Seniority isomerism in proton-rich $N=82$ isotones and its indication to stiffness of the $Z=64$ core

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The 10^+ and $27/2^-$ isomers of the $Z>64$, $N=82$ nuclei are investigated in the shell-model framework. We derive an extended seniority reduction formula for the relevant $E2$ transition strengths. Based on the extended formula, as well as on the approximate degeneracy among the $0h_{11/2}$, $2s_{1/2}$, and $1d_{3/2}$ orbits, we argue that the $B(E2)$ data require the ^{146}Gd core excitation. The energy levels of both parities and the $B(E2)$ values are simultaneously reproduced by a multi-j shell-model calculation with the modified surface-delta interaction, if the excitations from $(0g_{7/2}1d_{5/2})$ to $(0h_{11/2}2s_{1/2}1d_{3/2})$ are taken into account.

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I. INTRODUCTION

Through recent experiments on unstable nuclei, it has been recognized that the nuclear magic numbers are not rigorous and somewhat depend on Z and N [1]. The magicity observed around the β-stable line may disappear in a region far from the stability. For instance, the magicity of $N=8$ no longer holds in the neutron-rich nucleus ^{11}Be. Although there has been no clear evidence, it is also of interest whether new magic numbers emerge in proton- or neutron-rich region. So-called submagic numbers such as $Z=40$ and $Z=64$ have been known, which have been distinguished from the magic numbers partly because their magicity disappears as Z or N changes. However, we now know that even the usual magic numbers depend more or less on Z or N. A question should be recast: what is the difference between magic numbers and submagic numbers? In this respect, it is worthwhile reinvestigating the stiffness of the subshell closure.

The ^{146}Gd nucleus shows several indications of the $Z=64$ subshell closure (e.g., relatively high excitation energy of 2_1^+) [2]. In the $Z>64$, $N=82$ isotones, high-spin isomers with $J^\pi=10^+$ (for even-Z nuclei) and $27/2^-$ (for odd-Z nuclei) have systematically been observed [3–6]. In connection to these isomers, the single-j shell model with the $\pi 0h_{11/2}$ orbit was successfully applied to the $Z>64$, $N=82$ isotones [7]. In the single-j shell model, the seniority reduction formula (SRF) is available for the $E2$ decay strengths of the high-spin isomers. The SRF had predicted a strong hindrance for the decay strengths of the isomers around $Z=70$, which is in coincidence with the measured $E2$ properties of the 10^+ and $27/2^-$ isomers. At a glance, this seems to indicate that the $Z=64$ subshell is stiff enough for ^{146}Gd to be treated as an inert core. On the other side, the stiffness of the $Z=64$ core has been argued so far. For instance, by analyzing the excitation energy of the 10^+ state in ^{146}Gd as well as those in the $Z>64$ isotones, it was insisted that significant pair excitation across $Z=64$ exists [8].

In this paper, we investigate the 10^+ and $27/2^-$ isomers in the $Z>64$, $N=82$ nuclei, primarily focusing on the stiffness of the $Z=64$ core. For the decay strengths of the isomers, we extend the SRF so that it could apply to the multi-j cases. This formula shows that the decay strengths reflect the stiffness of the $Z=64$ core. If the approximate degeneracy among the $0h_{11/2}$, $2s_{1/2}$, and $1d_{3/2}$ orbits is taken into consideration, the hindrance of the $E2$ strengths of the isomers turns out to indicate the presence of the pair excitation across $Z=64$.

II. SINGLE-j SHELL MODEL FOR $Z>64$, $N=82$ ISOTONES

The proton-rich $N=82$ isotones have been explored experimentally. After the discovery of the $Z=64$ submagic nature at ^{146}Gd [2], several low-lying levels have been established up to ^{154}Hf [9]. In this region, the excitation energies of the yrast states are nearly constant from nucleus to nucleus, both for even-Z (^{148}Dy, ^{150}Er, ^{152}Yb, and ^{154}Hf) and odd-Z (^{149}Ho, ^{151}Tm, and ^{153}Lu) isotones. Furthermore, high-spin isomers were observed systematically; 10^+ isomers for the even-Z isotones around $E_x\sim 3$ MeV, and $27/2^-$ isomers for the odd-Z isotones around $E_x\sim 2.5$ MeV.

Whereas state-of-the-art shell-model calculations with a realistic effective interaction have been applied to the $Z\leqslant 64$, $N=82$ isotones [10], there have not been many theoretical studies in the $Z>64$ region. Lawson carried out a single-j shell-model calculation with $\pi 0h_{11/2}$ on top of the ^{146}Gd core [7]. The residual interaction was empirically determined from the experimental energy levels of ^{148}Dy. The levels of the $Z\geqslant 66$ isotones were reproduced to a certain extent, apart from the odd-parity levels for the even-Z nuclei and the even-parity ones for the odd-Z nuclei, which are outside the model space. It is noted that, while the measured excitation energies of the 2^+, 8^+, and 10^+ states gradually decrease as Z increases, this tendency is not reproduced in the single-j model.

In the single-closed nuclei, it has been known that the seniority v is conserved to a good approximation. This is true also in Lawson's results. The 10^+ and $27/2^-$ isomers decay via the $E2$ transition. The 10^+ isomers and their daughters 8^+ have the seniority $v=2$, which is carried by the $(0h_{11/2})^2$ configuration. Similarly, the $27/2^-$ isomers and

their daughters $23/2^-$ have $v=3$. The $E2$ transition is usually described by a one-body operator. The seniority reduction formula (SRF) is well-known in the single-j configuration. By representing the $0h_{11/2}$ orbit by ξ, the SRF for the seniority-conserving $E2$ transitions gives [11]

$$\langle \xi^n v J_f^\pi || T(E2) || \xi^n v J_i^\pi \rangle = \frac{\Omega_\xi - n}{\Omega_\xi - v} \langle \xi^v v J_f^\pi || T(E2) || \xi^v v J_i^\pi \rangle, \tag{1}$$

where $\Omega_\xi = j_\xi + 1/2$. In the present case $j_\xi = 11/2$ and $\Omega_\xi = 6$. In Lawson's model the particle number n should be $Z-64$. Equation (1) shows hindrance of the $E2$ strengths by the factor $[(6-n)/(6-v)]^2$ when n deviates from v. This hindrance factor gives the parabola behavior of $B(E2)$ as a function of Z and leads to a remarkably long lifetime around $Z=70$, i.e., ^{152}Yb. This stabilization mechanism is called *seniority isomerism*.

The experimental data on the $B(E2)$ values of the isomers fit well the parabola in the $66 \leqslant Z \leqslant 70$ region. Furthermore, both $E2$ strengths of the 10^+ and the $27/2^-$ isomers are described by a single effective charge ($e_{\rm eff} \sim 1.5e$). In particular, the strong hindrance of the $E2$ transition is actually detected for ^{152}Yb, with $B(E2;10^+ \rightarrow 8^+) = 0.9 \pm 0.1\ e^2{\rm fm}^4$ [6]. In comparison with the data, the $E2$ strengths are overestimated for ^{153}Lu and ^{154}Hf in the single-j model to a certain extent, as will be shown later. This discrepancy should be attributed to an effect of the orbits other than $0h_{11/2}$.

III. EXTENSION OF THE SENIORITY REDUCTION FORMULA

Despite its success in predicting the seniority isomerism, the single-j model will be too simple to be realistic, since the $\pi 0h_{11/2}$ orbit is not isolated. In the odd-Z $N=82$ isotones, $1/2^+$ and $3/2^+$ states are present in the vicinity of the $11/2^-$ states, indicating the approximate degeneracy among the proton orbits $0h_{11/2}$, $2s_{1/2}$, and $1d_{3/2}$. A certain number of levels with opposite parities (odd-parity levels for the even-Z isotones and even-parity ones for the odd-Z isotones) are also observed in the low-energy regime, which cannot be described in the $\pi 0h_{11/2}$ single-j model. We shall reinvestigate the 10^+ and $27/2^-$ isomers in the multi-j shell-model framework.

There is no evidence for a breakdown of the neutron magic number $N=82$ in the low-energy region, except for a few states relevant to the octupole collectivity. We, hereafter, maintain the $N=82$ inert core. For the proton degrees of freedom, the $50<Z<82$ major shell is considered.

While the seniority isomerism in this region has been discussed based on the SRF for the single-j orbit $\pi 0h_{11/2}$, in the following we show that the formula (1) can be extended to the multi-j model space with a simple modification.

Let us define the seniority in the multi-j space by the sum of the seniorities of each orbit, $v = \Sigma_j v_j$. The seniority is expected to be a good quantum number in single-closed nuclei, at least for their low-lying states. In the high-spin isomers under interest, the seniority is carried only by the $\pi 0h_{11/2}$ orbit, to a good approximation. In the $50<Z<82$ major shell, $J^\pi = 10^+$ with $v=2$ is uniquely formed by the $(0h_{11/2})^2$ configuration, and $J^\pi = 27/2^-$ with $v=3$ by $(0h_{11/2})^3$. The decays of the isomers occur via the $E2$ transition without changing the seniority. Within this major shell, the 8^+ state with $v=2$, the final state of the 10^+ decay, also has the $(0h_{11/2})^2$ configuration. The $23/2^-$ state having $v=3$, the daughter of the $27/2^-$ decay, is predominantly $(0h_{11/2})^3$. Although this state may have an admixture of $(0g_{7/2})^2(0h_{11/2})^1$ and $(0g_{7/2})^1(1d_{5/2})^1(0h_{11/2})^1$, the admixture will be small, because these configurations need excitation across $Z=64$ by two protons. Moreover, the remaining part consisting of 0^+ pairs is expected to have almost identical structure between the isomers and their daughter states. This is in accordance with the spherical BCS [12] or Talmi's generalized-seniority picture [13], where quasiparticles are defined on top of the coherent 0^+ pairs distributing over the valence orbits. Keeping this situation in mind, we derive an extended formula in a somewhat general manner.

