of the deuteron, which is the solution of the Schrödinger equation with the NR Graz-II separable potential.

In Fig. 6 we compare the above mentioned approaches with the exact relativistic calculation. One can see that the NR approach (dotted line) is a very crude approximation for the photon energies greater than 100 MeV. The minimally relativistic approach (dot-dashed line) slightly improves the situation. By no means is the NR approach reasonable to describe the deuteron photodisintegration at high photon energies above the pion threshold. At least one should include the minimally relativistic correction. Such a conclusion is in accordance with the discussion of Ref. [1], where it is shown that should such relativistic effects not be included, a theory gives too much peaking of the differential cross section at $\Theta_p = 0°$ and $180°$.

Finally, one can see that the overall sign of the relativistic contribution to the NR cross section is negative. At the same time it depends on a given photon energy in the range $\Theta_p = 60–120°$. Taking into account Lorentz deformation (dashed line in Fig. 6) results in a sizable effect. It is seen that the ET approximation is a quite good approximation at low photon energies. The discrepancy between the BS and ET calculations is practically isotropic reaching about 10% at $E_\gamma = 300$ MeV and 25% at the $E_\gamma = 500$ MeV.

C. Expansion of the relativistic model

It is well known that the habitual way to provide for some relativistic effects is the $|\mathbf{p}|/m$ and ω/m expansion of the exact relativistic model keeping the lowest order correction $|\mathbf{p}|^2/m^2$ and ω/m [1]. The procedure is valid at the photon energies E_γ up to a few hundred MeV. The additional terms to the NR amplitude seem to correspond to those that come from the inclusion of the spin-orbit interaction to the NR current operator.

We analytically performed the expansion of the relativistic expressions for the angular distributions in the limit $E_\gamma \ll m$. For the c.m. frame variables in the NR limit (corresponding to energies $E_\gamma \lesssim 100$ MeV), we have $|\mathbf{p}| \cong \sqrt{m(E_\gamma - \epsilon_d)}$, $\omega \cong E_\gamma$ and the boost parameter $\sqrt{\eta} \cong 0$. The matrix element is expanded in powers of $|\mathbf{p}|/m$ and ω/m with $|\mathbf{p}|/m \approx 0.3$ keeping the lowest order terms. The recoil effects and the boost effects are neglected as well. The result is that of the noncovariant description of the deuteron in the framework of the conventional NR models incorporating relativistic effects in the $|\mathbf{p}|/m$ expansion of the relativistic model [1,31].

The differential cross section of photoabsorption in the NR framework is given by

$$\frac{d\sigma_0}{d\Omega_p} = \frac{\alpha|\mathbf{p}|}{4\pi E_\gamma} \sum_{S=0,1} \overline{|X_0^S|^2}, \tag{46}$$

where the deuteron breakup matrix elements are expressed in terms of the NR deuteron wave function $\Psi_M(\mathbf{k})$, the wave function of the $2N$ scattering states $\Psi_{\mathbf{p}Sm_s}(\mathbf{k})$ as follows:

$$X_0^S = -\sum_{l=1,2} \int \frac{d\mathbf{k}}{(2\pi)^3} \overline{\Psi}_{\mathbf{p}Sm_s}(\mathbf{k}) F_l^\lambda(\mathbf{q}) \Psi_M(\mathbf{k}_l) \quad (\lambda = \pm 1), \tag{47}$$

where $\mathbf{k}_l = \mathbf{p} - (-1)^l \mathbf{q}/2$ and the EM nucleon form factor is given by

$$F_l^\lambda(\mathbf{q}) = (-1)^{(l+1)} \frac{1+\tau_z^{(l)}}{2} \frac{\mathbf{p}^\lambda}{m} + \frac{\kappa_s + \tau_z^{(l)}\kappa_v}{2} \frac{i[\boldsymbol{\sigma}^{(l)} \times \mathbf{q}]^\lambda}{2m}$$

with $\kappa_s = \frac{1}{2}(\kappa_p + \kappa_n)$ and $\kappa_v = \frac{1}{2}(\kappa_p - \kappa_n)$.

