Model ingredients and multifragmentation in symmetric and asymmetric heavy ion collisions

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The relative role of the momentum-dependent interactions and larger nucleon-nucleon cross section is discussed in multifragmentation using quantum molecular dynamical model. We find that the sensitivity of the larger cross section towards multifragmentation reduces in the presence of momentum-dependent interactions which makes it difficult to extract the magnitude of nucleon-nucleon cross section from multifragmentation. However, a large effect of different cross sections can be seen if a simple static equation of state is used.

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I. INTRODUCTION

The properties of nuclear matter at extreme conditions of density and temperature (i.e., the excitation energy) have been under study for the last couple of decades. It has been found in the experimental studies that the colliding nuclear matter can break into a large number of light particles and intermediate mass fragments (MF) depending upon the incident energy and the mass of the system $[1,2]$. This field (dubbed multifragmentation) is among a few branches which are aspirants of investigating the properties of nuclear matter at extreme conditions $[1-11]$. The multifragmentation has been studied using a variety of nuclei and over a wide domain of the incident energy. The experiments carried out for multifragmentation can be divided into symmetric and asymmetric reactions. The former leads to higher compression whereas the latter lacks the compression and therefore, a large part of the excitation energy is in the form of thermal energy $[1,2]$. One of the early attempts to study the multifragmentation was by Jakobsson et al . $[1]$ who measured the charge particle distribution (along with their kinetic energy spectra) in $16O/36$ Ar induced reaction between 25 MeV/ nucleon and 200 MeV/nucleon representing the fusion, fission, particle emission, and multifragmentation. These experiments make a stringent test for any theoretical model designed for multifragmentation $[1]$.

Theoretically, the semiclassical dynamical models are very useful in studying the evolution of the reaction and also extracting knowledge about the equation of state and nucleon-nucleon cross section $[3-14]$. In fact, the interaction among the nucleons (in a heavy ion reaction) can be studied within the *G* matrix where the real part of the *G* matrix represents the mean field and if the excitation energy is large enough (to allow on-shell real scattering), the G matrix becomes complex with the imaginary part defining the nucleon-nucleon cross section $[3,4,14,15]$. One should, therefore, use the same theory consistently to calculate the propagation and nucleon-nucleon collisions which takes care of the nucleon-nucleon correlations, density, and the environment of the medium. In other words, the mean-field (potential) and the cross section are related and cannot be changed independently. The potential is proportional to the gradient of the real part of the Bruckner *G* matrix whereas the cross section is proportional to $|G|^2$ [3,4,14,15].

There are, however, a few problems which forbid the use

of a full self-consistent *G* matrix in multifragmentation: First, the nuclei propagating under the influence of the *G* matrix are stable only for typical times of 60 fm/*c*, therefore, at present the *G* matrix is not adequate for studying the multifragmentation $[15]$. Second, a fully reliable self-consistent theory is still not in hand. The Bruckner Hartree-Fock theory cannot reproduce the saturation properties of the static nuclear matter and further the change in the nucleon-nucleon cross section in medium is not clear $[3]$. As noted in Ref. $[3]$, even the exact direction of the nucleon-nucleon cross section in medium is not known. Therefore, one often uses the parametrized forms both for the real and imaginary parts of the *G* matrix. It is now well accepted to use a density-dependent Skyrme-type interaction in place of the real part of the *G* matrix. By varying the parameters in the Skyrme parametrization, one can study the different kinds of equations of state. This point of view, however, has not been very successful mainly due to fact that the heavy ion dynamics depends not only on the static equation of state but also on the mean-field potentials in the whole ρ and p plane which, in general, is density and momentum dependent $[3,4,14,15,12,16]$. Though the static (momentumindependent) Skyrme potential is able to explain several observables in heavy ion collisions, the use of the momentumdependent interaction cannot be neglected primarily due to fact that in a typical heavy ion collisions, one observes a picture of two interpenetrating nuclei and as soon as the projectile and target begin to overlap, the interaction takes place among nucleons of different nuclei which have large relative momentum $[15,16]$. Naturally, due to large relative momentum, the projectile nucleons in the central collision region feel a strong repulsion from the neighboring target nucleons which can be represented by the self-consistent *G* matrix or by the momentum-dependent interactions. It is now well established that the momentum-dependent interactions affects the nuclear dynamics strongly. The parametrized form of the momentum-dependent interactions can be obtained by a fit to the nucleon optical potential on experimental data $[3,4,13]$.

