Mixing of the f_0 and a_0 scalar mesons in threshold photoproduction

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We examine the photoproduction of the light scalar mesons f_0-$a_0(980)$ with emphasis on isospin-violating f_0 and a_0 mixing due to the mass difference between neutral and charged kaons. General forms for the invariant amplitude for scalar meson photoproduction are derived, which yield expressions for the invariant mass distribution of the final meson pairs. A final state interaction form factor is obtained which incorporates f_0 and a_0 mesons, $K\bar{K}$ threshold effects, and f_0-a_0 mixing. This form factor is applied to predict the effective mass distribution of the $\pi\pi$ and $K\bar{K}$ pairs in the vicinity of the $K\bar{K}$ threshold. An estimate of the role of isospin mixing near threshold is provided. The potential role of polarized photons and protons is also discussed.

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I. INTRODUCTION

The f_0 and a_0 scalar mesons present a well-known puzzle for which several interesting, albeit controversial proposals have been made ranging from quark-antiquark makeup, to four-quark, and to $\bar{K}K$ molecular structure. Our goal is not to review these suggestions, but rather to investigate if new information on the makeup of these scalar mesons can be gleaned from threshold $f_0(980)$ and $a_0(980)$ photoproduction. Our hope is that threshold production of 0^+ $K\bar{K}$ and two-pion states that arise from the decay of scalar mesons will shed light on the nature of the f_0 and a_0 scalar mesons. For that purpose, we examine a potentially important final state isospin mixing effect.

The f_0 has been observed in a photoproduction experiment at Fermilab [1]. Measurements of exclusive f_0 photoproduction and electroproduction are being performed at Jefferson Laboratory [2]. Extensive theoretical studies of f_0 photoproduction have been presented in recent papers [3,4]. Our study differs from these works in that we assume f_0 and a_0 mesons are produced directly and play the role of doorway states, while in [3,4] these mesons enter via final state interactions in $K\bar{K}$ and $\pi\pi$ channels. The two descriptions are complementary and further studies are needed to clarify their interconnection. Another novel feature of our present work is the inclusion of possibly important final state f_0-a_0 mixing. This mixing is induced by scalar meson transitions into and out of K^+K^- and $K^0\bar{K}^0$ intermediate states, which generates f_0-a_0 mixing because of the 8 MeV splitting between the K^+K^- and $K^0\bar{K}^0$ thresholds. Thus scalar meson mixing arises from the difference between the neutral and charged kaon masses, e.g., from isospin breaking. This effect is fully included into our treatment. Calculations presented below rely on a very restricted number of parameters, most of which are fixed using recent experimental data on the $\phi \to \gamma f_0/a_0$ reaction [5,6].

We study the reaction

$$\gamma + p \to p' + f_0/a_0 \to p' + m_1 m_2, \tag{1.1}$$

using the f_0 and the a_0 as the possible doorway for subsequent decay meson production. Here $m_1 m_2$ denotes $\pi\pi$ or $K\bar{K}$ pairs or the $\pi\eta$ system. A thorough theoretical investigation of this reaction should include at least three essential points: (a) the general structure of the invariant scalar meson photoproduction amplitude, including expressions for the cross sections and spin observables, (b) effects of the final state interaction, including $K\bar{K}$ threshold phenomena, and (c) a dynamical model for the reaction mechanism, e.g., an effective Lagrangian and/or a set of leading diagrams. In our paper we concentrate on points (a) and (b), which are more universal and less model dependent than point (c) above. The structure of the basic production amplitude and spin observables presented below does not rely on any explicit reaction mechanisms (other than the doorway assumption) and are applicable to the photoproduction of any scalar meson. The final state interaction (FSI) form factor we present later is constructed in a model-independent way based on a theory of resonance mixture. Only a few parameters are needed which on the one hand are sensitive to the nature of the f_0/a_0 mesons, and on the other can be extracted from recent experimental data on the $\varphi \to \gamma f_0/a_0$ reaction [5,6]. Concerning the dynamical point (c), the dominant reaction mechanism is expected to depend upon the kinematical conditions of the explicit experiment. For example, at low photon momenta *s*-channel resonances might give a substantial contribution [7], while forward photoproduction at higher energies might be dominated by *t*-channel, ρ and ω meson exchanges [3]. An alternate, very promising approach making use of chiral Lagrangians was proposed recently [4]. Although we do not propose an explicit reaction mechanism, our driving idea is complementary to that of Refs. [3,4]. We take the f_0/a_0 to be ''doorway states'' for the reaction. Namely, we assume that f_0/a_0 scalar mesons are produced ''directly'' and then propagate under the strong influence of two nearby $K\bar{K}$ thresholds before decaying into $K\bar{K}, \pi\pi$, or $\pi\eta$. This

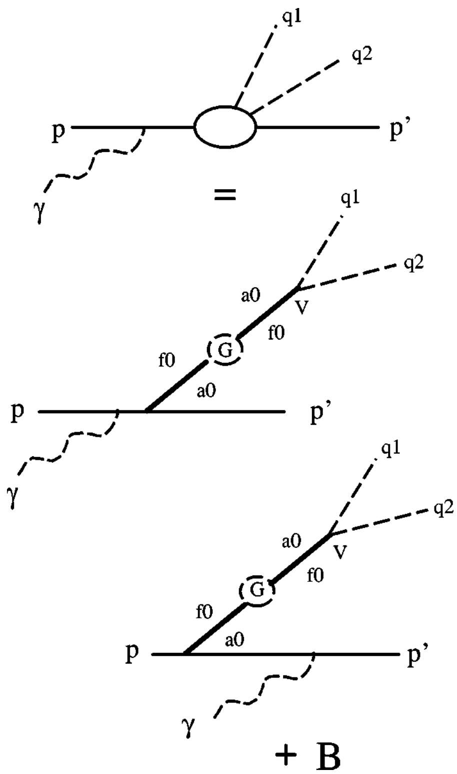

FIG. 1. Feynman diagrams describing the doorway production of the reaction $\gamma p \to p' m\bar{m}$. The total amplitude is a sum of these f_0/a_0 contributions plus the background *B*, which is described in Fig. 2. The solid line with the blob *G* insertion denotes the complete f_0/a_0 propagator [see Eq. (4.17)] with $f_0 \leftrightarrow a_0$ mixing. The decay mesons shown as dashed lines have momenta q_1 and q_2.

