Microscopic description of newly discovered mixed symmetry states

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The mixed symmetry states identified in a recent experiment on ^{94}Mo and successfully described within the interacting boson model (IBM) are studied microscopically within the quasiparticle-phonon model (QPM). The results are in agreement with experiments and consistent with the IBM picture of these states. New branches of the scissors mode are also predicted.

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Low-lying magnetic dipole ($M1$) transitions in heavy nuclei [1] have been under intensive investigations since the experimental discovery [2] of the scissors mode predicted in a semiclassical two-rotor model (TRM) [3] as a mode of rotational oscillation between proton and neutron deformed fluids. The same mode was predicted in a schematic model [4], a sum rule approach [5], and in the proton-neutron version of the interacting boson model (IBM-2) [6,7].

The IBM-2 predicts fully symmetric as well as mixed symmetry states with respect to the exchange between proton and neutron pairs. In the F-spin language [8], the symmetric states have the maximum F-spin value $F=F_{\max}$, while the nonsymmetric ones take smaller values. The scissors is a mixed symmetry state with F-spin quantum number $F=F_{\max}-1$. Also in IBM-2 deformation is essential for its excitation. In fact, the mode is predicted to be excited from the ground state in deformed nuclei [SU(3) limit] and, to a much less extent, in γ-unstable nuclei [0(6) limit]. The excitation of the scissors mode is forbidden in the spherical SU(5) limit [9].

Consistently with the theoretical predictions, systematic experiments [10,11] have ascertained that the mode arises with the onset of deformation and carries a strength which grows quadratically with the deformation parameter [12,13] while its energy remains fairly constant [14,15]. Such a behavior is to be ascribed to the superfluid character of the two rotors. Indeed, the many theoretical investigations have pointed out either explicitly, as, for instance, in Refs. [16,17], or implicitly, as in Refs. [18,19], the crucial role played by the pairing correlations in determining such a deformation law.

In IBM-2 the scissors is one of the several $F_{\max}-1$ mixed symmetry states predicted by the model. All of them are strongly coupled to the F-symmetric ones through enhanced $M1$ transitions. Until recently, however, the experimental information on $F_{\max}-1$ states other than the scissors was rather sparse. $J^{\pi}=2^{+}$ mixed symmetry states were observed in spherical and γ-soft nuclei through measurements of the $E2/M1$ mixing ratio [20,21], hadron scattering experiments [22], the measurements of the electron conversion coefficients in β decay [23] and, with more details, in lifetime measurements [24,25].

Direct and unambiguous evidence in favor of mixed symmetry states was gained only in a recent experiment on ^{94}Mo [26,27]. Such a nucleus was investigated by combining the (γ,γ') experiment on ^{94}Mo with the $\gamma\gamma$-coincidence measurements of transitions following the β decay of ^{94}Tc to ^{94}Mo. The combination of these two techniques has made possible the identification of several new mixed symmetry states with spins $J^{\pi}=1^{+}$, 2^{+}, and 3^{+}. Moreover, it has produced an almost exhaustive mass of information on low-lying levels and absolute transition strengths. All these new data have been analyzed using the IBM-2 and fit remarkably well within such an algebraic scheme. In the present paper we carry a fully microscopic study of the same experimental data within the QPM [28]. The calculation should give more detailed information on the structure of these states and, through a comparative analysis, should explain the impressive success of the IBM-2 approach.

The most general form of the QPM Hamiltonian is

$$H=H_{sp}+H_{\text{pair}}+H_{M}^{ph}+H_{\text{SM}}^{ph}+H_{M}^{pp}. \tag{1}$$

The term H_{sp} describes the motion of the independent nucleons in a self-consistent mean field, H_{pair} represents the proton-proton and neutron-neutron monopole pairing, H_{M}^{ph} is a sum of isoscalar and isovector separable multipole interactions in the particle-hole channel, H_{SM}^{ph} is its spin-multipole counterpart, and H_{M}^{pp} is the sum of the $J\neq 0$ multipole pairing interactions.

