# Inclusive reaction $^{40}$Ca$(p,p'x)$ at an incident energy of 392 MeV

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Emission-energy distributions were measured for the inclusive $^{40}$Ca$(p,p')$ reaction at an incident energy of 392 MeV and an angular range between 25° and 120°. A semiclassical distorted wave (SCDW) model gives a reasonably good description of the experimental data. The extent to which the observed yields are dominated by a two-particle rescattering emission contribution is estimated by a theoretical method that is known to describe experimental data for $^{40}$Ca$(p,p'p'')$ reasonably well. The energy distribution of the two-particle inclusive yield is found to agree well with the first-step contribution of the SCDW theory. Consequently this suggests strongly that the inclusive $(p,p')$ yield at forward angles and high emission energy consists mostly of two high-energy nucleons emitted in the first step of the collision.

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## I. INTRODUCTION

Investigations of the dominant processes which determine the characteristics of inclusive nucleon-induced reactions have been the subject of study for many years [1]. Although our fundamental quantum mechanical understanding of the mechanisms involved is now fairly well developed, the needs of applications in science and technology of these inclusive reactions require calculational procedures that are easily tractable. Towards this end, simplifications to the fundamental theories, development of simple semiclassical theories, or even of phenomenological approaches are useful.

At incident proton energies below about 200 MeV, numerous experimental results of inclusive $(p,p'x)$ reactions are available, and these have been scrutinized with a plethora of theoretical approaches. However, at higher incident energy there is a relative paucity of experimental data. The reason for this is simply that the experimental complexity increases with increasing incident and emission energy, and as a consequence considerable effort is currently devoted to developing simple and efficient new techniques for the detection of energetic protons. The present work, which uses a traditional although still cumbersome experimental procedure, is partly motivated by the desire to generate reliable data that could serve as a benchmark for a new generation of experiments using simpler detectors.

It seems reasonable to assume that multiparticle emission as a component of the inclusive $(p,p'x)$ reaction should become more important as the projectile energy is increased. In the past some attempts have been made to estimate the fraction of the yield which comes from multiparticle emission [2,3]. Of course, these estimates need to be based on the likely spatial trends of the angular distributions for multiparticle production. In our present study we utilize the insight gained from a previous investigation [4] of the two-proton emission reaction $^{40}$Ca$(p,p'p'')$ to calculate the relative importance of such a contribution to the inclusive $^{40}$Ca$(p,p')$ reaction.

## II. EXPERIMENTAL TECHNIQUE

The experiment was performed at an incident proton energy of 392 MeV at the accelerator facility of the Research Center for Nuclear Physics, Osaka, Japan. A large acceptance spectrometer (LAS) was operated at up to eight overlapping field settings to allow the measurement of the energy distributions of emitted protons in the $^{40}$Ca$(p,p')$ reaction from a threshold of approximately 50 MeV, up to the kinematic limit. Data were collected at scattering angles of 25.5°, 40°, 60°, 80°, 100°, and 120°. The horizontal acceptance angle, without use of defining slits, was $\pm 60$ mr. Standard focal plane detectors, electronics, and data taking systems were used.

The target was a self-supporting natural calcium (96.9% $^{40}$Ca) foil of normal grade chemical purity. Two targets were used and the results from both were checked for consistency; the thickness of one was measured as $12.8 \pm 0.2$ mg/cm$^2$, and the other as $16.0 \pm 0.2$ mg/cm$^2$. In order to evaluate the oxygen contamination on the Ca target, a comparison of the yield with a completely oxidized target was performed. This suggested that the oxygen content of the target used for the $(p,p')$ measurements contributed less than a few percent to our experimental cross sections and could therefore be neglected.

Momentum (kinetic energy) information was obtained from the data recorded with the focal plane detector system. As a consistency check on the extraction of data, cross sections generated from the overlap regions in the LAS field settings were compared. These cross sections agreed mostly to well within 10% over a sufficiently large region of overlap. The absolute experimental cross section scale is estimated to be accurate to within 10%.

## III. THEORETICAL CALCULATIONS

### A. Semiclassical distorted wave model

The semiclassical distorted wave (SCDW) model has been described in detail elsewhere (see, for example, Ref. [5]), so only a brief outline is presented here.

