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Isovector and isoscalar superfluid phases in rotating nuclei

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The subtle interplay between the two nuclear superfluids, isovector $T=1$, and isoscalar $T=0$ phases are investigated in an exactly soluble model. It is shown that $T=1$ and $T=0$ pair modes decouple in the exact calculations with the $T=1$ pair energy being independent of the $T=0$ pair strength and vice versa. In the rotating field, the isoscalar correlations remain constant in contrast to the well-known quenching of isovector pairing. An increase of the isoscalar $(J=1,T=0)$ pair field results in a delay of the band crossing frequency. This behavior is shown to be present only near the $N=Z$ line and its experimental confirmation would imply a strong signature for isoscalar pairing collectivity. The solutions of the exact model are also discussed in the Hartree-Fock-Bogoliubov approximation.

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There is overwhelming evidence that the isovector, $T=1$ pairing field among identical nucleons is an essential component of the nuclear mean-field potential. The bulk of nuclear ground-state properties, such as the odd-even mass differences and the moments of inertia of deformed nuclei, can be accounted for by considering nucleons to be in a superfluid $(T=1,J=0)$ paired phase [1]. These effects have been studied mostly in heavier nuclei with $N>Z$, where the Fermi surfaces of protons and neutrons lie in different major shells.

In recent years, however, due to substantial progress achieved in the sensitivity of the detecting systems it has been possible to study nuclei near the $N=Z$ line in the mass $A=70$ and 80 regions. Furthermore, with the availability of radioactive beams these studies are expected to reach even heavier $N=Z$ nuclei. For these nuclei, one expects the pairing between protons and neutrons to become important, since the Fermi surfaces of both protons and neutrons lie in the same major shell.

The role of the isovector $T=1$ pairing between protons and neutrons in the low-spin regime has been discussed in recent studies [2,3]. The importance of the isoscalar $T=0$ pairing can be inferred from masses [4] and studies of high-spin states [5–8]. However, most of these studies are based on the mean-field approximation which often predicts a transitional behavior for rotating nuclei for the $T=1$ and $T=0$ pair fields as a function of the rotational frequency and the strength of the $T=0$ interaction [4,5].

The purpose of the present Rapid Communication is to examine properties of the isoscalar and isovector correlations within an exactly soluble model of a deformed single-j shell and to compare with the predictions of the mean-field Hartree-Fock-Bogoliubov (HFB) approximation. The observable consequences of the $T=0$ pair field which have remained illusive are also discussed in the present study.

The model Hamiltonian consists of a cranked deformed one-body term and a scalar two-body interaction [9,10]:

$$H'=h'+V_2, \tag{1}$$

where

$$h'=h_{def}-\omega J_x, \tag{2}$$

with

$$h_{def}=-4\kappa\sqrt{\frac{4\pi}{5}}\sum_{ij}\langle j|Y_{20}|i\rangle\delta_{\tau_i\tau_j}\delta_{m_im_j}c_j^\dagger c_i. \tag{3}$$

The labels $i,j,\ldots$ denote the magnetic quantum-number (m) of the j shell and the isospin projection quantum-number τ [$\tau=1/2$ (neutron) and $-1/2$(proton)]. The deformation energy κ is equal to the usual deformation parameter β by $\kappa=0.16\hbar\omega(N+3/2)\beta$ in units of G (Ref. [11]). The two-body interaction in Eq. (1) is given by

$$V_2=\frac{1}{4}\sum_{ijkl}\langle ij|v_a|kl\rangle c_i^\dagger c_j^\dagger c_l c_k$$
$$=\frac{1}{2}\sum_{JMTT_z}E_{JT}A^\dagger_{JM;TT_z}A_{JM;TT_z}, \tag{4}$$

with $A^\dagger_{JM;TT_z}=(c^\dagger_{j\frac{1}{2}}c^\dagger_{j\frac{1}{2}})_{JM;TT_z}$ and $A_{JM;TT_z}=(A^\dagger_{JM;TT_z})^\dagger$. For the antisymmetric-normalized two-body matrix element (E_{JT}), we use the delta interaction which for a single j-shell is given by [12]

$$E_{JT}=-G\frac{(2j+1)^2}{2(2J+1)}\left\{\begin{bmatrix} j & j & J\\ \frac{1}{2} & -\frac{1}{2} & 0\end{bmatrix}^2 +\frac{1}{2}\{1+(-1)^T\}\begin{bmatrix} j & j & J\\ \frac{1}{2} & \frac{1}{2} & 1\end{bmatrix}^2\right\}, \tag{5}$$

where the bracket [ ] denotes the Clebsch-Gordon coefficient.

