Properties of the nonlocal *NN* interactions required for the correct triton binding energy

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In order to investigate their effect on the triton binding energy, a set of nonlocal $^1S_0$ and $^3SD_1$ nucleon-nucleon (NN) interactions is constructed in coordinate space. The properties which make possible the reproduction of the triton binding energy without a three-nucleon force are studied. For all of the investigated local and nonlocal interactions the dominant contribution to the triton binding energy comes from the pole terms of the $^1S_0$ and $^3SD_1$ states; consequently the singlet virtual bound state and deuteron properties have a dominant role. Low deuteron D-state and substantial S-state short-distance probabilities, which could not be produced by local NN interactions, are necessary requirements for the correct binding energy.

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I. INTRODUCTION

Because of the internal structure of nucleons, a short-range (up to 1–1.5 fm) nonlocality is undoubtedly present in NN interactions [1]. However, there are other sources of nonlocality with a longer range. The first theoretically sound nonlocal NN interaction was constructed by the Bonn group [2] in momentum space and called an r-space long-range nonlocal interaction [3]. The definition of the term ''long range'' [3] is $r \geqslant 1$ fm. Unfortunately there is no information available about the actual r-space region where the Bonn potentials [2,4] become local. But one expects a significantly longer range of nonlocality than the above-mentioned 1–1.5 fm estimate, since according to Ref. [4] already the simple mechanism of a one-boson exchange generates nonlocality, and more complicated processes can provide an even stronger one. Recent local potential models [5,6] give a considerable contribution to the Yukawa tail at 2–4 fm; therefore it would not be surprising if nonlocality were extended up to this region.

To gain some empirical information about the role, the effect, and the properties of the nonlocality of the NN interaction, phenomenological ''inside nonlocal, outside Yukawa (INOY) tail'' $^1S_0$ and $^3SD_1$ (this is a shorter notation for $^3S_1$-$^3D_1$) NN interactions were constructed in coordinate space and their effect on the triton binding energy was investigated [7–9] . The advantage in working in coordinate space is that the range of the locality and the nonlocality is explicitly controlled. It is vital, since a fundamental property of the NN interaction is that it is a short-range interaction (i.e., in coordinate space at long range it decreases as an exponential function). The aim was to find such nonlocal forms which reproduce the triton binding energy at the shortest possible range of nonlocality. The basic additional requirement on the constructed INOY interactions is, of course, that they must reproduce the known NN phase shifts [10] and the deuteron properties. An earlier separable form of the nonlocal part was changed to a more general and smoother nonseparable form.

The chosen shape of the nonlocality and the constructed interactions are described in Sec. II. The method of the triton calculations and the effect of the INOY interactions on the binding energy are given in Sec. III. The summary and conclusions are given in Sec. IV.

II. INOY *NN* INTERACTIONS

Although it is possible to define a nonlocal form which at large distance becomes a local Yukawa tail, in numerical calculations the handling of such a form would be extremely difficult. Therefore an explicit separation of the interaction into local and nonlocal parts was chosen:

$$V_{ll'}^{full}(r,r') = W_{ll'}(r,r') + \delta(r-r') F_{ll'}^{cut}(r) V_{ll'}^{Yukawa}(r), \tag{1}$$

where

$$F_{ll'}^{cut}(r) = \begin{cases} 1 - e^{-[\alpha_{ll'}(r-R_{ll'})]^2} & \text{for } r > R_{ll'}, \\ 0 & \text{for } r \leqslant R_{ll'}, \end{cases} \tag{2}$$

and $W_{ll'}(r,r')$ and $V_{ll'}^{Yukawa}(r)$ are the nonlocal part and the Yukawa tail (as it is defined in the Argonne v_{18} potential [6]), respectively.

The threshold behavior of the nonlocal part is separated:

$$W_{ll'}(r,r') = \left(\frac{\beta_l r}{\sqrt{1+\beta_l^2 r^2}} \right)^{l} \widetilde{W}_{ll'}(x,x') \left(\frac{\beta_{l'} r'}{\sqrt{1+\beta_{l'}^2 r'^2}} \right)^{l'}, \tag{3}$$

where

$$x = \frac{\gamma r^2}{\sqrt{1+\gamma^2 r^2}}.$$

The $\widetilde{W}_{ll'}(x,x')$ term is parametrized in the following way:

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TABLE I. Parameters of the INOY np and nn $^1S_0$ interactions.

	ISA	ISB	ISC	ISD
R_{00} (fm)	2.0	2.0	2.0	2.0
a_{00} (fm^{-1})	2.0	2.0	2.0	2.0
y_{00} (fm)	2.0	2.0	2.0	2.0
V_0 (MeV fm^{-3})	−240.3	−302.0	−308.1	−480.7
a_0 (fm^{-1})	2.298	2.351	2.507	3.050
a_0' (fm^{-1})	1.725	2.000	2.232	2.869
x_0 (fm)	0.0	0.0	0.0	0.0
x_0' (fm)	1.043	1.103	1.215	1.300
V_{00}^i (MeV fm^{-3})	8108.0	8175.0	8349.0	7679.0
	−231.480	−121.013	−88.1928	−102.247
	−0.8785	−0.8812	−0.8744	−1.420
V_{00}^2 (MeV fm^{-3}) for nn	−224.247	−115.096	−83.2243	−96.6726
b_{00}^i (fm^{-1})	1.553	1.602	1.673	1.727
	1.685	1.754	1.965	3.494
	0.5000	0.5000	0.5000	0.4140
c_{00}^i (fm^{-1})	1.604	2.008	2.356	1.987
	1.927	1.806	1.327	0.3091
	0.5000	0.5000	0.5000	0.6000
z_{00}^i (fm)	0.0	0.0	0.0	0.0
	0.6000	0.6488	0.6728	0.7000
	1.500	1.500	1.500	1.000