Suppose that (a) the seniority is a good quantum number, (b) for a seniority-conserving $E2$ transition, the seniority is carried by a single orbit (labeled by ξ) both for the initial and the final states of the transition, and (c) the wave functions of the paired particles are identical between the two states. Condition (c) will be given below in more definitive manner. We represent all valence orbits other than ξ by r. In the present $N=82$ case, $\xi = \pi 0h_{11/2}$ and $r = \pi(0g_{7/2}1d_{5/2}1d_{3/2}2s_{1/2})$. The shell-model bases are decomposed into the product of the ξ^{n_ξ} and r^{n_r} configurations, where the valence particle number is given by $n = n_\xi + n_r$. Because of (b), the seniorities of the ξ and r subspaces are $v_\xi = v$ and $v_r = 0$, respectively. The initial and final states are expanded as

$$|(\xi r)^n v J_i^\pi\rangle = \sum_{n_\xi (\geqslant v), \alpha} c_{n_\xi \alpha} |\xi^{n_\xi} v_\xi = v J_i^\pi\rangle |r^{n-n_\xi} \alpha v_r = 0 0^+\rangle, \tag{2}$$

and

$$|(\xi r)^n v J_f^\pi\rangle = \sum_{n_\xi (\geqslant v), \alpha} c_{n_\xi \alpha} |\xi^{n_\xi} v_\xi = v J_f^\pi\rangle |r^{n-n_\xi} \alpha v_r = 0 0^+\rangle. \tag{3}$$

Here α represents composition of the 0^+ pairs within the r^{n-n_ξ} configuration. For instance, α distinguishes $(0g_{7/2}1d_{5/2})^{14}(1d_{3/2})^2(2s_{1/2})^2$ from $(0g_{7/2}1d_{5/2})^{14}(1d_{3/2})^4$. The condition (c) is defined as the expansion coefficients ($c_{n_\xi \alpha}$) being equal between $|J_i^\pi\rangle$ and $|J_f^\pi\rangle$. For zero-range interactions like the surface-delta interaction (SDI), this condition results from (a) and (b), as is verified in the Appendix. The normalization yields

$$\sum_{n_\xi, \alpha} c_{n_\xi \alpha}^2 = 1. \tag{4}$$

Because of condition (c), the occupation number on the orbit ξ is equal between the initial and the final states:

$$\langle N_\xi\rangle=\sum_{n_\xi,\alpha} c^2_{n_\xi\alpha} n_\xi, \tag{5}$$

where N_ξ stands for the number operator on ξ.

Since the r subspace carries no seniority under condition (b), the $E2$ transition is forbidden within this subspace. Namely, in the seniority-conserving $E2$ transition, the r subspace behaves as a spectator. The $E2$ matrix element is then written as

$$\langle(\xi r)^n v J_f^\pi||T(E2)||(\xi r)^n v J_i^\pi\rangle = \sum_{n_\xi,\alpha} c^2_{n_\xi\alpha}\langle \xi^{n_\xi} v_\xi = v J_f^\pi||T(E2)||\xi^{n_\xi} v_\xi = v J_i^\pi\rangle. \tag{6}$$

This $E2$ transition is a noncollective one, contributed only by the ξ orbit. Substitution of the SRF for the orbit ξ [see Eq. (1)] into the right-hand side (rhs) yields

$$\langle(\xi r)^n v J_f^\pi||T(E2)||(\xi r)^n v J_i^\pi\rangle = \sum_{n_\xi,\alpha} c^2_{n_\xi\alpha}\frac{\Omega_\xi-n_\xi}{\Omega_\xi-v}\langle \xi^v v J_f^\pi||T(E2)||\xi^v v J_i^\pi\rangle, \tag{7}$$

with $\Omega_\xi\equiv j_\xi+1/2$. Because of Eqs. (4) and (5), we finally obtain

$$\langle(\xi r)^n v J_f^\pi||T(E2)||(\xi r)^n v J_i^\pi\rangle = \frac{\Omega_\xi-\langle N_\xi\rangle}{\Omega_\xi-v}\langle \xi^v v J_f^\pi||T(E2)||\xi^v v J_i^\pi\rangle. \tag{8}$$

Equation (8) links the $E2$ matrix element to $\langle N_\xi\rangle$, occupation number on the orbit ξ. If the effective charge parameter in $T(E2)$ is fixed in advance, the $E2$ matrix element is determined only from $\langle N_\xi\rangle$. Conversely, $\langle N_\xi\rangle$ can be extracted from the $E2$ matrix element. What determines $\langle N_\xi\rangle$ is $c_{n_\xi\alpha}$, which represents the configuration mixing due to the pairing correlation. Thus the $E2$ strengths of the isomers are a pairing property, sensitive to the mixing via the pairing interaction.

Compare formula (8) to the SRF for the single-j orbit (1). Although the multi-j matrix element is under discussion, the only difference in the rhs is that the particle number n is replaced by the expectation value $\langle N_\xi\rangle$. We call Eq. (8) *extended seniority reduction formula* (ExSRF). The hindrance of the transition strength occurs via the factor $[(\Omega_\xi-\langle N_\xi\rangle)/(\Omega_\xi-v)]^2$, in parallel to the argument in the single-j case, and extraordinarily long lifetime is expected if $\langle N_\xi\rangle\simeq\Omega_\xi$. The ExSRF (8) obviously contains the single-j formula (1) as a limiting case. Equation (8) reduces to Eq. (1) if $c_{n_\xi\alpha}=1$ for $n_\xi=n$ and 0 for the others. Still the difference from the single-j case should be remarked. Even when the seniority is conserved, there could be configuration mixing due to the pairing correlations. While the SRF (1) in the single-j model requires that any mixing should be negligible, the ExSRF (8) holds with the pairing mixing. The present conditions for the $E2$ hindrance are thereby much more realistic than in the single-j case, and the hindrance due to $\langle N_\xi\rangle$ may be found in a variety of the single-closed nuclei and their neighbors. We call this mechanism *extended seniority isomerism*.

Blomqvist suggested, without proof, that n in the SRF (1) can be reinterpreted as the occupation number [14]. Discussion based on the BCS approximation was given in Ref. [15]. The BCS argument leads to the factor $(u_\xi^2-v_\xi^2)$ in terms of the u and v coefficients [16], which is proportional to $(\Omega_\xi-\langle N_\xi\rangle)$. However, the degree of the approximation was not clear enough. The BCS approximation presumes coherent pairing and ignores some dependence on the seniority [e.g., the seniority dependence in the denominator of Eq. (8)]. On the other hand, we have derived the ExSRF in more rigorous and general manner, which is exact as far as the conditions (a)–(c) are satisfied.

We here comment on the relation of the ExSRF (8) to the multi-j quasispin (QS) formula for the degenerate single-particle orbits [17]. The multi-j QS formula is available when the pair distributes over all the valence orbits with equal amplitudes. We then have

$$\frac{n_\xi-v}{n-v}=\frac{\Omega_\xi-v}{\Omega-v}, \tag{9}$$

where we use the notation

$$\Omega=\sum_{j\in(\xi,r)}\Omega_j=\sum_{j\in(\xi,r)}(j+1/2). \tag{10}$$

By employing Eqs. (4) and (9), Eq. (7) reduces to

$$\langle(\xi r)^n v J_f^\pi||T(E2)||(\xi r)^n v J_i^\pi\rangle = \frac{\Omega-n}{\Omega-v}\langle \xi^v v J_f^\pi||T(E2)||\xi^v v J_i^\pi\rangle. \tag{11}$$

Because of condition (b), Eq. (11) is equivalent to the multi-j QS formula

$$\langle(\xi r)^n v J_f^\pi||T(E2)||(\xi r)^n v J_i^\pi\rangle = \frac{\Omega-n}{\Omega-v}\langle(\xi r)^v v J_f^\pi||T(E2)||(\xi r)^v v J_i^\pi\rangle. \tag{12}$$

We now return to the case of the $N=82$ isotones. Since $\xi=0h_{11/2}$ (thereby $\Omega_\xi=6$), the 10^+ or $27/2^-$ isomer has remarkably long life for a nucleus satisfying $\langle N_{0h_{11/2}}\rangle\simeq 6$. Namely, the observed long lifetime of the 10^+ isomer in ^{152}Yb implies $\langle N_{0h_{11/2}}\rangle\simeq 6$. As long as $2s_{1/2}$ and $1d_{3/2}$ lie closely to $0h_{11/2}$, there should be mixing among these orbits due to the pairing interaction, causing a decrease of $\langle N_{0h_{11/2}}\rangle$. However, it can be compensated by the excitation from $0g_{7/2}$ or $1d_{5/2}$ to $0h_{11/2}$, which increases $\langle N_{0h_{11/2}}\rangle$. As we discuss in the following sections, this should be what happens in the isomers in the $Z>64$, $N=82$ isotones.

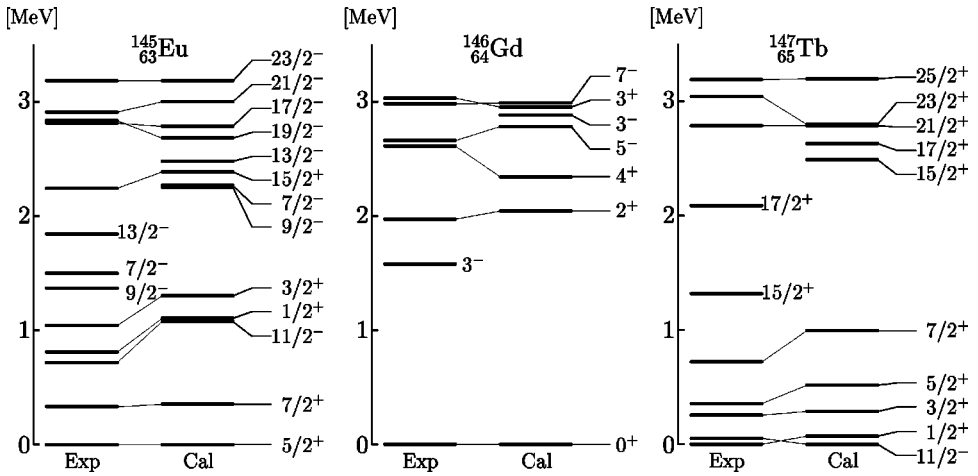

FIG. 1. Comparison of the observed and calculated energy levels for ^{145}Eu, ^{146}Gd, and ^{147}Tb. The levels of ^{146}Gd and the lowest five levels of ^{145}Eu and ^{147}Tb are employed to fit the parameters in the shell-model Hamiltonian. The experimental data are taken from Ref. [9].

