$E1$ transitions between spin-dipole and Gamow-Teller giant resonances

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The branching ratios for $E1$ transitions between the spin-dipole (SD) and Gamow-Teller (GT) giant resonances in ^{90}Nb and ^{208}Pb are evaluated. Assuming the main GT state has a wave function close to that for the ''ideal'' GT state, we reduced the problem to calculate the SD and GT strength functions. These strength functions are evaluated within an extended continuum-random phase approximation approach.

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Experimental and theoretical studies of direct-decay properties of various giant resonances (GRs) allow one to check their microscopic (particle-hole) structure in a quantitative way. Experimentally the partial branching ratios for the direct proton decay of both Gamow-Teller GR (GTR) and the spin-dipole GR (SDR$^{(-)}$) are obtained from the (^{3}He,t) and (^{3}He,tp) experiments. The data at $E(^3$He$) = 450$ MeV have been analyzed for the ^{208}Pb target nucleus [1,2] and have been rather successfully described within an extended continuum-RPA approach [3]. The data of the ^{90}Zr(^{3}He,tp) reaction are expected to be analyzed soon [4]. Another possibility to reveal the microscopic structure of the SDR$^{(-)}$ and GTR is to study γ transitions between these resonances. The branching ratios for the γ decay from the SDR$^{(-)}$ to the GTR can be deduced from the (^{3}He,$t\gamma$) coincidence experiments [5].

The intensity of the $E1$ γ transitions between GT and SD$^{(-)}$ states in ^{90}Nb and ^{48}Sc was evaluated within a TDA approach in Ref. [6]. However, the results obtained in this work are presented in a form which does not allow one to compare them directly with the experimental branching ratios. The aim of the present work is to evaluate the branching ratio for the $E1$ transitions between the SDR$^{(-)}$ and the GTR (main peak) in ^{208}Bi and ^{90}Nb within the approach given in Ref. [3]. In this approach we use (i) the continuum-RPA (CRPA), (ii) the phenomenological mean field and the Landau-Migdal particle-hole interaction together with some partial self-consistency conditions, (iii) a phenomenological description of the doorway-state coupling to many-quasiparticle configurations.

We start from consideration of the CRPA polarizabilities $\mathcal{P}_{JLS}^{(-,+)}(\omega)$ and the strength functions $S_{JLS}^{(-,+)}(\omega)$ corresponding to the external fields $V_{JLSM}^{(-,+)} = \Sigma_a V_{JLSM}^{(-,+)}(x_a)$. Here, $V_{JLSM}^{(-,+)}(x) = r^L T_{JLSM}(\vec{n}) \tau^{(-,+)}$ (with $J = S = 1; L = 0$ and $J = 0,1,2; L = S = 1$ for GT and SD excitations, respectively), $T_{JLSM}(\vec{n}) = \Sigma_m C_{Lm1M-m}^{JM} Y_{Lm}(\vec{n}) \sigma^{M-m}$ is the irreducible spin-angular tensor operator of the rank J, σ^μ and $\sqrt{2}\tau^{(\pm)}$ are the spherical spin and isospin Pauli matrices, respectively; ω is the excitation energy measured from the energy of the parent-nucleus ground state. For the considered spin GRs, the CRPA polarizabilities and the strength functions exhibit resonancelike behavior, corresponding to the excitation of isolated particle-hole type doorway states. In particu-

lar, using the Breit-Wigner parametrization of the polarizabilities and the strength functions

$$S_{J11}^{(-)}(\omega) = -\frac{1}{\pi} \operatorname{Im} \mathcal{P}_{J11}^{(-)}(\omega) = -\frac{1}{\pi} \operatorname{Im} \sum_s \frac{R_s}{\omega - \omega_s + (i/2)\Gamma_s^\uparrow}, \tag{1}$$

$$S_{101}^{(-)}(\omega) = -\frac{1}{\pi} \operatorname{Im} \mathcal{P}_{101}^{(-)}(\omega) = -\frac{1}{\pi} \operatorname{Im} \frac{R_g}{\omega - \omega_g + (i/2)\Gamma_g^\uparrow} \tag{2}$$

we can evaluate the doorway parameters: strength $R_{s(g)}$, energy $\omega_{s(g)}$, and total escape width $\Gamma_{s(g)}^\uparrow$. Similarly to the work of Ref. [3], we consider only the main GT doorway state with the maximal strength R_g.

