Extended quasiparticle random phase approximation at finite temperatures: Calculation of single β-decay Fermi transitions

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The formalism of the quasiparticle random phase approximation, extended to include scattering terms in the equation of motion, is used to describe allowed single β-decay transitions of Fermi type at finite temperatures. The calculations were performed by using a realistic single particle basis and a separable two body interaction in the proton-neutron channel. The behavior of the Ikeda sum rule is studied and it is found that this sum rule is strictly conserved in the presence of particle-particle and hole-hole correlations. As an example on the validity of the formalism the case of Fermi transitions in ^{76}Ge is considered.

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I. INTRODUCTION

An element of astrophysical interest is the calculation of single β-decay rates in stellar conditions [1]. The conventional procedure consists of large scale shell model (SM) and/or quasiparticle random phase approximation (QRPA) calculations, depending on the considered mass region, see Ref. [1], and references therein. These calculations describe single β-decay transitions feeding known sequences of β-stable nuclei or decay sequences leading to neutron or proton rich nuclei. A large number of uncertainties are introduced in the calculations due to several reasons, among them: (a) the low-energy spectrum of the participant nuclei are poorly known, (b) the parameters used in the calculations are fixed globally, and not on a case by case analysis; and (c) zero temperature strength distributions are used to compute decay rates at finite temperatures. For a recent compilation of results, see Ref. [1]. Large scale calculations of single β decay rates at finite temperatures can be performed by using the finite temperature QRPA formalism [2]. The use of this technique has the advantage that thermal occupation factors, excitation energies, and decay rates can be calculated as functions of the nuclear temperature. Previous experience with the finite temperature QRPA (FTQRPA) [3] indicates that complete expressions of the transition operators should be used to compensate for thermal blocking effects affecting transitions near the Fermi surface, that is to say that one should include particle-particle and hole-hole transitions in addition to transitions across the Fermi surface. Concerning the inclusion of particle-particle correlations it is known [4] that they are responsible for the hindrance of charge dependent transitions [5]. Furthermore these particle-particle correlations can induce instabilities of the QRPA vacuum and eventually be the source of the QRPA breakdown [6]. An undesired consequence of the presence of particle-particle correlations, in the QRPA wave functions near the breakdown, is the violation of sum rules. In this paper we aim at the description of single β-decay rates at finite temperature by using the FTQRPA and by including particle-particle and hole-hole terms in the proton-neutron interactions. We are also keeping all terms which appear in the expression of the transition operator to study the effect of thermal and particle correlations on the corresponding sum rule. We shall show that the strength distributions obtained by using the present formalism strictly obey the sum rule associated to the transition operator. Similar motivations about the use of an extended version of the QRPA to deal with all type of excitations around the Fermi surface can be found in Ref. [7], where a schematic interaction is treated in an extreme single particle model space. In the present work we are presenting a more general formalism without imposing restrictions on the configuration space and for the case of a separable interaction. As an example about the use of the formalism we have applied it to the calculation of single β-decay transitions of the Fermi type. As will be discussed in the text, in spite of the schematic structure of the interaction and of the relatively simple form of the transition matrix elements, the formalism illustrates the effect of small components of the wave functions upon the transition strength. We have taken the case of allowed Fermi transitions $(\Delta J=0, \Delta\pi=0, \Delta T_z=\pm 1)$, as a test case bearing in mind that more realistic calculations are needed in astrophysical applications [1]. The formalism is presented in Sec. II and the results of the calculations corresponding to Fermi transitions in a nucleus with $A=76$ are presented and discussed in Sec. III. Conclusions are drawn in Sec. IV.

