Effect of the α -nucleus potential on the ²⁸Si(α ,p)³¹P reaction

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The differential cross section of the ²⁸Si(α, p)³¹P reaction for 26 MeV incident energy has been analyzed in the distorted wave Born approximation (DWBA) with zero and full-finite range using a deep and shallow optical, Michel and molecular potentials in the incident channel, and a usual optical model potential for proton in the final channel. The parameters of potential in the entrance channel are determined from the elastic scattering data. The calculations done with the deep optical and Michel potentials reproduce the structure of the angular distributions reasonably well, but fail to account for the absolute magnitudes by a few orders. The shallow optical one is satisfactory up to about $\theta_{c.m.} = 100^{\circ}$. The molecular potential, on the other hand, reproduces both the absolute cross sections and the pattern of the angular distributions. Coupled-channels Born approximation calculations improve fits to the data over the DWBA predictions.

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I. INTRODUCTION

Anomalous large angle scattering (ALAS), observed in the elastic scattering of α particles by light and medium-light nuclei [1–3], have recently been analyzed successfully for ²⁴Mg, ²⁸Si, ^{30,32}S targets in terms of a complex molecular [4,5] and a special type of optical potential with a squared Woods-Saxon geometry, advocated by Michel and his collaborators [6,7]. The latter is, henceforth, referred to as the Michel potential. Both of these potentials describe equally well the elastic scattering data in a wide energy range with a small set of parameters changing systematically with energies.

The ALAS effects have also been observed in the nonelastic processes [2,3,8–13]. Schmittroth *et al.* [11] have established that the use of a complex molecular potential could enhance the back angle scattering in a single-nucleon transfer reaction involving heavy ion. Similarly, ALAS has also been observed by Jankowski *et al.* [14] in the two- and threeparticle transfer reactions on the ²⁸Si target. The large-angle behavior of the data in the latter work has been analyzed in terms of an incoherent sum of the distorted-wave Born approximation (DWBA) contribution calculated with the normal optical potentials and the compound nucleus contribution predicted on the basis of the Hauser-Feshbach model [15]. The method has, however, enjoyed limited success.

The three-nucleon transfer in (α, p) reactions is a complex process. In addition to probable contributions from compound nucleus, precompound and multistep sequential transfer processes, the direct part of the reaction mechanism may comprise triton stripping, knock-on and heavy-particle stripping [16–18]. Of these triton stripping has been found to be the dominant one [16]. Although calculations in DWBA using the usual optical potential in the incident channel can, quite often, reproduce the general pattern of angular distribution, but the absolute cross section is underestimated by two to three orders of magnitude [19]. The normalization problem also persists in (α, p) reactions [20–23]. The claim of Walz et al. [24] to reduce the discrepancy between the data and calculation to 20% for the (p,α) case using a double-folded α -nucleus potential, has been contradicted by Kajihara et al. [25], who failed to reproduce Walz et al.'s calculations and found an enhancement factor $\epsilon = 4$ instead of 1.2. The purpose of the present study is, therefore, to examine the extent to which the molecular, Michel, and optical potentials can account for the pattern, magnitude, and ALAS observed in the three-nucleon transfer (α, p) reaction on ²⁸Si. The study is also a part of our broader goal of finding the nature of α -nucleus potential capable of explaining a number of physical phenomena involving an α particle and a light nucleus, in this case ²⁸Si. As evidenced from a number of investigations including the single-nucleon (α, p) transfer reaction on ²⁸Si by Das et al. [12], it is important to select data having fairly complete angular distributions in order to differentiate the effects of α -nucleus potential on a reaction process. With this in mind, we have selected the experimental cross-section data of Jankowski et al. [14] for the ²⁸Si(α ,p)³¹P reaction covering a wide range in angular distribution including those at large angles, which are expected to be sensitive to the nature of α -nucleus potential.

The investigation has been carried out within the framework of zero-range (ZR) DWBA formalism with a simple process of triton-cluster transfer using shallow, deep, Michel, and molecular optical potentials in the incident α channel. The full finite-range (FFR) DWBA and the coupled-channels Born approximation (CCBA) calculations for the molecular potential have also been performed to determine the viability of the latter two potentials. This investigation further reinforces the past assertion that data having a wide range of angular distribution are important for understanding the physical process involving α -particle and light nuclei.

Section II discusses the α -nucleus potentials used in the analyses. The DWBA and CCBA analyses are furnished in Secs. III and IV, respectively. Section V deals with the discussion and the conclusions.

TABLE I. Parameters of the α -²⁸Si potentials given by Eqs. (1), (2), and (3) used in the calculations shown in Fig. 1 are given in columns 1–5. The parameters of proton optical-model potential, and bound states of $(t+{}^{28}Si)$ and (t+p) systems are noted in columns 6–8, respectively. *V* is adjusted to give the separation energy.