The SCDW model is based on a distorted wave Born approximation (DWBA) series expansion of $T$-matrix elements, the local semiclassical approximation to distorted waves, the eikonal approximation to intermediate state Green functions, and the local Fermi-gas model for the nuclear states. The inclusive double differential cross sections for one- and two-step processes, respectively, are expressed in the following closed forms:

$$\frac{\partial^2\sigma^{(1)}}{\partial E_f \partial\Omega_f} = \left(\frac{A}{A+1}\right)^2 \int d\mathbf{r} \frac{k_f/k_f(\mathbf{r})}{k_i/k_i(\mathbf{r})} |\chi_i^{(+)}(\mathbf{r})|^2 \times |\chi_f^{(-)}(\mathbf{r})|^2 \left(\frac{\partial^2\sigma}{\partial E_f \partial\Omega_f}\right)_{\mathbf{r}} \rho(\mathbf{r}), \tag{1}$$

and

$$\frac{\partial^2\sigma^{(2)}}{\partial E_f \partial\Omega_f} = \left(\frac{A}{A+1}\right)^4 \int dE_m \int d\mathbf{r}_1 \int d\mathbf{r}_2 \frac{k_f/k_f(\mathbf{r}_2)}{k_i/k_i(\mathbf{r}_1)} \times |\chi_i^{(+)}(\mathbf{r}_1)|^2 |\chi_f^{(-)}(\mathbf{r}_2)|^2 \left(\frac{\partial^2\sigma}{\partial E_f \partial\Omega_f}\right)_{\mathbf{r}_2} \times \rho(\mathbf{r}_2) \frac{\exp(-2\gamma_m|\mathbf{r}_2-\mathbf{r}_1|)}{|\mathbf{r}_2-\mathbf{r}_1|^2} \left(\frac{\partial^2\sigma}{\partial E_m \partial\Omega_m}\right)_{\mathbf{r}_1} \rho(\mathbf{r}_1), \tag{2}$$

where $A$ is the target mass number, $k_c$ and $k_c(\mathbf{r})$ ($c=i$ or $f$) the wave number at infinity and the local wave number in the initial ($i$) and final ($f$) channels, $\chi_i^{(+)}(\chi_f^{(-)})$ the distorted wave in the initial (final) channel, $\rho(\mathbf{r})$ the nucleon density, and $\gamma_m$ is the imaginary part of the local wave number of a leading particle in the intermediate channel $m$ with the energy $E_m$. The local average $N$-$N$ scattering cross section at the point $\mathbf{r}$, $(\partial^2\sigma/\partial E_c \partial\Omega_c)_{\mathbf{r}}$ ($c=m$ or $f$) in Eqs. (1) and (2) is given by

$$\left(\frac{\partial^2\sigma}{\partial E_f \partial\Omega_f}\right)_{\mathbf{r}} = \frac{4k_f(\mathbf{r})}{k_i(\mathbf{r})(4\pi/3)k_F(\mathbf{r})^3} \times \int\int_{k_\alpha<k_F(\mathbf{r})<k_\beta} d\mathbf{k}_\alpha d\mathbf{k}_\beta \left(\frac{\partial\sigma}{\partial\Omega_{\boldsymbol{\kappa}}}\right)_{NN} \times \delta[\mathbf{k}_\beta - \mathbf{k}_\alpha + \mathbf{k}_f(\mathbf{r}) - \mathbf{k}_i(\mathbf{r})]\,\delta(\varepsilon_\beta - \varepsilon_\alpha - \omega), \tag{3}$$

where $k_F(\mathbf{r})$ is the Fermi momentum at $\mathbf{r}$, $\mathbf{k}_\alpha(\mathbf{k}_\beta)$ the nucleon momentum for a single-particle state with energy $\epsilon_\alpha(\epsilon_\beta)$ below (above) the Fermi level, $(\partial\sigma/\partial\Omega_{\boldsymbol{\kappa}})_{NN}$ the two-nucleon scattering cross section, and $\omega$ the energy transfer. The Pauli blocking effect is taken into account by the limits of the integrations over $\mathbf{k}_\alpha$ and $\mathbf{k}_\beta$.

For $k$-step ($k \geqslant 3$), the cross sections are deduced straightforwardly to expressions of the same structure as Eq. (2), which contain the product of $k$ local average cross sections and nucleon densities at $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_k$, and $k-1$ propagators between the consecutive collision points.