The radiative width for the $E1$ transitions between SD$^{(-)}$ and GT doorway states is determined by the squared matrix elements of the electric dipole operator $D_\mu^{(3)} = -\frac{1}{2}e\Sigma_a r_a Y_{1\mu}(\vec{n}_a)\tau_a^{(3)}$ according to the expression

$$\Gamma_{s\to g}^\gamma = \frac{16\pi}{9}\left(\frac{\omega_s - \omega_g}{\hbar c}\right)^3 \sum_\mu \overline{|(D_\mu^{(3)})_{gs}|^2}. \tag{3}$$

Here, the bar denotes averaging over M_s and summation over M_g, where M are projections of the doorway-state total angular momentum. To describe the radiative width $\Gamma_{s\to g}^\gamma$ in terms of the SD$^{(-)}$ doorway-state strength R_s, we start from the assumption that the component of main GT state with projection M_G exhausts the total GT strength $R_G = (N-Z)/4\pi$. With this assumption the following equations, which are similar to those used in Ref. [6], are valid:

$$|G\rangle = R_G^{-1/2}V_{101M_G}^{(-)}|0\rangle, \quad V_{101M_G}^{(+)}|0\rangle = 0,$$

$$(D_\mu^{(3)})_{sG} = R_G^{-1/2}([D_\mu^{(3)}, V_{101M_G}^{(-)}])_{s0}. \tag{4}$$

Here, $|G\rangle$ is the wave function of the ''ideal'' GTR, $|0\rangle$ is the parent-nucleus ground-state wave function. Note that the commutator in Eq. (4) is a component of the operator $V_{J11M}^{(-)}$ divided by $\sqrt{4\pi}$. Assuming the main GT state has the wave

TABLE I. The calculated branching ratios for the γ decay from the SDR$^{(-)}$ to the GTR (main peak) in ^{90}Zr and ^{208}Pb. The mean-squared radii calculated by Eq. (6) and the $R^{(+)}$ to $R^{(-)}$ ratios B are also given together with the isoscalar mean field amplitude U_0, the Landau-Migdal parameters f' and g', and the calculated relative strengths x_g of the GT main peak.

Nucleus	U_0, MeV	f'	g'	x_g	$\langle r^2\rangle^{(-)}$, fm^2	B	$b_g^{\text{schem}}(\times 10^{-4})$	$b_g(\times 10^{-4})$
^{90}Zr	53.3	0.96	0.70	0.83	22.6	0.34	3.3	4.7
^{208}Pb	54.1	1.0	0.78	0.69	36.4	0.06	0.81	2.4

function close to that for the "ideal" GT state, we can derive from Eqs. (3) and (4) the expression for the radiative width $\Gamma_{s\to g}^{\gamma}$:

$$\Gamma_{s\to g}^{\gamma}=\frac{4e^2}{9R_G}\left(\frac{\omega_s-\omega_g}{\hbar c}\right)^3 x_g R_s, \tag{5}$$

where the factor $x_g=R_g/R_G$ is the strength of the main GT state related to the total one. Via this factor we take into account the difference between the "ideal" GT state and the main GT doorway state. The width $\Gamma_{s\to g}^{\gamma}$ for the $E1$ transitions from the SDR$^{(-)}$ to the GTR (main peak) can be schematically described with the use of Eq. (5). Assuming all the J components of the SDR$^{(-)}$ have the same energy ω_S, equal to the experimental SDR$^{(-)}$ energy, and both GRs have no spreading and escape widths, the radiative width can be expressed in terms of the nonenergy-weighted sum rule for spin-dipole transitions

$$\Gamma_{S\to g}^{\gamma}=\frac{4e^2}{9}\left(\frac{\omega_s-\omega_g}{\hbar c}\right)^3 x_g\frac{\langle r^2\rangle^{(-)}}{(1-B)},$$

$$\langle r^2\rangle^{(-)}=\frac{4\pi}{N-Z}\int \varrho^{(-)}(r)r^4dr, \quad B=\frac{R^{(+)}}{R^{(-)}}. \tag{6}$$

Here, B is the SDR$^{(+)}$ excitation strength related to that for the SDR$^{(-)}$, and $\varrho^{(-)}(r)$ is the neutron-excess density.