As mentioned in the introduction, one of the objectives of the present Rapid Communication is to investigate the HFB approximation. In the following, we present some basic HFB formulas, for details see, for instance, Ref. [13]. The HFB equations are given by

$$\mathcal{H}'\begin{pmatrix} U \\ V \end{pmatrix} = E_i'\begin{pmatrix} U \\ V \end{pmatrix}, \tag{6}$$

where

$$\mathcal{H}' = \begin{pmatrix} h' & \Delta \\ -\Delta^* & -(h')^* \end{pmatrix}, \tag{7}$$

with

$$h_{ij}' = \epsilon_{ij}' + \Gamma_{ij}, \tag{8}$$

$$\epsilon_{ij}' = \langle i | h_{def} | j \rangle - (\lambda_p Z + \lambda_n N + \omega m_i)\delta_{ij}, \tag{9}$$

$$\Gamma_{ij} = \sum_{kl} \langle ik | v_a | jl \rangle \rho_{lk}, \tag{10}$$

$$\Delta_{ij} = \frac{1}{2} \sum_{kl} \langle ij | v_a | kl \rangle \kappa_{kl}, \tag{11}$$

$$\rho = V^* V^T, \quad \kappa = V^* U^T = -UV^\dagger. \tag{12}$$

In order to evaluate the angular-momentum dependence of the pair energy, we define the coupled pair-field through

$$\Delta_{ij} = \sum_{JMTT_z} \begin{bmatrix} j & j & J \\ m_i & m_j & M \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & T \\ \tau_i & \tau_j & T_z \end{bmatrix} \Delta_{JT}, \tag{13}$$

with

$$\Delta_{JT} = E_{JT} \sum_{ij} \begin{bmatrix} j & j & J \\ m_i & m_j & m_i + m_j \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & T \\ \tau_i & \tau_j & \tau_i + \tau_j \end{bmatrix} \kappa_{ij}. \tag{14}$$

The pair energy can now be expressed in terms of the coupled pair-fields as

$$E_{pair} = \frac{1}{2} \sum_{ij} \Delta_{ij} \kappa_{ji}^* = \frac{1}{2} \sum_{JT} \frac{\Delta_{JT} \Delta_{JT}^*}{E_{JT}}. \tag{15}$$

The above expression is quite useful since in the exact calculations there is no gap parameter Δ, but one may associate ‘‘E_{pair}’’ with the expectation value of the two-body residual interaction, V_2. To obtain the Δ value from the exact analysis, Eq. (15) is then simply inverted.

The HFB solutions have been obtained by solving Eqs. (6)–(12) self-consistently. In order to treat both the $T=0$ and the $T=1$ pair fields simultaneously, it is necessary to define complex HFB potentials, since the symmetries of the $T=1$ and $T=0$ *n-p* fields are different [14]. The initial complex HFB wave functions have been constructed by using the expressions for real and imaginary V's and U's of the HFB transformation in terms of the pair gaps [14]. We would like to mention that no symmetry restrictions have been imposed on the HFB wave function since it is known that symmetries lead to the exclusion of particular correlations. For more details concerning the HFB transformation in the presence of both $T=1$ and $T=0$ pairing, we refer the reader to Refs. [15,16].

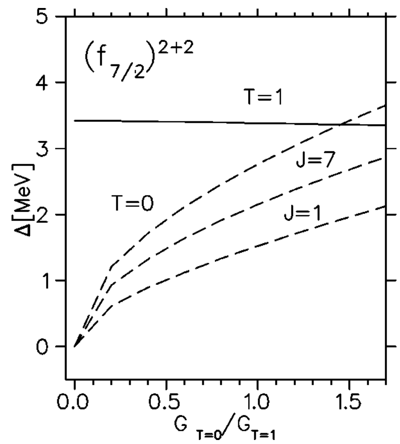

FIG. 1. The single-j exact shell model pairing gaps as a function of the $T=0$ strength for a system with two protons and two neutrons in $f_{7/2}$ shell.

Mean-field studies often show that the $T=0$ and $T=1$ pairing modes are exclusive in the BCS approximation [4,17]. Note, however that mixed solutions have been obtained in recent studies using a complex model space [6,16]. The question therefore arises as to what extent the different pairing modes survive in an exact model. Figure 1 depicts the size of the pair correlations as a function of increasing $T=0$ strength in the exact analysis for the $(2p+2n)$ system. The figure clearly shows that the two modes are essentially independent. There is no critical strength for either pairing mode and therefore one expects to have both modes present in nuclei. It also implies that atomic nuclei exhibit the unique possibility of exhibiting two different pairing condensates simultaneously.

