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Analyses of transverse momentum spectra in central C+C and C+Ta interactions at 4.2*A* GeV/*c* beam momentum within a collective flow model and a boundary model

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The transverse momentum spectra of secondary π^- mesons and participant protons in central C+C and C+Ta interactions at 4.2*A* GeV/*c* beam momentum have been analyzed within a collective flow model and a boundary model. All the particle spectra within a collective flow model at a given beam energy can be reproduced with a single set of intensive parameters for the initial state of fireball. As typical freeze-out parameters in this beam energy region we find a freeze-out temperature $T=(55\text{–}100)$ MeV, and an average transverse expansion velocity at freeze-out $\langle\beta\rangle=0.30\text{–}0.39$. The pion spectra within a boundary model including the quantum mechanical effects can be described well. To fit the proton spectra well, within a boundary model, one will need to introduce the transverse collective expansion.

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I. INTRODUCTION

Relativistic heavy-ion physics is at the interface between nuclear and particle physics. The aim is to create hadronic matter under extreme conditions where the quarks and gluons ‘‘are liberated’’ and quark-gluon-plasma is expected. An experimental question remains whether the appropriate conditions can be achieved in heavy-ion collisions.

Here we present a study of the experimental π^- mesons and participant protons spectra from central C+C and C+Ta collisions at a beam momentum of 4.2*A* GeV/*c* within a collective flow model and a boundary model. The goal is to find a simple, but realistic parametrization of the freeze-out stage in these collisions. In order to reach this goal an extensive experimental program has been done through studying heavy-ion collisions at various beam energies [1–6].

The extracted freeze-out parameters, temperature and the transverse flow velocity, will be compared with other collision systems and with collisions at different energies. Our results could be interesting because the beam energy in our experiment takes place between SIS and AGS energies. This should help to understand the collision dynamics at ultrarelativistic energies.

II. EXPERIMENTAL DATA

The experimental data have been obtained using a 2-m propane bubble chamber placed in a magnetic field of 1.5 T. The chamber was exposed to beams of light relativistic nuclei from the Dubna synchrophasotron. Three 1 mm thick tantalum plates were mounted inside the chamber. The general characteristics of the interactions and specific methods of data processing were published earlier in papers [7]. About 90% secondary charged particles emitted in 4π total solid angle have been detected in the chamber [8].

When scanning, all negative particles, except identified electrons, were considered as π^- mesons. The contaminations by misidentified electrons and negative kaons do not exceed 4% and 1%, respectively. The average minimal momentum for pion registration was about 70 MeV/*c* and momentum resolution $\langle\Delta p/p\rangle\simeq(5-10)\%$. The loss of slow π^- mesons vary about 5% for C+C to 10% for C+Ta interactions.

The protons and heavy nuclear fragments with the momentum value less than 0.2 GeV/*c* were not detected because of the short path in propane. The protons and positive pions were separated effectively up to $p_{\mathrm{lab}}\sim0.5$ GeV/*c* comparing the visible ionization density and path values. On the whole, the admixture of π^+ mesons among the protons was estimated as 10%. The problem of the misidentification can be resolved using the method of experimental spectra subtraction. Based on the isotopically symmetry of C+C collisions, the spectra of protons in the inelastic nuclear interactions were corrected for to satisfy the following expression $n_p=n|_{z=+1}-n_{\pi^+}(p<0.5\ \mathrm{GeV}/c)-n_{\pi^-}(p>0.5\ \mathrm{GeV}/c)$, where $n|_{z=+1}$ is the number of single charged positive particles (when the momenta, p, of these particles were measured they were considered as the protons). The similar expression was applied to the proton spectra in C+Ta interactions $n_p=n|_{z=+1}-n_{\pi^+}(p<0.5\ \mathrm{GeV}/c)-0.85n_{\pi^-}(p>0.5\ \mathrm{GeV}/c)$, where the factor of 0.85 reflects the deficiency of protons in the tantalum nucleus. The obtained spectra has to be modified also taking into account the losses of particles emitted at small angles relative to the camera optical axes and the particles absorbed by the tantalum plates. So, for C+Ta interactions the total corrections of the proton multiplicity were estimated to be about 14% and about (5–7)% for C+C collisions. The values of the proton