The nucleon-nucleon cross section, on the other hand, has been of great interest in recent years. In principle, this should be derived from the *G* matrix by solving the Bethe-Goldstone equation. At low energies, the Pauli blocking of the intermediate energies plays a role, whereas the medium cross section tends towards the free nucleon-nucleon cross section at higher energies. The problem with the in-medium

nucleon-nucleon cross section is that its size and direction is a controversial issue $[3]$. There are reports where a decrease as well as increase in the cross section is found in the medium environment. In view of this, a large number of calculations (and comparison with experiments) exist where several different varieties of nucleon-nucleon cross sections are used $[17]$. In a simple assumption of a hard-core radius of the nucleon-nucleon potential, one has often used a constant and isotropic cross section of 40 mb. Other calculations take an isotropic and constant cross section with magnitude between 20 and 55 mb. One has even suggested to use either a simple rescaling of the free nucleon-nucleon cross section or some form which depends on the density of the medium $[17]$.

Among different candidates, the disappearance of the flow (and the balance energy) has been found to be a good candidate for extracting the in-medium nucleon-nucleon cross section $[17]$. The method of comparing the calculations (with some assumed value and form of the nucleon-nucleon cross section) with experiments to extract the magnitude of the cross section suffers major problem because both the equation of state and nucleon-nucleon cross section are interrelated and also most of the observables depend on both. To give an example, most of the calculations reported for extracting the magnitude of nucleon-nucleon cross section use a static equation of state $[17]$. From the above discussion, it is clear that the momentum dependence of the equation of state is essential for a proper potential and in addition, its effect is not at all negligible in some of the observables. We discuss this point further by taking multifragmentation as an example where not many results are available on different nucleon-nucleon cross sections and momentum dependence of the equation of state.

As noted in Refs. $[3-5,8,10]$, a larger nucleon-nucleon cross section and momentum dependence of the equation of state has a strong role to play if the system is mildly excited. This study was carried out for symmetric collisions only. Now one is interested to see how different forms of the nucleon-nucleon cross section (e.g., the popular energy dependent, in-medium, and constant cross sections) affect the fragmentation in the case of asymmetric reaction where a large part of the energy is in the form of the thermal energy. Second, up to now, we and others have studied the effect of a larger nucleon-nucleon cross section in the presence of a static equation of state only. One is, naturally, tempted to understand the behavior of the dynamics of a reaction in the presence of a larger cross section and momentum dependence of the equation of state at the same time. The point to note here is that both the momentum-dependent interactions and larger nucleon-nucleon cross section tend to destroy the initial correlations among nucleons. This will naturally result in a further decrease in the fragments yield in the central collision (due to the fact that most of the correlations are already destroyed) whereas it will enhance the production at peripheral collisions (because the simple spatial correlation method is not able to detect the different closely placed or overlapping fragments). As noted above, the momentum dependence of the equation of state is essential, therefore, naturally one would like to know whether the response of different nucleon-nucleon cross sections remains the same in the

presence of the momentum-dependent equation of state or not. Further, we can use the multifragmentation to extract the magnitude of the nucleon-nucleon cross section or not. We plan to discuss the above questions by taking both the symmetric as well as asymmetric reactions into account. We use the well accepted *N*-body quantum molecular-dynamical (QMD) model to calculate the propagation of the nucleons and the clusterization will be made within the simple spatial correlation method where two nucleons share the same fragment if their centroids are closer than 4 fm [3,4,14]. This method is also known as the minimum spanning tree (MST) method. It is worth mentioning that the response of different forms (and sizes) of cross section also depends on the method of clusterization one is using. We first discuss briefly the QMD model and then present our analysis. The details of the model and its various different versions can be obtained from Refs. $[3,4,14]$.