IV. MULTI-j SHELL MODEL FOR $Z \gtrsim 64$, $N=82$ ISOTONES

A. Model space

In this section, we present how the properties of the high-spin isomers are described by a calculation in the multi-j shell-model framework. As discussed in the preceding section, the model space should include all the five orbits in the $50<Z<82$ major shell. Large-scale shell-model calculations were carried out for the $62 \leqslant Z \leqslant 65$, $N=82$ isotones with moderate truncation [18], as well as for the $Z \leqslant 64$, $N=82$ isotones in the full major shell [10]. On the other hand, our main purpose is to illustrate the extended seniority isomerism in the $Z>64$, $N=82$ isotones. In order to avoid time-consuming computations, we adopt a relatively small space by truncation.

The space for diagonalization is truncated as follows. Partially maintaining the $Z=64$ subshell structure, we restrict the excitation out of the $0g_{7/2}$ and $1d_{5/2}$ orbits to four particles. Furthermore, the total seniority is limited to $v \leqslant 3$ ($v \leqslant 2$) for the odd-Z (even-Z) nuclei. The seniorities of the 10^+ and the $27/2^-$ states are pure in this space as well as those of their decay daughters; condition (a) in Sec. III is satisfied. Condition (b) is exact for the 10^+ decay, while the final state $23/2^-$ of the $27/2^-$ decay has a small admixture of the $(0g_{7/2})^2(0h_{11/2})^1$ and $(0g_{7/2})^1(1d_{5/2})^1(0h_{11/2})^1$ configurations, as stated in Sec. III.

B. Energy levels

The shell-model Hamiltonian is written as

$$H=E_0+\sum_j \epsilon_j N_j+V. \tag{13}$$

Here E_0 is a constant shifting the origin of the energy, ϵ_j represents the single-particle energy of the orbit j, and N_j the number operator on j. The residual two-body interaction is denoted by V, for which we adopt the modified surface-delta interaction (MSDI),

$$V=-4\pi A_{T=1}\sum_\lambda Y^{(\lambda)}(\hat{\mathbf{r}}_1)\cdot Y^{(\lambda)}(\hat{\mathbf{r}}_2)+B. \tag{14}$$

There are eight parameters in the Hamiltonian, E_0, ϵ_j for the five orbits, $A_{T=1}$ and B. They can be classified into two groups. One is comprised of the differences of ϵ_j's (four parameters) and $A_{T=1}$. These five parameters are relevant to the excitation spectra for an individual nucleus. The other consists of E_0, B, and overall shift of ϵ_j's. They do not change excitation spectra, but affect the gross behavior of the binding energies. It is noticed that effects of the Coulomb repulsion between protons are principally contained in B. As is proven in the Appendix, the ExSRF (8) becomes exact for the 10^+ decay with the present seniority-truncated model space and the interaction.

In describing the extended seniority isomerism, it is important to reproduce the degree of the pair excitation out of the $Z=64$ core. In ^{147}Tb and ^{149}Ho, a $5/2^+$ and a $7/2^+$ level have been observed at very low energies ($E_x \lesssim 1$ MeV) [9]. These levels could be another manifestation of the core excitation. It is hard to reproduce these levels without including the $0g_{7/2}$ and $1d_{5/2}$ orbits. Analogously, ^{145}Eu has low-lying ($E_x \lesssim 1$ MeV) states with $11/2^-$, $1/2^+$, and $3/2^+$. The coupling constant $A_{T=1}$ and the ϵ_j differences are determined so as to reproduce the lowest levels of $E_x \lesssim 1$ MeV in ^{145}Eu and ^{147}Tb, as well as the $E_x \lesssim 3$ MeV low-lying levels of ^{146}Gd. The adopted value of $A_{T=1}$ is 0.210 MeV. The results of the fitting are depicted in Fig. 1, together with several higher-lying levels, in comparison with the experimental data.

There are a few levels which are not described by the calculation. The 3^- state of ^{146}Gd has been interpreted as an octupole collective mode including the neutron excitations [3]. Therefore, this state has been excluded from the fitting. The $9/2^-$, $7/2^-$, and $13/2^-$ states of ^{145}Eu are considered to be $\pi d_{5/2}^{-1}\otimes 3^-$ or $\pi g_{7/2}^{-1}\otimes 3^-$ [10]. Since they involve the octupole collective excitation, these states are beyond the model space in the present calculation as well. The $15/2^+$ and $17/2^+$ states of ^{147}Tb are also regarded as $\pi h_{11/2}\otimes 3^-$.

The remaining parameters, E_0, B, and the constant shift of the single-particle energies, are fixed from the binding energies of the $63 \leqslant Z \leqslant 74$, $N=82$ isotones [19]. We obtain $E_0=90.10$ MeV and $B=0.409$ MeV, representing the energies by relative values to the experimental ground-state energy of ^{146}Gd. The resultant single-particle energies are

TABLE I. Adopted values of the single-particle energies.

j	$0g_{7/2}$	$1d_{5/2}$	$0h_{11/2}$	$2s_{1/2}$	$1d_{3/2}$
ϵ_j (MeV)	-8.33	-7.73	-6.88	-6.73	-6.43

listed in Table I. The calculated binding energies are compared with the data in Fig. 2. We have sufficiently good agreement, with the largest discrepancy of 0.45 MeV for ^{145}Eu.

The ground-state wave function of ^{146}Gd holds the $Z=64$ closure only by 11% in this calculation. The core is broken due to the pairing correlation, with keeping the seniority a good quantum number. Having 53% excitation of a single pair and 36% of two pairs, the average number of protons excited out of the $Z=64$ core amounts to 2.5. This result is barely influenced even if we relax the seniority truncation to $v\leqslant 4$. As was pointed out in Ref. [8], the $Z=64$ core is broken to a sizable extent by the pair excitation.

We carry out a shell-model calculation with the above Hamiltonian for the $66\leqslant Z\leqslant 72$, $N=82$ nuclei. The calculated energy levels for the even-Z nuclei are compared with the observed ones [9] in Figs. 3 (for even-parity levels) and 4 (for odd-parity levels), up to $E_x\simeq 3$ MeV. Almost all levels in this energy range are in reasonably good agreement. Among them, the $E_x(2^+)$ values are somewhat higher than the data. This discrepancy seems mainly concerned with the quadrupole collectivity, and could be ascribed to the truncated model space or to the interaction which might be too simple. On the other hand, the $E2$ decay of the isomers has noncollective character, occurring via the transition within $0h_{11/2}$. Therefore, it is not quite relevant to the quadrupole collectivity. As presented in Fig. 4, the odd-parity levels are also reproduced, except for the octupole collective state 3^-, which is not shown in the figure. This is an obvious advantage over against the previous single-j calculation, since the odd-parity levels are out of the model space in the single-j calculation. As mentioned in Sec. II, the excitation energies of 2^+, 8^+, and 10^+ states slightly decrease as Z goes up. This behavior is well reproduced by the present calculation, while the energies slightly increase in the single-j model.

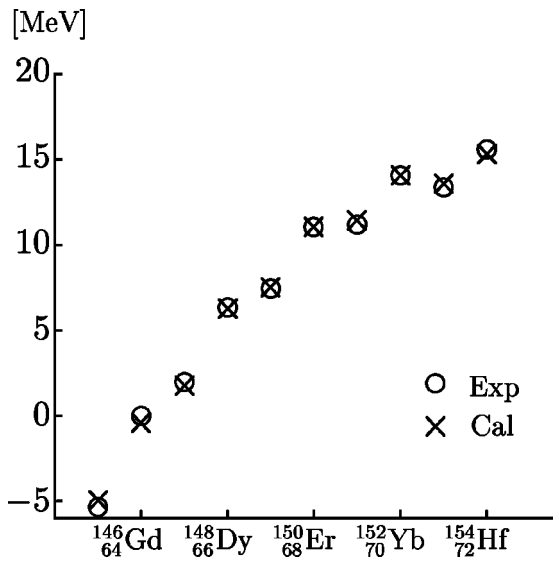

FIG. 2. Binding energies for $63\leqslant Z\leqslant 74$, $N=82$ isotones. All values are plotted relative to the experimental binding energy at ^{146}Gd. The data are taken from Ref. [19].

In Figs. 5 and 6, the calculated yrast levels are compared with the experimental data for the odd-Z nuclei, up to $E_x\simeq 3$ MeV. The energies relative to the $11/2^-$ state are presented both for the data and the calculated results. The agreement is sufficiently good, as in the even-Z nuclei. The even-parity states, which are beyond the space in the single-j model with $0h_{11/2}$, are also reproduced (Fig. 6). In all of the calculated levels presented in the figures, the seniority is conserved to an excellent extent. The $11/2^-$, $1/2^+$, $3/2^+$, $5/2^+$, and $7/2^+$ states lying in $E_x\lesssim 1$ MeV have $v=1$, while the others have $v=3$. It should be remarked that the $5/2^+$ and $7/2^+$ levels, the $1d_{5/2}$ and $0g_{7/2}$ states with the pair excitation, are also reproduced well, in ^{149}Ho and ^{151}Tm. The intruder level $15/2^+$ is not shown in the figure, which should be an octupole collective state with $\pi 0h_{11/2}\otimes 3^-$.

C. $E2$ strengths of the high-spin isomers

Let us turn to the $E2$ transition strengths of the high-spin isomers. The $E2$ operator is given by

$$T(E2)=e_{\rm eff}\sum_{j,j'}\frac{1}{\sqrt{5}}\langle j'||r^2Y^{(2)}(\hat{\mathbf{r}})||j\rangle[a_{j'}^{\dagger},\tilde{a}_j]^{(2)}, \quad (15)$$

where $\tilde{a}_{jm}=(-)^{j+m}a_{j-m}$. The single-particle matrix element $\langle j'||r^2Y^{(2)}(\hat{\mathbf{r}})||j\rangle$ is evaluated by using the harmonic oscillator single-particle wave functions with the oscillator parameter $\nu(=1/b^2)=M\omega/\hbar=0.98A^{-1/3}$ fm^{-2}.