In the PWOA one can reduce square of the amplitude (47) in Eq. (46) to ''textbook'' formula, which reproduces the shape of the angular distribution while missing its absolute size

$$\begin{aligned} d\sigma_0 = {} & \frac{\mathbf{p}^2}{2m^2} \sin^2\Theta_p (U_1^2 + W_1^2) \\ & + \frac{E_\gamma^2}{4m^2} \Bigg\{ \kappa_p^2 (U_1^2 + W_1^2) + \kappa_n^2 (U_2^2 + W_2^2) \\ & + \frac{\kappa_p \kappa_n}{3} \Bigg[2U_1U_2 - \frac{U_1W_2 + U_2W_1}{\sqrt{2}} (1 + 3\cos(2\Theta_p)) \\ & + \frac{W_1W_2}{2} (5 + 3\cos(2\Theta_p)) \Bigg] \Bigg\}. \end{aligned} \tag{48}$$

The *S*- and *D*-state amplitudes of the deuteron are denoted as *U* and *W*, respectively. The NR states are *formally* related to the corresponding states of the BS vertex function as

$$U = \frac{\sqrt{\pi m}}{2E_\mathbf{k} - M_d} g_0(0,|\mathbf{k}|; s = M_d^2),$$

$$W = \frac{\sqrt{\pi m}}{2E_\mathbf{k} - M_d} g_2(0,|\mathbf{k}|; s = M_d^2).$$

Subscripts 1 and 2 in Eq. (48) indicate the modulus of the proton and neutron three-momentum in the deuteron $\mathbf{k}_1$ and $\mathbf{k}_2$.

VI. CONCLUDING REMARKS

The objective of the present study is to evaluate the various relativistic contributions to the angular distributions of the deuteron photodisintegration. That is effected in a variety of theoretical frameworks and dynamics. The present paper has applied the fully relativistic formalism, based on the Bethe-Salpeter equation for the $2N$ elastic scattering amplitude and the deuteron bound state. Beyond the choice of the theoretical framework, which is manifestly covariant at every step of the calculation, the important issue is a dynamical model of the nucleon-nucleon interaction. Pursuing the aim to obtain a clear understanding and to conduct straightforward comparison with the nonrelativistic and minimally relativistic approaches, we have attracted the effective separable interactions in constructing the solvable dynamical model of the deuteron.

It is worthwhile to comment on the negative-energy states of the realistic deuteron vertex function. In order to analytically reach the ultimate results we have discarded channels containing negative-energy *P* states in the Bethe-Salpeter amplitude of the deuteron. So far, the numerical solution of the BS equation allowing *P* states can be obtained within the context of the one-boson exchange interaction kernel. The present version of the separable interaction kernel does not include the negative-energy states. The contributions of the *P* states to the cross section are expected to be significant within the considered interval of the photon energies. In this respect our present study is primarily of a comparative character. We also have not included the two-body contribution to the electromagnetic current operator and neglected the final state interaction in the outgoing np state. The last two limitations constitute the so-called plane wave one-body approximation.

Despite these specifications, the strongest advantage in our investigation concerns the fully covariant and rigorous description of the bound state and the deuteron electromagnetic current. The present approach accounts for the wealth of the relativistic effects to the differential cross section of the deuteron photodisintegration: the role of relativity in the transition matrix elements between different nucleonic states, the influence of the retardation in the deuteron vertex function, and the one-particle propagator, as well as changes in the amplitudes due to Lorentz deformation and the spin precession.

These relativistic effects together have a profound impact in the theoretical analysis of the deuteron photodisintegration even at intermediate laboratory photon energies for the forward and backward scattering. Here the most important contributions come from the boost in the arguments of the initial state vertex function and the boost on the relative energy in the one-particle propagator due to recoil. As one is concerned with the covariant approaches, the equal time approximation is a more or less reasonable approach from a pragmatic point of view.

Further, the following conclusions are confirmed by results of the present investigation. (1) The Bethe-Salpeter approach allows us to take into account Lorentz invariance and relativistic dynamical structure of the two-nucleon system in the most general form. (2) The novel feature brought by the Bethe-Salpeter approach is the retardation due to the dependence of the bound state amplitude on the relative energy of the nucleons. In the plane wave one-body approximation, the scattering amplitude bears the explicit dependence on this variable and its magnitude is measured by the photon energy. In our opinion, this turns out to be by far the most important fact enabling one to study recoil effects due to energy transfer to a nucleon by a photon. (3) The role of the boost transformation of the spin degrees of freedom becomes noticeable in increasing order of the photon energy at forward scattering.

Finally, the region of high photon energies (above $E_\gamma = 500$ MeV) calls for a more complete investigation. In this energy region one needs to construct a realistic interaction kernel in solving the Bethe-Salpeter equation. Moreover, extending the above calculations includes contributions due to *P* states and the two-body processes in the EM current operator matrix elements.

ACKNOWLEDGMENTS

We are indebted to V.V. Burov, S.G. Bondarenko, A.A. Goy, A.V. Molochkov, and A.V. Shebeko for many useful discussions and encouragement. We are grateful to O.V. Chelomina for the assistance rendered in preparation of the paper. This work has been partially supported by Grant No. 02.01.22/2000, ''Universities of Russia.''