II. FORMALISM

In this section we shall present the steps which we have followed in order to calculate wave functions and matrix elements of the Fermi operator $(\beta^{\pm}=\tau^{\pm})$ connecting the ground state of an even-even mass nucleus with $J^{\pi}=0^{+}$ excited states of a odd-odd mass nucleus. Since we aim at the study of the validity of the QRPA at finite temperature we have chosen the case of Fermi transitions as a test case, for convenience, but this choice does not introduce any signifi-

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cant restriction in the formalism or in the conclusions about its use. To start with let us introduce the Hamiltonian, which is the one proposed by Kuz'min and Soloviev [8] and more recently used in Refs. [9,10] in dealing with the calculations of double β-decay observables. The Hamiltonian includes a single-particle term, a separable monopole pairing interaction and a charge-dependent separable residual interaction with both particle-hole and particle-particle (hole-hole) proton(p)-neutron(n) channels. It is written as

$$H=\sum_{pj} e_{pj}N_{pj}+\sum_{j} e_{nj}N_{nj}-G_pS_p^\dagger S_p-G_nS_n^\dagger S_n +2\chi\beta^-\beta^+-2\kappa P^-P^+, \quad (1)$$

where

$$N_{qj}=\sum_m a_{qjm}^\dagger a_{qjm},$$

$$S_q^\dagger=\sum_{jm} a_{qjm}^\dagger a_{q\overline{jm}}^\dagger, \quad S_q=(S_q^\dagger)^\dagger, \quad q=p,n,$$

$$\beta^-=\sum_{jm} a_{pjm}^\dagger a_{njm}, \quad \beta^+=(\beta^-)^\dagger,$$

$$P^-=\sum_{jm} a_{pjm}^\dagger a_{n\overline{jm}}^\dagger, \quad P^+=(P^-)^\dagger, \quad (2)$$

are the number operator, the monopole pair operator, and the particle-hole and particle-particle creation operators, respectively. Proton and neutron single particle orbits, of angular momentum j and projection m, are denoted by the subindices (p) and (n) and $a_{qjm}^\dagger$ is a particle creation operator and $a_{q\overline{jm}}^\dagger=(-1)^{j-m}a_{qj-m}^\dagger$ its time reversal.

By transforming the Hamiltonian of Eq. (1) to the quasiparticle representation [11] one obtains

$$\begin{aligned}H=&\sum_{qj} E_{qj}N_{qj}\\&+\sum_{jj'}\Big\{\frac{1}{2}[r_{jj'}(A_j^\dagger A_{j'}+A_{j'}^\dagger A_j)+s_{jj'}(A_j^\dagger A_{j'}^\dagger+A_{j'}A_j)\\&+u_{jj'}(B_j^\dagger B_{j'}+B_{j'}B_j^\dagger)+v_{jj'}(B_j^\dagger B_{j'}^\dagger+B_{j'}B_j)]\\&+t_{jj'}(A_j^\dagger B_{j'}+B_{j'}^\dagger A_j)+w_{jj'}(A_j^\dagger B_{j'}^\dagger+B_{j'}A_j)\Big\}. \quad (3)\end{aligned}$$

In the above expression E_{qj} are the quasiparticle energies and, for simplicity, the index j indicates single particle states. The operators and matrix elements appearing in the same equation are defined by

$$A_j^\dagger=[\alpha_{pj}^\dagger\otimes\alpha_{nj}^\dagger]_{M=0}^{J=0},$$

$$B_j^\dagger=[\alpha_{pj}^\dagger\otimes\alpha_{\overline{nj}}]_{M=0}^{J=0},$$

$$N_{qj}=\sum_m \alpha_{qjm}^\dagger\alpha_{qjm}, \quad q=n,p,$$

$$r_{jj'}=2\chi(t_jt_{j'}+\overline{t_jt_{j'}})-2\kappa(p_jp_{j'}+\overline{p_j}\overline{p_{j'}}),$$

$$s_{jj'}=2\chi(t_j\overline{t_{j'}}+\overline{t_j}t_{j'})+2\kappa(p_j\overline{p_{j'}}+\overline{p_j}p_{j'}),$$

$$u_{jj'}=2\chi(p_jp_{j'}+\overline{p_j}\overline{p_{j'}})-2\kappa(t_jt_{j'}+\overline{t_jt_{j'}}),$$

