Proton angular distributions from oriented proton-emitting nuclei

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Angular distributions of protons emitted from ground and isomeric states in oriented nuclei are calculated. Odd-Z, odd-A proton emitting nuclei are considered in this work. The shape of the angular distributions shows strong dependence with the angular momentum carried away by the proton. They can be used potentially as a spectroscopic tool.

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I. INTRODUCTION

We have witnessed in the last few years a considerable amount of work in the field of proton radioactivity from ground state or low-lying isomers [1]. Among the highlights of this research, one could mention the experimental observation of proton emission from an intruder state [2], highly deformed proton emitters [3,4], fine structure from both deformed and spherical proton emitters [5,6], the derivation of excited state information from nuclei beyond the proton dripline [7,8], as well as the development of theoretical models [4,9–16].

A recent development in α decay is the measurement of angular distributions from oriented α emitters [17–19]. Briefly, α emitting nuclei, with spin $\vec{J}_i$, are implanted in a foil kept at very low temperatures and under a strong magnetic field $\vec{B}$. The Hamiltonian that describes the interaction between the nuclear spin and the magnetic field is given by $H=-g_n\beta_n\vec{J}_i\cdot\vec{B}$, where g_n is the nuclear gyromagnetic factor and β_n is Bohr's magneton. As a result, the nuclei will rotate to have, depending on the sign of g_n, $\vec{J}_i$ parallel or antiparallel to $\vec{B}$. Ideally, when alpha emission takes place, the orientation of the α-emitting nucleus is complete. With this experimental technique, only alpha emitters with nonzero value of spin can be studied, excluding in fact ground-state, even-even emitters.

Theoretical treatments of α angular distributions have been developed over the last 40 years [20–23]. The problem is not simple, and it basically requires to obtain the α particle wave function to then evaluate its evolution as the α particle leaves the parent nucleus. For the first part, the wave functions of the individual nucleons that make up the α particle have to be calculated, a task highly related to the evaluation of the so-called preformation factor; the second stage can be solved through a coupled-channel calculation. The comparison between theoretical and experimental values of the anisotropies of α angular distributions can be quite revealing [18,19], helping in fact to clarify the relative importance and validity of the several components in the theoretical models.

One of the most attractive and rewarding features of proton radioactivity is that from the experimental energy and half-life values of the proton peak, it is often possible to deduce the spin and parity of the parent nucleus [24]. As a result, a good amount of information about nuclei far away from the valley of stability has been gathered [1]. Research in this field may eventually include the measurement of proton angular distributions from oriented proton emitters. This possibility may be not too remote since one of the α emitters ^{189}Bi, studied in Ref. [19] is only four neutrons away from a known proton emitter ^{185}Bi. Additionally, the measurement of proton angular distributions from β-delayed proton emitters is underway [25].

From the experimental point of view, the study of proton angular distributions is immensely challenging. To start with, all the known proton emitters have $t_{1/2}$ that are shorter than a few seconds. Secondly, most proton emitters are formed by fusion-evaporation reactions, with cross sections in the μb range or smaller. On the positive side, and unlike α emitters, most observed proton emitters have a nonzero value of spin, allowing in principle to obtain information for a large number of them.

Two additional features distinguish proton from α radioactivity. (a) A large fraction of the known proton emitters corresponds to odd-Z, even-N nuclei, which populate the ground state $(0+)$ of the daughter nucleus; therefore, the proton is emitted with a given value of orbital angular momentum, contrasting with the kind of transitions that can be seen in α angular distributions from oriented nuclei, where the α decay connects states with nonzero values of spin and as a consequence, the α particle angular momentum will have a range of allowed values. (b) Proton radioactivity can be understood with a high degree of accuracy, in terms of a proton orbiting an inert core taking into account a spectroscopic factor, no preformation factor is needed.

The purpose of this work is to explore the kind of information that could be extracted from proton angular distributions from oriented proton emitters. The basic formalism is presented in Sec. II while its application, specially to recent findings, is done in Sec. III. Needless to say, the theoretical

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treatments of both proton and α angular distributions must run parallel to each other, in fact, many similarities between our work and those from Refs. [20,23] can be found. One will expect, however, that due to the simplicity of the daughter nucleus wave function, the more restricted range of angular momentum values, and the absence of preformation factors, proton angular distributions will be easier to model than α's.