It should be noticed that, in the $E2$ calculation, there remains only a single adjustable parameter $e_{\rm eff}$, the effective charge. It is found that $e_{\rm eff}=2.3e$ fits well to all of the 10^+ and $27/2^-$ decays. This value is significantly larger than the effective charge of $1.5e$ which was adopted in the single-j calculation [7]. This is in contrast to the collective transitions, where $e_{\rm eff}$ should be smaller as the model space is extended, since the matrix elements of $T(E2)/e_{\rm eff}$ tend to increase. For the $E2$ transitions of the isomers, which do not have collective character, the matrix elements of $T(E2)/e_{\rm eff}$ are smaller in the multi-j case than in the single-j case at ^{148}Dy and ^{149}Ho, as is recognized from Eq. (8).

While the effective charge of $1.4e$ was recommended in realistic calculations in the $Z<64$ region [10], several calculations in the $Z\geqslant 64$ region assumed $e_{\rm eff}=(2.0\sim 2.25)e$ [18,20]. The origin of the difference in the effective charge between $Z<64$ and $Z\geqslant 64$ is not clear, because in both cases the model space consists of all the five orbits in the $50<Z<82$ shell, and the orbital dependence of the effective charge is normally weak [21]. We just point out that our value seems consistent with those of the previous studies in the $Z\geqslant 64$ region.

In Fig. 7, we show the $B(E2;10^+\rightarrow 8^+)$ values for the $66\leqslant Z\leqslant 72$, $N=82$ isotones. The calculated values are compared with the measured ones, as well as with those obtained

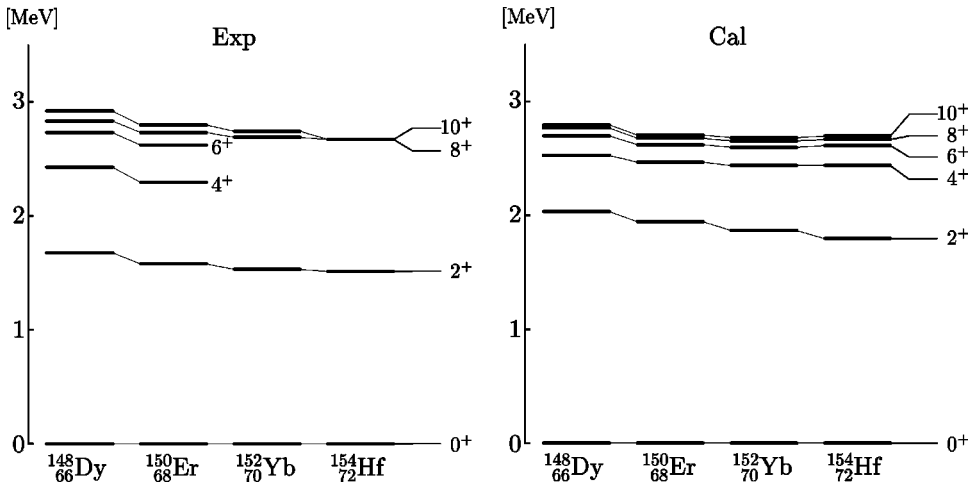

FIG. 3. Comparison between the experimental and calculated even-parity energy levels for even-Z, $N=82$ nuclei. The experimental data are taken from Ref. [9].

in the single-j calculation by Lawson [7]. The $E2$ hindrance at $Z=70$ (i.e., ^{152}Yb) occurs also in the present multi-j calculation. Our calculation gives $B(E2)=0.6\ e^2\mathrm{fm}^4$, in good agreement with the data $0.9\pm0.1\ e^2\mathrm{fm}^4$ [6].

As has been shown by the ExSRF (8), the $E2$ strengths of the 10^+ states are essentially determined from the occupation number $\langle N_{0h_{11/2}}\rangle$. In the present calculation, the wave functions of the 10^+ state and of 8^+ yield $\langle N_{0h_{11/2}}\rangle=5.7$ in ^{152}Yb. This occupation number close to $\Omega_{0h_{11/2}}=6$ gives rise to the strong $E2$ hindrance. We view this hindrance from another standpoint. See Eq. (6), recalling $\xi=0h_{11/2}$. By decomposing the wave functions as in Eqs. (2) and (3), we look into the contribution of each n_ξ component. Table II illustrates $\Sigma_\alpha c^2_{n_\xi\alpha}$, $\langle \xi^{n_\xi} v_\xi=v J_f^\pi||T(E2)||\xi^{n_\xi} v_\xi=v J_i^\pi\rangle$ and their product, for each $n_\xi(=2,4,6,8,10)$. As the SRF tells us, the matrix element $\langle \xi^{n_\xi} v_\xi=v J_f^\pi||T(E2)||\xi^{n_\xi} v_\xi=v J_i^\pi\rangle$ changes its sign at $n_\xi=6$, where it vanishes. The coefficient $\Sigma_\alpha c^2_{n_\xi\alpha}$ has the same sign and the same order of magnitude between n_ξ and $12-n_\xi$, causing a large cancellation. As a result, the $E2$ strength is significantly hindered for ^{152}Yb. Although the single-j picture discussed by Lawson [7] does not apply anymore, this mechanism, $\langle N_{0h_{11/2}}\rangle\simeq 6$ or in other words the cancellation of the matrix elements, explains why the $E2$ hindrance occurs in ^{152}Yb. Thus the 10^+ state of ^{152}Yb yields a typical example of the extended seniority isomerism.

The $E2$ strengths in the other even-Z isotones are also in remarkably good agreement with the data. We clearly view improvement over the single-j model in ^{154}Hf.

As is viewed in Fig. 3, the 4^+ and 6^+ states have not yet been detected in ^{152}Yb and ^{154}Hf. The ExSRF (8) approximately applies also to the $8^+\rightarrow 6^+$ and $6^+\rightarrow 4^+$ $E2$ transitions. These transitions are hindered by the same mechanism as in the $10^+\rightarrow 8^+$ transition. Hence it is not easy to populate the 4^+ and 6^+ states in the experiments.

The $E2$ strengths of the $27/2^-$ states are shown in Fig. 8, for the $67\leqslant Z\leqslant 71$, $N=82$ isotones. As in the 10^+ isomers in the even-Z isotones, the present calculation reproduces the measured values remarkably well. The hindrance at $Z=71$ (i.e., ^{153}Lu), which was not described well in the single-j model, is reproduced. In light of the ExSRF, this hindrance occurs because of $\langle N_{0h_{11/2}}\rangle=6.2$ in ^{153}Lu. It should be emphasized that the $E2$ properties of the isomers are naturally reproduced, by adjusting the energies relevant to the excitation out of the $Z=64$ core.

Figure 9 depicts the occupation number $\langle N_{0h_{11/2}}\rangle$ in the 10^+ or $27/2^-$ isomers, which corresponds to their $E2$ decay strengths via the ExSRF. We view the almost linear increase

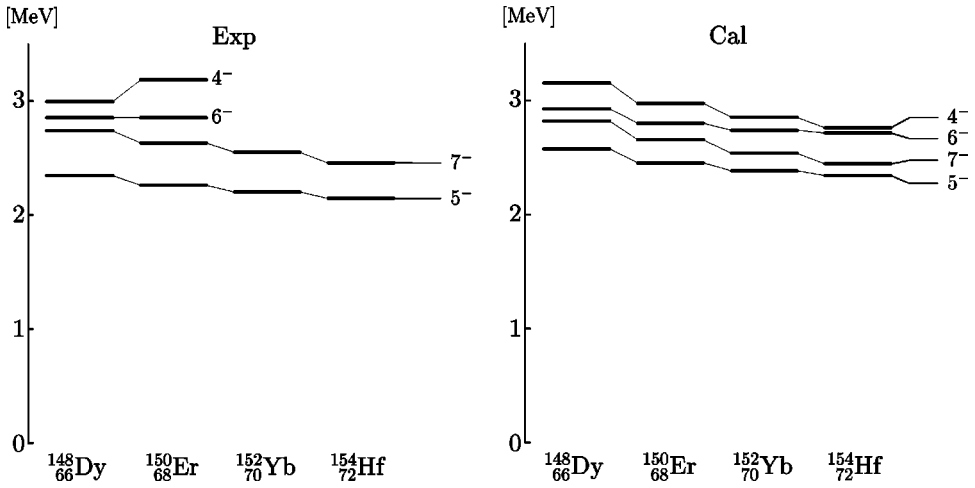

FIG. 4. Comparison between the experimental and calculated odd-parity energy levels for even-Z, $N=82$ nuclei. The experimental data are taken from Ref. [9].

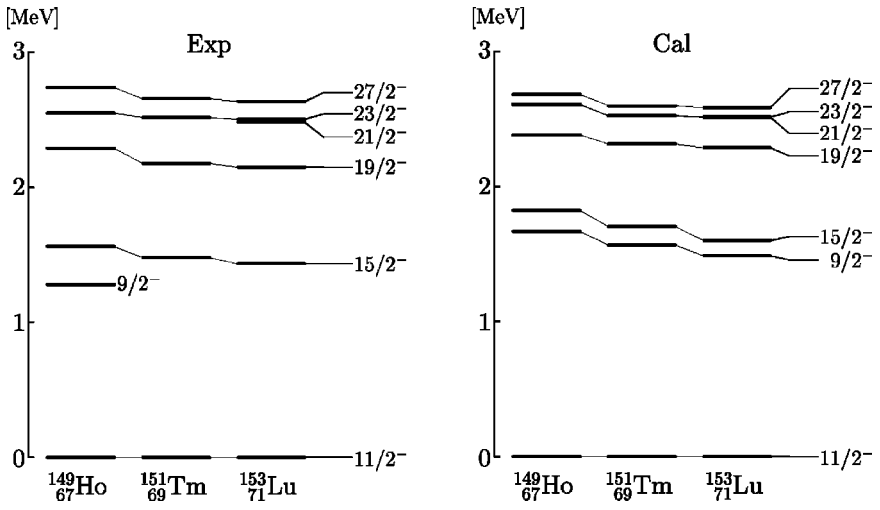

FIG. 5. Comparison between the experimental and calculated odd-parity energy levels relative to $11/2^-$, for odd-Z, $N=82$ nuclei. The experimental data are taken from Ref. [9].

of $\langle N_{0h_{11/2}}\rangle$ in the isomers, in coincidence with the schematic illustration by Blomqvist (Fig. 3-2 of Ref. [14]). The number of the particles excited out of the $Z=64$ core $\langle N_{\rm exc}\rangle \equiv 14 - (\langle N_{0g_{7/2}}\rangle + \langle N_{1d_{5/2}}\rangle)$ in the isomers is plotted as well, in the right panel of Fig. 9. It is found that the number of excited particles diminishes only gradually, as Z increases.