$$v_{jj'}=-2\chi(p_j\overline{p_{j'}}+\overline{p_j}p_{j'})-2\kappa(t_j\overline{t_{j'}}+\overline{t_j}t_{j'}),$$

$$t_{jj'}=2\chi(\overline{t_jp_{j'}}-t_jp_{j'})+2\kappa(\overline{p_jt_{j'}}-p_jt_{j'}),$$

$$w_{jj'}=2\chi(t_j\overline{p_{j'}}-\overline{t_j}p_{j'})+2\kappa(\overline{p_j}t_{j'}-p_j\overline{t_{j'}}), \quad (4)$$

where

$$t_j=u_{pj}v_{nj}, \quad \overline{t_j}=u_{nj}v_{pj},$$

$$p_j=u_{pj}u_{nj}, \quad \overline{p_j}=v_{nj}v_{pj}. \quad (5)$$

The creation and annihilation of quasiparticles are represented by the operators $\alpha_{qjm}^\dagger$ and α_{qjm}, respectively, u_{qj}, and v_{qj} are BCS occupations factors and all radial overlaps are taken as unity.

The QRPA method [12] prescribes that the Hamiltonian H can be diagonalized in the phonon basis $(\Gamma_k,\Gamma_k^\dagger)$. Usually, only the pair creation and pair annihilation operators $A_j^\dagger$ and A_j are included in the definition of the QRPA phonons. In the following we have generalized the standard QRPA to include the operators $B_j^\dagger$ and B_j in the definition of the phonons, as done in Ref. [2]. In the present case of proton-neutron excitations the extended QRPA phonon is written

$$\Gamma_k^\dagger=\sum_j [X_{kj}A_j^\dagger-Y_{kj}A_j+Z_{kj}B_j^\dagger-\overline{Z}_{kj}B_j], \quad (6)$$

where extra terms $Z_{kj}B_j^\dagger-\overline{Z}_{kj}B_j$, are added to the conventional definition of the phonon operator. As shown in Ref. [2] vacuum expectation values can be replaced by thermal averages in order to account for temperature dependent effects. The resulting QRPA matrix equation can be written as

$$\begin{pmatrix}\tilde{A} & \tilde{B}\\ \tilde{B}^* & \tilde{A}^*\end{pmatrix}=\omega\begin{pmatrix}\tilde{S} & 0\\ 0 & -\tilde{S}\end{pmatrix}\begin{pmatrix}\tilde{X}\\ \tilde{Y}\end{pmatrix}. \quad (7)$$

The forward ($\tilde{A}$) and backward ($\tilde{B}$) matrices, the metric matrix ($\tilde{S}$), and the amplitudes ($\tilde{X}$ and $\tilde{Y}$) are defined as

$$\tilde{A}=\begin{pmatrix}A & C\\ E & G\end{pmatrix},$$

$$\tilde{B}=\begin{pmatrix}B & D\\ F & H\end{pmatrix},$$

$$\tilde{S}=\begin{pmatrix} S & 0 \\ 0 & T \end{pmatrix},$$

$$\tilde{X}=\begin{pmatrix} X \\ Z \end{pmatrix},$$

$$\tilde{Y}=\begin{pmatrix} Y \\ \bar{Z} \end{pmatrix}. \tag{8}$$

The corresponding matrix elements, in the basis of quasiproton-quasineutron pairs, of the above matrices are written

$$A_{ij}=\langle[A_i,[H,A_j^\dagger]]\rangle,$$

$$B_{ij}=-\langle[A_i,[H,A_j]]\rangle,$$

$$C_{ij}=\langle[A_i,[H,B_j^\dagger]]\rangle,$$

$$D_{ij}=-\langle[A_i,[H,B_j]]\rangle,$$

$$E_{ij}=\langle[B_i,[H,A_j^\dagger]]\rangle,$$

$$F_{ij}=-\langle[B_i,[H,A_j]]\rangle,$$

$$G_{ij}=\langle[B_i,[H,B_j^\dagger]]\rangle,$$

$$H_{ij}=-\langle[B_i,[H,B_j]]\rangle,$$

$$S_{ij}=\langle[A_i,A_j^\dagger]\rangle,$$

$$T_{ij}=\langle[B_i,B_j^\dagger]\rangle. \tag{9}$$

The explicit expressions of these matrix elements, in terms of quasiparticle energies, quasiparticle occupation factors, and matrix elements of the residual interaction, are obtained after evaluation of the commutators and double commutators. They are given by