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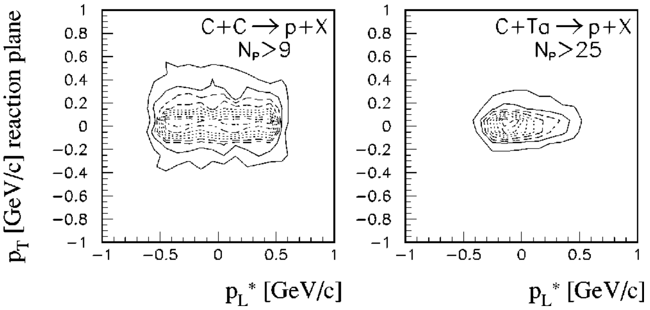


FIG. 1. Projection onto the reaction plane of the momenta of participant protons in the c.m.s. of participants in C+C collisions with $N_p>9$ (a) and C+Ta interactions with $N_p>25$ (b).

momenta were corrected calculating the average energy losses of the particle inside the tantalum plate. An average error of the proton momenta was obtained to be $\langle \Delta p/p\rangle \simeq 11\%$.

The stripping protons and charged fragments with the momentum of $p_{\rm lab}>3$ GeV/c and the emission angle of $\theta \leqslant 4°$, were excluded from the analysis as the projectile spectators, also the protons with the momentum of $p_{\rm lab}<0.3$ GeV/c, were excluded from the analysis as the target spectators. The remaining protons were considered as the participants.

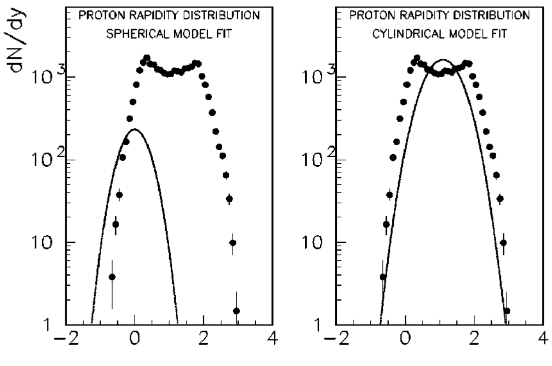
The admixture of deuterons (d) and tritons (t) to the secondary protons was studied in Ref. [9] in detail. The admixture does not alter the transverse momentum spectra and was not taken into account in the presented analysis.

III. ANALYSIS OF THE EXPERIMENTAL RESULTS WITH THE COLLECTIVE FLOW MODEL

The expanding fireball model [10] was developed earlier to describe the transverse momentum invariant spectra of secondary hadrons. The radial expansion of thermal fireball with a radially increasing expansion velocity was considered in the model and this spherical fireball picture failed to describe the longitudinal momentum distributions at the SPS energies [10]. So, this model had been improved upon a previous analysis and the flow model with cylindrically symmetry was presented [11].

We have tested both of the models to see which of them was appropriate for Dubna energies. It was quite important since two and three dimensional phase spaces look quite different.

First, we have carried out analysis based on the experimental determination of the reaction plane. This was done by means of the event shape. For each event the longitudinal momenta of protons were Lorentz-transformed into the rest frame of participant protons. The reaction plane was determined for each event and rotated by azimuthal angle ϕ in such a way that the individual reaction planes coincided with the x-z plane, where the z axis is the beam one. In other words, the reaction planes were superposed for all events. It is seen from Fig. 1 that the contour plot $p_L^*-p_T$ reaction plane exhibits a clear elyptic shape which helps to clarify the question of whether the cylindrical model is more appropriate for our energies.



(a) (b)

FIG. 2. Rapidity distribution for participant protons from central C+C collisions. The theoretical spectrum uses the parameters from spherical model (a) and from cylindrical model (b).

Second, the rapidity distribution from spherical flow model fails to describe experimental rapidity distribution, Fig. 2(a). Much wider experimental rapidity distribution is an indication for much stronger flow occurring along the beam axis than in the transverse direction and again requires the cylindrical model, as Fig. 2(b) proves.