V. DISCUSSION — NECESSITY OF $Z=64$ CORE EXCITATION

The ExSRF derived in Sec. III accounts for the seniority mechanism to hinder the $E2$ decay of a certain class of isomers. In Sec. IV, we have demonstrated that the strong $E2$ hindrance in ^{152}Yb is reproduced by taking the $Z=64$ core excitation into account. However, the ExSRF itself does not exclude the possibility of the single-j solution to the hindrance for ^{152}Yb. In this section we argue that the $E2$ properties of the isomers exclusively indicate the presence of the excitation across $Z=64$.

According to the ExSRF (8), the vanishing $E2$ strength at ^{152}Yb indicates $\langle N_{0h_{11/2}}\rangle \simeq 6$. In respect to the stiffness of the $Z=64$ core, the following two possibilities result: (i) $\pi 0h_{11/2}$ couples to the surrounding orbits very weakly and the single-j picture holds to a good approximation, or (ii) the pair excitation across $Z=64$ compensates the pairing mixing of $0h_{11/2}$ with $(2s_{1/2}1d_{3/2})$, the possibility first suggested by Blomqvist [14]. We have shown in Sec. IV that (ii) is plausible, by reproducing the energy levels and the $E2$ strengths simultaneously. We here discuss whether (i) is possible or not.

For this purpose we consider the $E2$ strength of ^{152}Yb in the $3j$ model of $0h_{11/2}$, $2s_{1/2}$, and $1d_{3/2}$, keeping the $Z=64$ closure. The major point will be the amount of mixing of $0h_{11/2}$ with the surrounding orbits $2s_{1/2}$ and $1d_{3/2}$ due to the pairing interaction. The possibility (i) requires that the mixing should be negligibly small. The valence particle number n is 6 for ^{152}Yb in the $3j$ model. In order for the strong hindrance to be reproduced, the wave function of ^{152}Yb should have $n_\xi=6$ ($\xi=0h_{11/2}$) as the main component, with a small admixture of the $n_\xi=4$ component. By using the effective charge of Ref. [7], the measured $B(E2)$ leads to the admixture of the $n_\xi=4$ component by no greater than 10%.

For the sake of simplicity, let us first consider mixing between two configurations. In reality, this mixing could be either of the $0h_{11/2}$-$2s_{1/2}$ or the $0h_{11/2}$-$1d_{3/2}$ pairing mixing.

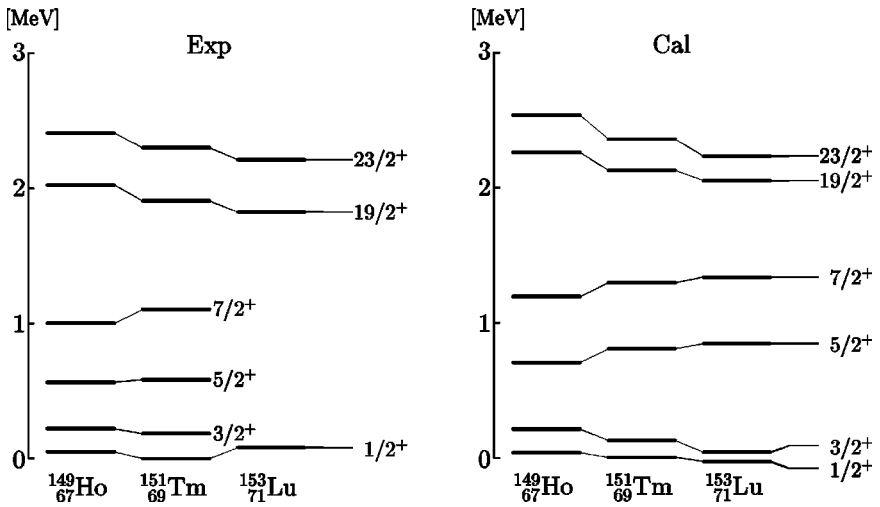

FIG. 6. Comparison between the experimental and calculated even-parity energy levels relative to $11/2^-$, for odd-Z, $N=82$ nuclei. The experimental data are taken from Ref. [9].

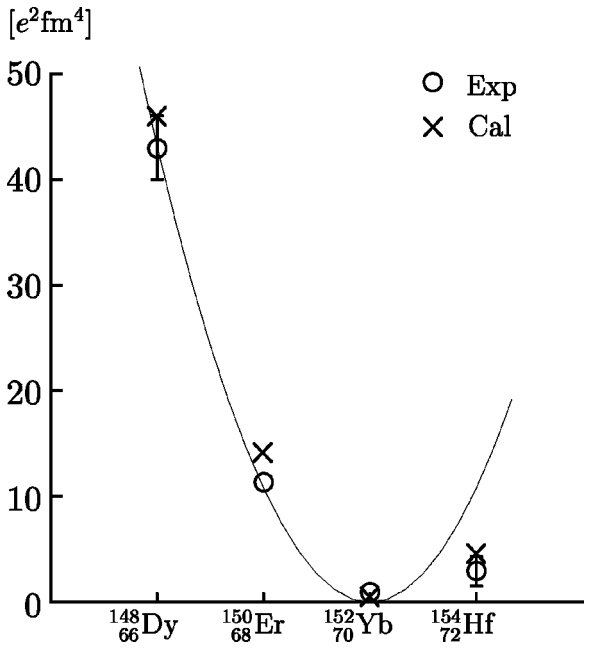

FIG. 7. $B(E2;10^+\rightarrow 8^+)$ for even-Z, $N=82$ nuclei. The crosses show the results of the present calculation, while the thin solid line those of the $\pi 0h_{11/2}$ single-j calculation of Ref. [7]. The circles stand for the experimental data taken from Ref. [3] for ^{148}Dy, Ref. [5] for ^{150}Er, and Ref. [6] for ^{152}Yb and ^{154}Hf.

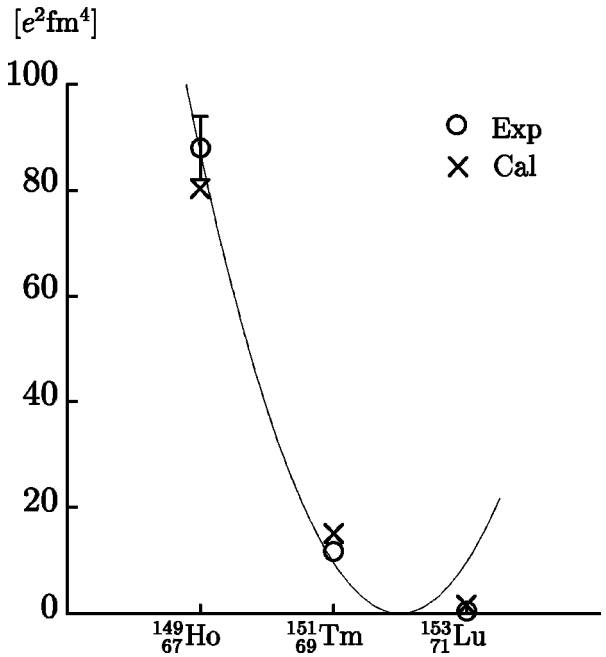

FIG. 8. $B(E2;27/2^-\rightarrow 23/2^-)$ for odd-Z, $N=82$ nuclei. See Fig. 7 for symbols. The experimental data are taken from Ref. [5] for ^{149}Ho and ^{151}Tm, and from Ref. [6] for ^{153}Lu.

The degree of the mixing is connected to the ratio of the off-diagonal matrix elements of the pairing interaction (denoted by $\langle V_{\rm pair}^{\rm off}\rangle$) to the energy difference of the relevant orbits (denoted by ΔE). The mixing probability is given by $\langle V_{\rm pair}^{\rm off}\rangle^2/[(2\Delta E)^2+\langle V_{\rm pair}^{\rm off}\rangle^2]$. The above 10% mixing indicates $\langle V_{\rm pair}^{\rm off}\rangle/2\Delta E=0.45$. If the mixing among the three orbits is considered, this ratio should be regarded as the upper limit for each of the $0h_{11/2}$-$2s_{1/2}$ and the $0h_{11/2}$-$1d_{3/2}$ mixing. The level scheme of the odd-Z isotones ^{147}Tb, ^{149}Ho, and ^{151}Lu implies that the three orbits keep nearly degenerate within the 0.2 MeV accuracy in this region. Hence we can set $\Delta E<0.2$ MeV. We thus find that, in order for the possibility (i) to be realized, $\langle V_{\rm pair}^{\rm off}\rangle<0.18$ MeV is necessary.

Generally speaking, interactions with the shorter range yield the larger off-diagonal pairing matrix elements. We estimate $\langle V_{\rm pair}^{\rm off}\rangle$ in the SDI and in the Yukawa form with the range of the one-pion exchange, as representatives of short-range and long-range interactions. After fitting their strengths to the observed energy levels in the $3j$ model, the SDI and the Yukawa interaction give $\langle V_{\rm pair}^{\rm off}\rangle\sim 1$ and 0.5 MeV, respectively [22]. As a consequence of the weak coupling between $0h_{11/2}$ and $(2s_{1/2}1d_{3/2})$, the long-range interaction gives very low 0_2^+ states, low enough to be first excited states, in ^{148}Dy and ^{150}Er. Such 0_2^+ levels have not been observed. Nevertheless, even the long-range interaction gives significantly larger $\langle V_{\rm pair}^{\rm off}\rangle$ than required in (i). It is practically impossible to avoid a considerable mixing of $(2s_{1/2}1d_{3/2})$ due to the pairing interaction. Indeed, in both calculations with the SDI and the Yukawa interaction in the $3j$ model space, the three orbits mix well with one another via the pairing interaction. Thereby the $E2$ strengths of the high-spin isomers are almost described by formula (11), the QS formula for the degenerate single-particle orbits, with $\Omega=9$. This is obviously inconsistent with the measurement.