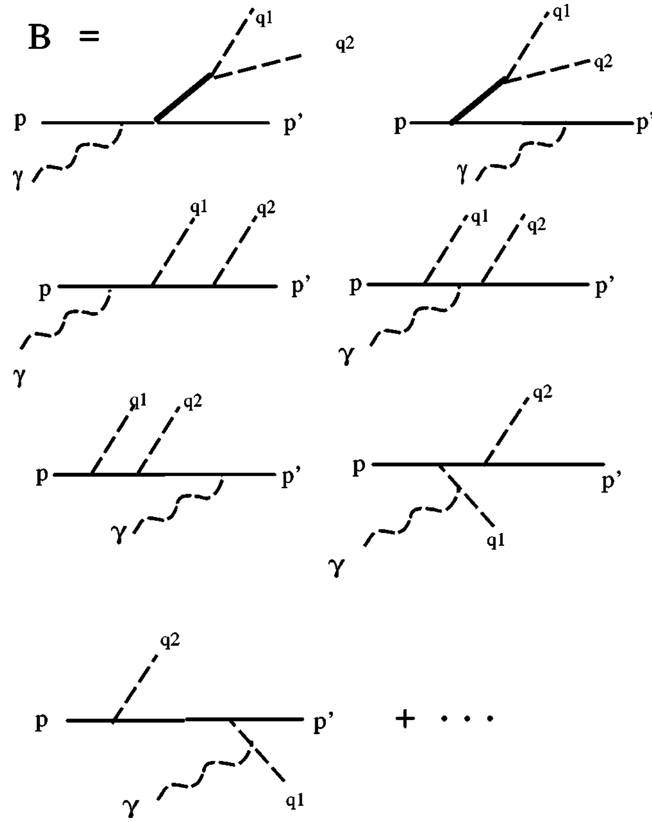

FIG. 2. Feynman diagrams describing the possible background contributions *B* for the the reaction $\gamma p \to p' m\bar{m}$. Dark solid lines stand for the contribution of resonances other than f_0/a_0.

propagator, or FSI form factor, has a simple form and is applicable to any reaction involving f_0/a_0.

We begin by analyzing the structure of the photoproduction of scalar mesons using this 0^+ doorway model; then we formulate a description of the final state interaction. The general behavior of the cross section as a function of the invariant mass of the kaon or pion pair is then generated to gauge the importance of the mixing near threshold and for comparison with experiment. Speculations about the role of scalar-meson nucleon *P* waves are presented, especially as they relate to the possible observation of spin observables.

II. DIAGRAMS AND CROSS SECTIONS

Before focusing on f_0/a_0 photoproduction in reaction (1.1), we comment on possible background effects.

The amplitude for two-meson photoproduction $\gamma p \to p' m_1 m_2$ can be represented symbolically by the series of graphs depicted in Fig. 1. Although all diagrams entering into the background term[1] B_α (Fig. 2) might be explicitly calculated (some of them were calculated in [3]), our main point is that B_α is a smooth function of the energy in the vicinity of the $K\bar{K}$ threshold, whereas dramatic energy-dependent effects in the invariant mass distribution of the $m_1 m_2$ pair (see below) are generated mainly by the first two diagrams in Fig. 1. We neglect the pion loop diagram shown in Fig. 3 (see [3,4]) for two reasons. One reason is that near the $K\bar{K}$ threshold the pion loop by itself, i.e., without adding a resonant rescattering vertex, has smooth energy behavior and has to be included into the renormalized vertex of the f_0 production. The second reason for neglecting pion loops is based on Weinberg's power counting theorem [8], which states that pion loops are suppressed compared to the tree diagrams depicted in Fig. 1. According to these arguments the pion loops are of the same order as higher terms in the effective Lagrangian. However, the above reasoning is far from being a rigorous proof of the direct production dominance. The theoretical and experimental study of this prob-

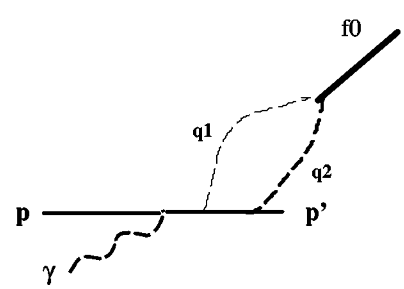

FIG. 3. Production of f_0 via the pion loop as discussed in the text.

[1]The subscript α denotes the $\pi\pi$ or $K\bar{K}$ channels.

lem is at an early stage and we hope that comparison of complementary approaches will advance the investigation.

Consider now the graphs on the top of Fig. 1. We first consider only the f_0 meson; a_0 production will be included later. Then these diagrams under consideration are described by the following equation:

$$T_\alpha = V_{\alpha f} G_f F_f + B_\alpha, \tag{2.1}$$

where F_f is the f_0 photoproduction amplitude, G_f is the f_0 propagator (also called the FSI form factor), and $V_{\alpha f}$ is the amplitude for f_0 decay into the final meson pair channel α ($\alpha \equiv \pi\pi, K\bar{K}$); here, B_α denotes the background terms for channel α which we presume to be weakly energy dependent.

As shown below, the form factor G_f contains dynamical information on the nature of the f_0 meson. This form factor also reflects the interplay of the f_0 with the nearby $K\bar{K}$ threshold. The simple kinematical fact that K^+K^- and $K^0\bar{K}^0$ thresholds are split by 8 MeV induces significant f_0/a_0 mixing, as first noticed by Achasov *et al.* [9]. Taking account of this f_0/a_0 mixing, Eq. (2.1) is now extended to include a sum over intermediate f_0 *and* a_0 states:

$$T_\alpha = \sum_{i,k} V_{\alpha i} G_{ik} F_k + B_\alpha, \tag{2.2}$$

with $i,k = f_0, a_0$; i.e., the form factor G_{ik} becomes a 2×2 matrix in f_0/a_0 space. As pointed out earlier by Stodolsky [11] measurement of the invariant mass distribution of the (m_1+m_2) system in reaction (1.1) enables one to determine the nonorthogonality of the decaying states, i.e., the probability for $f_0 \leftrightarrow a_0$ transitions.