II. FORMALISM

In this paper, expressions for angular distributions from odd-Z, odd-A proton emitters are derived and results are discussed. We will assume that we are dealing with pointlike proton radioactivity source. The description will be done in the center-of-mass reference frame. The parent nucleus angular momentum operators will be noted as J_i^2 and J_{iz}, with associated quantum numbers J_i and M_i, respectively. The eigenvalue of the parity operator will be noted as π_i. Similarly, the final state of the daughter nucleus will be described with quantum numbers J_f, M_f, and π_f. For the escaping proton, the total angular momentum quantum numbers will be noted by j and m, the orbital ones by l and m_l and the spin quantum number by the z projection m_s of the spin-1/2 spin operator. The total decay width to a state $J_f^{\pi_f}$ of the daughter nucleus, $\Gamma_{J_f^{\pi_f}}$, is written as

$$\Gamma_{J_f^{\pi_f}}=\sum_{j,l}\Gamma_{J_f^{\pi_f}jl}, \tag{1}$$

where $\Gamma_{J_f^{\pi_f}jl}$ is the decay width for given values of proton total and orbital angular momenta [9,10], and the sum includes all values of j and l that conserve angular momentum and parity.

For large distances, the wave function of the proton, that left the daughter nucleus in the state defined by the quantum numbers $J_f^{\pi_f}$ and M_f, has the following form:

$$\phi_{J_f^{\pi_f},M_f}(\vec{r},\vec{\sigma})=\sum_{j,l,m_s}\sqrt{\frac{\Gamma_{J_f^{\pi_f}jl}k_p}{2Q_p}}C^{J_iM_i}_{J_fjM_fm}C^{jm}_{l1/2m_lm_s}\times\frac{e^{i(k_pr-l\pi/2+\delta_l^{\text{Coul}})}}{r}Y_{lm_l}(\Omega)\chi_{1/2m_s}(\vec{\sigma}), \tag{2}$$

where k_p is the proton wave number, Q_p is the proton Q value, $m=M_i-M_f$, $m_l=M_i-M_f-m_s$, and δ_l^{Coul}'s are the Coulomb phase shifts; $C^{jm}_{l_1l_2m_1m_2}$, Y_{lm}, and $\chi_{1/2m_s}$ are the usual Clebsch-Gordan coefficients, spherical harmonics, and 1/2 spinors, respectively.

The width $\Gamma_{J_f^{\pi_f}}$ can be written as

$$\Gamma_{J_f^{\pi_f}}=\int d\Omega\frac{d\Gamma_{J_f^{\pi_f}}}{d\Omega}(\Omega). \tag{3}$$

The angular distribution, defined as the probability distribution $F_{J_f^{\pi_f}}(\Omega)$, is given by

$$F_{J_f^{\pi_f}}(\Omega)=\frac{1}{\Gamma_{J_f^{\pi_f}}}\frac{d\Gamma_{J_f^{\pi_f}}}{d\Omega}(\Omega), \tag{4}$$

which clearly satisfies $\int d\Omega F_{J_f^{\pi_f}}(\Omega)=1$.

From the asymptotic form of the wave function, we can obtain

$$\frac{d\Gamma_{J_f^{\pi_f}}}{d\Omega}(\Omega)=\sum_{M_i}P(M_i)\sum_{M_f,m_s}\left|\sum_{j,l}\sqrt{\Gamma_{J_f^{\pi_f}jl}}C^{J_iM_i}_{J_fjM_fm}\times C^{jm}_{l1/2m_lm_s}e^{i(-l\pi/2+\delta_l^{\text{COUL}})}Y_{lm_l}(\Omega)\right|^2, \tag{5}$$

where $P(M_i)$ is the probability of finding the parent nucleus with angular momentum projection M_i on the Z axis. For instance, if all magnetic substates are equally probable, $P(M_i)=1/(2J_i+1)$.