According to the cylindrical flow model [11], the following expression was proposed to approximate the p_T spectra:

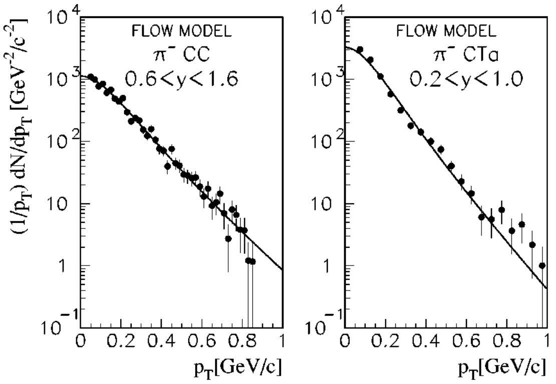
$$\frac{dN}{p_T dp_T}=N_0\sqrt{p_T^2+m^2}\int_0^R r dr K_1 \times(\gamma\sqrt{p_T^2+m^2})I_0\left(\frac{\gamma(r)\beta(r)p_T}{T}\right), \qquad (1)$$

where $y=0{,}5\ln[(E+p_L)/(E-p_L)]$, $\gamma(r)=1/\sqrt{1+\beta^2(r)}$ and N_0 is a normalization factor. K_1 and I_0 are the modified Bessel functions. The velocity profile in the transverse coordinate r for the expanding fireball is $\beta(r)=\beta_s\cdot(r/R)^n$, where β_s is the surface velocity and n fixes the shape of the profile. Using values of $n=1/2$ or 2, better fit of our data was obtained with $n=2$. As freeze-out values of R we used $R_{\rm CC}=R_{\rm CTa}\sim 3$ fm obtained by the interferometric analysis [12] for the radius of secondary particles emission region.

The data are from the most central events which were selected by the participant protons multiplicity N_p [13]. The statistics of selected events was 902 C+C interactions with $N_p>9$ and 378 C+Ta collisions with $N_p>25$. In these subsets the p_T spectra of π^- mesons and participant protons in a narrow midrapidity interval were analyzed. For the presented data midrapidity corresponds to $y_{\rm lab}=1.1$ in C+C collisions and $y_{\rm lab}=0.6$ in C+Ta collisions.

Figures 3 and 4 represents the transverse momentum invariant spectra of the π^- mesons and participant protons in C+C collisions with $N_p>9$ in rapidity interval $0.6<y<1.6$ [Figs. 3(a) and 4(a)] and in C+Ta interactions with $N_p>25$ and $0.2<y<1$ [Figs. 3(b) and 4(b)].

The solid line at Figs. 3 and 4 shows a fit with function (1). The fit was made for $p_T>50$ MeV/c for π^- mesons and a satisfactory agreement with the experimental data in



(a) (b)

FIG. 3. Invariant transverse momentum spectra of negative pions in C+C collision with $N_p>9$ (a) and C+Ta interactions with $N_p>25$ (b). Approximations by Eq. (1): T and β_s are free parameters (solid line).

C+C collisions is observed. In C+Ta interactions p_T spectrum for π^- mesons was fitted with $\chi^2/N_{\mathrm{DOF}}\sim 2$.

Also the fit for participant protons was made for $p_T > 300$ MeV/c in C+Ta interactions. In C+Ta interactions p_T spectrum for participant protons was fitted with $\chi^2/N_{\mathrm{DOF}}\sim 3$. It could be seen that the fits satisfactory reproduced the participant protons transverse momentum spectra in C+C collisions.

The results of the fit (1) are listed in Table I. The values of the χ^2/N_{DOF} are written among values of parameters T and β_s. The proton spectra consistently yield, as could be seen from the Table I, a greater temperature and lower transverse expansion velocity than the same for pions.

IV. ANALYSIS OF THE EXPERIMENTAL RESULTS WITH THE BOUNDARY MODEL

As could be seen from Figs. 3 and 4 a very interesting feature exhibited by the data is the increase in pion and proton yields at large transverse momentum, particularly for the pion distribution from the C+Ta system and for the proton distributions from both systems. This increase cannot be satisfactorily explained by the flow model used in the present analysis. Also the proton distribution from the C+Ta system within flow model was fitted for $p_T>300$ MeV/c.

TABLE I. Approximation results of p_T spectra for π^- mesons and participant protons in central C+C and C+Ta interactions. Values T and corresponding χ^2/N_{DOF} in brackets obtained by boundary model.