In QPM one constructs the RPA phonon operators first, then expresses the separable Hamiltonian in terms of these phonons obtaining a new Hamiltonian composed of a quasiparticle and a RPA phonon terms plus a quasiparticle-phonon coupling piece. One finally diagonalizes this new Hamiltonian in a set of multiphonon states. The phonon operators have the form

$$Q_{\lambda\mu i}^{\dagger}=\frac{1}{2}\sum_{\tau jj'}\{\psi_{jj'}^{\lambda i}[\alpha_{j}^{\dagger}\alpha_{j'}^{\dagger}]_{\lambda\mu}-(-1)^{\lambda-\mu}\varphi_{jj'}^{\lambda i}[\alpha_{j'}\alpha_{j}]_{\lambda-\mu}\}, \tag{2}$$

where $\tau=p,n$ and $\alpha_{jm}^{\dagger}$ (α_{jm}) are creation (annihilation) quasiparticle operators. They fulfill the commutation relations

$$[Q_{\lambda\mu i},Q^{\dagger}_{\lambda'\mu' i'}]_{-}=\delta_{\lambda,\lambda'}\delta_{\mu,\mu'}\delta_{i,i'}-\sum_{\substack{jj'j_2\\ mm'm_2}}\alpha^{\dagger}_{jm}\alpha_{j'm'}\times\{\psi^{\lambda i}_{j'j_2}\psi^{\lambda' i'}_{jj_2}C^{\lambda\mu}_{j'm'j_2m_2}C^{\lambda'\mu'}_{jmj_2m_2}-(-)^{\lambda+\lambda'+\mu+\mu'}\varphi^{\lambda i}_{jj_2}\varphi^{\lambda' i'}_{j'j_2}\times C^{\lambda-\mu}_{jmj_2m_2}C^{\lambda'-\mu'}_{j'm'j_2m_2}\},$$

where $C^{\lambda\mu}_{jmj'm'}$ are Clebsch-Gordan coefficients. The second term, usually neglected in RPA (quasiboson approximation), accounts for the internal fermion structure of phonons and is crucial in QPM in order to enforce the antisymmetrization on the multiphonon components of the states. These QPM states are superposition of one, two, and three RPA phonons [29,30]. The diagonalization of the Hamiltonian (1) in this multiphonon space yields the QPM energies and eigenstates.

The electromagnetic transition operators are also written in terms of quasiparticle and phonon operators and assume the form

$$M(X\lambda\mu)=\sum_{\tau jj'}\frac{\langle j||X\lambda||j'\rangle}{\sqrt{2\lambda+1}}\left\{\frac{u^{(\pm)}_{jj'}}{2}\sum_{i}(\psi^{\lambda i}_{jj'}+\varphi^{\lambda i}_{jj'})\times(Q^{\dagger}_{\lambda\mu i}+(-)^{\lambda-\mu}Q_{\lambda-\mu i})+v^{(\mp)}_{jj'}\times\sum_{mm'}C^{\lambda\mu}_{jmj'm'}(-)^{j'+m'}\alpha^{\dagger}_{j'm'}\alpha_{j'-m'}\right\},\quad(3)$$

where $\langle j||X\lambda||j'\rangle$ is a reduced single-particle transition matrix element and

$$u^{(\pm)}_{jj'}=(u_j v_{j'}\pm v_j u_{j'}),\quad v^{(\pm)}_{jj'}=(u_j u_{j'}\pm v_j v_{j'}),$$

u_j and v_j being the particle occupation amplitudes of the Bogoliubov transformation. The first term in Eq. (3) promotes a one-phonon exchange between initial and final states. The second connects components which differ by zero or, in general, by an even number of phonons. Such a term, usually neglected, is of crucial importance in our context. It gives, in fact, the main contribution to the direct transitions between ground and two-phonon states [29,30].