We thus conclude that the possibility (i) cannot be realistic. As has been discussed in Sec. III, in order to reproduce the $E2$ hindrance for ^{152}Yb the admixture of the $2s_{1/2}$ and $1d_{3/2}$ orbits must be compensated by the $Z=64$ core excitation. Therefore, with the near degeneracy between $0h_{11/2}$ and $(2s_{1/2}1d_{3/2})$ taken into consideration, the $E2$ properties of the $N=82$ isomers are exclusive evidence for the presence of the excitation across $Z=64$, not indicating the stiff $Z=64$ core. The seniority isomerism in this region is a probe sensitive to the $Z=64$ core excitation due to the pairing correlations. It is difficult to handle the influence of the $Z=64$ core breaking on the $E2$ properties of the isomers by renormalization. The presence of substantial pair excitation is a clear difference of the submagic number $Z=64$ from the ordinary magic numbers.

It is commented here that, in contrast to the SDI adopted in Sec. IV, Wenes *et al.* applied a finite-range interaction

TABLE II. Contribution of each n_ξ component ($\xi=0h_{11/2}$) to the $E2$ matrix element of the $10^+\rightarrow 8^+$ transition in ^{152}Yb [see text and Eq. (6)]. The third row shows the matrix element $\langle \xi^{n_\xi};8^+||T(E2)||\xi^{n_\xi};10^+\rangle/e_{\rm eff}$, which is evaluated by using the harmonic-oscillator single-particle wave functions with $\nu=0.18$ fm^{-2}.

n_ξ	2	4	6	8	10	Sum
$\Sigma_\alpha c^2_{n_\xi\alpha}$	0.028	0.282	0.498	0.181	0.010	1.00
Matrix element (fm^2)	21.9	11.0	0	−11.0	−21.9	
Product (fm^2)	0.614	3.09	0	−1.98	−0.219	1.50

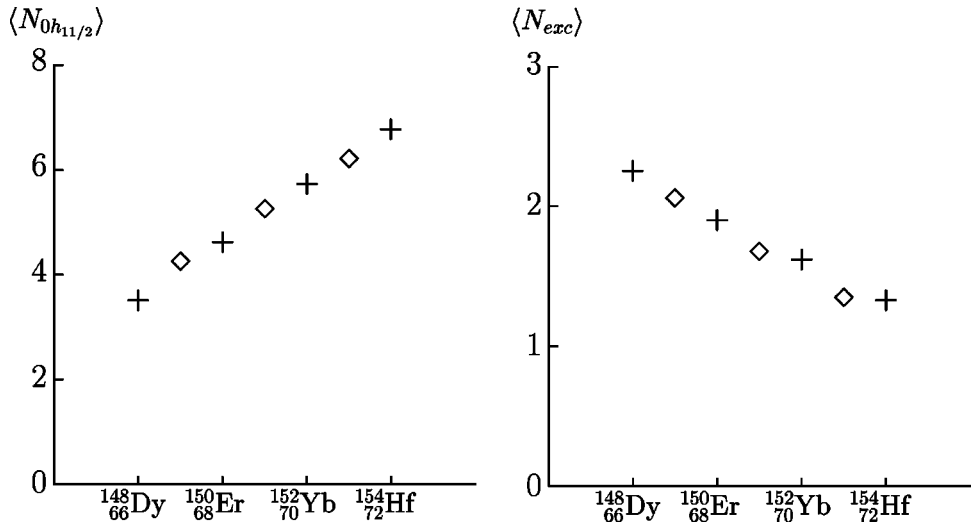

FIG. 9. Left: occupation numbers $\langle N_{0h_{11/2}}\rangle$. Right: numbers of particles excited out of the $Z=64$ core, $\langle N_{\rm exc}\rangle \equiv 14-(\langle N_{0g_{7/2}}\rangle + \langle N_{1d_{5/2}}\rangle)$. The plus symbols show the expectation values for the 10^+ states, and the diamonds those for the $27/2^-$ states.

with the Gaussian form to the nuclei in this region [23]. As a result of the weak coupling between $0h_{11/2}$ and $(2s_{1/2}1d_{3/2})$, their calculation predicted 0_2^+ states at unusually low energies in ^{148}Dy and ^{150}Er, quite similar to the above $3j$ case with the Yukawa interaction.

Wildenthal proposed an effective Hamiltonian for the $N=82$ isotones [24]. Starting from the SDI+QQ interaction, the interaction matrix elements are fitted to the $50<Z\leqslant 72$ nuclei. While Wildenthal's Hamiltonian (with the assumed truncation) nicely describes the energy levels, it does not reproduce the Z dependence of the $E2$ properties of the isomers; in particular, the strong hindrance around ^{152}Yb. This is because the pair excitation out of the $Z=64$ core is too small, at least for the high-spin isomers.

The above possibilities (i) and (ii) can also be judged by future experiments. Though the ExSRF (8) connects the $E2$ strength to the occupation number $\langle N_{0h_{11/2}}\rangle$, the ambiguity in $e_{\rm eff}$ prohibits us from extracting $\langle N_{0h_{11/2}}\rangle$ directly from $B(E2)$ of the isomers, in practice. The two possibilities (i) and (ii) give somewhat similar $E2$ strengths for the high-spin isomers on account of the difference in $e_{\rm eff}$, in the discussions so far. However, this does not apply to the transition from the lowest 2^+ state to the ground 0^+ state. In Table III, the $B(E2;2^+\rightarrow 0^+)$ values calculated in the multi-j model are compared with those in the single-j model. Without state dependence of the effective charges, we predict 2.5–5 times larger $E2$ strengths in the multi-j space [i.e., (ii)] than in the single-j space [i.e., (i)]. In the multi-j model of Sec. IV, there seems to be a problem with respect to the quadrupole collectivity. Hence we should not expect an excellent precision on the multi-j prediction; indeed, by a slight variation of the interaction $B(E2;2^+\rightarrow 0^+)$ can deviate by 30% without influencing the $E2$ strengths of the isomers. Still the big difference between the single-j and multi-j models would enable us to judge which of the two possibilities (i) or (ii) is reliable.

VI. SUMMARY AND OUTLOOK

We have investigated the 10^+ and $27/2^-$ isomers of the $Z>64$, $N=82$ nuclei. The extended seniority reduction formula has been derived for the $E2$ decay strengths of the isomers, under reasonable assumptions. This formula links the $E2$ strength to the occupation number on the $\pi 0h_{11/2}$ orbit, apart from the ambiguity in the effective charge. The extended formula accounts for the mechanism of the $E2$ hindrance, which we have called extended seniority isomerism.

By taking into account the excitations from $(0g_{7/2}1d_{5/2})$ to $(0h_{11/2}2s_{1/2}1d_{3/2})$, the binding energies, the energy levels of both parities, and the $B(E2)$ values have simultaneously been reproduced in a multi-j shell-model calculation with the MSDI. The $E2$ hindrance in ^{153}Lu as well as in ^{152}Yb has been described quite well. Combined with the approximate degeneracy among the $0h_{11/2}$, $2s_{1/2}$, and $1d_{3/2}$ orbits, the strong $E2$ hindrance around $Z=70$ exclusively indicates the presence of the pair excitation out of the $Z=64$ core. Thus the $Z=64$ core is not very stiff. It is not always justified to assume the ^{146}Gd inert core, even for the relatively low-lying states in the $N=82$ isotones. In this respect, the number $Z=64$ should be distinguished from magic numbers like $N=82$, though it would be fair to call it a *submagic* number.

The extended seniority isomerism may exist in other single-closed nuclei and their neighbors. While we have restricted our discussion to the $Z>64$, $N=82$ nuclei, focusing on the stiffness of the $Z=64$ core, it is of interest to apply similar approaches to nuclei in other mass regions. Work along this line is in progress.

ACKNOWLEDGMENT

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TABLE III. Calculated $B(E2;2^+\rightarrow 0^+)$ values (e^2fm^4). The $0h_{11/2}$ single-j results using the parameters of Ref. [7] and those of the present work (PW) are compared.

	$^{148}_{66}$Dy	$^{150}_{68}$Er	$^{152}_{70}$Yb	$^{154}_{72}$Hf
Ref. [7]	187	298	335	298
PW	967	960	886	787

APPENDIX: ARGUMENT ON CONDITION (c) IN SEC. III

Condition (c) in Sec. III is expected to be a good approximation. Indeed, it is exactly derived from (a) and (b) if the interaction within the ξ subspace has zero-range character. We prove it in this Appendix.

1. General argument

With assuming conditions (a) and (b), let us consider matrix elements of the general shell-model Hamiltonian between the bases appearing in Eqs. (2) and (3). We first prove the relation

$$(\langle \xi^{n'_\xi} v_\xi = v J_f^\pi | \langle r^{n-n'_\xi} \alpha' v_r = 00^+ |) H (| \xi^{n_\xi} v_\xi = v J_f^\pi \rangle | r^{n-n_\xi} \alpha v_r = 00^+ \rangle) - (\langle \xi^{n'_\xi} v_\xi = v J_i^\pi | \langle r^{n-n'_\xi} \alpha' v_r = 00^+ |)$$

$$\times H (| \xi^{n_\xi} v_\xi = v J_i^\pi \rangle | r^{n-n_\xi} \alpha v_r = 00^+ \rangle) = \delta_{n_\xi, n'_\xi} \delta_{\alpha,\alpha'} (\langle \xi^{n_\xi} v J_f^\pi | V_\xi | \xi^{n_\xi} v J_f^\pi \rangle - \langle \xi^{n_\xi} v J_i^\pi | V_\xi | \xi^{n_\xi} v J_i^\pi \rangle), \tag{A1}$$

as long as the Hamiltonian consists of single-particle energies and of a two-body residual interaction. Here V_ξ stands for the two-body interaction within the ξ subspace. The left-hand side (lhs) of Eq. (A1) obviously vanishes for the single-particle energy term of H. It is sufficient to focus on matrix elements of the two-body interaction.