As seen in Eqs. (1)–(3), the input data for SCDW calculations are the distorted potentials, the two-nucleon scattering cross sections, and the nucleon density distribution. Since these quantities can be determined either empirically or theoretically, no free adjustable parameters are involved in the SCDW model.

### B. Calculation of two-particle emission

The explicit calculation of the contribution to the inclusive yield from two-particle emission is based on the successful description [4] of the reaction $^{40}$Ca$(p,p'p'')$ at an incident energy of 392 MeV. Complete details of these theoretical ideas are presented in Refs. [6,7] therefore, also in this case, only a brief summary is provided here.

It is assumed that the two-particle yield originates from a collision of the projectile with a target nucleon (either proton or neutron) bound in a shell model orbital. Apart from this collision initiating a knockout process, it is assumed that an additional contribution to the two-proton yield comes from a rescattering of the struck nucleon with the spectator part of the target nucleus. Thus the cross section for this latter process, indicated as the reaction $(p,p'p'')$, can be expressed as

$$\frac{d^4\sigma}{d\Omega_1\ d\Omega_2\ dE_1\ dE_2} = \sum_N \left[\int d\Omega_N \sum_\lambda \frac{d^3\sigma}{d\Omega_1\ d\Omega_N\ dE_1} \times \frac{d^2\sigma}{d(\Omega_2-\Omega_N)\ dE_2}\right], \tag{4}$$

where the cross section $d^3\sigma/d\Omega_1 d\Omega_N\ dE_1$ describes the knockout of nucleons $N$ from each orbital $\lambda$ and $d^2\sigma/d(\Omega_2 - \Omega_N)\ dE_2$ represents the (properly normalized, see Cowley *et al.* [4]) rescattering cross section.