$$\begin{aligned}\tilde{W}_{ll'}(x,x') = {} & e^{-[a_{ll'}F(y,y_{ll'})(x-x')]^2} \\ & \times\Bigg[\delta_{ll'}V_l\{e^{-[a_l(x-x_l)]^2-[a_l'(x'-x_l')]^2} \\ & + e^{-[a_l(x'-x_l)]^2-[a_l'(x-x_l')]^2}\}+(1-\delta_{ll'}) \\ & \times\{V_{10}e^{-[a_{1l}(x-x_{1l})]^2-[a_{1l'}(x'-x_{1l'})]^2} \\ & + V_{20}e^{-[a_{2l}(x-x_{2l})]^2-[a_{2l'}(x'-x_{2l'})]^2}\} \\ & + \sum_{i=1}^{n_{ll'}} V_{ll'}^i e^{-[b_{ll'}^i(x+x'-2z_{ll'}^i)]^2-[c_{ll'}^i(x-x')]^2}\Bigg],\end{aligned} \tag{4}$$

where

$$F(y,y_{ll'}) = \begin{cases} y/y_{ll'}-1 & \text{for } y>y_{ll'}, \\ 0 & \text{for } y\leq y_{ll'}, \end{cases} \quad y^2=x^2+x'^2.$$

To satisfy the required symmetry of the interaction the following conditions must be fulfilled:

$$a_{ll'}=a_{l'l}, \quad y_{ll'}=y_{l'l}, \quad V_{ll'}^i=V_{l'l}^i,$$

$$b_{ll'}^i=b_{l'l}^i, \quad z_{ll'}^i=z_{l'l}^i, \quad c_{ll'}^i=c_{l'l}^i.$$

The introduction of the x,x' variables assures an even behavior of the potential $\tilde{W}_{ll'}(x,x')$ to the $r\to -r$ or $r'\to -r'$ transformation. Therefore at the origin of three-dimensional coordinate space the derivative exists.

It has to be emphasized that within this framework the central parts of the INOY interactions already include the diagonal part of the tensor force (contrary to the usual formulation of local potentials).

The first term $e^{-[a_{ll'}F(y,y_{ll'})(x-x')]^2}$ of formula (4) was chosen to secure the cutoff of the nondiagonal part of the interaction at larger r,r' values. The first terms inside the squared brackets (outside the sum) are the off-diagonal centered terms. The second term (i.e., the sum) contains the diagonal centered terms: these terms have their maximum absolute values on the $r=r'$ diagonal and decrease with increasing $|r-r'|$ values.

Since the $^1S_0$ and $^3SD_1$ interactions play a dominant role in binding the triton, and since the local or nonlocal character of the higher partial-wave components of the NN interaction has no considerable effect on the triton binding energy [7], at present only $^1S_0$ and $^3SD_1$ INOY interactions were constructed.

Some parameters were fixed for all potentials. These are $\beta_l=\gamma=2.0$ fm^{-1} and $\alpha_{ll'}=1.0$ fm^{-1}. The cutoff parameter $R_{ll'}$ for the Yukawa tail is chosen in most cases to be 2 fm, which seems to be rather large. $R_{00}=1$ fm is an acceptable choice for the S-wave interactions; however, a low deuteron D-state probability requires a higher $R_{ll'}$ value. Therefore except for one case (ITA $^3SD_1$ force) an $R_{ll'}=2$ fm value is used. This means that suppression of the Yukawa tail starts at 4 fm and it turns to zero at 2 fm. Correspondingly, the nonlocality stretches out to the 3–4 fm region (although at 4 fm it is already very small). The overlap region of the local and nonlocal parts of the interactions is therefore the 2–4 fm region.

TABLE II. Parameters of the *S*-wave part of the INOY $^3SD_1$ interactions.

	ITA	ITB	ITC	ITD	ITE	ITF
R_{00} (fm)	1.0	2.0	2.0	2.0	2.0	2.0
a_{00} (fm^{-1})	1.5	2.0	2.0	2.0	2.0	2.0
y_{00} (fm)	1.5	2.0	2.0	2.0	2.0	2.0
V_0 (MeV fm^{-3})	−242.6	−253.0	−258.1	−264.3	−151.1	−100.0
a_0 (fm^{-1})	2.938	2.873	2.790	2.497	2.473	2.744
a_0' (fm^{-1})	2.176	1.804	1.811	2.262	2.543	2.773
x_0 (fm)	0.0	0.0	0.0	0.0	0.0	0.0
$x_0'(fm)$	1.200	1.050	1.050	1.050	1.000	1.000
V_{00}^i (MeV fm^{-3})	9094.0	9529.0	9512.0	8488.0	9184.0	8547.0
	−119.993	−235.533	−245.815	−149.037	−190.600	−257.249
	−0.8660	−0.7290	−0.7271	−1.133	−1.154	−1.168
b_{00}^i (fm^{-1})	1.712	1.760	1.748	1.739	1.746	1.748
	1.469	1.878	1.858	1.699	1.497	1.507
	0.5500	0.4391	0.4437	0.5000	0.5044	0.5065
c_{00}^i (fm^{-1})	1.671	1.809	1.840	2.003	1.853	0.9872
	0.9988	1.670	1.657	1.398	1.295	1.509
	0.8000	0.5059	0.5006	0.7000	0.7000	0.7000
z_{00}^i (fm)	0.0	0.0	0.0	0.0	0.0	0.0
	0.4858	0.6000	0.6000	0.6523	0.6214	0.6198
	1.400	1.200	1.200	1.200	1.193	1.189