In our calculation we adopted a Woods-Saxon potential with the parameters taken from [31,32]. The corresponding single-particle spectra for the $A=90$ region can be found in Ref. [33]. The radial component of the multipole fields entering into the particle-hole and particle-particle separable interaction is $f(r)=dU(r)/dr$, where $U(r)$ is the central part of the Woods-Saxon potential. The strength κ_2 and κ_3 of the quadrupole-quadrupole and octupole-octupole interactions were fixed by a fit of the lowest 2^+ and 3^+ levels. The strengths κ_λ of the other multipole pieces were adjusted so as to leave unchanged the energy of the computed two-quasiparticle states [33]. The strength of the spin-spin interaction was adjusted so as to reproduce the position of the $M1$ resonance in ^{90}Zr as given in Ref. [34]. The same strength was adopted for the spin-quadrupole interaction. Only quadrupole and octupole pairing interactions in addition to monopole pairing were used. An unambiguous choice of their strengths is not possible. We chose $G_\lambda=0.8\kappa_\lambda$. This set of parameters was widely used and gave an overall descriptions of the low-lying as well the high-energy states of nuclei in this mass region [33].

TABLE I. Energy and structure of selected low-lying excited states in ^{94}Mo. Additional explanations are given in the text.

J^π	E (keV) Exp.	E (keV) QPM	Structure
2^+_1	871	910	$91\%[2^+_1]_{RPA}$
2^+_2	1864	1912	$77\%[2^+_1\otimes 2^+_1]_{RPA}$
2^+_3	2067	1953	$93\%[2^+_2]_{RPA}$
2^+_4	2393	2819	$9\%[2^+_1\otimes 2^+_2]_{RPA}$
2^+_5	2740	3146	$75\%[2^+_1\otimes 2^+_2]_{RPA}$
4^+_2	1573	1970	$68\%[2^+_1\otimes 2^+_1]_{RPA}$
1^+_1	3129	3079	$91\%[2^+_1\otimes 2^+_2]_{RPA}$
3^+_2	2965	3083	$80\%[2^+_1\otimes 2^+_2]_{RPA}$

Our first step consisted in solving the RPA eigenvalue equations for phonons of multipolarity $\lambda^\pi=1^+,\ 2^+,\ 3^+,\ 3^-,\ 4^+,\ 5^-$, and 6^+. The first and second 2^+_{RPA} states are the most important for our purposes. The first $[2^+_1]_{RPA}$ is isoscalar or, in the IBM language, F-spin symmetric. In fact, the proton and neutron ψ amplitudes [Eq. (2)] of its main component are in phase. Also, it carries a very large $E2$ strength and, therefore, is collective. The second $[2^+_2]_{RPA}$ is slightly collective and isovector or of mixed symmetry with respect to F spin. In fact it carries a much smaller but non negligible $E2$ strength and the proton and neutron amplitudes of its main component are in opposition of phase. A detailed description of the $[2^+_1]_{RPA}$ and $[2^+_2]_{RPA}$ states can be found in Refs. [35,36]. The 1^+_{RPA} states are very few with a strength concentrated mainly around the region of the $M1$ giant resonance (~ 8 MeV). For this reason, they play a minor role in our calculation.

We then proceeded with solving the QPM eigenvalue problem. To this purpose we included all one- and two-phonon states up to 5 MeV. We checked that the high-lying configurations affect very little the structure of the low-lying states up to 3 MeV. Only in the case of 1^+ states, the one-phonon space was extended up to an energy which includes the $M1$ resonance. The tree-phonon states have been ascertained to fall at too high energy and to be completely ineffective. They have therefore been ignored.