$$A_{ij}=\delta_{ij}2\Omega_j(1-f_{nj}-f_{pj})(E_{nj}+E_{pj}) + r_{ij}2\Omega_j(1-f_{nj}-f_{pj})2\Omega_i(1-f_{ni}-f_{pi}),$$

$$B_{ij}=s_{ij}2\Omega_j(1-f_{nj}-f_{pj})2\Omega_i(1-f_{ni}-f_{pi}),$$

$$C_{ij}=t_{ij}2\Omega_j(f_{nj}-f_{pj})2\Omega_i(1-f_{ni}-f_{pi}),$$

$$D_{ij}=w_{ij}2\Omega_j(f_{nj}-f_{pj})2\Omega_i(1-f_{ni}-f_{pi}),$$

$$E_{ij}=C_{ji},$$

$$F_{ij}=D_{ji},$$

$$G_{ij}=\delta_{ij}2\Omega_j(f_{nj}-f_{pj})(E_{pj}-E_{nj}) + u_{ij}2\Omega_j(f_{nj}-f_{pj})2\Omega_i(f_{ni}-f_{pi}),$$

$$H_{ij}=v_{ij}2\Omega_j(f_{nj}-f_{pj})2\Omega_i(f_{ni}-f_{pi}),$$

$$S_{ij}=\delta_{ij}2\Omega_j(1-f_{nj}-f_{pj}),$$

$$T_{ij}=\delta_{ij}2\Omega_j(f_{nj}-f_{pj}), \tag{10}$$

where f_{qj} are thermal occupations factors

$$f_{qj}=[1+\exp E_{qj}/T]^{-1}. \tag{11}$$

The expectation values which appear in Eq. (9) have been calculated at finite temperature and the quantity T, which appears in the quasiparticle occupation factor of Eq. (11), represents the nuclear temperature in units of energy.

The normalization condition for the phonons is

$$\langle[\Gamma_k,\Gamma_{k'}^\dagger]\rangle=\delta_{kk'}\sum_j 2\Omega_j[(f_{nj}-f_{pj})(Z_{kj}^2-\bar{Z}_{kj}^2) + (1-f_{nj}-f_{pj})(X_{kj}^2-Y_{kj}^2)], \tag{12}$$

where the sum runs over proton-neutron two quasiparticle configurations. Next, we shall write the transition operators $\beta^\pm$, which are the isospin rising and lowering operators, in the quasiparticle basis. The explicit expressions are

$$\beta^-=\sum_j (t_jA_j^\dagger+\bar{t}_jA_j-p_jB_j^\dagger+\bar{p}_jB_j),$$

$$\beta^+=(\beta^-)^\dagger. \tag{13}$$

Using inversion formulas one can express these transition operators in the QRPA phonon basis. They are written as

$$\beta^-=\sum_k (a_k\Gamma_k^\dagger+b_k\Gamma_k),$$

$$\beta^+=\sum_k (a_k\Gamma_k+b_k\Gamma_k^\dagger). \tag{14}$$

The amplitudes a_k and b_k

$$a_k=\langle[\Gamma_k,\beta^-]\rangle,$$

$$b_k=\langle[\Gamma_k,\beta^+]\rangle \tag{15}$$

are the thermal expectation values of the commutator of the transition operators with the QRPA phonons. These amplitudes can be written in terms of the quasiparticle pair and scattering amplitudes of the QRPA phonons, namely,

$$a_k=a_k^{(\mathrm{pair})}+a_k^{(\mathrm{scatt})},$$

$$b_k=b_k^{(\mathrm{pair})}+b_k^{(\mathrm{scatt})}. \tag{16}$$

Explicit expressions of the pair and scattering contributions entering in the a_k and b_k amplitudes are

$$a_k^{(\mathrm{pair})}=\sum_j (t_jX_{kj}+\bar{t}_jY_{kj})2\Omega_j(1-f_{nj}-f_{pj}),$$

$$a_k^{(\mathrm{scatt})}=\sum_j (p_jZ_{kj}-\bar{p}_j\bar{Z}_{kj})2\Omega_j(f_{nj}-f_{pj}),$$

