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Monte Carlo mixing simulation of the decay out of superdeformed bands in ^{194}Hg

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A simple two level mixing model is employed to perform a Monte Carlo simulation for the decay out of the superdeformed bands SD-1 and SD-3 in ^{194}Hg. An exponential spin dependence of the interaction between superdeformed and normally deformed states is applied. Average intraband intensities and transition quadrupole moments are determined at each spin. The simple two level mixing approach is consistent with more sophisticated theoretical models. The Monte Carlo approach allows one to determine probabilities for the reproduction of the experimental intensity patterns and transition quadrupole moments. An interaction of $v \approx 0.5$ eV was determined for spin $12\hbar$ in the yrast SD band. The results show that the decay out of the yrast and excited SD bands in ^{194}Hg can be described with the same spin dependent interaction.

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In recent years various theoretical models have been put forward to describe the mechanism that governs the decay out of superdeformed (SD) bands in the mass-190 region [1–8]. The models are based either on the interaction of the low-spin superdeformed states with the surrounding normally deformed (ND) states or the tunneling of the SD wave function through the potential barrier separating the SD and ND wells in the deformation dependent nuclear energy. Recently the energies of SD bands in ^{194}Hg [band SD-1 (3) is 4.407 (4.322) MeV above yrast at spin 12 (13) $\hbar$] and ^{194}Pb (band SD-1 is 2.889 MeV above yrast at spin $12\hbar$) were experimentally established [9–12]. Due to the high excitation energies of the SD bands the properties of the ND states are determined by statistical model calculations.

Experimentally the decay mechanism has been studied by the measurement of level lifetimes [13–16], which are sensitive to the admixture of ND wave functions into the SD states, and the study of the quasicontinuum of γ rays linking SD and ND states [17–19]. The experimental transition quadrupole moments Q_t for the intraband transitions at the bottom of the SD bands were found to be consistent with the average values for Q_t of the higher spin states [20], which are not mixed with ND states. It was therefore concluded that the interaction between the SD and ND states has to be weak and the amount of the ND admixture was estimated to be less than a few percent [13,14,21]. This weak mixing scenario allowed an estimate of the maximum interaction strength between SD and ND states. Since the upper limit for the interaction strength (typically on the order of a few eV) was found to be significantly smaller than the average level spacing of the ND states it was also concluded that the decay out is a result of the weak mixing between the SD states and their energetically nearest neighboring ND states [13].

Recently, a model of the decay out of the SD bands was presented based on the mixing of SD and ND states and a complete statistical treatment of the ND states [6]: The ND states are described in terms of the Gaussian orthogonal ensemble (GOE) of random matrices. This model considers a very broad range of parameters. However, for the description of the decay out of SD bands in the mass-190 region only the limit $\Gamma_N \ll D$ is relevant. Here Γ_N is the decay width of ND states and D is their average level spacing. In this limit the predicted intraband intensities depend explicitly on the relative position of the SD and ND states [6,8], which leads to a large variance of the result when the ensemble average is calculated.

Another recent approach [7] used retarded Green's functions to describe the decay out within a two-level model. In this model ''the electromagnetic decay and the tunneling through the barrier separating SD and ND states were treated on equal footing'' [7]. For the range of parameters relevant for the mass-190 region the models of Refs. [2,6,7] all lead to similar results [8].

It is the purpose of this Rapid Communication (i) to show that a Monte Carlo (MC) simulation based on the application of a simple two-level mixing model provides a good description of the decay out of SD bands and (ii) to extract the probability distributions of the intraband intensities and transition quadrupole moments using this approach. This allows us to deduce probabilities for reproducing the experimental data in our simulation. We also show the equivalence between our approach and the statistical model [6,8] in the limit of small Γ_N/D and the model of Ref. [7].

The approach presented here is based on the mixing at each spin between the SD state and its nearest neighbor ND state. At each spin the relative spacing x between the mixing SD and ND states is randomly varied. A spin dependent interaction with an exponential spin dependence is used. The interaction matrix elements are not randomly varied but only the average value is used. We recognize that this is a significant simplification compared to the full statistical model but we believe that this simplicity is very instructive.