We will now investigate angular distributions from oriented nuclei, i.e., we will assume that due to the presence of a magnetic field, at the moment of the emission $\vec{J_i}$ is parallel to the Z axis. Hence, $P(M_i)=1$ if $M_i=J_i$ and $P(M_i)=0$ for any other values. Under this condition, we obtain

$$F_{J_f^{\pi_f}}(\theta)=\frac{1}{\Gamma_{J_f^{\pi_f}}}\sum_{M_f,m_s}\left|\sum_{j,l}\sqrt{\Gamma_{J_f^{\pi_f}jl}}C^{J_iJ_i}_{J_fjM_fm}C^{jm}_{l1/2m_lm_s}\times e^{i(-l\pi/2+\delta_l^{\text{Coul}})}Y_{lm_l}(\Omega)\right|^2, \tag{6}$$

where the symmetry around the azimuthal angle has been explicitly taken into account.

The overwhelming majority of odd-odd proton emitters populate the ground state (0^+) of the daughter. Hence, when $J_f=0$, $M_f=0$, and $j=J_i$, Eq. (6) reduces to

$$F_{0^+}(\theta)=\sum_{m_s}|C^{J_iJ_i}_{l1/2m_lm_s}Y_{lm_l}|^2. \tag{7}$$

More explicit formulas can be obtained for the two coupling possibilities between the proton spin and orbital angular momenta. If $j=l+1/2$, Eq. (7) reduces to

$$F_{0^+}(\theta)=|Y_{ll}|^2, \tag{8}$$

and from usual expressions for the spherical harmonics, we obtain

$$F_{0^+}(\theta)=\frac{(2l+1)!}{\pi4^{l+1}l!^2}\sin^{2l}(\theta), \tag{9}$$

or alternatively, $|Y_{ll}|^2$ can be expanded in terms of Y_{k0} for even values of k to get

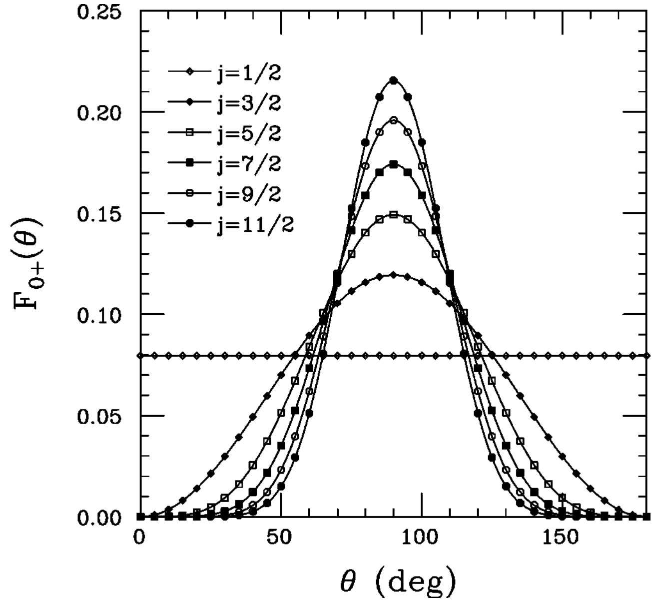

FIG. 1. Proton angular distributions for different values of proton angular momentum, for decays that populate the ground state (0^+) of the daughter nucleus. As explained in the text, the angular distributions do not depend on the proton parity value.