The cross section for the reaction $\gamma p \to p' m_1 m_2$ (see Fig. 1), where the meson pair is in channel α, is

$$\sigma_\alpha = \frac{m^2}{(2\pi)^3} \frac{1}{\lambda^{1/2}(s,0,m^2)} \int dE_{m_1}\, dE_{m_2} \frac{d\Omega_{m_1}}{4\pi} \frac{d\varphi_{m_2}}{2\pi} |T_\alpha|^2, \tag{2.3}$$

where $\lambda(x,y,z) \equiv (x-y-z)^2 - 4yz$, $s = (k+p)^2 = (p' + q_{m_1} + q_{m_2})^2$, m is the proton mass, Ω_{m_1} is the angle of decay meson 1 with respect to the incident photon beam $\vec{k}$, and $\Omega_{m_2} = (\cos\theta_{m_1 m_2}, \varphi_{m_2})$ is the angle of decay meson 2 with respect to Ω_{m_1}. The decay mesons are coplanar as described by φ_{m_2}.

To observe the f_0/a_0 meson and to investigate its properties use is made of the double differential cross section

$$\frac{d\sigma_\alpha}{dt\, dm_x} = \frac{m^2}{(2\pi)^3} \frac{\sqrt{\lambda(m_x^2, m_{m_1}^2, m_{m_2}^2)}}{4\ m_x\ \lambda(s,0,m^2)} \frac{1}{4\pi} \int d\Omega^*\ |T_\alpha|^2 \tag{2.4}$$

or of the invariant mass m_x distribution

$$\frac{d\sigma_\alpha}{dm_x} = \frac{m^2}{(2\pi)^3} \frac{\sqrt{\lambda(s, m_x^2, m^2)\ \lambda(m_x^2, m_{m_1}^2, m_{m_2}^2)}}{4\ s\ m_x\ (s-m^2)} \frac{1}{2} \int_{-1}^{+1} \times d\cos\theta \frac{1}{4\pi} \int d\Omega^*\ |T_\alpha|^2, \tag{2.5}$$

where $t = (p-p')^2, m_x = s_1^{1/2}, s_1 = (p_{m_1} + p_{m_2})^2$, and Ω^* is the angle between the relative momentum of the two decay mesons in their c.m. system. The angle θ refers to the scalar meson production angle. Proton spinors are normalized according to $\bar{u}u = 1$. We will use this last expression to display the cross section as a function of the invariant mass m_x, once we have described the amplitude T_α of Eq. (2.2).

The initial and final helicity summations are suppressed above in $|T_\alpha|^2$. For unpolarized beam, target, and recoil proton experiments, it is therefore understood that one sums over final and averages over initial helicities. However, if one has a polarized beam or polarized target, the associated cross sections can be expressed, using the bold assumption that B_α interference can be neglected, by

$$|T_\alpha|^2 = V_{\alpha i} G_{ik} \operatorname{Tr}[(F_k \rho_I F_{k'}^\dagger)]\ G_{i'k'}^* V_{\alpha i'}^*, \tag{2.6}$$

where ρ_I is the density matrix which describes the spin state of the initial beam and target. Since the decay $V_{\alpha i}$ and the propagator G_{ik} are both independent of helicities, the inner term above $\mathcal{S}_{kk'} = \operatorname{Tr}[(F_k \rho_I F_{k'}^\dagger)]$ includes a trace over the helicity quantum numbers in the usual way.[2] The function $\mathcal{S}_{kk'}$ is a coupled-channel version of the usual ensemble average expression. For a polarized recoil proton experiment, $\mathcal{S}_{kk'}$ generalizes to $\mathcal{S}_{kk'} = \operatorname{Tr}[\rho_F(F_k \rho_I F_{k'}^\dagger)]$, where ρ_F describes the final recoil polarization measurement. With this expression, it is possible to extract the effect of having a polarized beam and/or target on the cross section. It is of interest to see if such measurements, especially as meson-nucleon P waves appear, can shed light on the structure of the scalar mesons.

We are now ready to examine the structure of scalar meson photoproduction in order to specify the scalar meson production amplitude F.

III. INVARIANT AMPLITUDES AND SPIN OBSERVABLES FOR THE SCALAR MESON

The spin structure of the scalar meson photoproduction amplitude F and the corresponding spin observables are discussed in this section. Again k, p, and p' denote the four-momenta of the photon and the initial and final protons, respectively, while the scalar meson's four-momentum is denoted by q. The decomposition of the *scalar* meson photoproduction amplitude as invariant amplitudes has the following [Chew-Goldberger-Low-Nambu (CGLN) [12]] form

[2] Note that $\mathcal{S}_{kk'}^* = \mathcal{S}_{k'k}$ which assures real observables and that $\mathcal{S}$ is Hermitian in the f_0/a_0 channel space, which should not be confused with the Hermiticity of the density matrix in spin space.

$$F=\overline{u}(p')\sum_{j=1}^{4} M_j A_j(s,t,u)u(p). \tag{3.1}$$

We follow the $\overline{u}u=1$ normalization conditions of [13]. For the scalar meson, the four invariant amplitudes M_j are

$$M_1=-\not\varepsilon\not k,$$

$$M_2=2(\varepsilon p)(kp')-2(\varepsilon p')(kp),$$

$$M_3=\not\varepsilon\ (kp)-\not k\ (\varepsilon p),$$

$$M_4=\not\varepsilon\ (kp')-\not k\ (\varepsilon p'), \tag{3.2}$$

where ε_μ is the photon polarization four-vector. The four amplitudes M_i differ from the 0^- meson photoproduction only in the omission of the γ_5 pseudoscalar factor.