The energies of the low-lying QPM states together with the square amplitudes of the components corresponding to the IBM states are given in Table I. The close correspondence with the IBM scheme is to be noticed. Indeed, the first 2^+ state is dominated by the isoscalar $[2^+_1]_{RPA}$ and can be therefore associated to the first 2^+ IBM symmetric state. The second 2^+ is mainly a two-phonon state composed of isoscalar phonons and, therefore, corresponds to the IBM two-phonon symmetric 2^+_2 state. The table shows also its QPM 4^+_2 partner. This corresponds to the measured 4^+_1 in Ref. [26]. Our first, mainly one-phonon, 4^+_1 is of no interest for

TABLE II. $B(E2)$ values for $E2$ transitions connecting some excited states in ^{94}Mo calculated in QPM. The experimental data as well as the IBM results are taken from Refs. [26,27].

$B(E2;i\rightarrow f)(e^2\,\mathrm{fm}^4)$	Exp.	QPM	IBM
$B(E2;\mathrm{g.s.}\rightarrow 2_1^+)$	2030(40)	2066	2333
$B(E2;\mathrm{g.s.}\rightarrow 2_2^+)$	32(7)	21	0
$B(E2;\mathrm{g.s.}\rightarrow 2_3^+)$	230(30)	100	151
$B(E2;\mathrm{g.s.}\rightarrow 2_4^+)$	27(8)	4.5	0
$B(E2;\mathrm{g.s.}\rightarrow 2_5^+)$	83(10)	43	0
$B(E2;2_2^+\rightarrow 2_1^+)$	720(260)	617	592
$B(E2;2_3^+\rightarrow 2_1^+)$	<150	1.0	0
$B(E2;2_4^+\rightarrow 2_1^+)$		142	
$B(E2;2_5^+\rightarrow 2_1^+)$		84	
$B(E2;2_3^+\rightarrow 2_4^+)$		72	
$B(E2;2_3^+\rightarrow 2_5^+)$		572	
$B(E2;4_2^+\rightarrow 2_1^+)$	670(100)	726	592
$B(E2;1_1^+\rightarrow 2_1^+)$	30(10)	76	49
$B(E2;1_1^+\rightarrow 2_2^+)$		9	
$B(E2;1_1^+\rightarrow 2_3^+)$	<690	740	556
$B(E2;3_2^+\rightarrow 2_1^+)$	9^{+25}_{-8}	70	38
$B(E2;3_2^+\rightarrow 2_2^+)$	<100	9	0
$B(E2;3_2^+\rightarrow 2_3^+)$	250^{+310}_{-210} $(1.5^{+1.2}_{-0.6})\times 10^3$	646	371
$B(E2;3_2^+\rightarrow 4_2^+)$	<17	0.18	0

our purposes and was ignored. The third 2^+ is almost entirely an isovector one-phonon state and is, therefore, the counterpart of the first IBM one-phonon mixed symmetry 2_3^+. The first 1_1^+ is mainly a two-phonon state with the mixed symmetry $[2_2^+]_{\mathrm{RPA}}$ phonon on top of the symmetric $[2_1^+]_{\mathrm{RPA}}$, exactly as in IBM. This is a branch of the scissors state. The other branches are the 2_5^+ and 3_2^+ states which have the same phonon content.

The simple structure of the states leads to regularities in the systematic of the $E2$ and $M1$ transition strengths which enforce the consistency with experiments as well as with the IBM scheme. The $E2$ transition probabilities were computed using the same effective charge $e_{\mathrm{eff}}=0.25$ for both protons and neutrons. This value reproduces reasonably well the strength of the $E2$ transition between the ground and the first 2^+ states.