The two-body interaction is expressed, in the second-quantized form, by the sum of the terms composed of $a_{j_1}^\dagger a_{j_2}^\dagger a_{j_3} a_{j_4}$ operators (with coupling constants). According to which of the j's belong to ξ or r, all the possible terms contributing to the matrix elements are classified into the following categories: (i) $a_\xi^\dagger a_\xi^\dagger a_r a_r$ terms and their Hermitian conjugates, (ii) $a_r^\dagger a_r^\dagger a_r a_r$ terms, (iii) $a_\xi^\dagger a_r^\dagger a_r a_\xi$ terms, and (iv) $a_\xi^\dagger a_\xi^\dagger a_\xi a_\xi$ terms. It is noted that, since $v_r = 0$, the terms having an odd number of $(a_r^\dagger, a_r)$ operators vanish. We decompose the operators into the ξ part and the r part, and denote them by $\hat{h}_\xi$ and $\hat{h}_r$. The matrix elements are also decomposed into the ξ and r part,

$$(\langle \xi^{n'_\xi} v_\xi = v J^\pi | \langle r^{n-n'_\xi} \alpha' v_r = 00^+ |) \hat{h}_\xi \hat{h}_r (| \xi^{n_\xi} v_\xi = v J^\pi \rangle | r^{n-n_\xi} \alpha v_r = 00^+ \rangle)$$

$$= \langle \xi^{n'_\xi} v_\xi = v J^\pi | \hat{h}_\xi | \xi^{n_\xi} v_\xi = v J^\pi \rangle \langle r^{n-n'_\xi} \alpha' v_r = 00^+ | \hat{h}_r | r^{n-n_\xi} \alpha v_r = 00^+ \rangle. \tag{A2}$$

It is obvious that the $\langle r^{n-n'_\xi} \alpha' v_r = 00^+ | \hat{h}_r | r^{n-n_\xi} \alpha v_r = 00^+ \rangle$ part does not depend on J^π. Therefore,

$$(\langle \xi^{n'_\xi} v_\xi = v J_f^\pi | \langle r^{n-n'_\xi} \alpha' v_r = 00^+ |) \hat{h}_\xi \hat{h}_r (| \xi^{n_\xi} v_\xi = v J_f^\pi \rangle | r^{n-n_\xi} \alpha v_r = 00^+ \rangle) - (\langle \xi^{n'_\xi} v_\xi = v J_i^\pi | \langle r^{n-n'_\xi} \alpha' v_r = 00^+ |)$$

$$\times \hat{h}_\xi \hat{h}_r (| \xi^{n_\xi} v_\xi = v J_i^\pi \rangle | r^{n-n_\xi} \alpha v_r = 00^+ \rangle)$$

$$= \langle r^{n-n'_\xi} \alpha' v_r = 00^+ | \hat{h}_r | r^{n-n_\xi} \alpha v_r = 00^+ \rangle (\langle \xi^{n'_\xi} v_\xi = v J_f^\pi | \hat{h}_\xi | \xi^{n_\xi} v_\xi = v J_f^\pi \rangle - \langle \xi^{n'_\xi} v_\xi = v J_i^\pi | \hat{h}_\xi | \xi^{n_\xi} v_\xi = v J_i^\pi \rangle). \tag{A3}$$

For the matrix elements between the $v_r = 0$ bases, $\hat{h}_r$ cannot carry angular momentum, and therefore $\hat{h}_\xi$ cannot either. Then $\hat{h}_\xi$ and $\hat{h}_r$ for each category are defined as, without loss of generality, (i) $\hat{h}_\xi = [a_\xi^\dagger a_\xi^\dagger]^{(0)}$, $\hat{h}_r = [\tilde{a}_r \tilde{a}_r]^{(0)}$, (ii) $\hat{h}_\xi = 1$, $\hat{h}_r = [a_r^\dagger a_r^\dagger \tilde{a}_r \tilde{a}_r]^{(0)}$, (iii) $\hat{h}_\xi = [a_\xi^\dagger \tilde{a}_\xi]^{(0)}$, $\hat{h}_r = [a_r^\dagger \tilde{a}_r]^{(0)}$, and (iv) $\hat{h}_\xi = [a_\xi^\dagger a_\xi^\dagger \tilde{a}_\xi \tilde{a}_\xi]^{(0)}$, $\hat{h}_r = 1$. V_ξ in Eq. (A1) represents the collection of the $\hat{h}_\xi$'s belonging to (iv). We discuss the matrix elements of $\hat{h}_\xi$ in the right-hand side (rhs) of Eq. (A3), respective to the above four categories.

Category (i) leads to $n'_\xi = n_\xi \pm 2$ off-diagonal elements. The $\hat{h}_\xi = [a_\xi^\dagger a_\xi^\dagger]^{(0)}$ operator in this case is proportional to a generator of the quasispin in the orbit ξ. Thus $\langle \xi^{n_\xi + 2} v_\xi = v J^\pi | \hat{h}_\xi | \xi^{n_\xi} v_\xi = v J^\pi \rangle$ depends only on n_ξ and v_ξ, not on J^π. Namely,

$$\langle \xi^{n'_\xi} v_\xi = v J_f^\pi | [a_\xi^\dagger a_\xi^\dagger]^{(0)} | \xi^{n_\xi} v_\xi = v J_f^\pi \rangle = \langle \xi^{n'_\xi} v_\xi = v J_i^\pi | [a_\xi^\dagger a_\xi^\dagger]^{(0)} | \xi^{n_\xi} v_\xi = v J_i^\pi \rangle, \tag{A4}$$

and the rhs of Eq. (A3) vanishes.

Since $\hat{h}_\xi = 1$ in category (ii), the relevant matrix elements of the ξ part are

$$\langle \xi^{n'_\xi} v_\xi = v J_f^\pi | \xi^{n_\xi} v_\xi = v J_f^\pi \rangle = \langle \xi^{n'_\xi} v_\xi = v J_i^\pi | \xi^{n_\xi} v_\xi = v J_i^\pi \rangle = \delta_{n_\xi, n'_\xi}. \tag{A5}$$

For category (iii), $\hat{h}_\xi = [a_\xi^\dagger \tilde{a}_\xi]^{(0)} \propto N_\xi$, leading to

$$\langle \xi^{n'_\xi} v_\xi = v J_f^\pi | [a_\xi^\dagger \tilde{a}_\xi]^{(0)} | \xi^{n_\xi} v_\xi = v J_f^\pi \rangle = \langle \xi^{n'_\xi} v_\xi = v J_i^\pi | [a_\xi^\dagger \tilde{a}_\xi]^{(0)} | \xi^{n_\xi} v_\xi = v J_i^\pi \rangle \propto \delta_{n_\xi, n'_\xi} n_\xi . \tag{A6}$$

Both terms do not contribute to the rhs of Eq. (A3).

From Eqs. (A4), (A5), and (A6), only the terms of the category (iv) may contribute to the rhs of Eq. (A3). Equation (A1) follows, because $\hat{h}_r = 1$ for category (iv), with replacing the sum of $\hat{h}_\xi$'s by V_ξ. The argument is now reduced to the single-j matrix elements within the ξ orbit. Note again that we have not imposed any restriction on the Hamiltonian in the discussion so far, apart from it consisting of the single-particle energies and two-body interaction.

2. Property of V_ξ

We next consider the property of V_ξ. If $\langle \xi^{n_\xi} v J^\pi | V_\xi | \xi^{n_\xi} v J^\pi \rangle$ is independent of n_ξ or J, we have

$$\langle \xi^{n_\xi} v J_f^\pi | V_\xi | \xi^{n_\xi} v J_f^\pi \rangle - \langle \xi^{n_\xi} v J_i^\pi | V_\xi | \xi^{n_\xi} v J_i^\pi \rangle = \langle \xi^{v} v J_f^\pi | V_\xi | \xi^{v} v J_f^\pi \rangle - \langle \xi^{v} v J_i^\pi | V_\xi | \xi^{v} v J_i^\pi \rangle. \tag{A7}$$

Substituting it into the rhs of Eq. (A1), we obtain

$$\begin{aligned} &(\langle \xi^{n'_\xi} v_\xi = v J_f^\pi | \langle r^{n-n'_\xi} \alpha' v_r = 00^+ |) H (| \xi^{n_\xi} v_\xi = v J_f^\pi \rangle | r^{n-n_\xi} \alpha v_r = 00^+ \rangle) \\ &\quad - (\langle \xi^{n'_\xi} v_\xi = v J_i^\pi | \langle r^{n-n'_\xi} \alpha' v_r = 00^+ |) H (| \xi^{n_\xi} v_\xi = v J_i^\pi \rangle | r^{n-n_\xi} \alpha v_r = 00^+ \rangle) \\ &= \delta_{n_\xi, n'_\xi} \delta_{\alpha,\alpha'} (\langle \xi^{v} v J_f^\pi | V_\xi | \xi^{v} v J_f^\pi \rangle - \langle \xi^{v} v J_i^\pi | V_\xi | \xi^{v} v J_i^\pi \rangle). \end{aligned} \tag{A8}$$

The Hamiltonian matrix can be separated according to J, because of the angular momentum conservation. Moreover, since seniority v has been assumed to be a good quantum number, it is sufficient to consider submatrices of H for a fixed v. The space to be diagonalized is spanned by $|\xi^{n_\xi} v_\xi = v J^\pi\rangle | r^{n-n_\xi} \alpha v_r = 00^+\rangle$ with various n_ξ and α [see Eqs. (2) and (3)]. Equation (A8) implies that the submatrices of H are identical between $|(\xi r)^n v J_i^\pi\rangle$ and $|(\xi r)^n v J_f^\pi\rangle$, except for a constant shift of the diagonal elements. Diagonalized by the same unitary matrix, the lowest eigenstates $|J_i^\pi\rangle$ and $|J_f^\pi\rangle$ have equal coefficient $c_{n_\xi \alpha}$ to each other; condition (c) is exactly satisfied. It is now clear that Eq. (A7) is crucial to condition (c).