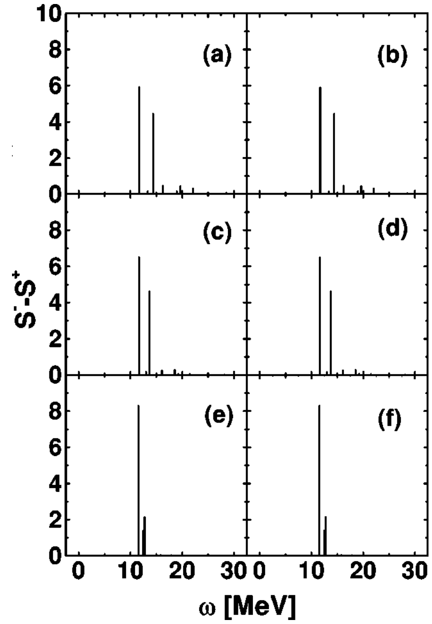

FIG. 1. Energy distribution of the Ikeda sum rule, Eq. (20). The contribution to the quantity $S^- - S^+$, for each phonon of energy ω, is shown as a function of the phonon energy. The calculations were performed for the temperature $T=0.0$ MeV and for the particle-hole coupling interaction $\chi=0.3$ MeV. Cases (a) and (b) correspond to values of $\kappa=0.0$ MeV, cases (c) and (d) correspond to values of $\kappa=0.025$ MeV, and cases (e) and (f) correspond to values of $\kappa=0.05$ MeV. The insets (a), (c), and (e) show the results obtained when the scattering terms are not included, while the insets (b), (d), and (f) show the results obtained with scattering terms included.

$$b_k^{(\mathrm{pair})}=\sum_j \,(t_j Y_{kj}+\overline{t_j}X_{kj})2\Omega_j(1-f_{nj}-f_{pj}),$$

$$b_k^{(\mathrm{scatt})}\sum_j \,(\overline{p_j}Z_{kj}-p_j\overline{Z_{kj}})2\Omega_j(f_{nj}-f_{pj}). \tag{17}$$

In the present version of the QRPA eigenvalue problem the transition strength is defined by

$$S^{\pm}=\sum_k \,|\langle \Gamma_k \beta^{\pm}\rangle|^2, \tag{18}$$

and the Ikeda sum rule is given by the difference $S^- - S^+$, i.e.,

$$\begin{aligned} S^- - S^+ &= \langle [\beta^+,\beta^-]\rangle \\ &= \sum_j \,2\Omega_j[(t_j^2-\overline{t_j^2})(1-f_{nj}-f_{pj}) \\ &\quad +(p_j^2-\overline{p_j^2})(f_{nj}-f_{pj})] \\ &= N-Z. \end{aligned} \tag{19}$$

This sum rule can also be written in terms of quasiparticle pair and scattering amplitudes [see Eq. (16)] and the result is

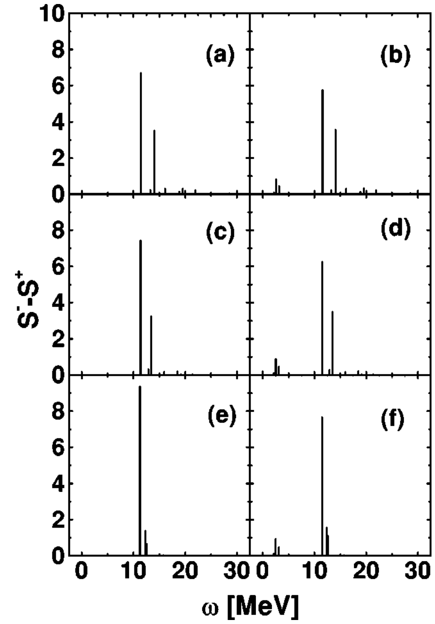

FIG. 2. Same as Fig. 1 for the temperature $T=0.5$ MeV.