The amplitude F can be expressed in terms of the two-component spinors χ. For this purpose we write

$$F=\sqrt{\frac{\omega(p)\omega(p')}{4m^2}}\langle\chi(p')|R|\chi(p)\rangle, \tag{3.3}$$

where $\omega(p)=E_p+m$. Substitution of the invariant amplitudes, Eq. (3.3), into Eq. (3.1) leads to the following CGLN-type representation for the amplitude R:

$$R=i\vec{\varepsilon}\cdot(\vec{\sigma}\times\hat{k})\ R_1+(\vec{\varepsilon}\cdot\vec{\sigma})(\vec{\sigma}\cdot\hat{q})\ R_2+i(\vec{\varepsilon}\cdot\hat{q})(\hat{q}\times\hat{k})\cdot\vec{\sigma}\ R_3 +(\vec{\varepsilon}\cdot\hat{q})\ R_4, \tag{3.4}$$

where $\hat{k},\hat{q}$ are unit vectors. The four amplitudes R_j are related to the four amplitudes A_j of Eq. (3.1) via

$$R_1=-(\sqrt{s}-m)\left\{A_1+\frac{1}{2}(\sqrt{s}-m)A_3+\frac{(kp')}{\sqrt{s}+m}A_4\right\},$$

$$R_2=\frac{|\vec{q}\,|}{E_{p'}+m}\left\{-(\sqrt{s}-m)A_1+\frac{1}{2}(s-m^2)A_3+(kp')A_4\right\},$$

$$R_3=|\vec{q}\,|^2\left(\frac{\sqrt{s}-m}{E_{p'}+m}\right)\{(\sqrt{s}-m)A_2-A_4\},$$

$$R_4=-(\sqrt{s}-m)\ |\vec{q}\,|\ \left\{A_2(\sqrt{s}+m)\left[1-\frac{\vec{k}\cdot\vec{q}}{(E_p+m)(E_{p'}+m)}\right] +A_4\left[1+\left(\frac{\sqrt{s}+m}{\sqrt{s}-m}\right)\frac{\vec{k}\cdot\vec{q}}{(E_p+m)(E_{p'}+m)}\right]\right\}. \tag{3.5}$$

The (kp') factor denotes a four-vector product. To recast Eqs. (3.5) into a fully relativistic invariant form, the following simple kinematical equations may be used:

$$2(\vec{k}\cdot\vec{q})=t-\mu^2+\frac{1}{2s}(s-m^2)(s-m^2+\mu^2),$$

$$2kp'=m^2-u,$$

$$|\vec{q}\,|=\frac{1}{2\sqrt{s}}\lambda^{1/2}(s,\mu^2,m^2),$$

$$E_p=\frac{1}{2\sqrt{s}}(s+m^2),\quad E_{p'}=\frac{1}{2\sqrt{s}}(s+m^2-\mu^2), \tag{3.6}$$

where μ is the mass of either f_0 or a_0. Note that an alternative form of the amplitude (3.5) may be found in Ref. [14].

Note that at the scalar meson production threshold all of the R_i amplitudes vanishes except for R_1. We conclude that at the scalar meson production threshold the operator F is given by

$$F=\frac{[\omega(p)\omega(p')]^{1/2}}{2m}R_1\ i\vec{\varepsilon}\cdot(\vec{\sigma}\times\hat{k}). \tag{3.7}$$

The value of the coefficient R_1 is discussed below in Sec. V.

As meson-nucleon P waves turn on, the associated $J=\frac{1}{2}^-,J=\frac{3}{2}^-$ amplitudes contribute with an initial linear dependence on the momentum $|\vec{q}\,|$; hence, it is likely that the terms R_2 and R_4 will appear along with their operators. That unfolding of P waves has implications concerning which spin observables assume nonzero values as the energy rises beyond the threshold value of $E_\gamma(\text{lab})=1.5$ GeV.

IV. FORM FACTOR OF THE SCALAR MESONS NEAR THE $K\overline{K}$ THRESHOLD

In this section, we consider the FSI form factors G_f (pure 0^+ propagation) and G_{ik} (coupled 0^+ propagation) introduced in Eqs. (2.1) and (2.2). These form factors describe the propagator of an unstable particle in the energy region overlapping some of the decay thresholds (K^+K^- and $K^0\overline{K}^0$). We are mainly interested in the coupled-channel form factor G_{ik}, which includes f_0-a_0 mixing, but to simplify the discussion, we start with the simple one-channel propagator G_f, which describes the f_0 meson and its decay channels. Later we generalize that discussion to the coupled-channel G_{ik} case.

There exists an overwhelming number of approaches to describe an unstable particle and particle mixtures. We follow the general phenomenological approach developed by Stodolsky [15] and Kobzarev, Nikolaev, and Okun [16], and in a slightly modified form presented in the book of Terent'ev [17]. This approach has already been applied to the f_0/a_0 system in [18]. We now briefly outline the derivation of the basic equations for propagation of an unstable particle; see [15–18] for details.

First consider f_0 and its decay channels (a_0 will be incorporated later) and let us introduce a set of state vectors $|i\rangle$ describing the scalar meson. One of these states, often denoted as $|f\rangle$, is a discrete state; continuum states are also included in the set $|i\rangle$. The discrete state couples to the continuum states and thereby acquires a width and a shift in mass; e.g., it becomes an unstable state. Let the whole scalar meson system be described by the Hamiltonian $H=H_0+V$, where H_0 has a multichannel continuous spectrum $H_0|i\rangle$

$=E_i|i\rangle$ and a discrete state $H_0|f\rangle=E_f|f\rangle$. The interaction V is responsible for the transitions between the above channels, so that with V "turned on," $|f\rangle$ becomes a resonance. Consider the transition amplitude

$$A_{if}(t)\equiv\langle i|f;t\rangle, \tag{4.1}$$

where $|f;t\rangle=\exp[-i(H_0+V)t]|f\rangle$. "Diagonal" transitions $A_{ff}(t)$ are also included in this definition. The amplitude $A_{if}(t)$ satisfies the equation

$$i\frac{\partial}{\partial t}A_{if}(t)=E_i\,A_{if}(t)+\sum_j\ V_{ij}\,A_{jf}(t), \tag{4.2}$$

where $V_{ij}=\langle i|V|j\rangle$, and the initial condition is $A_{if}(0)=\delta_{if}$. The equivalent integral equation reads

$$A_{if}(t)=e^{-iE_it}A_{if}(0)-i\sum_j\int_0^t dt'\ V_{ij}\ e^{-iE_i(t-t')}A_{jf}(t'). \tag{4.3}$$