Following the experience gained by the application of a separable form of the nonlocal *NN* interaction earlier [7], forms which produce higher zero-energy singlet or deuteron *S*-state wave functions at short distance (similarly to the wave functions of the Bonn potentials [2,4]) were selected. For this requirement a strong off-diagonal attraction has to be built into the nonlocal potentials. The terms outside of the sum of formula (4) represent this off-diagonal part. The center of this attractive off-diagonal part was kept at $r=0$ ($x_0=0.0$ or $x_{01}=0.0$) which is not unconditionally necessary. This type of the off-diagonal term is suppressed by the threshold behavior for $l\neq 0$ values; therefore it is not present in the higher partial-wave components of the interactions ($V_2=0.0$ and $V_{20}=0.0$).

A technical remark: usually four digits are given for the parameters, and the calculations are insensitive to a better accuracy. However, the V_{00}^2 values are used for the fine-tuning of the singlet scattering length and the deuteron binding energy; therefore here more digits are required.

To compare the results provided by INOY interactions with those of a a local potential, the Argonne v_{18} potential [6] was chosen as a reference local potential. However, the electromagnetic terms were omitted and small correctional Gaussian terms were added in the inside region (at 0.8 fm for the $^1S_0$ and at 1.5 fm for the $^3S_1$ potential) in order to reproduce the *np*, *nn* scattering lengths and the deuteron binding energy. This reference Argonne potential is denoted as ARG throughout the present paper.

Four $^1S_0$ and six $^3SD_1$ interactions were constructed and labeled as ISA, ISB, ISC, ISD and ITA, ITB, ITC, ITD, ITE, ITF, respectively. Their parametrization is shown in Tables I–IV. All of these interactions fit the Nijmegen phase shifts [10] with high accuracy, and therefore they are nearly on-shell equivalent (together with the Argonne v_{18} and Bonn-CD [4] potentials).

The INOY $^1S_0$ interactions differ mainly in the short-distance behavior of the zero-energy wave functions (normalized to 1 at $r=5$ fm). The *np* zero-energy INOY $^1S_0$ wave functions are shown in Fig. 1. The scattering lengths and effective ranges (Table V) are practically the same for all of the INOY $^1S_0$ interactions and coincide with those of the reference Argonne v_{18} potential (although the effective ranges are slightly different). The *np* INOY interactions were fitted; the parametrizations of the *nn* interactions are the same, except for the V_{00}^2 values, which were adjusted to the *nn* scattering length of the Argonne potential. These values of V_{00}^2 for the *nn* pair are shown in a separate line (Table I).

There are two groups of the INOY $^3SD_1$ interactions: in the first one the deuteron *D*-state probability is varied, while the short-distance deuteron *S*-state wave function was kept high. These are the ITA ($P_D=5.38\%$), ITB ($P_D=4.20\%$), ITC ($P_D=3.90\%$), and ITD ($P_D=3.60\%$) interactions; the deuteron *S*-state wave functions produced by them are shown in Fig. 2. In the second group the deuteron *D*-state probability is kept the same ($P_D=3.60\%$) and the short-range deuteron *S*-state wave function is varied. These are the ITD, ITE, and ITF interactions and their deuteron *S*-state wave functions are shown in Fig. 3. The deuteron and low-energy properties produced by the Argonne and INOY tensor interactions are listed in Table VI.

The fit of the ITA interactions was not too difficult with a shorter range of nonlocality (the $R_{ll'}$ and $y_{ll'}$ values are smaller), because the deuteron *D*-state probability was allowed to be higher (it was not fitted to a given value). How-

TABLE III. Parameters of the tensor part of the INOY $^3SD_1$ interactions.