Similar calculations but for the HFB approximation are presented in Fig. 2. For the normal strength, $G_{T=0}=1$, the solution corresponds to a $T=1$ pair field. With increasing $G_{T=0}$ the HFB energy remains constant which is obvious

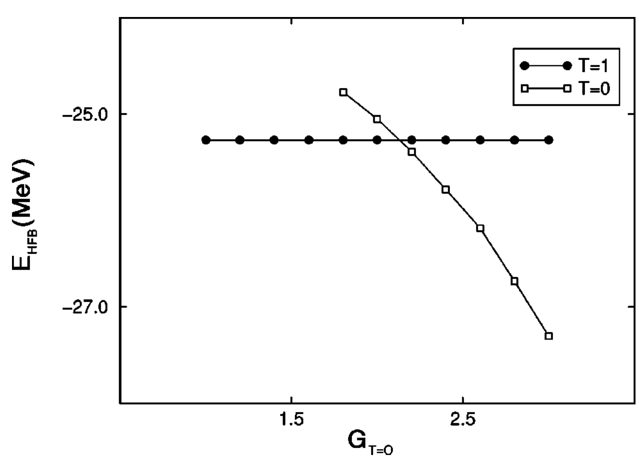

FIG. 2. The HFB pairing energy for four protons and four neutrons as a function of the $T=0$ strength. The label for $T=1$ (0) indicates the symmetry of the HFB pairing field.

since the solution has only the $T=1$ component and there is no $T=0$ component. The $T=0$ solution shown in Fig. 2 has been obtained by solving the HFB equations for a very large value of $G_{T=0}$ ($G_{T=0}=2.8$) and then using this solution for lower values of $G_{T=0}$. In this manner, it was possible to obtain a $T=0$ solution also below the critical point, see Fig. 2. We note from Fig. 2 that the two solutions coexist for most of the $G_{T=0}$ values. They represent two different solutions of the HFB equations. Isospin is of course not preserved in the HFB approximation and the label for $T=0$ (1) in Fig. 2 is merely used to indicate the kind of pairing field obtained in the calculations.

The exact solution, presented in Fig. 1 contains both the $T=0$ and $T=1$ pair modes, whereas HFB gives two solutions that are decoupled, corresponding to either $T=0$ or $T=1$ pair fields. The difference between the two models resides in the fact that in the exact model, the two-body interaction is a scalar whereas in the HFB aproximation, the pairing potential is either a $T=0$ or $T=1$ field, with the corresponding symmetry. Our analysis shows that starting from a certain solution, with a given symmetry, this symmetry propagates to the next solution (with different $G_{T=0}$), analogous to other self-consistent symmetries of the HFB Hamiltonian, see, e.g., the discussion in [15]. The different pair fields appear as independent of each other. Our results further indicate, that for a certain strength of the $G_{T=0}$ pair field, energy can be gained. This conforms with earlier results to associate the Wigner energy with $T=0$ pair correlations [4].

Hence, the reason why the mean-field approximation often avoids mixed solutions relates to several facts. (i) Whereas in the exact model the two fields are fully decoupled, in the BCS approximation they are linked through the number constraint, $N=\Sigma[v_i^2(T=0)+v_i^2(T=1)]$. If one pair-field is increasing, the other has to decrease. (ii) For simple interactions, such as constant G, the system is always choosing the mode that generates the lowest energy, resulting in either $T=0$ or a $T=1$ pairing [7]. In the presence of approximate particle-number projection, the two modes coexist, but only above a critical strength [7,4].

As a next step, we consider the response of the nuclear pair-potential to the rotating fields. In Fig. 3, we show the total Δ-parameter obtained from the pair field of Eq. (15) as well as selected individual (J,T) contribution as a function of the rotational frequency ($\hbar\omega$) for four particles (two protons and two neutrons) in the $f_{7/2}$ shell. First of all, we may note the distinct difference between the $T=1$ and $T=0$ pairing fields. Whereas the $T=1$ field is dominated by one component with $J=0$, the $T=0$ mode is dominated by the $J=1$ and $J=2j$ part of the interaction, but also the intermediate spins $J=3,5$ play a role. This already indicates that a discussion of a pairing force restricted to $L=0$ may be appropriate for the $T=1$ part of the interaction, but not for $T=0$, see also the analogous discussion in Refs. [18,19].