Next we introduce the propagator for the interacting scalar meson system in the energy representation

$$G_{if}(E+i\delta)=\int_0^\infty dt\ e^{i(E+i\delta)t}A_{if}(t). \tag{4.4}$$

Applying a time integration similar to Eq. (4.4) to all of Eq. (4.3) yields

$$G_{if}(E+i\delta)=\frac{iA_{if}(0)}{E-E_i+i\delta}+\sum_j\ V_{ij}\ \frac{G_{jf}(E+i\delta)}{E-E_i+i\delta}. \tag{4.5}$$

For $i\neq f$, taking account of the initial condition, one gets

$$G_{if}(E+i\delta)=V_{if}\ \frac{G_{ff}(E+i\delta)}{E-E_i+i\delta}+\sum_{j\neq f}V_{ij}\ \frac{G_{jf}(E+i\delta)}{E-E_i+i\delta}, \tag{4.6}$$

while for $i=f$ the equation has the form

$$G_{ff}(E+i\delta)=\frac{i}{E-E_f+i\delta}+V_{ff}\ \frac{G_{ff}(E+i\delta)}{E-E_f+i\delta}+\sum_{j\neq f}V_{fj}\ \frac{G_{jf}(E+i\delta)}{E-E_f+i\delta}. \tag{4.7}$$

Now we return to Eq. (4.6) and assume that V only connects $|f\rangle$ with its decay channels, while direct coupling between decay channels is absent, i.e., $V_{ij}=0$ in Eq. (4.6) (this assumption may be dropped without changing the results significantly; see Ref. [15]). Then we take G_{if} given by the left-hand side of Eq. (4.6) with $V_{ij}=0$ on the right-hand side, change the index i in G_{if} into j, and substitute this G_{jf} into Eq. (4.7). Thus we obtain

$$\left(E-E_f-V_{ff}-\sum_{j\neq f}V_{fj}\ \frac{1}{E-E_j+i\delta}V_{jf}+i\delta\right)G_{ff}(E+i\delta)=i. \tag{4.8}$$

This is the result for a pure f_0-meson case. The physical meaning of this propagator is that there is a shift in energy of the interacting meson due to a self-interaction, plus a complex contribution from transitions to intermediate continuum states. Some of the flux into the intermediate state returns to the discrete state $|f\rangle$, and some flows away, thereby creating a shift in the width as well as the real part of the energy. Note that we often designate the interacting propagator G_{ff} simply as G_f. We now examine the role of the scalar mesons coupling to pion and kaon pairs in the intermediate states.

Some remarks are now in order. Since we are interested in $K\bar{K}$ channels with thresholds close to the f_0 mass, the nonrelativistic derivation and in particular the nonrelativistic kinematics used above are justified; the relativistic generalization is straightforward. Keeping in mind that $\pi^+\pi^-$ and $\pi^0\pi^0$ thresholds are far away from the f_0 position, we rewrite Eq. (4.8) in terms of "renormalized" eigenvalues instead of the "bare" ones. By that we mean that E_f is redefined to include both the self-energy contribution V_{ff} and the part of the sum in Eq. (4.8) that arise from channels other than the $K\bar{K}$ channels. Hence, effects of the $\pi^+\pi^-$ and $\pi^0\pi^0$ are incorporated by redefining or renormalizing E_f. In the vicinity of the $K\bar{K}$ threshold, the terms arising from intermediate pion pairs are almost energy independent. They shift E_f by the following real and imaginary parts:

$$\sum_{\pi\pi}V_{f\pi\pi}\frac{1}{E-E_{\pi\pi}+i\delta}V_{\pi\pi f}=M_f-i\frac{\Gamma_f}{2}. \tag{4.9}$$

The width Γ_f is thus generated by the imaginary part of the transitions to the pion pair continuum. For the a_0-meson propagator analogous real and imaginary energy shifts arise from $\eta\pi$ intermediate states. The sum in Eq. (4.9) implies both a sum over different channels ($\pi^+\pi^-$ and $\pi^0\pi^0$) and integration over the energies $E_{\pi\pi}$ in each channel. For large values of $E_{\pi\pi}$ the integral diverges and standard renormalization has to be performed. In the simplified approach presented here we may even argue that the integral is convergent (or cut off) due to the matrix elements $V_{f\pi\pi}$, $V_{\pi\pi f}$. We keep the same notation E_f for the real part of the f_0 energy renormalized in the above sense, i.e., after V_{ff} and M_f are absorbed into it. Then the f_0 propagator takes the following simple form:

$$G_f(E+i\delta)=\frac{i}{E-E_f+i\frac{\Gamma_f}{2}-\sum_{K\bar{K}}V_{fK\bar{K}}\frac{1}{E-E_{K\bar{K}}+i\delta}V_{K\bar{K}f}}, \tag{4.10}$$

where summation is now only over K^+K^- and $K^0\bar{K}^0$ channels and the sum also implies integration over energy. Being interested in the $K\bar{K}$ near threshold energy region, one cannot neglect the splitting between K^+K^- and $K^0\bar{K}^0$ thresholds equal to $2\,(m_{K^0}-m_{K^\pm})=8$ MeV. This mass difference induces isospin violating f_0-a_0 mixing. Such "kinematical" isospin violation was carefully studied earlier using effective range theory [19]. For the f_0-a_0 system the effect was probably first pointed out in [9], where it was shown that mixing is enhanced, i.e., is of the order of $[(m_{K^0}-m_{K^\pm}]/m_{K^0}]^{1/2}$, instead of $[(m_{K^0}-m_{K^\pm}]/m_{K^0}]$ as might be naively expected. As we shall see, this enhancement also follows in our approach and motivates us to include this effect in scalar meson production. Isospin violating f_0-a_0 mixing was also studied earlier by Barnes [10].