	ITA	ITB	ITC	ITD	ITE	ITF
R_{02} (fm)	1.0	2.0	2.0	2.0	2.0	2.0
a_{02} (fm^{-1})	1.5	2.5	2.5	2.5	2.5	2.5
y_{02} (fm)	1.5	2.5	2.5	2.5	2.5	2.5
V_{10} (MeV fm^{-3})	0.0	0.0	0.0	−141.6	−150.0	−150.0
a_{10} (fm^{-1})				2.085	2.000	2.000
a_{12} (fm^{-1})				1.230	1.250	1.250
x_{10} (fm)				0.0	0.0	0.0
x_{12} (fm)				1.000	1.000	1.000
V_{20} (MeV fm^{-3})	0.0	0.0	0.0	0.0	0.0	0.0
V^i_{02} (MeV fm^{-3})	−16930.0	−4713.0	−5236.0	−3477.0	−3576.0	−3583.0
	−369.1	−178.6	−151.0	−63.16	−57.09	−57.75
	−8.951	−16.56	−13.39	−8.579	−8.142	−8.444
		−2.085	−1.966	−4.966	−4.925	−4.950
				−0.6651	−0.6596	−0.6557
b^i_{02} (fm^{-1})	2.000	1.582	1.640	1.785	1.922	1.921
	1.418	2.162	2.073	2.029	2.070	2.039
	0.8093	1.282	1.450	1.632	1.579	1.566
		0.6781	0.7075	0.9915	0.9717	0.9720
				0.7000	0.7098	0.7080
c^i_{02} (fm^{-1})	2.000	0.5425	0.4913	1.009	0.9822	0.9793
	1.411	0.5577	0.4840	0.4694	0.4700	0.4662
	1.218	0.7041	0.4821	0.4934	0.4945	0.4967
		1.285	0.7974	0.6596	0.6627	0.6613
				0.9000	0.9000	0.9000
z^i_{02} (fm)	0.0	0.0	0.0	0.0	0.0	0.0
	0.6487	7.0000	0.7000	0.7000	0.7000	0.7000
	1.300	1.300	1.300	1.100	1.100	1.100
		2.000	2.000	1.600	1.600	1.600
				2.400	2.400	2.400

TABLE IV. Parameters of the *D*-wave part of the INOY $^3SD_1$ interactions.

	ITA	ITB	ITC	ITD	ITE	ITF
R_{22} (fm)	1.0	2.0	2.0	2.0	2.0	2.0
a_{22} (fm^{-1})	1.5	2.0	2.0	2.0	2.0	2.0
y_{22} (fm)	1.5	2.0	2.0	2.0	2.0	2.0
V_2 (MeV fm^{-3})	0.0	0.0	0.0	0.0	0.0	0.0
V^i_{22} (MeV fm $^{-3}$)	6000.0	4679.0	4876.0	5010.0	5169.0	5172.0
	339.0	366.2	360.8	435.3	397.0	394.9
	1.959	15.53	15.96	20.36	20.81	21.14
		0.8488	0.8344	1.650	1.685	1.697
b^i_{22} (fm^{-1})	1.500	1.117	1.100	2.000	2.000	2.000
	1.76	2.039	1.960	1.433	1.443	1.444
	1.017	0.9533	0.9215	1.247	1.255	1.252
		0.7613	0.8015	0.9000	0.9076	0.9102
c^i_{22} (fm $^{-1}$)	2.500	1.648	1.652	2.000	2.000	2.000
	2.000	0.6183	0.6337	1.605	1.574	1.585
	1.500	1.752	1.718	1.705	1.714	1.719
		0.9111	0.9084	1.100	1.100	1.100
z^i_{22} (fm)	0.0	0.0	0.0	0.0	0.0	0.0
	0.7182	0.7000	0.7000	0.6900	0.7000	0.7000
	1.382	1.300	1.300	1.300	1.300	1.300
		2.000	2.000	2.000	2.000	2.000

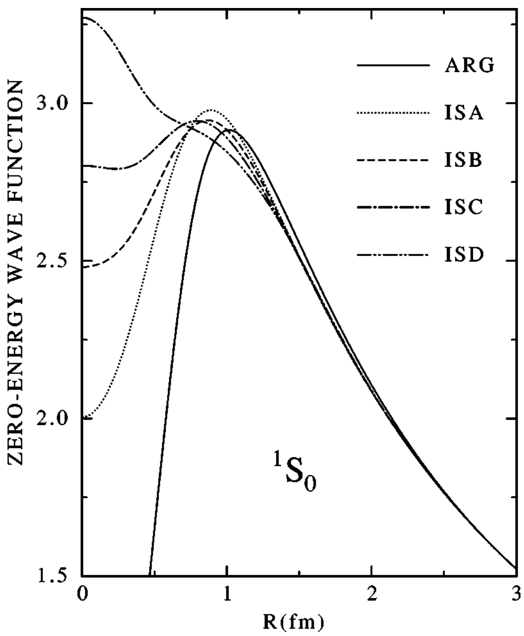

FIG. 1. Zero-energy $^1S_0$ wave functions. ARG denotes the Argonne [5] potential, while ISA, ISB, ISC, and ISD denote different versions of the INOY potentials.

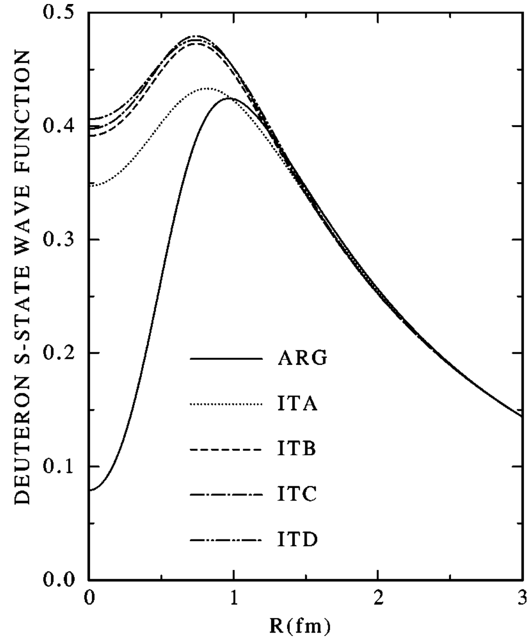

FIG. 2. Deuteron *S*-state wave functions. ARG denotes the Argonne [5] potential, while ITA, ITB, ITC, and ITD denote the different versions of the INOY potentials.