Interactions	Selection criteria	T (MeV)	χ^2/N_{DOF}	β_s
π^- mesons				
C + C	$N_p>9$	73 ± 7	0.91	0.69 ± 0.09
	$0.6<y_\pi<1.6$	(98 ± 3)	(0.88)	
C + Ta	$N_p>25$	55 ± 1	2.23	0.69 ± 0.03
	$0.2<y_\pi<1.0$	(62 ± 4)	(2.4)	
participant protons				
C + C	$N_p>9$	103 ± 18	1.43	0.49 ± 0.09
	$0.6<y_p<1.6$	(147 ± 2)	(1.49)	
C + Ta	$N_p>25$	70 ± 9	3.37	0.57 ± 0.08
	$0.2<y_p<1.0$	(114 ± 2)	(4)	

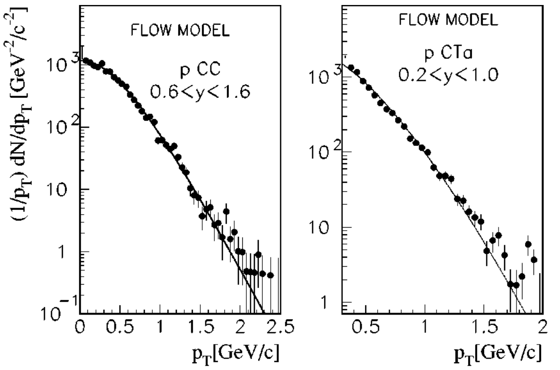
In order to explain this feature we have done calculations based on the effects of a boundary with the assumption of thermal equilibration [14].

Figures 5 and 6 represents the transverse momentum invariant spectra of the π^- mesons and participant protons in C+C collisions with $N_p>9$ in rapidity interval $0.6<y<1.6$ [Figs. 5(a) and 6(a)] and in C+Ta interactions with $N_p>25$ and $0.2<y<1$ [Figs. 5(b) and 6(b)].

The solid line at Figs. 5 and 6 shows a fit with function from boundary model [14]. The fit was made for $p_T > 50$ MeV/c for π^- mesons and for the whole p_T region for protons in both interactions.

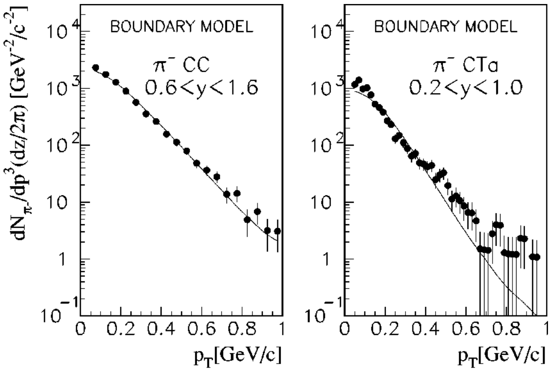
The results of the boundary model fit: The temperatures and corresponding χ^2/N_{DOF} are written in brackets in Table I. The obtained temperatures from boundary model fit, as could be seen from the Table I, are consistently higher than the same from the collective flow model.

We found that boundary fit works well for pions, but for



(a) (b)

FIG. 4. Invariant transverse momentum spectra of participant protons in C+C collision with $N_p>9$ (a) and C+Ta interactions with $N_p>25$ (b). Approximations by Eq. (1): T and β_s are free parameters (solid line).



(a) (b)

FIG. 5. Invariant transverse momentum spectra of negative pions in C+C collision with $N_p>9$ (a) and C+Ta interactions with $N_p>25$ (b). Approximations by boundary model: T is free parameter (solid line).

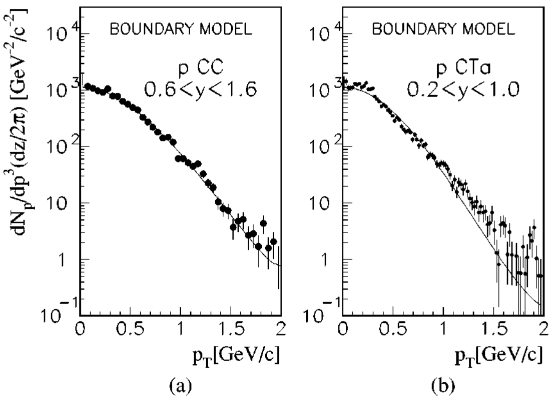


FIG. 6. Invariant transverse momentum spectra of participant protons in C+C collision with $N_p>9$ (a) and C+Ta interactions with $N_p>25$ (b). Approximations by boundary model: T is free parameter (solid line).

protons, the temperature is quite high for our energy. If we want to describe the system consistently as a single source of temperature, then we will need to introduce the transverse collective expansion. The effect of the transverse expansion is small for pions, but is much more important for protons. Hence, the neglect is tolerable for pions but not for protons.