$$\begin{aligned} S^- - S^+ &= \sum_k \,(a_k^2-b_k^2), \\ &= [(a_k^{\mathrm{pair}})^2-(b_k^{\mathrm{pair}})^2]+[(a_k^{\mathrm{scatt}})^2-(b_k^{\mathrm{scatt}})^2] \\ &\quad +2(a_k^{\mathrm{pair}}a_k^{\mathrm{scatt}}-b_k^{\mathrm{pair}}b_k^{\mathrm{scatt}}). \end{aligned} \tag{20}$$

This result should be compared with the conventional one which contains only pair contributions. The cancellation of the interference between scattering and pair terms [the last term of Eq. (20)] is guaranteed by the orthonormalization of the QRPA phonons.

Before ending this section, we would like to make a few comments on the scope of the above presented formalism. In spite of its schematic structure, the Hamiltonian of Eq. (1) describes a good amount of the correlations which are specific of charge-exchange $J^{\pi}=0^+$ channels. It illustrates the main mechanism leading to the hindrance of low-energy charge-exchange transitions, namely, the repulsion induced by particle-hole (χ), and the attraction induced by pairing (G_p and G_n) and particle-particle (κ) interactions. Naturally, a more realistic treatment would require the use of an effective two-body interaction, but the formalism is able to deal with such a force. Concerning the use of thermal averages, we have so far described the case of excited final states belonging to the double-odd mass nucleus, assuming that the initial nucleus is in its ground state and that the thermal occupancies are the ground state (BCS) ones [see Eq. (11)]. As explained before we have proceeded in this manner because we wanted to study the distribution and conservation of the transition strength in ground state to excited state transitions. One can also consider the case of decays from excited states of the initial nucleus by performing a finite temperature QRPA calculation for all J^{π} states of the even-even mass nucleus, as described in Ref. [2]. One can then connect the ground and excited states of the initial double-even mass

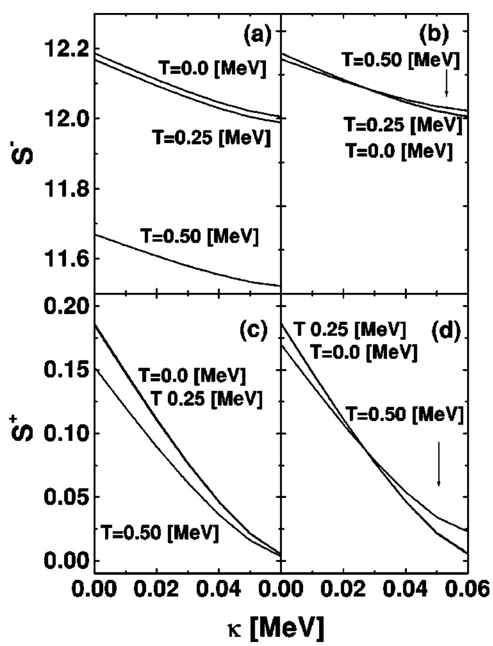

FIG. 3. Strength distributions S^- and S^+. Insets (a) and (b) show the behavior of the strength function S^-, as a function of the coupling constant κ for $\chi=0.3$ MeV and for different values of the temperature T given on the curves. Insets (c) and (d) show the strength function S^+. Cases (a) and (c) show the results obtained when the scattering terms are not included while (b) and (d) show the results obtained by taken into account scattering terms in the QRPA equation of motion and in the transition operator.

nucleus with all states of the final double-odd mass nuclei which are allowed by the transition rules of the β-decay operator. The expressions corresponding to the transition matrix elements are more involved but still they can be computed in the same way as the ground state to excited state transitions.