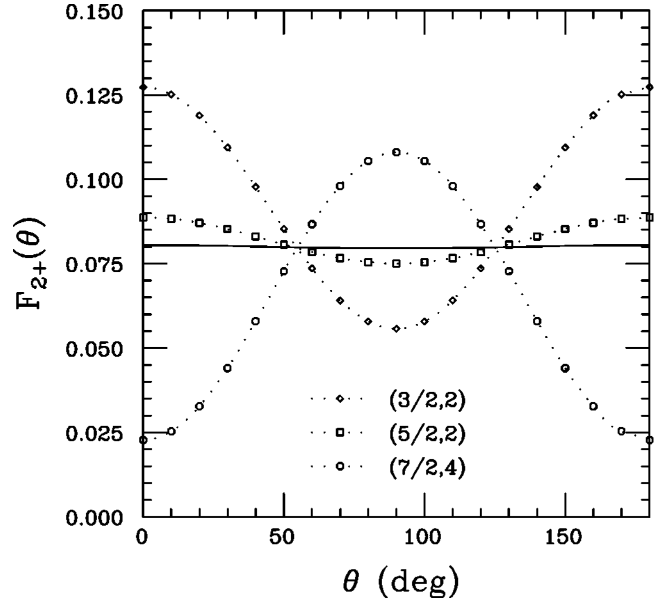

FIG. 2. Proton angular distributions for the $3/2^+ \to 2^+$ decay in ^{131}Eu. The full curve corresponds to a calculation using Eq. (15), while the remaining curves are meant to facilitate the understanding of this result, as explained in the text.

$$F_{0^+}(\theta)=\sum_{k\geq 0,\mathrm{even}}\sqrt{\frac{2k+1}{4\pi}}C^{l0}_{lk00}C^{ll}_{lkl0}Y_{k0}(\theta). \qquad (10)$$

For the cases in which $j=l-1/2$:

$$F_{0^+}(\theta)=|C^{(l-1/2)(l-1/2)}_{l1/2l-1/2}Y_{ll}|^2+|C^{(l-1/2)(l-1/2)}_{l1/2(l-1)1/2}Y_{ll}|^2, \qquad (11)$$

as we did before, this equation could be expressed as a function of sin(θ) and cos(θ). Instead, we would like to remember that

$$C^{(l-1/2)(l-1/2)}_{l1/2l-1/2}=\sqrt{\frac{2l}{2l+1}}, \quad C^{(l-1/2)(l-1/2)}_{l1/2(l-1)1/2}=-\sqrt{\frac{2l}{2l+1}}, \qquad (12)$$

$$\frac{2l}{2l+1}|Y_{ll}|^2+\frac{1}{2l+1}|Y_{l(l-1)}|^2=|Y_{(l-1)(l-1)}|^2, \qquad (13)$$

to show that protons with $j=l-1/2$ exhibit the same angular distribution as protons with $j=(l-1)+1/2$. For instance, protons escaping from a $g_{9/2}$ orbital will show the same angular distribution as those from a $h_{9/2}$ orbital. This means that the shape of the angular distribution depends only on j, not on the parity value.

III. DISCUSSION

Examples of angular distributions are plotted in Fig. 1 for different values of j. It can be seen that as the angular momentum increases, the angular distributions become narrower and taller. One can understand this feature by thinking that an increase in angular momentum requires the proton to spend more time in the equatorial areas to accommodate for an increase in m. This result could sound counterintuitive for deformed nuclei, since one may think that emission from the polar areas should be somehow favored due to a lowering of the Coulomb field.

Due to relatively large values of the production cross section and large half-lives, it is likely that the angular distributions from spherical proton emitters like ^{147}Tm or ^{151}Lu will be the first ones to be investigated. For these nuclei, the choice of proton orbitals narrows to the $2s_{1/2}$, $1d_{3/2}$, and $0h_{11/2}$ ones [1], which in light of Fig. 1, will have very different experimental signatures. The angular distributions shape dependence with j makes them particularly useful to reveal the angular momentum of the proton. For instance, we can define the anisotropy $A(\theta)$ as

$$A(\theta)=\frac{F_{0^+}(\theta)}{F_{0^+}(\pi/2)} \qquad (14)$$

and in particular, $A(\pi/4)=(1/2)^{(j-1/2)}$, i.e., $A(\pi/4)=1,0.5,0.25,0.125, \ldots$, for $j=1/2$, 3/2, 5/2, 7/2, etc.