As shown in Table II, the large $B(E2;\mathrm{g.s.}\rightarrow 2_1^+)$ qualifies the first 2^+ as a quadrupole collective state. Such a state is also strongly coupled by the $E2$ operator to the states 2_2^+ and 4_2^+ which contain a large $[2_1^+\otimes 2_1^+]_{\mathrm{RPA}}$ two-phonon component. Equally strong strengths were obtained for the $E2$ transitions between the mixed symmetry 2_3^+ state and the $\{1_1^+,2_5^+,3_2^+\}$ scissors multiplet. As pointed out already, they are all two phonon states characterized by a dominant $[2_1^+\otimes 2_2^+]_{\mathrm{RPA}}$. According to our calculations, in agreement with experiments, the mixed symmetry 2_3^+ is excited from the ground state with a moderately large $E2$ transition probability. One has therefore to expect moderately enhanced $E2$ transitions between the two-phonon $\{1_1^+,2_5^+,3_2^+\}$ scissors multiplet and the symmetric 2_1^+. This is confirmed by our calculations. All the above theoretical results are supported by the experimental data reported so far in the literature.

TABLE III. $B(M1)$ values for $M1$ transitions connecting some excited states in ^{94}Mo calculated in QPM. The experimental data as well as the IBM results are taken from Refs. [26,27].

$B(M1;i\rightarrow f)(\mu_N^2)$	Exp.	QPM			IBM
		$g^s_{\mathrm{eff}}=0.7$	$g^s_{\mathrm{eff}}=0.3$	$g^s_{\mathrm{eff}}=0.0$	
$B(M1;1_1^+\rightarrow \mathrm{g.s.})$	0.16(1)	0.32	0.16	0.08	0.16
$B(M1;1_1^+\rightarrow 2_1^+)$	$0.007^{+0.006}_{-0.002}$	0.01	0.006	0.003	0
$B(M1;1_1^+\rightarrow 2_2^+)$	0.43(5)	1.44	0.78	0.42	0.36
$B(M1;1_1^+\rightarrow 2_3^+)$	<0.05	0.01	0.006	0.003	0
$B(M1;3_2^+\rightarrow 2_1^+)$	$0.01^{+0.012}_{-0.006}$	0.019	0.008	0.004	0
$B(M1;3_2^+\rightarrow 2_2^+)$	$0.24^{+0.14}_{-0.07}$	0.55	0.31	0.17	0.18
$B(M1;3_2^+\rightarrow 2_3^+)$	$0.21^{+0.035}_{-0.014}$ $0.09^{+0.07}_{-0.03}$	0.024	0.01	0.006	0
$B(M1;3_2^+\rightarrow 4_2^+)$	$0.074^{+0.044}_{-0.019}$	0.43	0.2	0.12	0.13
$B(M1;2_2^+\rightarrow 2_1^+)$	0.06(2)	0.033	0.024	0.016	0
$B(M1;2_3^+\rightarrow 2_1^+)$	0.48(6)	0.61	0.36	0.20	0.30
$B(M1;2_4^+\rightarrow 2_1^+)$	0.07(2)	0.09	0.034	0.018	0
$B(M1;2_5^+\rightarrow 2_1^+)$	0.03(1)	0.06	0.02	0.008	0
$B(M1;2_5^+\rightarrow 2_2^+)$		1.97	1.0	0.56	

The QPM $E2$ strengths are in overall good agreement with the IBM values. We get large or small $E2$ strengths for transitions which are respectively strongly allowed or forbidden in IBM. They are also in good agreement with the experimental data. Most of them fall within the experimental errors. Only the moderately large strength of the transition from the ground to the 2_3^+ mixed symmetry state is underestimated. On the other hand, we did not make any effort to improve the agreement by properly choosing, for instance, the neutron effective charge.

The $M1$ transition probabilities were computed for several values of the spin gyromagnetic factors and are shown in Table III. For any value of g_s, we got large $M1$ strengths for transitions between mixed symmetry and symmetric states and small strengths for transitions between states of equal F-spin symmetry. In particular, we obtain large $M1$ strengths for transitions between the one-phonon mixed symmetry 2_3^+ and the symmetric 2_1^+. Equally strong are the $M1$ transitions between the two-phonon $\{1_1^+,2_5^+,3_2^+\}$ scissors multiplet and the symmetric 2_2^+. Of comparable magnitude is the strength of the $M1$ transition between the 3_2^+ and the two-phonon symmetric 4_2^+ which, as has been pointed out, is the partner of the 2_2^+. We can infer from this analysis that the data on $M1$ transitions support the consistency of the QPM scheme with experiments and the IBM. In addition, the QPM predicts a strong $M1$ transition between the scissors branch 2_5^+ and the symmetric 2_2^+ state. No experimental data on such a transition are reported yet in the literature.