Channel			α + ²⁸ Si	$p + {}^{31}P$	$t + {}^{28}Si$	t+p	
potential type	Molecular ^a Michel ^a Deep optical Shallow ^b optica		Shallow ^b optical	Optical ^c	Bound state ^b	Bound state ^b	
V ₀ (MeV)	26.0	21.0	216.0	55.0	$53.3 - 0.55E_p$	V	V
R_0 (fm)	5.35	5.00	3.70	5.16	X		
r_0 (fm)					1.25	0.929	1.05
a_0 (fm)	0.34	0.60	0.67	0.505	0.65	0.921	0.50
V_1 (MeV)	42.0						
R_1 (fm)	2.80						
α		8.39					
ρ (fm)		6.25					
W_0 (MeV)	14.5	33.1	22.4	8.64			
R_I (fm)		3.85	3.98	5.16			
a_I (fm)		0.65	0.67	0.505			
R_W (fm)	4.00						
W_D (MeV)					13.5		
r_D (fm)					1.25		
a_D (fm)					0.47		
R_C (fm)	9.35	3.95	4.07	3.95			
r_C (fm)					1.30	1.30	1.25

^aReference [5].

^bReference [14].

^cReference [31].

II. α -NUCLEUS POTENTIALS

 $V_m(r)$, the real and $W_m(r)$, the imaginary parts of the complex molecular potential, which has its root in the energy-density functional study of the reaction [4,26] are given by

$$V_m(r) = -V_0 [1 + \exp\{(r - R_0)/a_0\}]^{-1} + V_1 \exp\{-(r^2/R_1^2)\},$$

$$W_m(r) = -W_0 \exp\{-(r^2/R_W^2)\}.$$
 (1)

The real $V_M(r)$ and imaginary $W_M(r)$ parts of the Michel potential, which is an approximate form of the nonlocal potential expected from the resonating group method (RGM) as applied to the α -cluster system [27,28] are given by

$$V_{M}(r) = -V_{0}[1 + \alpha \exp\{-(r^{2}/\rho^{2})\}] \\ \times [1 + \exp\{(r - R_{0})/2a_{0}\}]^{-2}, \\ W_{M}(r) = -W_{0}[1 + \exp\{(r - R_{1})/2a_{1}\}]^{-2}.$$
(2)

The real and imaginary parts of the normal optical potential V(r) and W(r) are given, respectively, by

$$V(r) = -V_0 [1 + \exp\{(r - R_0)/a_0\}]^{-1},$$

$$W(r) = -W_0 [1 + \exp\{(r - R_I)/a_I\}]^{-1}.$$
 (3)

The Coulomb part for all three types of potentials is given by

$$V_{C}(r) = \left[\frac{Z_{1}Z_{2}e^{2}}{2R_{C}}\right] \left[3 - \frac{r^{2}}{R_{C}^{2}}\right] \text{ for } r \leq R_{C}, \qquad (4)$$

$$= \frac{Z_1 Z_2 e^2}{r} \quad \text{for } r > R_C.$$
 (5)

In the case of Michel and optical potentials, R_C is quite often written as $= r_C A_T^{1/3}$, where A_T is the target mass number. On the other hand, in case of the molecular potential, R_C is the sum of the α and ²⁸Si radii when they barely touch each other.

Although the normal optical model has not been very successful in reproducing the elastic scattering data over the energy range investigated by Jankowski *et al.* [14] and Jarczyk *et al.* [10], it is possible to find a set of parameters producing a reasonable fit to the elastic scattering data at 26 MeV. Observing that the energy-density functional approach in the special adiabatic approximation may lead to a shallow optical potential [26], a search for such a potential has also been made and included in the study.

The parameter search has been carried out using the code SCAT2 [29] modified by us to incorporate molecular and Michel potentials. The parameters obtained from the best fit to the elastic scattering data of the α particle by ²⁸Si at 26 MeV incident energy are listed in Table I. The fits to the elastic data are shown in Fig. 1. In general, the fits with all



FIG. 1. Solid lines represent calculated angular distributions for elastic scattering 26 MeV particles by ²⁸Si using four types of potentials noted above them and parameters listed in Table I. Open circles are data from [14].

four potentials are reasonable, although the shallow optical potential fit is somewhat poorer than those of the rest. Parameters of the molecular and Michel potentials are the same as the ones in Ref. [5].