ever, if the deuteron *D*-state probability was forced to be lower, the Yukawa tail had to be cutoff at a larger distance ($R_{ll'}=2.0$ fm) and longer-range nonlocality was necessary to fit other quantities (mostly the mixing parameters). The deuteron binding energy, the asymptotic normalization A_S, and the quadrupole momentum Q_D were fit to 2.224 575 MeV, 0.8848 fm$^{-1/2}$, and 0.275 fm^2 values, respectively. For the quadrupole momentum the 0.275 fm^2 value was chosen, because the meson exchange current contribution of the pions increases this value near the experimental 0.286 fm^2 value. The other asymptotic normalization constant A_D/A_S and the root mean square radius $r_{\rm rms}$ are results without any fit.

It has to be noted that there seems to be a relation between the deuteron *D*-state probability (P_D) and the asymptotic normalization constant A_D/A_S. A decrease of the deuteron *D*-state probability leads to an increase of the A_D/A_S ratio (see Fig. 4). It was stated in Refs. [11,12] that the one-pion exchange tail has a dominant effect on the A_D/A_S value. Indeed, a higher A_D/A_S value at large distances means a lower *D*-state wave function in the $r=3$–4 fm region. This is the manifestation of the strong coupling between the *S*- and *D*-state wave functions generated by the tensor force. Therefore a higher A_D/A_S value makes a lower *D*-state probability feasible: the wave function in the internal region is suppressed. But this dependence is a tendency only; there are important additional parameters to be described. In fact, in Fig. 4 all the INOY interactions are nearly on a common line, while the Argonne v_{18} and Bonn-CD interactions are below this line. The reason could be the lower value for the quadrupole momentum (0.275 fm^2 for the INOY interac-

TABLE V. $^1S_0$ *nn* and *np* scattering lengths and effective ranges.

	ARG	ISA	ISB	ISC	ISD
a_{nn} (fm)	18.487	18.487	18.487	18.487	18.487
r_{nn} (fm)	2.839	2.801	2.801	2.801	2.801
a_{np} (fm)	23.748	23.748	23.748	23.748	23.748
r_{np} (fm)	2.696	2.680	2.679	2.679	2.680

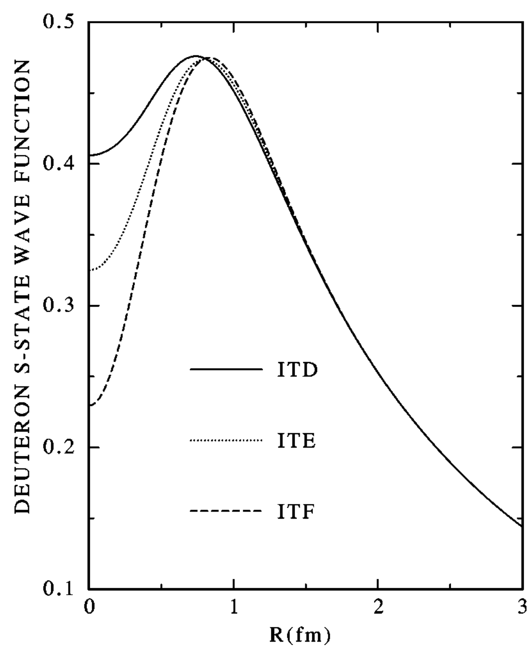

FIG. 3. Deuteron *S*-state wave functions. ITD, ITE, and ITF denote different versions of the INOY potentials with the same deuteron *D*-state probability ($P_D=3.60\%$).

TABLE VI. Deuteron properties, triplet scattering lengths, and effective ranges given by different interactions.

	ARG	ITA	ITB	ITC	ITD	ITE	ITF
ε_D (MeV)	−2.22458	−2.22458	−2.22458	−2.22459	−2.22457	−2.22457	−2.22456
P_D (%)	5.764	5.381	4.202	3.899	3.604	3.601	3.600
Q_D (fm^2)	0.2699	0.2746	0.2748	0.2747	0.2747	0.2752	0.2751
A_S (fm$^{-1/2}$)	0.8851	0.8849	0.8849	0.8849	0.8849	0.8849	0.8848
A_D/A_S	0.02509	0.02571	0.02636	0.02658	0.02678	0.02682	0.02682
$r_{\rm rms}$ (fm)	1.9674	1.9666	1.9654	1.9652	1.9655	1.9658	1.9660
a_t (fm)	5.4192	5.4190	5.4190	5.4190	5.4190	5.4190	5.4190
r_t (fm)	1.7532	1.7538	1.7533	1.7534	1.7538	1.7539	1.7540

tions and 0.270 fm^2 for the Argonne and Bonn-CD potentials), since the Bonn OBEPQ and OBEPR potentials which produce a $Q_D=0.274$ fm^2 lie on the line of the INOY interactions (although these potentials are different in the sense that they do not fit the Nijmegen phase shifts). It indicates that the dominant parameter which labels the P_D-A_D/A_S nearly linear dependence is the quadrupole momentum.