We use the case of ^{194}Hg as a test ground for our simulation and below we will outline in detail the procedure for the yrast SD band in ^{194}Hg. The decay width Γ_N of the ND states in the direct vicinity of the SD states is calculated under the assumption that at the high excitation energy of the SD states statistical $E1$ transitions will dominate the decay of the ND states ($\Gamma_N \approx \Gamma_{E1}$). We use the approach outlined by Døssing and Vigezzi [22] to calculate the level density $\rho(U)$ and the $E1$ width Γ_{E1} at the excitation energy $U = E_x - 2\Delta$. The excitation energy E_x is calculated with reference to a parabolic ND yrast line, which is obtained by a fit to the experimental yrast levels in the spin range from $14\hbar$ to

TABLE I. Experimental transition quadrupole moments Q_t^{exp} (average of results from Refs. [15,16]) compared to the average transition quadrupole moments for all simulated bands (second column) and those bands that are in agreement with the experimental intensities and quadrupole moments (third column). An average quadrupole moment of 18.3 e b was assumed in the simulation for the pure SD configuration. The uncertainties shown for the calculated quadrupole moments represent the variance of the calculated distributions. The last column shows the average ND squared mixing amplitudes a_n^2 for those bands that reproduce the experimental intensities and quadrupole moments

J [$\hbar$]	Q_t^{exp} [e b]	$Q_t^{calc,all}$ [e b]	$Q_t^{calc,fit}$ [e b]	a_n^2 [%]
14	17.2 ± 1.1	17.7 ± 1.1	18.2 ± 0.1	0.10 (6)
12	18.7 ± 1.7	16.2 ± 1.4	17.2 ± 1.3	1.2 (2)
10		13.6 ± 1.0	14.3 ± 0.7	11 (5)

$26\hbar$. A band-head energy of 6.02 MeV is used for the yrast SD band and the energies of the SD states are calculated with respect to the band head by using a parabolic fit to the experimental SD level energies [9,10]. The $E1$ width Γ_{E1} is approximated by the analytical expression [22] $\Gamma_{E1}=c_{E1}\cdot T^5$, with $T\approx\sqrt{a/U}$. The parameters in Ref. [13] are used to calculate the factor c_{E1} [22] and the level density parameter $a=22.58\ \mathrm{MeV}^{-1}$. A back-shift parameter of $2\Delta=1.4$ MeV is used. The decay width Γ_S of the SD states is calculated by using the average experimental SD quadrupole moment of $Q_{SD}=17.3\pm1.0$ e b for ^{194}Hg [15,16,20].

The final quantity needed for the simulations is the interaction v between the SD and ND states. We use the exponential parametrization

$$v(J)=v_0\cdot e^{-\alpha\cdot J}, \tag{1}$$

and we start with the parameters $v_0=50$ keV and $\alpha=1$ [8] in order to compare our simple model directly with the predictions of Refs. [6,8]. Later on we are going to discuss other possible values for the parameters v_0 and α.

After calculating values for Γ_S, Γ_{E1}, v, and D at each spin (values for the low-spin states can be found in Ref. [8]) mixing calculations for 10^5 bands are performed where at each spin the distance x between the SD state and its nearest ND neighbor is randomly varied between $x=0$ and $x=D/2$. The SD-ND spacing x at each spin is independent from the spacing at any other spin in each band. Thus we simulate the decay pattern of 10^5 bands with the overall properties of the known SD yrast band in ^{194}Hg. The results are used to determine the average intraband intensity I_{av} and its variance $\sigma_{I_{av}}$ as well as the average transition quadrupole moments Q_t and its variance σ_{Q_t}. In Table I the average quadrupole moments from our simulation are compared to the experimental values.

Figure 1 shows the average intraband intensity determined by the MC simulation in comparison to the experimental data for band SD-1 in ^{194}Hg and the results of Ref. [8]. The two models agree well with each other and closely follow the experimental data. The agreement of our simple model with the full statistical model of Ref. [6] gives us confidence to use the MC simulation for a more detailed study of the decay out.