III. DWBA ANALYSIS

The zero-range DWBA calculations have been performed using the code DWUCK4 [30] which has been modified to include the Michel potential in the distorting channels. The potential parameters in the distorting incident channel used in the DWBA calculations are noted in Table I for all four potentials. The bound state wave function for the transferred triton, considered as a point cluster, has been generated by assuming a real Woods-Saxon well with its depth adjusted to reproduce the separation energy. These parameters along with the proton optical potential are also noted in Table I. Corrections due to nonlocality [31] of potential in the conventional form have been applied using the nonlocality ranges $\beta(\alpha) = 0.2$, $\beta(p) = 0.85$, and $\beta(t) = 0.2$ fm. The correction in the triton-bound state form factor is found to produce little effect on the cross section. The calculations using all four potentials for the ²⁸Si(α , p)³¹P reaction leading to the $1/2^+$ ground, 1.266 MeV $3/2^+$ and 2.234 MeV $5/2^+$ states are compared with the data of Jankowski et al. [14] in Fig. 2.

To test the validity of using the molecular potential, the full finite-range DWBA calculations have been carried out using the code DWUCK5 [22]. The (t+p) bound state geometry for the FFR calculations is shown in Table I. The FFR predictions are compared to the data in Fig. 3. The spectro-

²⁸Si(α ,p)³¹P, E_{α} = 26 MeV



FIG. 2. Calculated angular distributions in zero-range DWBA for the (α, p) reaction using four potentials noted in the upper right-hand corner. The data, marked as open circles, are from [14].

scopic factors S for the cluster transfer have been deduced from the expression [30]

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{expt}} = \frac{2J_f + f}{2J_i + 1} C^2 Ss \left(\frac{d\sigma}{d\Omega}\right)_{\text{DWUCK5}}.$$
 (6)

Here $(d\sigma/d\Omega)_{expt}$ and $(d\sigma/d\Omega)_{DWUCK5}$ are, respectively, the experimental cross section and that predicted by DWUCK5. J_f and J_i are the total spins of the final and initial nuclei, respectively. s = 2.0 is the light particle spectroscopic factor. C^2 is the isospin Clebsch-Gordon coefficient. The deduced S-values are listed in Table II. The normalization constant D_0^2 for the *t*-cluster transfer in the ZR calculations has been estimated from the expression [22]

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{expt}} = \frac{(2J_f + 1)}{(2J_i + 1)(2j + 1)} D_0^2 C^2 S\left(\frac{d\sigma}{d\Omega}\right)_{\text{DWUCK4}}.$$
 (7)

Here $(d\sigma/d\Omega)_{\text{expt}}$ and $(d\sigma/d\Omega)_{\text{DWUCK4}}$ are, respectively, the experimental cross section and that predicted by DWUCK4. The deduced D_0^2 values and the average D_0^2 = $2.25 \times 10^4 \text{ MeV}^2 \text{ fm}^3$ have been shown in Table II. It is evident from Fig. 3 that the FFR calculations do not improve fits over the ZR predictions and reduce the cross sections at larger reaction angles even more. Nevertheless, the FFR calculations allow us to extract the spectroscopic factors.



FIG. 3. The same as in Fig. 2 using CCBA instead of DWBA.

IV. CCBA ANALYSIS

The CCBA calculations using the molecular potential have been carried out using the code CHUCK3 [30], with the coupling scheme shown in Fig. 4 and the deformation parameters $\beta_2 = -0.18$ and $\beta_4 = +0.08$ for ²⁸Si. In the CCBA calculations, the depth of the imaginary part of the molecular potential (Table I) has been decreased to 10.5 MeV in order to reproduce the angular distribution for the elastic scattering. All possible relative phases and various relative transition amplitudes a_R in the rearrangement paths have been tried in the simplest possible coupling scheme. The transition strength in a two-step path is proportional to the square of βa_R . The CCBA predictions using the relative spectroscopic amplitudes given in Table II for the $1/2^+$ ground, 1.266 MeV $3/2^+$, 2.234 MeV $5/2^+$, and 3.415 MeV $7/2^+$ state transitions



FIG. 4. The coupling scheme for the CCBA used in calculations shown in Fig. 3.

have been compared to the data in Fig. 3. The CCBA calculations improve the fits over the ZR and FFR calculations. The inelastic 4⁺ state at E_x =4.618 MeV in ²⁸Si plays a major role in the CCBA calculations in reproducing the ground state data. The coupling to the inelastic 2⁺ state to the ground state of ²⁸Si is also significant in improving the fits to the data for the 1.266 and 2.234 MeV states of ³¹P. The CCBA calculations seem to confirm the deformed shape of the ²⁸Si nucleus.