As we increase the rotational frequency, the $T=1$ pairing correlations (solid line) reveal the well-known drop due to particle alignment from the $f_{7/2}$ shell at around $\hbar\omega=0.7$ G.

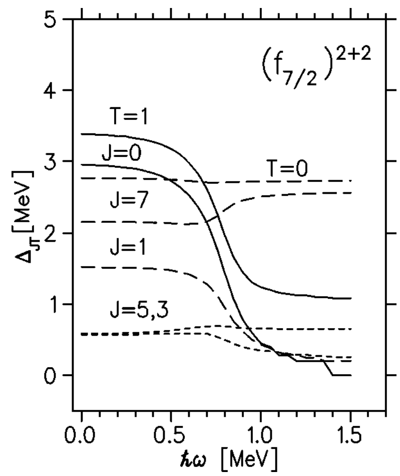

FIG. 3. Behavior of the exact shell model pairing gaps as a function of rotational frequency $\hbar\omega$ for 2+2 particles in the $f_{7/2}$ shell. The solid (dashed) lines represent the $T=1$ ($T=0$) part of the pairing energy. For the case of $T=0$, we show all individual components of the force, clearly demonstrating the importance of the different parts. In contrast, the $T=1$ force is dominated by the $J=0$ part.

At this crossing point, the yrast band changes character from the paired ($J=0$) configuration to the aligned ($J=M_x=6+6$) state.

Similar calculations were also performed for the case of the (4+2) and (4+4) systems. Qualitatively, they all show the same trend, where of course the size of the drop in correlation energy depends on the number of particles present in the single-j shell. For the (4+2) system, the correlations of the $J=0$ component only for one pair disappear whereas the drop for the 4+4 particles is less pronounced. This is due to the fact that only one-proton and one-neutron pair have aligned at the first crossing. Hence, the $J=0$ correlations are still active for the remaining two pairs. For higher frequencies, the next pair will align, and then the $J=0$ (and in consequence) the $T=1$ correlations will drop in a similar fashion as for the system with one-proton and one-neutron pair only. The important message remains, as is evident from Fig. 1, that the $T=1$ field is largely built up from the $J=0$ pair correlations, that are diminished in the process of particle alignment. Although, the components with higher J contribute at higher values of angular momentum, the $T=1$ correlations are strongly reduced by the rotational motion.

In contrast, the $T=0$ correlations evolve quite differently with rotational frequency. The contribution of the coupling to low J, such as the $J=1$ pairs, behave similarly to the coupling to $J=0$. This is quite natural, since they are built up by pairs of $L=0$ and $L=2$. However, although the contribution of the $J=1$ to the $T=0$ correlations drops in a similar fashion as the $J=0$, the value of the total $T=0$ correlations remains essentially unchanged. Apparently, the part that is lost by $J=1$ and $J=3$ is gained by $J=7$ and $J=5$. This implies that the high-J components of the $T=0$ correlations compensate for the loss of the low J. This feature appears to be independent of the number of particles in the system. It means that for a given interaction in the pp-channel, the total

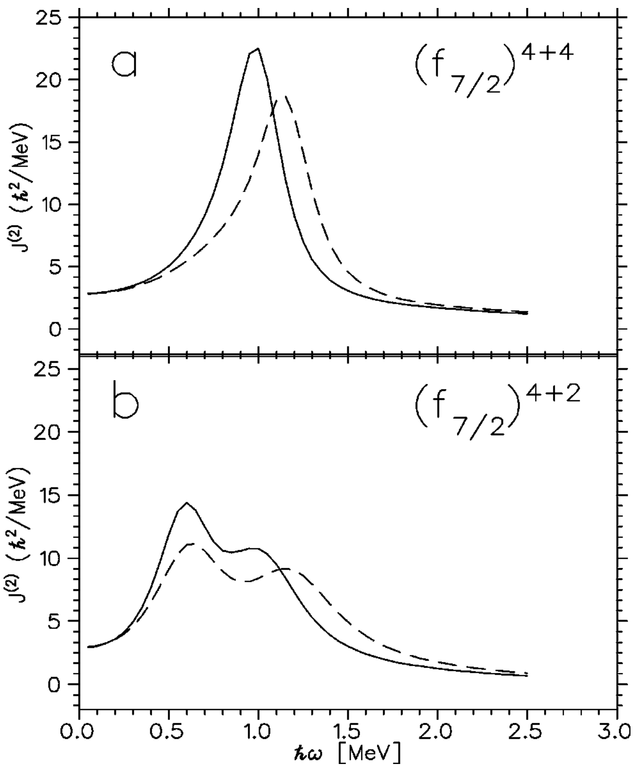

FIG. 4. The dynamical moment of inertia, $J^2 = dI/d\omega$, as a function of frequency. Solid line corresponds to standard single-j shell calculations, whereas the dashed line depicts the case where the $J=1$ part of the interaction is increased by a factor of 2. Note the difference between the 4+4 and the 4+2 systems.