III. TRITON CALCULATIONS

The Faddeev code which was used for solving the bound state problem is based on a separable expansion of the two-body interactions; therefore a sufficiently accurate separable expansion of the ARG and INOY interactions was constructed using the Ernst-Shakin-Thaler (EST) [13] method. To check the effect of the pole terms on the triton binding energy, rank-1 separable expansions were made at $E_1=0.0$ MeV and at $E_1=\varepsilon_D\approx-2.22475$ MeV for the $^1S_0$ and $^3SD_1$ interactions, respectively; i.e., the unitary pole approximation [14] was constructed. Otherwise the EST expansions were performed at zero and negative energy values.

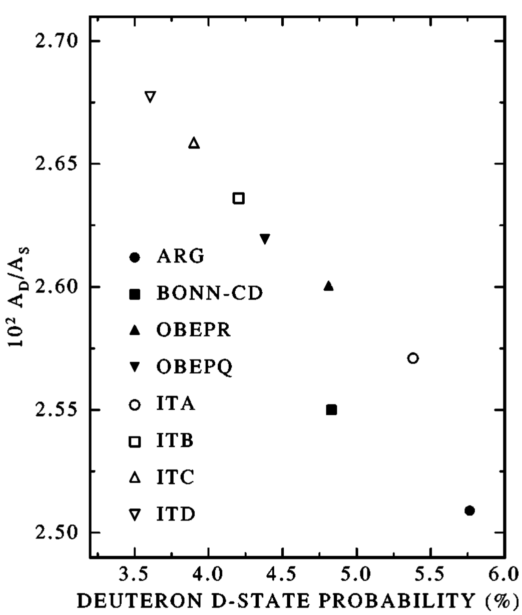

FIG. 4. Dependence of the asymptotic normalization constant A_D/A_S on the deuteron D-state probability.

The on-shell momentum values for the expansion were chosen as the square root of the absolute values of the energy points.

The necessary accuracy of the separable expansion of the original NN interactions was reached with a rank-5 expansion for the $^1S_0$, $^1P_1$, $^3P_0$, $^3P_1$, $^1D_2$, and $^3D_2$ interactions, rank-11 expansion for the $^3SD_1$ interaction, rank-10 expansion for the $^3PF_2$ and $^3DG_3$ interactions, and rank-4 expansion for the $^1F_3$, $^3F_3$, and $^3F_4$ interactions. The effect of the G-wave interactions were checked with rank-4 $^1G_4$ and $^3G_4$ and rank-3 $^3G_5$ expansions. Besides these separable expansions rank-4 $^1S_0$, rank-9, and rank-15 $^3SD_1$ separable expansions were also constructed for checking the accuracy. The EST energy points for the different interactions are shown in Table VII.

The dependence of the triton binding energy on the included partial-wave components is shown in Table VIII. Since the effect of the G-wave interactions is very small, the binding energy values are accurate up to three digits without them. The triton binding energy produced by the Argonne v_{18} potential (although the potential is slightly modified) agrees very well with the reported values (7.62 MeV in Ref. [4] or 7.61 MeV in Ref. [15]).

The triton binding energies for the different $^1S_0$ (ARG, ISA, ISB, ISC, and ISD) and $^3SD_1$ (ARG, ITA, ITB, ITD, ITC, ITD, ITE, and ITF) interactions in the presence of the Argonne P-, D-, and F-wave interactions (but note that the F-H coupling is neglected) are shown in Table IX.

It is evident that the chosen nonlocality of the $^1S_0$ and $^3SD_1$ interactions produces the correct triton binding energy only together. The first row (the $^1S_0$ interaction is the Argonne one) or the first column (the $^3SD_1$ interaction is the Argonne one) of Table IX clearly shows an underbound triton. The calculations with the Argonne $^3SD_1$ interaction produce nearly the same results for local or nonlocal $^1S_0$ interactions (the first column of Table IX). The situation is different if the local $^1S_0$ interaction is fixed (first row of the Table IX): there is a variation of the triton binding energy due to the different deuteron D-state probabilities of the different interactions, although the best result is nearly half an MeV less than the experimental value.

The situation changes significantly if both $^1S_0$ and $^3SD_1$ interactions are nonlocal, although the different INOY $^1S_0$ interactions have varied effect. Depending on the deuteron

TABLE VII. Energy points of the EST separable expansions. For tensor forces the energy points are doubled, except for the bound-state energy (ε_D) point.

	E_i (MeV)							
Rank-4 ARG $^1S_0$	0	−20	−120	−300				
Rank-5 ARG $^1S_0$	0	−15	−50	−150	−350			
Rank-4 INOY $^1S_0$	0	−30	−120	−500				
Rank-5 INOY $^1S_0$	0	−15	−50	−125	−500			
Rank-9 $^3SD_1$	ε_D	−20	−60	−130	−350			
Rank-11 $^3SD_1$	ε_D	0	−20	−60	−130	−350		
Rank-15 $^3SD_1$	ε_D	0	−20	−50	−90	−150	−250	−500
Rank-5 $^1P_1$	0	−12	−55	−125	−250			
Rank-5 $^3P_0$	0	−10	−30	−90	−250			
Rank-5 $^3P_1$	0	−15	−50	−125	−250			
Rank-10 $^3PF_2$	0	−10	−30	−100	−300			
Rank-5 $^1D_2$	0	−20	−75	−250	−800			
Rank-5 $^3D_2$	0	−20	−60	−150	−400			
Rank-10 $^3DG_3$	−5	−20	−50	−150	−400			
Rank-4 $^1F_3$	0	−30	−100	−300				
Rank-4 $^3F_3$ and $^3F_4$	0	−25	−80	−200				
Rank-4 $^1G_4$ and $^3G_4$	0	−25	−75	−160				
Rank-3 $^3G_5$	0	−25	−100					

D-state probability produced by the different INOY $^3SD_1$ interactions, the increase of the singlet S-state zero-energy wave function at an $r=0$ value (see Fig. 1) slightly increases the triton binding energy. However, this is not a one-to-one correspondence: the maximum of the triton binding energy with the ITA interaction is reached with the ISB interaction, while the maximum with the ITD interaction is reached with the ISD interaction (this is the maximum value of the triton binding reached with these interactions).