This problem has been analyzed recently by Ayala *et al.* [15]. They applied the formalism with simultaneously expansion and boundary effects in a phenomenological calculation of pion spectra in relativistic heavy-ion collisions. They found a very good agreement with data on central Au+Au reactions at 11.6 GeV/c.

Our further calculations with transverse expansion within the boundary model will be discussed in an upcoming work.

V. SYSTEMATICS OF FLOW

A few interesting things can be learned from a systematic study of flow for different collision systems and bombarding energies. The summary of the available data in the energy range from 0.15–160 A GeV is presented in Fig. 7. Our results are shown in comparation to data from FOPI [1], EOS [2], AGS [3], and SPS [4]. Plotted are the average transverse velocities and the accompanying temperatures that are obtained from the collective flow model fit. Figure 7 shows that the freezeout temperature rises continuously as the bombarding energy is increased. The temperatures obtained by collective flow model follow the general trend found by other experimentalists in a wide range of beam energies.

The average transverse expansion velocities obtained at our energy are the same for the π^- mesons in C+C and C+Ta interactions, $\langle\beta\rangle=0.39$. Also for the participant protons obtained the average transverse expansion velocities are the same in both interactions, $\langle\beta\rangle\sim0.30$ within the statistical errors. The average transverse expansion velocities are rising from 0.1A GeV to AGS energies and maximum is reached at energies between SIS and SPS. The average transverse velocities seem to be limited to $\langle\beta\rangle\leqslant0.5$. A maximum is expected, where the lifetime of the system is largest such that a high degree of collectivity in the subsequent expansion can be achieved—one of the predictions of hydrodynamics with a phase transition [16,17].

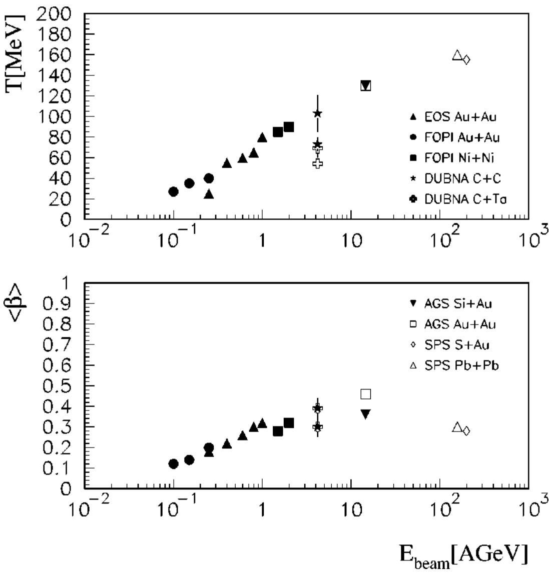


FIG. 7. Energy dependence of the temperature and average transverse expansion velocity.

VI. CONCLUSIONS

The transverse momentum spectra of secondary π^- mesons and participant protons in central C+C and C+Ta interactions at 4.2A GeV/c beam momentum have been analyzed within a cylindrical collective flow model. The values of freeze-out temperature $T=(55–100)$ MeV, and an average transverse expansion velocity at freeze-out $\langle\beta\rangle=0.30–0.39$ have been determined as the free parameters from the fit. The proton spectra consistently yield a greater temperature and lower transverse expansion velocity than the same for pions. The fits satisfactorily reproduced the transverse momentum spectra in C+C collisions. The extracted freeze-out parameters, temperature, and the transverse flow velocity were compared with other collision systems and with collisions at different energies and satisfactory agreement with general trend was found.

The transverse momentum spectra of secondary π^- mesons and participant protons in central C+C and C+Ta interactions at 4.2A GeV/c beam momentum have been analyzed within a boundary model.

The boundary model including the quantum mechanical effects can describe the pion spectra well. However, to fit the proton spectra, one needs to use a higher temperature, and there is a need to introduce an additional degree of freedom such as the transverse collective flow if one wants to have the same temperature for pions and protons.

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