It should also be pointed out that there is not a clear cut distinction between mixed symmetry and scissorslike states. Both kinds of states have the same signature, a strong $M1$ coupling with the symmetric states. Moreover, the amplitudes of the $M1$ transitions between the scissors and the 2_2^+ state can be expressed in terms of the amplitude of the transition between the mixed symmetry 2_3^+ and the symmetric

2_1^+ states. The equivalence between the two kinds of states can be proved by the following heuristic argument. We can obtain a mixed symmetry state from a symmetric one by replacing one symmetric quadrupole operator $Q_s = Q_p + Q_n$ with the antisymmetric one $Q_m = Q_p - Q_n$. Such a transformation is induced by the action of scissors operator $S_\mu = J^p_\mu - J^n_\mu$ on symmetric states in spherical or nearly spherical nuclei.

Quantitatively, the $M1$ strengths resulting from using the standard values for the quenching factor ($g_s = 0.6$–0.7) are larger than the experimental ones by a factor of 2 and even 3, in one case. The set of data which approach more the IBM values is obtained if we suppress completely the spin contribution ($g_s = 0$). The best overall agreement with experiments is reached for $g_s = 0.3$. It is to be pointed out that, even for $g_s = 0.6$, the spin contribution is considerably smaller than the orbital one and is about half the orbital strength in the transition $1_1^+ \rightarrow 2_2^+$ for which the largest discrepancy between theory and experiments occurs. The large total $M1$ strengths are the result of a coherent sum between spin and orbital $M1$ amplitudes.

Although both experiments and IBM results point toward a suppression of the spin contribution to the $M1$ transitions at low energy, it is not obvious to identify the appropriate quenching mechanism in our microscopic approach. In fact, the low-lying spin transitions are very sensitive to small components of the wave functions. Since there are several mechanisms which may bring changes on these small components, it is necessary to gain more detailed experimental information on spin excitations in this region in order to find the right solution to the problem.

Apart from the puzzle concerning the spin contribution, the QPM results are consistent with experiments and confirm the IBM predictions. By using only two out of the many RPA states, namely the isoscalar $[2_1^+]_{\mathrm{RPA}}$ and the isovector $[2_2^+]_{\mathrm{RPA}}$, we have obtained a set of states which can be classified according to the IBM scheme. More specifically, we have a set of F-spin symmetric states connected among themselves through strong $E2$ transitions and a set of mixed symmetry and scissorslike states coupled to the symmetric ones through strong $M1$ transitions. In addition to the IBM results presented in Refs. [26,27], our calculation predicts a 2_5^+ state as a new branch of the two-phonon scissors multiplet. Such a prediction is not yet supported by the experiments reported in the literature.