III. RESULTS AND DISCUSSION

As an application of the previously introduced formalism we have calculated allowed Fermi transitions in the mass region $A=76$. The single particle basis includes all single particle states of the $N_{\rm osc}=3$ and 4 mayor harmonic oscillators shells and the $l=5$ levels from $N_{\rm osc}=5$, both for protons and neutrons (where $N_{\rm osc}$ is the oscillator principal quantum number). The single particle energies around the Fermi surface have been shifted, respect to their harmonic oscillator values [13], to reproduce observed low-energy levels belonging to the spectra of odd-even (even-odd) mass nuclei around ^{76}Ge. The pairing coupling constants, for neutrons (G_n) and for protons (G_p), were fixed at the values $19/A$ MeV and $21/A$ MeV, respectively. The BCS equations [11] were solved by taking $N=Z=20$ as shell closure, for protons and neutrons, respectively. The obtained proton and neutron gaps and quasiparticle energies, calculated at $T=0$ MeV, were found to be in reasonable agreement with data. Temperature dependent BCS equations [14] were solved by varying the temperature in the interval 0 MeV$\leqslant T \leqslant$0.5 MeV. The critical temperatures, associated to the collapse of the proton

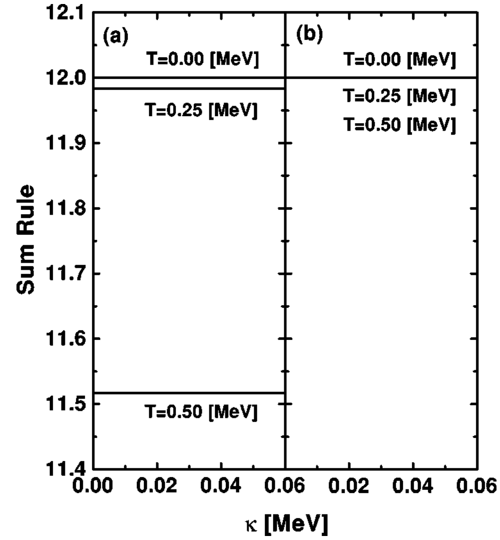

FIG. 4. Ikeda sum rule as a function of the coupling constant κ, for $\chi=0.3$ MeV and for different values of the temperature T. Case (a) corresponds to the results obtained without including scattering terms. Case (b) corresponds to the results obtained with the inclusion of scattering terms.

and neutron gaps, were obtained at values of the order of 0.7 and 0.8 MeV, respectively. Excited $J^\pi=0^+$ states of the double odd mass nucleus ^{76}As were described as QRPA one phonon states. The QRPA equations, Eq. (7), were solved as functions of the temperature T and by taking the strength parameter κ as a free parameter. The parameter χ of the Hamiltonian of Eq. (1) was fixed at the value $\chi=0.3$ MeV, as indicated in Ref. [15]. The contributions to the Ikeda sum rule, for each phonon, are shown in Figs. 1

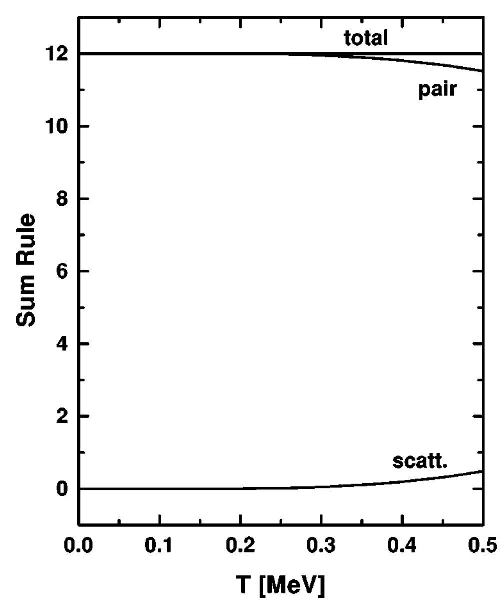

FIG. 5. Partial contributions to the sum rule of Eq. (20), and its total value, as a function of the temperature T. These are the results obtained by using the extended QRPA method. The permanence of the total value of the strength sum, Ikeda sum rule (ISR), is verified for all values of κ below collapse ($\kappa \leqslant 0.06$ MeV).