[1] H. Arenhövel and M. Sanzone, Few-Body Syst., Suppl. **3**, 1 (1991).
[2] D. Babusci *et al.*, Nucl. Phys. **A633**, 683 (1998).
[3] F. Adamain (unpublished).
[4] J.E. Belz, Phys. Rev. Lett. **74**, 646 (1995).
[5] H. Ejiri (private communication).
[6] E89-012 Collaboration, C. Bochna *et al.*, Phys. Rev. Lett. **81**, 4576 (1998).
[7] R. Crawford *et al.*, Nucl. Phys. **A603**, 303 (1998).
[8] S. Wartenberg *et al.*, Few-Body Syst. **26**, 213 (1999).
[9] P. Wilhelm and H. Arenhövel, Phys. Lett. B **318**, 410 (1993).
[10] J.M. Laget, Nucl. Phys. **A579**, 333 (1994).
[11] W. Jaus, B. Bofinger, and W.S. Woolock, Nucl. Phys. **A562**, 477 (1993).
[12] J.M. Zuilhof and J.A. Tjon, Phys. Rev. C **22**, 2369 (1980).
[13] A.A. Logunov and A.N. Tavkhelidze, Nuovo Cimento **29**, 370 (1963).
[14] R. Blankenbecler and R. Sugar, Phys. Rev. **142**, 1051 (1966).
[15] J. Adam, Jr., J.W. Van Orden, and F. Gross, Nucl. Phys. **A640**, 391 (1998).
[16] L.P. Kaptari, B. Kämpfer, S.M. Dorkin, and S.S. Semikh, Phys. Rev. C **57**, 1097 (1998).
[17] A.Yu. Umnikov, F.C. Khanna, K.Yu. Kazakov, and L.P. Kaptari, Phys. Lett. B **334**, 163 (1994).
[18] C. Ciofi degli Atti, L.P Kaptari, and S. Scopetta, Eur. Phys. J. A **5**, 191 (1999).
[19] N. Honzawa and S. Ishida, Phys. Rev. C **45**, 17 (1992).
[20] L.P. Kaptari, A.Yu. Umnikov, S.G. Bondarenko, K.Yu. Kazakov, F.C. Khanna, and B. Kämpfer, Phys. Rev. C **54**, 986 (1996).
[21] A.V. Anisovich and V.A. Sadovnikova, Eur. Phys. J. A **2**, 199 (1998).
[22] A.E.L. Dieperink and S.I. Nagorny, Phys. Lett. B **456**, 9 (1999).
[23] A.Yu. Korchin and A.V. Shebeko, Sov. J. Nucl. Phys. **54**, 214 (1991).
[24] E. Hummel and J.A. Tjon, Phys. Rev. C **49**, 21 (1994).
[25] D.R. Phillips and S.J. Wallace, Few-Body Syst. **24**, 175 (1998).
[26] V. Pascalutsa and J.A. Tjon, Few-Body Syst., Suppl. **10**, 105 (2000).
[27] D.R. Phillips, S.J. Wallace, and N.K. Devine, nucl-th/9906086.
[28] W. Jaus and W.S. Woolock, Helv. Phys. Acta **57**, 644 (1984).
[29] S.G. Bondarenko, V.V. Burov, M. Beyer, and S.M. Dorkin, Phys. Rev. C **58**, 3143 (1998).
[30] L.P. Kaptari, B. Kämpfer, S.M. Dorkin, and S.S. Semikh, Few-Body Syst. **27**, 189 (2000).
[31] M. Anastasio and M. Chemtob, Nucl. Phys. **A364**, 219 (1981).
[32] G. Rupp, Nucl. Phys. **A508**, 131 (1990).
[33] N. Nakanishi, *Graph Theory and Feynman Integral* (Gordon and Breach, New York, 1971).
[34] K. Kusaka, K.M. Simpson, and A.G. Williams, Phys. Rev. D **56**, 5071 (1998).
[35] G. Rupp and J.A. Tjon, Phys. Rev. C **41**, 472 (1990).

[36] G. Rupp and J.A. Tjon, Phys. Rev. C **45**, 2133 (1992).
[37] K. Schwarz, J. Haidenbauer, and J. Fröhlich, Phys. Rev. C **33**, 456 (1986).
[38] G. Rupp and J.A. Tjon, Phys. Rev. C **37**, 1729 (1988).
[39] S.G. Bondarenko, V.V. Burov, and S.M. Dorkin (unpublished).
[40] S.I. Nagorny and A.E.L. Dieperink, Eur. Phys. J. A **5**, 417 (1999).