$T=0$ correlations remain almost unaffected by rotation. The presence of increasing L-values in the pairing field will affect deformation properties. This is what one expects in a fully self-consistent approach, which of course is beyond our present model analysis. Note that a recent analysis within the Monte Carlo shell model shows that at high angular momenta, the $T=0$ correlations with $2j$ increase [20].

From the above analysis, one may conclude that the $T=0$ correlations are not able to affect rotational properties, since the increase in the stretched $J=2j$ component is exactly nullified by the decrease of the $J=1$ part, see also the discussion in Ref. [21]. Indeed, these are the results, e.g., for the $f_{7/2}$ shell where one is dealing with a ''single-j'' shell. However, in heavier nuclei, when $Z>28$, the active shell is composed of, e.g., $p_{3/2}$, $f_{5/2}$, $p_{1/2}$, and $g_{9/2}$. For those cases, the $J=1$ part of the $T=0$ interaction becomes more coherent, since every subshell can contribute. In contrast, the $J=2j$ components become fragmented, since they have a different value for each subshell. Therefore, one may expect a different response of the $T=0$ pair field to rotation in heavier nuclei. Since we are dealing in our model with a single-j shell it is not possible to deal with such a case. One may, however, mimic in an ad hoc way the region beyond $Z=28$ by increasing the strength of the $J=1$ part of the interaction.

The effect of a redistributed strength of the $T=0$ correlations, where the $J=1$ part has been increased by a factor of 2, is shown in Fig. 4. Indeed, the crossing frequency is shifted. In other words, a coherence of $J=1, T=0$ pairs results in a change of the crossing frequency. What is even more striking is that this effect is suppressed when $N\neq Z$. In Fig. 4, we also show the case of (2+4) nucleons in the $f_{7/2}$ shell and, indeed, the first crossing frequency remains essentially unchanged. This feature persists also in the HFB approximation. Although our model is highly simplistic, one can certainly conclude that $(T=0, J=1)$ collectivity results in a shift of the crossing frequency to higher values and that this property is expected to be present also in more realistic calculations. Of course, as discussed above, there can be other factors that may affect the crossing frequency, such as the deformation that certainly is influenced by the $T=0$ pairing field [6].

A shift of the crossing frequency has been reported for the case of the $N=Z$ nucleus ^{72}Kr [22]. There have been several attempts to explain this shift of the crossing in terms of $T=1$ np pairing [23]. Assuming that the nuclear force depends only on total $J(T)$ and not on the projection of isospin (T_z), analogous to the assumption that it does not depend on the angular-momentum projection J_z, one finds that the $T=1$ pair gaps are not affected by rotation in isospace [4,21]. In other words, these very basic assumptions imply that $(\Delta_{nn}^2+\Delta_{pp}^2+\Delta_{np}^2)$ is an invariant quantity. In an attempt of Ref. [23] to account for the shift of the crossing frequency, the pairing $T=1$ pair gap was simply adjusted by increasing Δ_{np}^2 from 0 to a value of 2.5 MeV. Such an increase strongly violates isospin symmetry. Following the arguments given above, one could as well increase the nn- or pp-pairing gap. Of course, any increase of the $T=1$ pairing energy will result in a shift of the crossing frequency but this has nothing to do with np pairing.

In summary, we have studied the competition between the $T=0$ and $T=1$ pair fields in an exactly soluble deformed single-j shell model. It is shown that the HFB approach gives rise to two decoupled solutions corresponding to $T=1$ and $T=0$ modes. Although, in the exact shell model analysis, the solution contains both $T=0$ and $T=1$ modes, the two modes are independent with $T=1$ pair energy independent of the strength of the $T=0$ correlations and vice versa. The $T=0$ correlations have a rather complicated structure where the total amount is not affected by rotation. For realistic cases in heavy nuclei $(Z>28)$, with several j-shells, the $J=1$ part will effectively acquire a larger strength. It has been demonstrated that increasing the value of the $(T=0, J=1)$ pair strength results in a shift of the band crossing frequency. Such a shift of the crossing frequency in heavy $N=Z$ nuclei, therefore, may be an indication of the collective $(T=0, J=1)$ correlations.

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