We will not repeat the derivation including the a_0 meson since it proceeds along the same lines. The propagator now becomes a 2×2 matrix in the f_0-a_0 basis. Instead of Eqs. (4.8) and (4.10), one gets

$$\sum_{m=f,a}\left[\left(E-E_n+i\frac{\Gamma_n}{2}\right)\delta_{nm}-\sum_{K\bar{K}}V_{nK\bar{K}}\frac{1}{E-E_{K\bar{K}}+i\delta}V_{K\bar{K}m}\right]G_{mk}=i\delta_{nk}, \tag{4.11}$$

where $n,k=f,a$. The a_0 meson entering into this equation has been also "renormalized," this time the $\pi\eta$ channel playing the role of the $\pi\pi$ channel for f_0.

Our next task is to present explicit expressions for the sums over $K\bar{K}$ entering into Eqs. (4.10) and (4.11). This problem was considered in [18] in detail including the Coulomb interaction in the K^+K^- channel. The Coulomb interaction results in spectacular effects including the formation of the K^+K^- atom, but the typical energy scale for these phenomena is of the order of a few keV which requires an experimental resolution that is probably inaccessible in the near future. Neglecting the Coulomb interaction, the above sums are reduced to expressions of the type

$$\sum_{K\bar{K}}V_{nK\bar{K}}Y(m_x,m_K)V_{K\bar{K}m},$$

where we replaced E by m_x to be consistent with expressions (2.4) and (2.5) for the cross sections; here, $Y(m_x,m_K)$ are integrals of the type

$$Y(m_x,m_K)=\frac{1}{2m_x^3}\int\frac{d^3p}{(2\pi)^3}\frac{1}{m_x-2m_K-\frac{p^2}{m_K}+i\delta}, \tag{4.12}$$

with $m_K=m_{K^\pm},m_{K^0}$. As already mentioned, the integral (4.12) is formally divergent which is, however, neither dangerous nor important as soon as we focus on the m_x region close to the $K\bar{K}$ threshold. Close to the $K\bar{K}$ threshold the leading energy-dependent contribution is

$$Y(m_x,m_K)=-i\frac{1}{32\pi m_x}\sqrt{\frac{m_x-2m_K}{m_K}+i0}, \tag{4.13}$$

where $+i0$ means that for $m_x<2\,m_K$ the square root acquires a positive imaginary part. From Eqs. (4.11) and (4.13), it follows that for $m_x>2\,m_K$ the decay width into the $K\bar{K}$ channel is given by $(n=f,a)$

$$\Gamma_{nK\bar{K}}=\frac{|V_{nK\bar{K}}|^2}{16\pi m_x}\sqrt{\frac{m_x-2m_K}{m_K}}. \tag{4.14}$$

Comparison of Eq. (4.14) with the parametrization used in [5] shows that $V_{nK\bar{K}}=g_{nK\bar{K}}$, where $g_{nK\bar{K}}$ are the coupling constants used in [5]. Recall that Eq. (4.11) is written in the basis of f_0 and a_0 states having definite isospins, $I_{f_0}=1,I_{a_0}=0$. Therefore,

$$V_{K^+K^-f}=V_{K^0\bar{K}^0f},\quad V_{K^+K^-a}=-V_{K^0\bar{K}^0a}. \tag{4.15}$$

Next we introduce the notation

$$\mathcal{D}\equiv\frac{|V_{K^+K^-f}|^2}{32\pi m_x},\quad \zeta\equiv\frac{V_{K^+K^-a}}{V_{K^+K^-f}}, \tag{4.16}$$

where ζ can be complex with phase ϕ; see later. In this notation, the explicit form of Eq. (4.11) can be expressed as a matrix in f_0-a_0 space as

$$i\hat{G}^{-1}=\begin{pmatrix}m_x-E_f+i\Gamma_f/2 & 0\\ 0 & m_x-E_a+i\Gamma_a/2\end{pmatrix}+i\mathcal{D}\begin{pmatrix}1 & \zeta\\ \zeta^* & |\zeta|^2\end{pmatrix}\sqrt{\frac{m_x-2m_{K^+}}{m_{K^+}}+i0}+i\mathcal{D}\begin{pmatrix}1 & -\zeta\\ -\zeta^* & |\zeta|^2\end{pmatrix}\sqrt{\frac{m_x-2m_{K^0}}{m_{K^0}}+i0}. \tag{4.17}$$

Here we see that the isospin violating f_0-a_0 mixing is really proportional to $\sqrt{(m_x-2m_K)/m_K}$ and is indeed enhanced as was pointed out in Ref. [9]. We also see that far beyond the region $|m_x-2m_K|\approx 8$ MeV of the $K\bar{K}$ thresholds, the nondiagonal elements cancel each other and thereby extinguish the f_0-a_0 mixing. Away from the threshold region one should also take into account corrections to Eq. (4.13), i.e., include the next terms in the expression for the renormalized kaon loop.

Now we have at our disposal all quantities needed to calculate the near threshold amplitudes and cross sections using equations presented in Sec. II. For example, the amplitude for the process $\gamma+p\to p'+f_0\to p'+\pi\pi$ reads

$$T_{\pi\pi}=V_{\pi\pi f}G_{ff}F_f+V_{\pi\pi f}G_{fa}F_a+B_{\pi\pi}. \tag{4.18}$$

Here we see the doorway production of the f_0, followed by its subsequent propagation via G_{ff} and then its decay $V_{f\pi\pi}$. The second term includes the doorway production of the a_0, followed by its transition into a f_0 via isospin violation and then the f_0 decays into a pion pair. Other pion pair processes are included in the background term $B_{\pi\pi}$. For final kaon production the corresponding result is

$$T_{K^+K^-}=V_{K^+K^-f}\,G_{ff}\,F_f+V_{K^+K^-a}\,G_{aa}\,F_a+V_{K^+K^-f}\,G_{fa}\,F_a+V_{K^+K^-a}\,G_{af}\,F_f+B_{K^+K^-}. \quad (4.19)$$

For final $\pi\eta$ production the corresponding result is

$$T_{\pi\eta}=V_{\pi\eta a}\,G_{aa}\,F_a+V_{\pi\eta a}\,G_{af}\,F_f+B_{\pi\eta}. \quad (4.20)$$

The values of the parameters are discussed in the next section.