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[1] N. Lo Iudice, Prog. Part. Nucl. Phys. **28**, 556 (1997), for a review and references.
[2] D. Bohle, A. Richter, W. Steffen, A.E.L. Dieperink, N. Lo Iudice, F. Palumbo, and O. Scholten, Phys. Lett. **137B**, 27 (1984).
[3] N. Lo Iudice and F. Palumbo, Phys. Rev. Lett. **41**, 1532 (1978).
[4] R.R. Hilton, Z. Phys. A **316**, 121 (1984).
[5] E. Lipparini and S. Stringari, Phys. Lett. **130B**, 139 (1983).
[6] F. Iachello, Nucl. Phys. **A358**, 89c (1981).
[7] A.E.L. Dieperink, Prog. Part. Nucl. Phys. **9**, 121 (1983).
[8] T. Otsuka, A. Arima, and F. Iachello, Nucl. Phys. **A309**, 1 (1978).
[9] P. van Isacker, K. Heyde, J. Jolie, and A. Sevrin, Ann. Phys. (N.Y.) **171**, 253 (1986), and references therein.
[10] A. Richter, Prog. Part. Nucl. Phys. **34**, 261 (1995), and references therein.
[11] U. Kneissl, H.H. Pitz, and A. Zilges, Prog. Part. Nucl. Phys. **37**, 349 (1996), and references therein.
[12] W. Ziegler, C. Rangacharyulu, A. Richer, and C. Spieler, Phys. Rev. Lett. **65**, 2515 (1990).
[13] C. Rangacharyulu, A. Richter, H.J. Wörtche, W. Ziegler, and R.F. Casten, Phys. Rev. C **43**, R949 (1991).
[14] N. Pietralla, P. von Brentano, R.-D. Herzberg, U. Kneissl, N. Lo Iudice, H. Maser, H.H. Pitz, and A. Zilges, Phys. Rev. C **58**, 184 (1998).
[15] J. Enders, H. Kaiser, P. von Neumann-Cosel, C. Rangacharyulu, and A. Richter, Phys. Rev. C **59**, R1851 (1999).
[16] I. Hamamoto and C. Magnusson, Phys. Lett. B **260**, 6 (1991).
[17] K. Heyde and C. De Coster, Phys. Rev. C **44**, R2262 (1991).
[18] J.N. Ginocchio, Phys. Lett. B **265**, 6 (1991).
[19] L. Zamick and D.C. Zheng, Phys. Rev. C **44**, 2522 (1991).
[20] W.D. Hamilton, A. Irback, and J.P. Elliott, Phys. Rev. Lett. **53**, 2469 (1984).
[21] G. Molnár, R.A. Gatenby, and S.W. Yates, Phys. Rev. C **37**, 898 (1988).
[22] R. De Leo *et al.*, Phys. Rev. C **53**, 2718 (1996).
[23] A. Giannatiempo, A. Nannini, A. Perego, P. Sona, and D. Cutoui, Phys. Rev. C **53**, 2718 (1996).
[24] P.E. Garrett, H. Lehmann, C.A. McGrath, Minfang Yeh, and S.W. Yates, Phys. Rev. C **54**, 2259 (1996).
[25] N. Pietralla *et al.*, Phys. Rev. C **58**, 796 (1998).
[26] N. Pietralla *et al.*, Phys. Rev. Lett. **83**, 1303 (1999).
[27] N. Pietralla, C. Fransen, P. von Brentano, A. Dewald, A. Filzler, C. Friesner, and J. Gableske, Phys. Rev. Lett. **84**, 3775 (2000).
[28] V.G. Soloviev, *Theory of Atomic Nuclei: Quasiparticles and Phonons* (IOP, Bristol and Philadelphia, 1992).
[29] M. Grinberg and Ch. Stoyanov, Nucl. Phys. **A535**, 231 (1994).
[30] V.Yu. Ponomarev, Ch. Stoyanov, N. Tsoneva, and M. Grinberg, Nucl. Phys. **A635**, 470 (1998).
[31] V.A. Chepurnov, Sov. J. Nucl. Phys. **6**, 955 (1967).
[32] K. Takeuchi and P.A. Moldauer, Phys. Lett. **28B**, 384 (1969).
[33] S. Gales, Ch. Stoyanov, and A.I. Vdovin, Phys. Rep. **166**, 125 (1988).
[34] E. Lipparini and A. Richter, Phys. Lett. **144B**, 13 (1984).
[35] R. Nikolaeva, Ch. Stoyanov, and A.I. Vdovin, Europhys. Lett. **8**, 117 (1989).
[36] M. Grinberg, Ch. Stoyanov, and N. Tsoneva, Fiz. Elem. Chastits At. Yadra **29**, 1456 (1998).