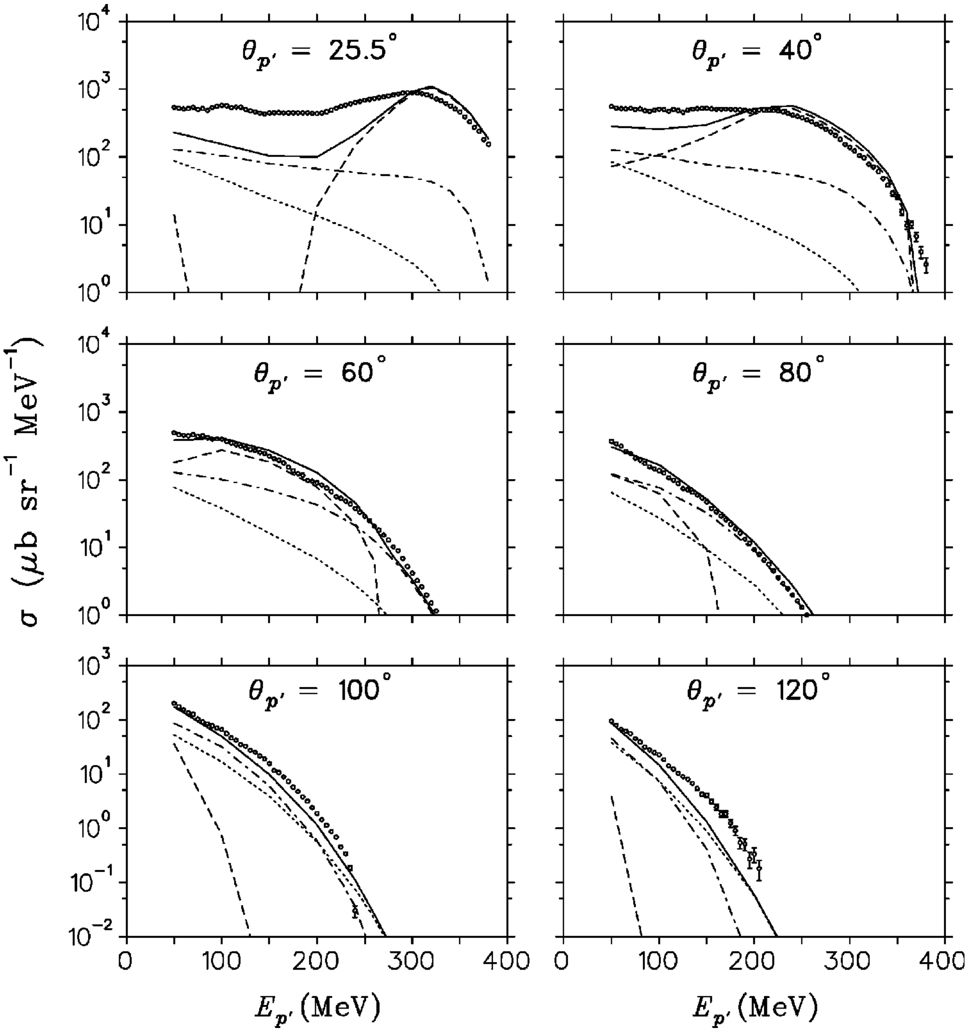


FIG. 1. Differential cross sections for the inclusive reaction $^{40}$Ca$(p,p')$ as a function of emission energy $E_{p'}$, for various scattering angles $\theta_{p'}$. Experimental data with statistical error bars where these exceed the symbol size, are given in the laboratory coordinate system. Results of the SCDW calculations are shown for the first (dashed curve), second (dot-dashed curve), and third (dotted curve) steps. The total SCDW contributions are shown as continuous curves.

The inclusive yield from such a process is found by integrating over the energy and solid angle of the particle that is unobserved in an inclusive $(p,p')$ experiment, therefore

$$\frac{d^2\sigma}{d\Omega_1\ dE_1}=\int\int dE_2\ \ d\Omega_2\frac{d^4\sigma}{d\Omega_1\ \ d\Omega_2\ \ dE_1\ \ dE_2}\ . \tag{5}$$

The total inclusive two-particle yield then consists of the above-mentioned rescattered part plus the discrete knockout components from $^{40}$Ca$(p,2p)^{39}$K and $^{40}$Ca$(p,pn)^{39}$Ca added incoherently. It should be noted that in this approach we implicitly assume that the (observed) proton of interest is one that does not participate in a rescattering process. Clearly, in the spirit of our assumptions, this only applies to protons that have relatively high energy, and which are scattered through small angles only. In other words, we attempt to reproduce only the high-energy, small-angle, component of the inclusive spectra.

The knockout reactions are calculated in the distorted wave impulse approximation (DWIA) [8] with spectroscopic factors extracted from the work of Cowley *et al.* [4]. As was mentioned before, this procedure of reconstructing the inclusive $(p,p')$ spectra relies on the ability of the rescattering model to predict $(p,p'p'')$ spectra. Consequently, another component from an analogous $(p,p'n)$ reaction, in which the neutron results as an ejectile from the nucleon rescattering mechanism, has to be added to the inclusive yield. For this we simply assume that the rescattered yields from $(p,p'p'')$ and $(p,p'n)$ are the same in magnitude and angular distribution. This amounts to treating all nucleons on an equal footing.

## IV. RESULTS

Experimental cross section distributions for the inclusive $^{40}$Ca$(p,p')$ reaction, at an incident energy of 392 MeV, are shown in Figs. 1 and 2. The results are shown as a function of emission energy for various scattering angles, and the statistical error bars are indicated where these exceed the symbol size.

In Fig. 1 the theoretical cross section distributions, calculated with the semiclassical distorted wave formalism with nonrelativistic kinematics, are compared with the experimental quantities. The input data used for the calculation are the same as in Ref. [5], except for the global optical potential of Madland [9]. Although the model underpredicts the cross section for low emission energies at 25.5°, the results of the