V. DISCUSSION AND CONCLUSION

The present work reports, for the first time, the analyses of a three-nucleon transfer reaction using the molecular type potential. While the patterns of the angular distributions for the reaction to the ground $(1/2^+)$, 1.27 MeV $(3/2^+)$, and 2.23 MeV $(5/2^+)$ states of the final nucleus, are reasonably reproduced by the DWBA calculations using the deep optical and Michel potentials, the predicted cross sections are off by 2 to 4 orders of magnitudes in each case. This agrees with the results of Refs. [21,23] for the calculation with the deep optical potential and those of Xiumin *et al.* [36] who failed to reproduce the data for the ⁴⁰Ca(α , p)⁴³Sc reaction with the squared WS potential used by the Michel group [32]. However, the DWBA and CCBA calculations using the molecular potential and assuming a simple triton-cluster transfer

TABLE II. Cluster transfer configurations (*n*: number of nodes, *L*: angular momentum) used in the CCBA are shown in columns 3–6. Column 7 indicates the relative spectroscopic amplitudes used in calculations shown in Fig. 3. Columns 8 and 9 are, respectively, the spectroscopic factors deduced from the FFR calculations and the normalization constant for the (α, p) reaction for the DWBA calculations.

E_x (³¹ P)	J^{π}	Cluster transfer configuration One-step Two-step Relative spect.					Spect. factor	$D_0^2 \times 10^4$ MeV ² fm ³
MeV		nL_d	nL_{t_1}	nL_{t_2}	nL_{t_3}	amplitudes	S	
0.0	1/2 +	3S	1G			+01:+15	0.070	2.00 ± 0.50
1.266	$3/2^{+}$	2D	2D	3S		+01:+05:-05	0.031	2.56 ± 0.64
2.234	$5/2^{-}$	2D	2D	3S	1G	+01:+01:+02:-01	0.004	
3.415	$7/2^{-}$	2G	3D	4S		+01:+06:+02	0.003	

mechanism, reproduce not only the angular oscillations more satisfactorily, but also the correct order of absolute cross sections for each of the four final states including the one at 3.42 MeV excitation of ³¹P. The calculation using the shallow potential reproduces the magnitudes up to 100° or so, but then decreases sharply at large angles. Thus, the molecular potential is the only one to account for the data for the ground $(1/2^+)$, 1.27 MeV $(3/2^+)$, and 2.23 MeV $(5/2^+)$ final states over the entire angular distributions. Furthermore, the present analysis indicates that the data for the reaction can be successfully described without any compound nucleus contribution, as included by Jankowski *et al.* [14], which is highly improbable at the incident energy considered here.

A pertinent question arises as to why the Michel potential, which has been so successful in accounting for ALAS in the elastic scattering on many targets [6,7,32,33] including ²⁸Si in the present work, fails to reproduce the data of the ²⁸Si(α ,p)³¹P reaction. The Michel potential has also been found to be inadequate for the one-nucleon transfer reaction [12]. Aside from the fact that the Michel potential is monotonic, whereas the molecular is nonmonotonic, one may note that the two potentials differ significantly in defining the Coulomb radius. In case of the molecular potential, the Coulomb radius R_C is the distance where ²⁸Si barely touches the α particle. The observed density distribution, $\rho(r)$ for ²⁸Si is given by [34]

$$\rho(r) = \rho(0) \left[1 + \exp \frac{r - c}{d} \right]^{-1} \tag{8}$$

with c=3.14 fm and d=0.537 fm. Thus, at r=6 fm, $\rho(r) = 0.005\rho(0)$. A reasonable density distribution for α particle is $4(\gamma/\pi)^{3/2} \exp(-\gamma r^2)$ with $\gamma=0.5$ [35]. This is about 0.001 at r=3.35 fm. Thus, a reasonable value of R_C is (6.00+3.35) =9.35 fm, which is used in the molecular potential. The

Michel, on the other hand, uses $R_C = 3.95$ fm. At this distance, the two nuclei have interpenetrated each other substantially. In the DWBA theory, the stripped particles from the projectile are assumed to drop on the nuclear surface and hence, the treatment may be somewhat sensitive to the actual value of R_C .

One may summarize from the displays in Figs. 1–3 that, while the molecular, the Michel with the squared WS geometry, and the normal optical potentials produce more or less the similar quality of fits to the elastic data, their use in describing the transfer data for the (α, p) reactions leads to significantly different results, with only the molecular one accounting for the observed data in terms of both absolute cross sections and angular distribution. This supports Satchler's contention [37] that the real test of a potential set generated from the analysis of elastic scattering data lies in its ability in reproducing the nonelastic data. The present work seems to suggest preference for the molecular potential over other forms of the α -nucleus potential in describing the angular distribution of the (α, p) reaction on ²⁸Si at 26 MeV. The finding demands further investigation with other targets.

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