V. PARAMETERS OF THE MODEL

The theoretical investigation of the light scalar mesons f_0 and a_0 has a long history. A most thoughtful study of f_0/a_0 has been performed by the Novosibirsk group (see [5,6] and references therein). Although our approach differs from other authors, including the Novosibirsk group, the key parameters are similar in various treatments. Thus, in order to fix the parameters we shall relate them to those used by the Novosibirsk group. This would still leave the set of parameters incomplete; namely, the photoproduction amplitude F_k and the background term B_α [see Eq. (2.2)] have no analogs in the Novosibirsk set, and to get at least an educated guess about them, we resort to the recent paper [4]. We stress that the characteristics of f_0/a_0 are at present far from being perfectly established. For example, the Novosibirsk group presents several solutions with some of the parameters ranging over rather wide limits [5]. In order to determine the parameters of the model we use the recent experimental data presented in [6]. These data do not allow us to fix all the parameters and some parameters will be set from different consideration (see below).

The model under consideration can be described by several equivalent sets of parameters. We used the following set: $E_f, E_a, \Gamma_f, \Gamma_a, V_{fK^+K^-}$, and ζ. The quantities $V_{fK^0\bar{K}^0}$, $V_{aK^0\bar{K}^0}$, and $\mathcal{D}$ are then expressed according to Eqs. (4.15) and (4.16), while $V_{f\pi\pi}$ and $V_{a\pi\eta}$ are determined via Γ_f and Γ_a according to the equation

$$\Gamma_{jm_1m_2}=\frac{|V_{jm_1m_2}|^2}{16\pi m_x}\rho_{m_1m_2}, \quad (5.1)$$

where $j=f,a$, and $\rho_{m_1m_2}$ is the two-body phase space. At this point we remind the reader that since we treat the $K\bar{K}$ channels explicitly, their contribution does not enter into E_j and $\Gamma_j(j=f,a)$. We also mention that the ''visible'' widths of f_0/a_0 are smaller than the widths Γ_f and Γ_a [5]. Finally, we recall that our parameters $V_{jm_1m_2}$ are equivalent to $g_{jm_1m_2}$ used by the Novosibirsk group [5,6].

In [6] we find the following parameters relevant for our purposes (experimental errors are omitted):

$$\frac{|V_{f\pi\pi}|^2}{4\pi}=0.44\ \text{GeV}^2,\quad R=\frac{|V_{fK\bar{K}}|^2}{|V_{f\pi\pi}|^2}=3.77. \quad (5.2)$$

Using Eq. (5.1) and the isotopic relations

$$|V_{f\pi^+\pi^-}|^2=\frac{2}{3}|V_{f\pi\pi}|^2,\quad |V_{fK^+K^-}|^2=\frac{1}{2}|V_{fK\bar{K}}|^2, \quad (5.3)$$

we get, from Eq. (5.2),

$$\Gamma_f=0.1\ \text{GeV},\quad V_{fK^+K^-}=3.23\ \text{GeV}. \quad (5.4)$$

Next setting in Eq. (4.17) m_x equal to the physical mass of the f_0 meson, i.e., $m_x=0.98$ GeV, we find $E_f=0.947$ GeV. Unfortunately, we cannot rely on well-established experimental data for the parameters of the a_0 meson. Therefore we assume $\Gamma_a\equiv\Gamma_f$ and $E_a\equiv E_f$ (these assumptions are in line with the solutions proposed in [5]). From Γ_a we get $V_{a\pi\eta}=3.08$ GeV. Finally we set $\zeta=1$, which is true in both four-quark and molecular models of f_0/a_0 [5]. Thus we arrive at the following parameter set (set A):

$$E_f=E_a=0.947\ \text{GeV},\quad \Gamma_a=\Gamma_f=0.1\ \text{GeV},$$
$$V_{fK^+K^-}=3.23\ \text{GeV},\quad \zeta=1. \quad (5.5)$$

Note that the value $|V_{fK^+K^-}|^2/4\pi=0.83\ \text{GeV}^2$ lies in between the values typical for four-quark and molecular models of f_0 meson [5].

In order to display the effect of f_0-a_0 mixing, we consider also set B whose only difference from set A is that in B the splitting of the $K^0\bar{K}^0$ and K^+K^- thresholds is ignored and the kaon mass is taken equal to the average value $m_K=\frac{1}{2}(m_{K^0}+m_{K^\pm})$ and hence mixing is thereby turned off. This results in cancellation of the nondiagonal terms in Eq. (4.17).

Within our approach we are not in a position to determine the value of the photoproduction amplitude $F_j(j=f,a)$. Its invariant structure is given by Eqs. (3.2) and (3.4), but the magnitudes of the invariant amplitudes remain unknown. In a recent paper [4] the f_0/a_0 photoproduction has been considered in the near-threshold region, namely, at $k=1.7$ GeV. Close to threshold the dominant contribution stems from the amplitude R_1 of Eq. (3.4). Its value can be considered a free parameter, but we prefer to take for R_1 the value proposed in [4].[3] The reasons for adopting this value for R_1 are on the one hand because it enables a direct comparison of our results with that of [4] and on the other hand, as already argued in Sec. II, we may consider the pion loops included into the renormalized vertex of f_0 photoproduction while in the approach of [4] the two-pion photoproduction is the driving mechanism that determines the value of R_1.

The calculation or even estimate of the background term B_α in Eq. (2.1) is beyond the scope of this article. Some of the diagrams entering into B_α are depicted in Fig. 2 and in general may be calculated. In Ref. [4] the background for $d\sigma_{\pi^+\pi^-}/dm_x$ [see Eq. (2.5)] was estimated to be around 55 μb/GeV.