and 2. The strength distributions of Fig. 1, which corresponds to the case of zero temperature, show the effect of particle-particle correlations for increasing values of the coupling constant κ. The influence of scattering terms, at $T=0$ MeV, is a minor one. Figure 2 shows the results corresponding to $T=0.5$ MeV. While the effects due to increasing particle-particle interactions [insets (a), (c), and (e) of Fig. 2] are very much the same, as compared to the $T=0$ results, the effects of the scattering terms are noticeable particularly in the low-energy region of the spectrum. In Fig. 3 the values of S^- and S^+, of Eq. (18), are shown as functions of the temperature and of the coupling constant κ without [insets (a) and (c)] and with [insets (b) and (d)] the inclusion of scattering terms in the QRPA phonons and transition operators. From the results shown in Fig. 3 it is clearly seen that the contributions coming from scattering terms tends to compensate the decrease of particle-hole transitions due to thermal blocking. The main effect is obviously reflected on the dependence of S^- upon T [inset (a) and (b) of Fig. 3]. The curves shown in Fig. 3 illustrate the main trend of the results, concerning symmetry cancellations and thermal effects. The global symmetry of the Hamiltonian of Eq. (1), as a function of the coupling constant κ, was discussed in Refs. [16,17]. The suppression of the S^+ strength at $T=0$, for $\kappa \geqslant 0.06$ MeV is a consequence of the isospin symmetry restoration. The isospin violation which is inherent to mean field BCS calculations reflects upon the small values of the strength S^+, as compared to the values of the strength S^-, for values of κ which are smaller than the symmetry one ($\kappa \approx 0.06$ MeV). In consequence, for the present case, the nonvanishing values of S^+ for Fermi β^+ transitions from the ground state of ^{76}Ge is a consequence of the use of the BCS mean field, which moderates the complete suppression of the decay branch due to the Pauli principle. Figure 4 shows total values of the Ikeda sum rule, Eq. (20), as functions of the temperature T and of the coupling constant κ, without [inset (a)] and with [inset (b)] the inclusion of the scattering terms. It is evident that the inclusion of the scattering terms plays a crucial role in preserving the value of the sum rule at finite temperature and in the presence of particle particle correlations. Finally, in Fig. 5, we show the contributions of pair and scattering terms to the Ikeda sum rule. It is seen that the decrease of the pair contribution is balanced by the increase of the particle-hole ones. In the standard QRPA treatment of the interaction the decrease of the pair contributions to the sum rule cannot be avoided by the renormalization of the strength κ, which is itself a source of manifest violations of the sum rule.

IV. CONCLUSIONS

In this work we have presented an extended version of the QRPA equations which incorporates scattering terms in the definition of the QRPA phonons. The effect of these terms becomes particularly significant when finite temperature QRPA equations are solved. As an example of these effects we have calculated strength distributions and the Ikeda sum rule for single β-decay transitions of the Fermi type in the mass region $A=76$; i.e., $^{76}\mathrm{Ge}(0^+\mathrm{g.s.}) \rightarrow \beta^- \rightarrow {}^{76}\mathrm{As}(0^+\mathrm{exc})$. We found that the sum rule is preserve only if the $B^\dagger$ and B terms (scattering terms) of the operator and phonons are accounted for in the QRPA equations. The results reported above about the temperature dependence of the transition strength may be significant in the context of the calculation of β-decay rates in stellar conditions [1]. The results concerning the inclusion of scattering terms in the QRPA equation of motion, described in the previous sections, are in agreement with recently reported results by other authors [7]. Although the results presented in this paper have been obtained by using a schematic interaction they illustrate rather well the effects of small components of the wave functions upon the sum rules at finite temperatures. Work is in progress [18] concerning the use of this technique in the systematic calculation of single β-decay rates of astrophysical interest [1].

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