We will discuss a few more cases, being perfectly aware that some of the half-lives and production cross sections involved are quite small and experimental studies may lie beyond the possibilities of the current techniques, but which are nevertheless interesting and may help reveal a few more intrinsic qualities of the angular distributions. We begin with ^{141}Ho, which exhibits ground-state and isomeric proton radioactivity [3,4]. Based on a precise knowledge of the proton energy and half-life, the [523]$7/2^-$ and [411]$1/2^+$ orbitals have been assigned to ground state and isomer, respectively. While angular distributions from the ground state proton group will be strongly peaked around 90° ($j=7/2$ in Fig. 1), that from the isomer should be isotropic.

The nucleus ^{145}Tm is another interesting case, a fast proton peak has been observed and the proton energy and half-life are consistent with a $h_{11/2}$ spherical decay [26]. On the other hand, this nucleus is predicted to have a permanent quadrupole deformation $\beta_2=0.249$ [27]. In fact, from systematic trends, the energy of the first 2^+ state can be estimated to be $\sim$0.3 MeV, which if Grodzins relation [28] is used, translates into a β_2 value of $\sim$0.2. We have calculated the decay width using the formalism of Ref. [9] and found that the orbital $[404]7/2^+$ can account for the experimental values. Here we have a case where the proton radioactivity information is consistent with both a spherical and a deformed shape, but the agreement with one of them is simply fortuitous. Proton angular distributions could be used to discern between these two cases, since $j=11/2$ protons will exhibit a more strongly peaked angular distribution than $j=7/2$ protons as can be seen from Fig. 1. Needless to say, an experimental determination of the excited states of either ^{145}Tm or ^{144}Dy would also reveal the quadrupole deformation of these nuclei and help to settle the value of spin and parity for ^{145}Tm.

Finally, we will consider the case in which the first 2^+ state of the ground state rotational band in the daughter nucleus is populated in the decay of a odd-Z, odd-A nucleus. This situation has been so far observed in the decay of ^{131}Eu [5]. From Eq. (6) we can obtain

$$F_{2^+}(\theta)=\frac{1}{\Gamma_{2^+}}\sum_{M_f,m_s}\left|\sum_{j,l}\sqrt{\Gamma_{2^+jl}}C^{J_iJ_i}_{2jM_fm}C^{jm}_{l(1/2)m_lm_s}\times e^{i(-l\pi/2+\delta_l^{\mathrm{Coul}})}Y_{lm_l}(\Omega)\right|^2. \tag{15}$$

Two interesting conclusions can be drawn from this expression: (a) the calculated angular distributions now depend on the approximations one applies to obtain the Γ_{2^+jl} values, (b) the number of independent Y_{lm_l} functions contributing to the angular distribution will generally be larger than in the $J_f=0$ case.

As was mentioned in Ref. [5], the observed 2^+ branching ratio in ^{131}Eu can be explained if the proton is emitted from the $[411]3/2^+$ Nilsson orbital. For this case, Γ_{2^+} is dominated by the $\Gamma_{2^+,5/2,2}$ term, accounting for $\sim$95% of it, while $\Gamma_{2^+,3/2,2}$ represents $\sim$5% and $\Gamma_{2^+,7/2,4}$ gives a negligible contribution. The resulting angular distribution can be seen as the full line in Fig. 2, an almost isotropic behavior $[A(0)=1.012]$, which reaches a minimum at 90°. This angular dependence contrasts nicely with that of the ground-state to ground-state angular distribution, which corresponds to $j=3/2$ in Fig. 1. To better understand this result we have also plotted in Fig. 2 the angular distribution for the hypothetical cases in which one of the Γ_{2^+jl} is equal to one and the others are equal to zero, these calculations are labeled in Fig. 2 by the (j,l) values of the nonvanishing Γ_{2^+jl}. As expected, the angular dependence of the full line can be traced back to the behavior of the $\Gamma_{2^+,5/2,2}=1$ curve.

In summary, it is shown that proton angular distributions have the potential to clearly determine the spin carried away by the proton, which can be used to deduce the spin and parity of proton emitting nuclei. These results could be compared with what is already known from the study of proton Q values and half-lives. We hope that this work will convince experimental groups that there is interesting physics in proton angular distributions and will encourage them to surmount the considerable technical challenges that such a measurement imposes.

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