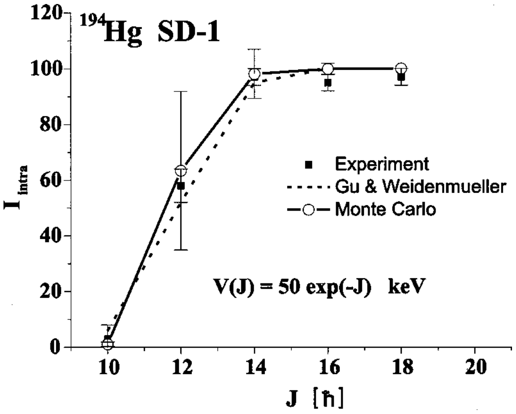

FIG. 1. Experimental intraband intensity (filled squares) of the band SD-1 in ^{194}Hg compared to the average intraband intensity calculated by using the Monte Carlo approach of this work (open circles connected with solid line) and the model by Gu and Weidenmüller [6,8] (dashed line). The error bars for the Monte Carlo results indicate the variance of the results (see text). For both calculations an exponential parametrization of the interaction strength v was chosen (see text).

Recently Stafford and Barrett [7] have presented a two-level model for the decay out of SD bands using retarded Green's functions to describe the dynamics of the system. This approach differs from our ansatz mostly by explicitly taking into account the width of the levels involved. In order to determine the importance of the intrinsic width of SD and ND levels we have also performed a MC simulation for the yrast SD band in ^{194}Hg using the formalism of Ref. [7] with the same spin dependent interaction as above. Figure 2 shows the intensity distributions at spin 10, 12, and 14 $\hbar$ for both approaches. There is little difference between the approaches and we conclude that the intrinsic width of the levels is for the most part not important since the level widths Γ_N and Γ_S are small compared to the interaction v and the level spacing D in this spin range.

Figure 2 also shows that the intensity distributions are not symmetric and as a consequence the average value may not coincide with the most likely value. Thus, comparing the average intensities determined by a model to the experimental data may not be the best choice. The MC approach offers a better way to compare the model results with the experimental data by enabling us to directly determine the percentage of simulated bands that reproduce the experimental data. Additionally, our approach will compare our model results not only with experimental intensities but, for the first time, also with experimental quadrupole moments.

We have determined the probability to reproduce the experimental data as a function of the parameters (α, v_0) and have found a class of parameters which best agree with the data. Figures 3(a) and 3(b) show the percentage of simulated SD bands that reproduce the experimental transition quadrupole moments and experimental intensities, respectively, as a function of the parameters v_0 and α. Figure 3(c) shows the

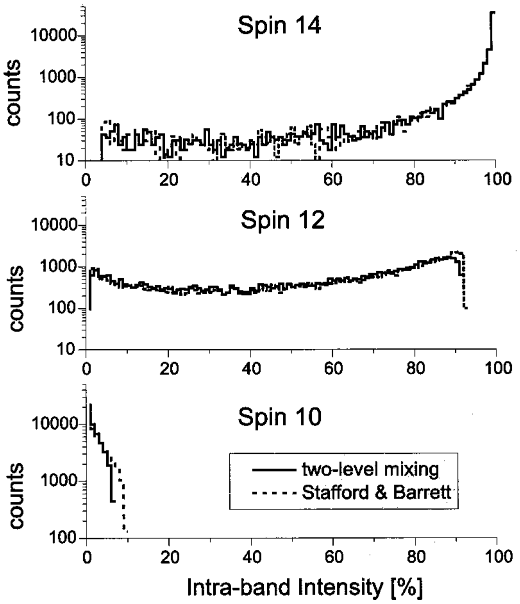

FIG. 2. Distribution of the intraband intensity at spins 10, 12, and 14 $\hbar$ from the Monte Carlo calculation using our simple two level mixing ansatz (solid line) and the mixing of two resonances [7] (dashed line).