The SD$^{(-)}$ strength distribution and the doorway-state coupling to many-quasiparticle configurations are taken into account within the approach of Ref. [3]. Similarly to the SD$^{(-)}$ polarizability (to the "forward-scattering amplitude") of Eq. (1) we can also use the Breit-Wigner parametrization for the "reaction amplitude" $M_g^J(\omega)$, corresponding to both the excitation of J^- doorway states and their $E1$ decay to the main GT state

$$M_g^J(\omega)=\frac{1}{\sqrt{2\pi}}\sum_s \frac{R_s^{1/2}(\Gamma_{s\to g}^{\gamma})^{1/2}}{\omega-\omega_s+(i/2)\Gamma_s^{\uparrow}}$$

$$=\frac{\alpha_g^{1/2}}{\sqrt{2\pi}}\sum_s \frac{R_s\sqrt{(\omega_s-\omega_g)^3}}{\omega-\omega_s+(i/2)\Gamma_s^{\uparrow}}, \tag{7}$$

where $\Gamma_{s\to g}^{\gamma}$ is the radiative width of Eq. (5) and $\alpha_g=[4e^2/9(\hbar c)^3](x_g/R_G)$. Thus, the structure of the resultive amplitude M_g^J is found to be close to that of the SD polarizability of Eq. (1).

Then the doorway-state coupling to many-quasiparticle configurations is phenomenologically taken into account. To get the expressions for the energy-averaged "reaction amplitudes" we substitute the escape widths $\Gamma_s^{\uparrow}$ in Eqs. (1), (7) and the width $\Gamma_g^{\uparrow}$ in Eq. (2) by $\Gamma_s^{\uparrow}+\Gamma_S^{\downarrow}$ and $\Gamma_g^{\uparrow}+\Gamma_G^{\downarrow}$, respectively. The mean doorway-state spreading width $\Gamma_S^{\downarrow}$ is found from the condition that the total width Γ of the SD$^{(-)}$ energy-averaged strength function $\bar{S}_{\text{SD}}^{(-)}(\omega)=\Sigma_{J=0,1,2}(2J+1)\bar{S}_{J11}^{(-)}(\omega)$ coincides with the total width Γ^{exp} of the SDR$^{(-)}$ in the experimental inclusive reaction cross section. The same procedure is used to evaluate $\Gamma_G^{\downarrow}$. Because the doorway-state spreading widths $\Gamma_S^{\downarrow}$ and $\Gamma_G^{\downarrow}$ are found to be rather large, we take approximately into account a variation of factor E_γ^3 over the doorway-state resonances, using in the expression for the squared energy-averaged "reaction amplitude" $\bar{M}_J(\omega)$ the corresponding averaged value

$$\overline{(\omega_s-\omega_g)^3}=(\omega_s-\omega_g)^3+3(\omega_s-\omega_g)\sigma_{gs}^2,$$

$$\sigma_{gs}=\sqrt{(\Gamma_s^{\uparrow}+\Gamma_S^{\downarrow})^2+(\Gamma_g^{\uparrow}+\Gamma_G^{\downarrow})^2}/2.35. \tag{8}$$

The ratio of the integrated energy-averaged "cross sections"

$$b_g=\int\sum_{J=0,1,2}(2J+1)|\bar{M}_g^J(\omega)|^2d\omega \Big/ \int \bar{S}_{\text{SD}}^{(-)}(\omega)d\omega, \tag{9}$$

can be considered as the partial branching ratio for the $E1$ decay from the SDR$^{(-)}$ to the GTR (main peak). The branching ratio described schematically is determined by using the width Γ_{gS}^{γ} of Eq. (6) as $b_g^{\text{schem}}=\Gamma_{gS}^{\gamma}/\Gamma_S^{\downarrow}$.