The triton binding energy depends on the deuteron D-state probability in a well-defined, characteristic way (see Fig. 5). There are two different curves: the lower one is produced by the Argonne v_{18} $^1S_0$ potential, the upper one by the INOY $^1S_0$ and $^3SD_1$ interactions simultaneously. It has to be noted that the triton binding energy produced by the Bonn-CD potential [4] lies on the curve given by the INOY interactions.

The fact that the triton binding energy is predominantly determined by the zero-energy $^1S_0$ and the deuteron bound state wave functions is demonstrated by using a simple unitary pole approximation (UPA) model [14] for the $^1S_0$ and $^3SD_1$ interactions. In this model the form factors of the rank-1 separable interactions are those of the the pole terms (for the singlet interaction the zero-energy wave function produces nearly the pole term because the virtual bound state pole is in the vicinity of the zero-energy value). Performing the triton calculations with these rank-1 separable interactions, the basic features of the results (see in Table X) are practically the same as those obtained with the full interaction. It is not a new result (e.g., [16]), but it was forgotten when the separable expansion method was abandoned for a more accurate handling of the full realistic NN interactions.

IV. SUMMARY AND CONCLUSIONS

The assumption of nonlocality itself does not solve the triton binding energy problem. A wide variation of nonlocal potentials can be constructed without having a noticeable effect on the binding. To be able to reproduce the correct triton binding energy without an additional $3N$ force, a special set of phenomenological INOY NN interactions had to be constructed. It was found, however, that at least for the present interactions a price had to be payed for it: (i) the Yukawa tail, especially the tensor part, had to be cut off at a relatively far distance (between 2 and 4 fm) and consequently a long-range (up to 3–4 fm) nonlocality had to be used, and (ii) a strong off-diagonal attraction centered at $r=0$, $r'\approx 1$ fm had to be built in.

On the whole, the nonlocal interactions used to produce

TABLE VIII. Calculated triton binding energies (in MeV) for various included P-, D-, F-, and G-wave Argonne v_{18} interactions.

	Rank-5 $^1S_0$, rank-11 $^3SD_1$				
		P waves	P, D waves	P, D, F waves	P, D, F, G waves
ARG $^1S_0$, $^3SD_1$	7.446	7.383	7.615	7.618	7.624
ISA $^1S_0$, ITD $^3SD_1$	8.614	8.433	8.488	8.488	8.490

TABLE IX. The calculated triton binding energies (in MeV) with the inclusion of the *S*-, *P*-, *D*-, and *F*-wave interactions. The *P*-, *D*-, and *F*-wave interactions are the Argonne v_{18} potentials.

		$^3SD_1$						
		ARG	ITA	ITB	ITC	ITD	ITE	ITF
	ARG	7.618	7.681	7.878	7.936	8.015	8.034	8.021
	ISA	7.632	7.871	8.255	8.361	8.488	8.473	8.438
$^1S_0$	ISB	7.603	7.874	8.279	8.390	8.524	8.502	8.460
	ISC	7.566	7.863	8.287	8.402	8.542	8.514	8.467
	ISD	7.535	7.846	8.283	8.401	8.547	8.513	8.459

the correct triton binding energy have a relatively simple form: from an attractive nonlocal sea a strong short-range nonlocal repulsion emerges near the origin.

Before a more detailed discussion of the results it has to be emphasized again that all of the interactions used or referred to in the present work (Argonne, Bonn-CD, INOY) are practically on-shell equivalent, since all of them fit the Nijmegen phase shifts. Therefore the effect of the nonlocal interactions on the triton binding energy is an effect of the different off-shell properties of these interactions as compared to the off-shell properties of the local potentials.

The results (Table IX) allow one to make a cautious quantitative statement: in the presented model roughly half of the missing binding energy of the triton, as compared to the local potential model, comes from the low (3.6%) deuteron *D*-state probability (the largest triton binding with the ARG $^1S_0$ interaction is 8.034 MeV with the ITE tensor force), and the other half comes from the structure of the *S*-state wave functions. These features both are connected with the nonlocality of the interactions.

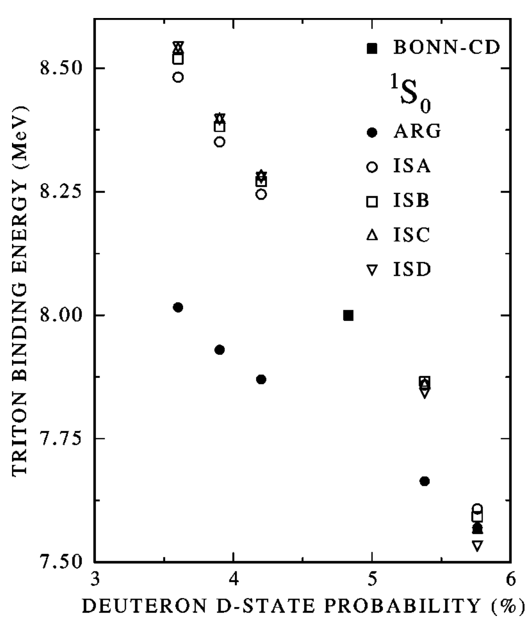

FIG. 5. Dependence of the triton binding energy on the deuteron *D*-state probability. The P_D values define the applied tensor force (see Table VI), except for $P_D = 3.60\%$. Here the ITD interaction is used.