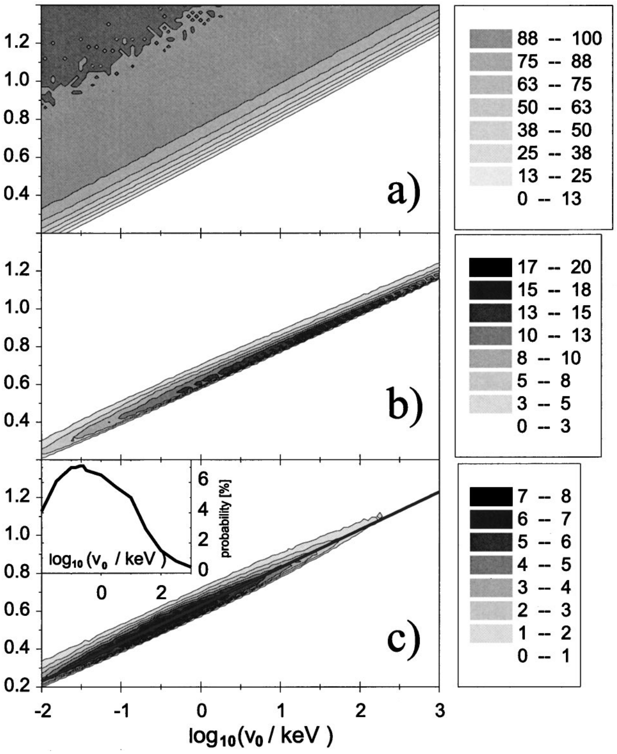

FIG. 3. Percentage of MC simulated SD bands that reproduce the experimental transition quadrupole moments (a), experimental intensities (b), and intensities *and* quadrupole moments (c) as a function of the parameters v_0 and α in Eq. (1). Panel (c) also shows the line along the maximum probability values, described by Eq. (2). A quadrupole moment of $Q_{SD}=18.3$ e b was assumed for the pure SD states. The inset in the bottom panel shows the percentage of matching bands along the line in panel (c).

percentages of simulated bands that reproduce the intensities *and* quadrupole moments simultaneously. A logarithmic scale was used for the x axis. One finds very large probabilities to reproduce the quadrupole moments for very small interactions [upper left corner of Fig. 3(a)] where the ND admixtures are so small that the quadrupole moments are barely reduced compared to the value for the pure SD states. However, such small interactions do not lead to sufficiently large ND admixtures to produce a drop of the intraband intensities, leading to zero probability to reproduce the experimental intensities [Fig. 3(b)]. Intraband intensities and quadrupole moments are reproduced simultaneously within a narrow range of the parameter space with the highest probabilities around $\log_{10}(v_0/\mathrm{keV})\approx-0.5$ [see inset in Fig. 3(c)]. We have used a quadrupole moment of 18.3 e b for the pure SD states, which is within one standard deviation of the average of $Q_{SD}=17.3\pm1.0$ e b of the experimental data. This larger value was used since the percentage of simulated bands that agreed with the experimental data was about twice as large as compared to the results for $Q_{SD}=17.3$ e b. The parameter range for which the experimental data is reproduced is the same for both values of Q_{SD}. This larger quadrupole moment is in good agreement with the experimental data, in particular in the light of the systematic uncertainties of up to $\approx$15% of the DSAM results [20], which were not taken into account in the determination of the average experimental value of Q_{SD}.

The region of the maximum probabilities follows a line described by the relation

$$\alpha=0.63+0.2\log_{10}(v_0/\mathrm{keV}), \tag{2}$$

which is also shown in Fig. 3(c). At each value of v_0 one can define an uncertainty for α of $+0.10$ and -0.05. Outside this range the probability to reproduce the experimental data essentially drops to zero. For the parameters described by Eq. (2) one obtains an interaction at spin $12\hbar$ between 0.35 eV and 0.65 eV when v_0 is varied over five orders of magnitude. Using Eq. (2) and the very conservative uncertainty estimate for α we can determine the interaction strength v at spin $12\hbar$ to be 0.55 ± 0.42 eV. The maximum probability to reproduce the full experimental properties of the yrast SD bands in ^{194}Hg is about 7%.

Our MC simulation also allows us to determine the average squared ND mixing amplitudes a_n^2 in the SD wave function, which are presented in the last column of Table I. The values for a_n^2 increase by about one order of magnitude from state to state. The value for spin $12\hbar$ is consistent with the estimate of Refs. [15,16].