The partial self-consistency conditions and choice of model parameters are described in Ref. [3]. In particular, the isoscalar mean field amplitude U_0 and the amplitude f' of the isovector part of the Landau-Migdal particle-hole interaction are chosen for each nucleus to reproduce in calculations the experimental proton and neutron separation energies. The values of U_0 and f' are listed in Table I. The ability of the model to describe the single-neutron-hole spectrum of ^{207}Pb has been demonstrated in Ref. [3].

The spin-dipole sum rule $\int\varrho^{(-)}(r)r^4dr$ is evaluated for ^{90}Zr and ^{208}Pb in the same way as described in Ref. [3]. The sum rule is determined by the mean-squared radius $\langle r^2\rangle^{(-)}$ of Eq. (6) (the corresponding calculated values are listed in Table I). The amplitude g' of the spin-isospin part of the Landau-Migdal particle-hole interaction is chosen for each nucleus to reproduce in calculations the experimental GTR

energy. The values of g' are listed in Table I along with the relative strengths of the GT main peak x_g.

The basic assumption used in this work is the substitution of the RPA creation operator, corresponding to the main GT state, by the operator $R_G^{-1/2}V_{101M_G}^{(-)}$ with taking x_g as the correction factor [see Eqs. (4),(5)]. Although the accuracy of this assumption is not too high because the x_g values are not too close to unity (Table I), it seems enough to make a reasonable comparison of the calculated branching ratios with coming experimental data.

The partial branching ratios b_g^{schem} for the γ decay of the SDR$^{(-)}$ to GTR are calculated within the framework of the schematic description. In the calculations, the experimental energy 21.1 MeV [1] (17.9 MeV [7]) is used for the SDR$^{(-)}$ in ^{208}Bi (^{90}Nb) along with the experimental energy 15.5 MeV [1] (8.7 MeV [7]) for the GTR. The calculated values of b_g^{schem} are given in Table I.

To take into account the distribution of the spin-dipole particle-hole strength over the SDR$^{(-)}$, the doorway-state spreading widths for both the SDR$^{(-)}$ and GTR, we calculated partial radiative branching ratios b_g by Eqs. (7)–(9) within the framework of the more refined description. Along with the spin-dipole doorway-state parameters calculated in the same way given in Ref. [3], we used in the calculations for ^{208}Bi (^{90}Nb) (i) the mean spin-dipole doorway-state spreading width $\Gamma_{\text{SD}}^{\downarrow}=4.7$ MeV found in Ref. [3] ($\Gamma_{\text{SD}}^{\downarrow}$

$=5.0$ MeV to reproduce the experimental total SDR width $\Gamma_{\text{SDR}}^{\text{exp}}=7.8$ MeV [7]) and (ii) the experimental total width of the GTR $\Gamma_{\text{GTR}}^{\text{exp}}=(\Gamma_{\text{GTR}}^{\downarrow}+\Gamma_{\text{GTR}}^{\uparrow})^{\text{exp}}=3.72$ MeV [1] ($\Gamma_{\text{GTR}}^{\text{exp}}=4.4$ MeV [7]). The calculated values of b_g are given in Table I.

The partial branching ratios b_g^{schem} and b_g for the γ decay from the SDR$^{(-)}$ to the GTR in ^{208}Pb are rather different (Table I). The difference is mainly due to variation of the factor E_γ^3 over the SDR$^{(-)}$ doorway states taken into account within the realistic description. It is also noteworthy that the calculated mean SDR$^{(-)}$ energy 23.1 MeV [3] is higher than the experimental value of 21.1 ± 0.8 MeV [1] (for ^{90}Nb the corresponding values are 19.2 MeV and 17.9 MeV [7], respectively).

In conclusion, we evaluate the branching ratios of the γ decay from the SDR$^{(-)}$ to the GTR in ^{208}Bi and ^{90}Nb within the extended continuum-RPA approach. These predictions are expected to be appropriate for a comparison with the corresponding experimental data.

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