In Fig. 4, we present the invariant mass distribution $d\sigma(K^+K^-)/dm_x$ for K^+K^- pair production calculated according to Eqs. (2.5), (4.17), and (4.19). In order to display the role of isospin breaking f_0-a_0 mixing, we plot in the

[3]This threshold value for R_1 is basically the pseudoscalar photoproduction Kroll-Ruderman limit scaled by a factor $k/(2M)$ due to the switch to a scalar meson.

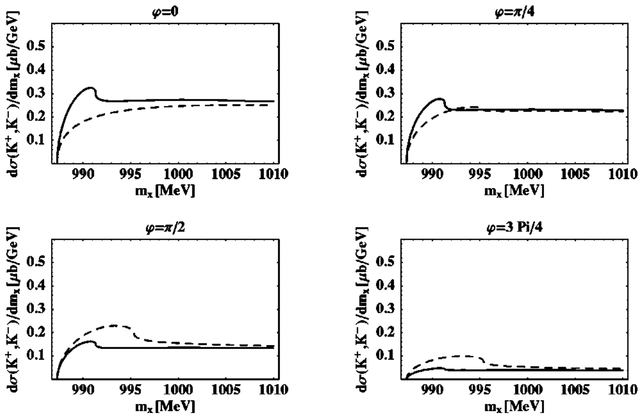

FIG. 4. The invariant mass distribution $d\sigma(K^+K^-)/dm_x$ in μb/GeV is plotted versus the invariant mass of the K^+K^- pair resulting from scalar meson photoproduction. The dashed curve includes the $f_0 \leftrightarrow a_0$ transitions caused by isospin breaking, whereas the solid curve is for case B for which the charged and neutral kaon masses are set equal to their average and the isospin effect is thus turned off. The dependence on the phase of the ratio $\zeta = |\zeta| \exp i\phi \equiv V_{K^+K^-a}/V_{K^+K^-f}$ is also shown, including the associated phase on the decay amplitudes $V_{K^+K^-a}$ and $V_{K^+K^-f}$.

same figure the curve obtained with set B parameters, i.e., with f_0-a_0 mixing switched off (which is achieved by using the same mass for the charged and neutral kaons). The two curves are different, in line with ''enhanced'' mixing in the sense described above. (The scalar meson mixing can be as large as a 70% enhancement.) The structure in the invariant mass distribution for K^+K^- pair production in this near-threshold region arises from the isospin breaking difference in mass between charged and neutral kaons.

The a_0-f_0 interference pattern turns out to be very sensitive to the phase of the parameter ζ [see Eq. (4.16)]. The sensitivity to that phase is clearly displayed in Fig. 4. One should, however, keep in mind that in both $q\overline{q}$ and $q^2\overline{q}^2$ constituent quark models this parameter is predicted to be real [6,20].

In Fig. 5, the $\pi^+\pi^-$ invariant mass distribution $d\sigma(\pi^+\pi^-)/dm_x$ is presented. We now use Eqs. (2.5), (4.17), and (4.18). Here the deviation from a simple Breit-Wigner resonance structure due to the influence of the $K\overline{K}$ thresholds is clearly seen. We do not insist on the absolute values of the cross sections in Figs. 4 and 5, because of backgound effects, but we do believe the change due to mixing is of some predictive value.

VI. CONCLUSIONS

The analysis presented herein illustrates that threshold photoproduction can provide insight into the nature of the f_0 and a_0 scalar mesons. Valuable information may be obtained from the $K\overline{K}$ mass distribution in the threshold region if measured with a few MeV resolution. In our approach, this distribution has been described in a model-independent way, once the doorway idea is invoked. Most parameters can be deduced from $\phi \to \gamma(a_0 + f_0) \to K\overline{K}$ experiments.

The invariant amplitude formalism for the photoproduction of scalar mesons presented in Sec. III enables us to consider higher energy regions which are accessible at JLab. Going to higher energies increases the number of CGLN amplitude parameters by involving all of the R_i terms in Eq. (3.4). Also, with increasing energy more partial waves come into play, giving rise to nonzero spin observables. The role of spin observables and how they evolve with increasing energy will be discussed in a future paper.

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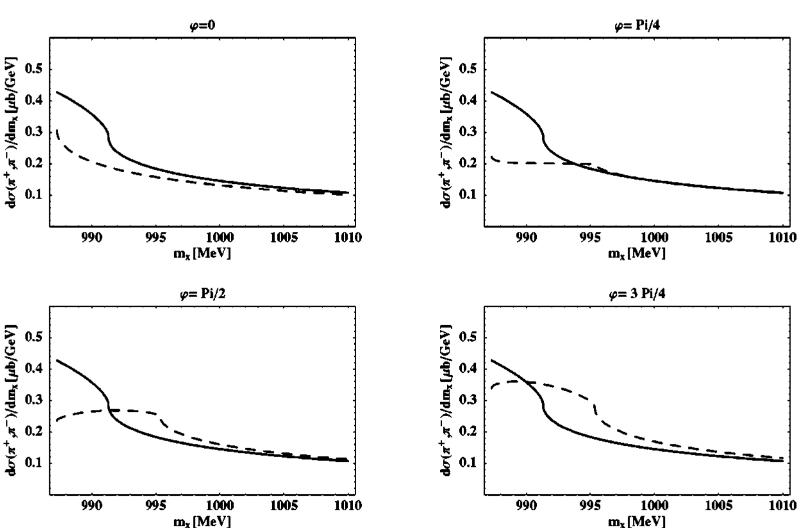

FIG. 5. The invariant mass distribution $d\sigma(\pi^+\pi^-)/dm_x$ in μb/GeV is plotted versus the invariant mass of the $\pi^+\pi^-$ pair resulting from scalar meson photoproduction. The dashed curve includes the $f_0 \leftrightarrow a_0$ transitions caused by isospin breaking, whereas the solid curve is for set B for which the charged and neutral kaon masses are set equal to their average and the isospin effect is thus turned off. The dependence on the phase of the ratio $\zeta = |\zeta| \exp i\phi \equiv V_{K^+K^-a}/V_{K^+K^-f}$ is also shown. Here $V_{\pi^+\pi^-f}$ is taken as real. The same value of R_1 is used as in the kaon pair case.

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