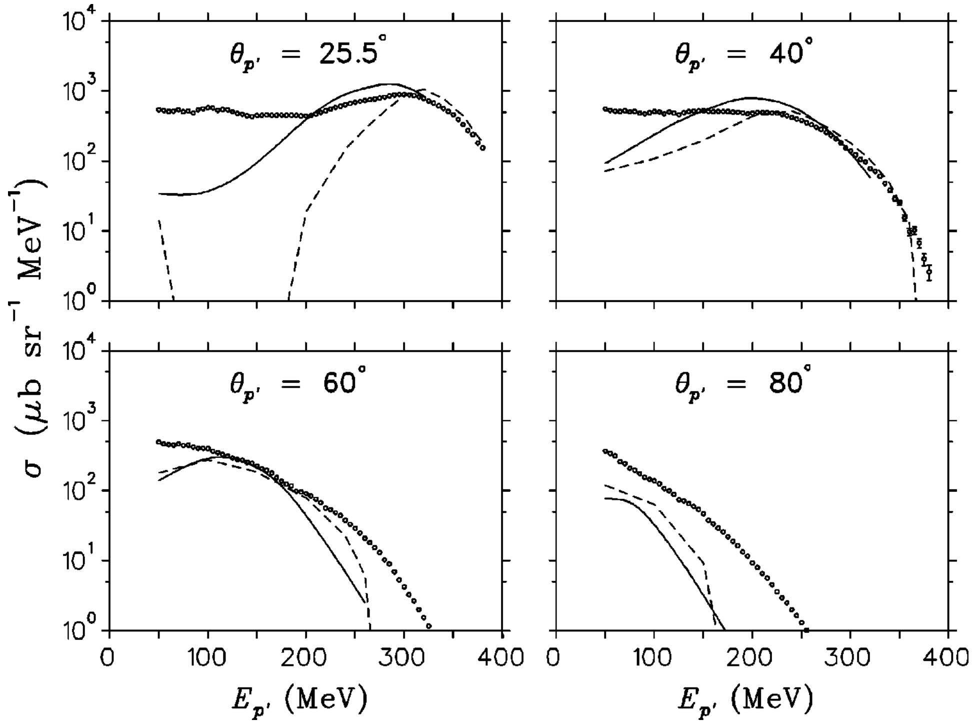


FIG. 2. Comparison between the first-step SCDW calculations for the inclusive reaction $^{40}$Ca$(p,p')$, and calculations based on integration of the two-particle emission, as described in the text. The first step SCDW results correspond to the dashed curve. The experimental data are also shown.

theory are generally in reasonable agreement with the observed quantities. We also find that the first step dominates at high emission energy at scattering angles smaller than 80°. Beyond that scattering angle, the higher steps rapidly grows in importance, and at the largest angle investigated there is already an indication that more than three steps would be needed to reproduce the absolute magnitude of the observed yield. Moreover, the underprediction seen at backward angles would be due to the use of the local Fermi-gas model for the nuclear states, as investigated in Ref. [10].

From perusal of the various simplistic assumptions that enter into our calculation of two-particle emission, it is clear that this component, when compared to the semiclassical distorted wave treatment would be expected to correspond to the first step of the latter theory. (Of course, the other rescattered proton, which is treated as an ''unobserved'' particle, would appear at lower emission energy and generally at larger angles.) In Fig. 2 we show that this expectation is realized. The unexpected result is that the agreement in absolute magnitude between the observed yield and the two-particle emission calculations implies that there are almost always two high-energy nucleons emitted in a first-step collision. This result is very different from the situation at lower incident energy (100–200 MeV), where estimates indicate only 20–30 % two-particle emission [2,3]. In principle it is also possible to calculate the rescattered inclusive yield with the two-particle formalism. However, in that case application of the method to a primary proton that does not suffer rescattering, yet has a large energy transfer, and which is consequently observed at the same energy and angular range as the secondary particles, becomes questionable.

## V. SUMMARY AND CONCLUSION

The inclusive $^{40}$Ca$(p,p')$ reaction was studied at an incident energy of 392 MeV and an angular range between 25° and 120°. The emission-energy distributions were modeled with a semiclassical distorted wave (SCDW) formalism. The results of the theory agree reasonably well with the experimental quantities.

The two-particle component, with both ejectiles having fairly high energies (tens to hundreds of MeV) above the evaporation peak, was estimated by means of extrapolation with a theoretical method that is known to describe existing experimental data for $^{40}$Ca$(p,p'p'')$ well. The extent to which the two-particle emission dominates the inclusive singles yield is remarkable, and it appears that almost all of the high energy, small angle spectra consist of ejectiles of such a nature. These calculations are in agreement, not only with the experimental data, but naturally also with the first step of the SCDW theory, as would be required for consistency.

Clearly this work provides an incentive for further investigation of the incident energy dependence of the extent to which multiparticle emission influences the appearance of inclusive reactions. Towards this end, a refined model of the coincident particle mechanism would be desirable.

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