TABLE X. Calculated triton binding energies (in MeV) with rank-1 $^1S_0$ and rank-1 $^3SD_1$ interactions.

		$^3SD_1$						
		ARG	ITA	ITB	ITC	ITD	ITE	ITF
	ARG	7.638	7.840	8.064	8.116	8.165	8.147	8.124
	ISA	7.744	7.986	8.300	8.375	8.448	8.425	8.400
$^1S_0$	ISB	7.744	7.989	8.300	8.390	8.467	8.443	8.413
	ISC	7.739	7.986	8.310	8.397	8.478	8.454	8.423
	ISD	7.732	7.981	8.311	8.399	8.483	8.459	8.428

For separable potentials the dominant dependence of the triton binding energy on the deuteron *D*-state probability has been known for more than 30 years [17], and a similar dependence seems to be valid for the INOY interactions and for the Bonn potentials too. At least, the position of the Bonn-CD potential on the triton binding energy versus deuteron *D*-state probability plot (Fig. 5) strongly supports this statement. The low triton binding energy of the local *NN* interaction also fits into this picture, since all of these potentials produce a deuteron *D*-state probability higher than 5% which seems to lead inevitably to an underbound triton.

All of the sensitivity of the triton binding energy to the off-shell properties of the *NN* interactions is rooted in the pole terms (the deuteron bound state and the zero-energy singlet *S*-state form factors). This is true for all combinations of the presently used interactions (see Table X): in all cases already the pole terms approximately determine the binding energy. In this way the strong correlation between the properties of these pole terms (for example the sensitivity to the deuteron *D*-state probability) and the triton binding energy can be understood.

It is interesting to note without a plausible explanation that the effect of the higher partial-wave components of the *NN* interaction is different for the local Argonne and for the nonlocal INOY interactions (see Table VIII). The mostly repulsive *P*-wave interactions have a significantly larger effect in the case of INOY interactions, while the attractive *D*-wave interactions have a larger effect in the case of the Argonne potential. As a consequence, the overall effect of the *P*-, *D*-, *F*-, and *G*-wave interactions is attractive for the Argon potential, while it is repulsive for the INOY interactions. It was shown [7] that the same effect arises if the *P*- and *D*-wave interactions are also changed to nonlocal ones.

The nonlocality range of the INOY interactions is much larger than nowadays commonly accepted. On the other hand, at present no convincing information seems to exist about the allowed *r*-space range for the nonlocality. If the complete one-pion exchange contribution to the *NN* interaction is already nonlocal [4], then nonlocality in the region of the two-pion exchange is perhaps not entirely infeasible.

In the presented work an attempt was made to define those features which allows the reproduction of the triton binding energy without the introduction of three-nucleon forces. It seems to be evident that for this purpose a nonlocal *NN* interaction is necessary, because (i) the low deuteron *D*-state probability required for the correct triton binding energy could not be produced with a local potential [3,15] (provided the known *NN* data have to be also reproduced), and (ii) the zero-energy singlet and deuteron *S*-state wave functions at small *r* values have to be enhanced, which is possible only with a reasonably strong off-diagonal attraction of a nonlocal interaction. All other tested nonlocality structures (which does not mean that all possible ones were tested) were unable to reproduce the triton binding energy.

It is clear that the nonlocal interactions have to be checked in other processes too. For example, the electromagnetic properties of the deuteron can be studied. Also, there is a feature of the INOY interactions which indicates that an earlier discussion has to be reopened: the connection of the deuteron *D*-state probability and the A_D/A_S ratio (see Fig. 4) indicates that a high value for A_D/A_S is preferable. There were analyses [18] which predicted this A_D/A_S value; nevertheless, later the value of Ref. [19] was accepted. This lower value is more acceptable for the local potentials, which all produce a high deuteron *D*-state probability.

The results of this work do not prove that the *NN* interaction must have the proposed form and that 3*N* force does not exist. Although the INOY interactions are purely phenomenological ones (except for their Yukawa tail), some of their properties seem to be similar to those of the Bonn potentials: (i) the deuteron *S*-state wave functions of the Bonn OBEPQ potential [2] and of the Bonn-CD potential [4] are also enhanced at a small distance, and (ii) the triton binding energy produced by the Bonn-CD potential lies on the same line in the binding energy versus the deuteron *D*-state probability plot (Fig. 5). Therefore it is quite possible that the proposed phenomenological potentials belong to the same family where the Bonn potentials are. Using coordinate space makes the extent of the nonlocality visually more controlled; nevertheless, one has too much freedom defining the potentials and the connection to a more sound theoretical base can be lost. Still, using this freedom the requirement of a low deuteron *D*-state probability was found. It is an open question whether the Bonn model is able to lower even further the deuteron *D*-state probability or whether such an attempt leads to unrealistic parameter values.

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