In the quasispin (QS) regime within the single orbit ξ, $[a_\xi^\dagger a_\xi^\dagger \tilde{a}_\xi \tilde{a}_\xi]^{(0)}$ can be QS-scalar, vector or tensor, in general. If V_ξ is purely QS scalar, the matrix element $\langle \xi^{n_\xi} v_\xi = v J_i^\pi | V_\xi | \xi^{n_\xi} v_\xi = v J_i^\pi \rangle$ is independent of n_ξ, and Eq. (A7) is fulfilled. Equation (A7) is also satisfied if QS-vector and QS-tensor parts of V_ξ are J independent. This is indeed attained when the QS vector and tensor parts can be expressed by the QS generators ($[a_\xi^\dagger a_\xi^\dagger]^{(0)}$, $[\tilde{a}_\xi \tilde{a}_\xi]^{(0)}$, and $[a_\xi^\dagger \tilde{a}_\xi]^{(0)}$, besides appropriate constant factors). An immediate example is the monopole pairing ($[a_\xi^\dagger a_\xi^\dagger]^{(0)} [\tilde{a}_\xi \tilde{a}_\xi]^{(0)}$). A sufficient condition to Eq. (A7) is that V_ξ consists only of QS-scalars and of the QS generators.

The general form of V_ξ can be represented by

$$V_\xi = -\sum_{\lambda = \text{even}} \frac{g_\lambda}{2} [a_\xi^\dagger a_\xi^\dagger]^{(\lambda)} \cdot [\tilde{a}_\xi \tilde{a}_\xi]^{(\lambda)}. \tag{A9}$$

The corresponding ''particle-hole'' interaction is defined by [17]

$$\overline{V}_\xi = \sum_\lambda f_\lambda [a_\xi^\dagger a_\xi^\dagger]^{(\lambda)} \cdot [\tilde{a}_\xi \tilde{a}_\xi]^{(\lambda)}, \tag{A10}$$

with

$$f_\lambda = \sum_{\lambda' = \text{even}} (2\lambda' + 1) W(j_\xi j_\xi j_\xi j_\xi; \lambda' \lambda) g_{\lambda'}. \tag{A11}$$

In Eq. (A10), only $\lambda =$ even terms remain owing to the antisymmetrization. According to the QS argument [17], we have

$$\begin{aligned} \langle \xi^{n_\xi} v J^\pi | V_\xi | \xi^{n_\xi} v J^\pi \rangle = &\left\{ \frac{(\Omega_\xi - 2v)(2\Omega_\xi - n_\xi - v)}{4(\Omega_\xi - v)(\Omega_\xi - v - 1)} \times (g_0 + 2f_0) - f_0 \right\} (n_\xi - v) \\ &+ \frac{(\Omega_\xi - v)(\Omega_\xi - v - 2) + (n_\xi - \Omega_\xi)^2}{2(\Omega_\xi - v)(\Omega_\xi - v - 1)} \times \langle \xi^{v} v J^\pi | V_\xi | \xi^{v} v J^\pi \rangle \\ &+ \frac{(\Omega_\xi - v)^2 - (n_\xi - \Omega_\xi)^2}{2(\Omega_\xi - v)(\Omega_\xi - v - 1)} \times \langle \xi^{v} v J^\pi | \overline{V}_\xi | \xi^{v} v J^\pi \rangle. \end{aligned} \tag{A12}$$

By subtracting out the n_ξ- and J-independent terms, Eq. (A12) derives

$$(\langle \xi^n \xi v J_f^\pi | V_\xi | \xi^n \xi v J_f^\pi \rangle - \langle \xi^n \xi v J_i^\pi | V_\xi | \xi^n \xi v J_i^\pi \rangle)$$
$$- (\langle \xi^v v J_f^\pi | V_\xi | \xi^v v J_f^\pi \rangle - \langle \xi^v v J_i^\pi | V_\xi | \xi^v v J_i^\pi \rangle)$$
$$= \frac{(n_\xi - \Omega_\xi)^2}{2(\Omega_\xi - v)(\Omega_\xi - v - 1)} \{ \langle \xi^v v J_f^\pi | (V_\xi - \overline{V}_\xi) | \xi^v v J_f^\pi \rangle$$
$$- \langle \xi^v v J_i^\pi | (V_\xi - \overline{V}_\xi) | \xi^v v J_i^\pi \rangle \}. \quad \text{(A13)}$$

The n_ξ dependence is eliminated if we have

$$\langle \xi^v v J_f^\pi | (V_\xi - \overline{V}_\xi) | \xi^v v J_f^\pi \rangle = \langle \xi^v v J_i^\pi | (V_\xi - \overline{V}_\xi) | \xi^v v J_i^\pi \rangle. \quad \text{(A14)}$$

Equations (A9) and (A10) lead to

$$V_\xi - \overline{V}_\xi = - \sum_{\lambda = \text{even}} \frac{g_\lambda + 2 f_\lambda}{2} [a_\xi^\dagger a_\xi^\dagger]^{(\lambda)} \cdot [\tilde{a}_\xi \tilde{a}_\xi]^{(\lambda)}. \quad \text{(A15)}$$

As long as the seniority is not large, only a limited number of λ's in Eq. (A15) contribute to the rhs of Eq. (A13). For instance, only the $\lambda = J_i$ and J_f terms are relevant to the $v = 2$ case, and Eq. (A14) then derives $g_{J_i} + 2 f_{J_i} = g_{J_f} + 2 f_{J_f}$. When V_ξ is QS scalar, $V_\xi = \overline{V}_\xi$ and $g_\lambda + 2 f_\lambda = 0$ for any even λ.

3. Zero-range interaction

We here verify that Eq. (A7) is exactly fulfilled if V_ξ is a zero-range interaction. The one-body operator $[a_\xi^\dagger \tilde{a}_\xi]^{(\lambda)}$ is QS scalar for an odd λ [17]. An easy way to construct a QS-scalar interaction is to take

$$\sum_{\lambda = \text{odd}} q_\lambda [a_\xi^\dagger \tilde{a}_\xi]^{(\lambda)} \cdot [a_\xi^\dagger \tilde{a}_\xi]^{(\lambda)}, \quad \text{(A16)}$$

where q_λ is an arbitrary constant.

Suppose that V_ξ is a zero-range interaction, which we here define as

$$V_\xi^S = u(r_1, r_2) \delta(\hat{\mathbf{r}}_1 - \hat{\mathbf{r}}_2), \quad \text{(A17)}$$

with the exchange symmetry $u(r_1, r_2) = u(r_2, r_1)$. The SDI adopted in the text is of this type $[u(r_1, r_2) \propto \delta(r_1 - r_2) \delta(r_1 - R)/R^2]$. Since V_ξ is under discussion, the radial part of the interaction is unimportant, giving only an overall factor to the matrix elements. Expanding the angular part of V_ξ^S by the Legendre polynomials, we obtain

$$V_\xi^S = u(r_1, r_2) \sum_\lambda \frac{2\lambda + 1}{2} P_\lambda(\cos \theta_{12})$$
$$= 2\pi u(r_1, r_2) \sum_\lambda Y^{(\lambda)}(\hat{\mathbf{r}}_1) \cdot Y^{(\lambda)}(\hat{\mathbf{r}}_2). \quad \text{(A18)}$$

On the other hand, the zero-range interaction given in Eq. (A17) acts on the spatially symmetric two-body states. Therefore, if the two-body states are antisymmetrized, the zero-range interaction automatically picks up the spin-singlet two-body states for identical fermion systems. This leads to [11]

$$V_\xi^S = - \frac{4}{3} (\mathbf{s}_1 \cdot \mathbf{s}_2) V_\xi^S = - \frac{8\pi}{3} u(r_1, r_2)$$
$$\times (\mathbf{s}_1 \cdot \mathbf{s}_2) \sum_\lambda Y^{(\lambda)}(\hat{\mathbf{r}}_1) \cdot Y^{(\lambda)}(\hat{\mathbf{r}}_2). \quad \text{(A19)}$$

With the angular momentum recoupling, we rewrite it as

$$V_\xi^S = - \frac{8\pi}{3} u(r_1, r_2) \sum_{\lambda, \kappa} [Y^{(\lambda)}(\hat{\mathbf{r}}_1) \mathbf{s}_1]^{(\kappa)} \cdot [Y^{(\lambda)}(\hat{\mathbf{r}}_2) \mathbf{s}_2]^{(\kappa)}. \quad \text{(A20)}$$

We now switch to the second-quantized representation. The equivalent one-body operator to $[Y^{(\lambda)}(\hat{\mathbf{r}}) \mathbf{s}]^{(\kappa)}$ in the ξ subspace is

$$\frac{1}{2\kappa + 1} \langle \xi || [Y^{(\lambda)}(\hat{\mathbf{r}}) \mathbf{s}]^{(\kappa)} || \xi \rangle [a_\xi^\dagger \tilde{a}_\xi]^{(\kappa)}. \quad \text{(A21)}$$

The single-particle matrix element in Eq. (A21) is evaluated by

$$\langle \xi || [Y^{(\lambda)}(\hat{\mathbf{r}}) \mathbf{s}]^{(\kappa)} || \xi \rangle = \sqrt{2\kappa + 1} (2 j_\xi + 1) \begin{Bmatrix} l_\xi & 1/2 & j_\xi \\ \lambda & 1 & \kappa \\ l_\xi & 1/2 & j_\xi \end{Bmatrix}$$
$$\times \langle l_\xi || Y^{(\lambda)}(\hat{\mathbf{r}}) || l_\xi \rangle \langle 1/2 || \mathbf{s} || 1/2 \rangle. \quad \text{(A22)}$$

In order for $\langle l_\xi || Y^{(\lambda)}(\hat{\mathbf{r}}) || l_\xi \rangle$ in the rhs not to vanish, λ must be even (parity selection rule). On the other hand, owing to the symmetry of the $9j$-symbol in Eq. (A22), the above matrix element vanishes if $\lambda + 1 + \kappa$ is odd. This is a consequence of the time reversality. Therefore, the single-particle matrix element of Eq. (A22) vanishes for even κ. Back to Eq. (A21), we find that $[Y^{(\lambda)}(\hat{\mathbf{r}}) \mathbf{s}]^{(\kappa)}$ is QS scalar because κ is always odd. Hence V_ξ^S is also QS scalar via Eq. (A20). Thus, the zero-range V_ξ has $g_\lambda + 2 f_\lambda = 0$ in Eq. (A15) for any possible λ, and therefore satisfies Eq. (A7).

In reality, V_ξ will not fully be zero range. However, as far as the short-range interaction dominates, the matrix elements of V_ξ are not very different from the zero-range interaction, having $g_\lambda + 2 f_\lambda \simeq 0$. Condition (c) is therefore expected to be a good approximation.

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