We have performed the same procedure for the excited band SD-3 in ^{194}Hg, for which the excitation energy is 7.716 MeV at spin $13\hbar$ [10]. We find that the interaction parameters determined by Eq. (2) also reproduce the experimental data for bands SD-3 with the largest probability. Thus, our results confirm the result of Ref. [8] that the interaction in the region of the decay out is the same for the yrast and excited SD bands in ^{194}Hg. As already stated in Ref. [8] this result seems inconsistent with the interpretation of Ref. [5] that the enhancement of the decay out, or in other words the

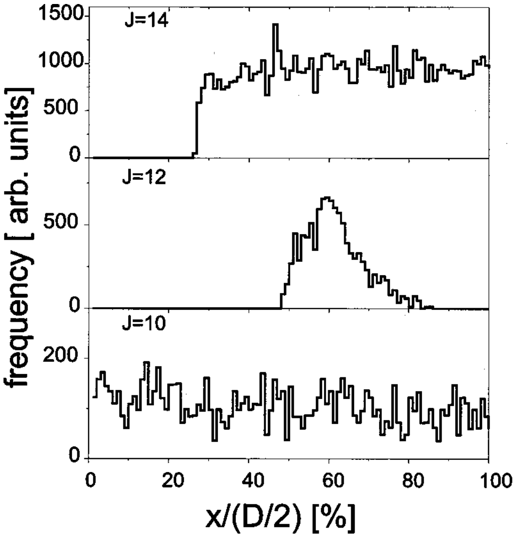

FIG. 4. Distribution of the ND-SD distances *x* (in percent of *D*/2) at spins 10, 12, and 14 $\hbar$ of those bands that match the experimental data.

exponential spin dependence of the interaction, is due to the onset of chaos. For a chaos-enhanced decay out one would expect that the interaction is a function of the excitation energy. This is not supported by our results.

Using the MC simulation we have also addressed the question of which spacings between SD and ND states contribute to the decay out of the band. Figure 4 shows the distribution of distances *x* between the mixing SD and ND states for spins 10, 12, and 14 $\hbar$ of all bands that reproduce the experimental data. One can see that at spin 14$\hbar$ all distances above $x=0.25\cdot D/2$ contribute about equally. The middle of Fig. 4 shows that there is only a rather narrow distribution of distances, centered around $x=0.6\cdot D/2$. At spin 10$\hbar$ the distribution is again flat and all spacings contribute equally. Note that the number of counts in the middle and bottom panel are equal. Clearly, the exact decay pattern is most sensitive to the SD-ND spacing at spin 12$\hbar$. It will be interesting to see if the spacing distributions in the decay out of other SD bands are similar or if there is no correlation.

In summary, we have presented results of a Monte Carlo simulation of the decay out of the SD bands SD-1 and SD-3 in ^{194}Hg based on a nearest neighbor mixing model with random spacing between the mixing SD and ND states. We have shown that the approach presented here leads to very similar results as compared to the statistical model used in Ref. [8]. The comparison with the two resonance mixing model of Ref. [7] shows that both models are in almost perfect agreement and we conclude that the intrinsic width of the mixing states can be safely neglected. Our Monte Carlo simulation compares model results for the first time directly with measured intensities *and* transition quadrupole moments. With Eq. (2) we have established a correlation between the parameters α and v_0 of Eq. (1) for which the experimental data is reproduced with the largest probability. This allowed us to establish an interaction strength of $v(12)=0.55(42)$ eV at spin 12$\hbar$, where the intraband intensity of the yrast SD band has dropped to almost 50%. We have found that the observed intensity pattern for the yrast SD band is most sensitive to the SD-ND spacing at spin 12$\hbar$, while the spacings at 10 and 14 $\hbar$ are not as relevant. The admixtures of ND components in the SD wave function changed by one order of magnitude from state to state and was on the order of about 1% at spin 12$\hbar$. The results also show that the decay out of the yrast and excited SD bands in ^{194}Hg can be described with the same spin dependent interaction.

We conclude that the very basic two-level mixing approach provides a good description of the decay out of SD bands in the mass-190 region. The Monte Carlo simulation provides a powerful tool to gain further insight into the decay out process. As further experimental excitation energies as well as better measurements of quadrupole moments and intraband intensities become available one can use these types of simulations to obtain a systematic picture of the decay out in the mass-190 region.

We recognize that further studies are needed that take into account Gaussian distributed ND-SD interactions as well as Wigner distributed nearest neighbor spacings of the ND levels and Porter-Thomas distributed decay strength of the ND levels. This approach will introduce additional parameters, which makes the study of the reproduction probabilities much more complex. We hope that the present work also encourages further theoretical studies that try to uncover the origin of the spin dependence of the SD-ND interaction.

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