II. THE MODEL

The nucleons in the molecular-dynamics picture interact via two- and three-body interactions. The explicit two- and three-body interactions lead to the preservation of fluctuations and correlations which are important for *N*-body phenomena like multifragmentation. This is in contrast to onebody dynamical models which are good for studying the onebody observables only $[3,4,14]$.

In the QMD model, nucleons are represented by the Gaussian distribution and the equation of motion for their centriods $(\mathbf{r}_i \text{ and } \mathbf{p}_i)$ is given by

$$
\frac{d\mathbf{r}_i}{dt} = \frac{dH}{d\mathbf{p}_i},\tag{1}
$$

$$
\frac{d\mathbf{p}_i}{dt} = -\frac{dH}{d\mathbf{r}_i},\tag{2}
$$

where the Hamiltonian is given by

$$
H = \sum_{i} \frac{\mathbf{p}_i^2}{2m_i} + V^{\text{tot}},\tag{3}
$$

with

$$
V^{\text{tot}} = V^{\text{loc}} + V^{\text{Yuk}} + V^{\text{Coul}} + V^{\text{MDI}}.\tag{4}
$$

Here V^{loc} , V^{Yuk} , V^{Coul} , and V^{MDI} , represent, respectively, the Skyrme, Yukawa, Coulomb, and momentum-dependent (MDI) parts of the interaction. The interaction without the MDI part is called the static interaction. The different values of the compressibility in the Skyrme force give the possibility of looking for the role of different equations of state termed soft and hard equations of state. The inclusion of the momentum-dependent interactions are labeled as soft momentum-dependent (SMD) and hard momentumdependent interactions (HMD), respectively.

The *G* matrix at higher excitation energies becomes complex in nature and its imaginary part acts like the collision term. We use here the energy-dependent cross section fitted by Cugnon and also the in-medium cross section derived from the *G* matrix. The limits of the possible size of the cross section will also be discussed by taking an isotropic and constant cross section with the magnitude between 20 and 55 mb. For the discussion on the different cross section in the present context, we refer the reader to Ref. $[8]$.

III. RESULTS AND DISCUSSION

We present here the analysis for symmetric and asymmetric reactions. One of the complete measurements for asymmetric reactions was done by Jakobsson *et al.* [1] who measured the charge distribution of all kinds of fragments for incident energies between 25 and 200 MeV/nucleon. This energy domain provides the possibility of fusion, fission, and total disassembly of the matter. We, therefore, also compare our results with the emulsion data. As noted by several authors, the emulsion experiments do not have very high statistics. In addition, the lower threshold of the fragment detection is 0.5 MeV which also makes the exact measurements more difficult. We do not apply any cut in our calculations, and therefore the calculated protons may overestimate the observed number. We have also checked the stability of the nuclei propagating under the momentumdependent interactions. The nuclei within MDI start (artificial) emission of nucleons after $150 \text{ fm}/c$. Whereas, the emission of IMF's is unaffected by the destabilization of the nuclei with MDI. Therefore, this comparison should, at least, give us the trend of the relative role of the momentumdependent interaction and larger cross section in the presence of each other. The emulsion analysis selects the highest 10% events which represents the central collisions. It has been shown by a number of authors that the multiplicity of the light charge particles and IMF's remains constant for the central impact parameters $[4,9]$. In view of these calculations [4,9], we use $b=0$ for the reaction of O+Ag and O+Br. As far as symmetric reactions are concern, we shall examine the reaction of $Xe+Sn$ at $b=8$ fm and at 400 MeV/nucleon. Note that the maximum effect of the larger cross section and momentum-dependent interaction was obtained for this reaction and hence it is an ideal case to study the comparative role of larger cross section and momentum-dependent interactions. In a recent report, the comparison of the ALADIN experiment with QMD $(+\text{MST})$ revealed that the model underestimates the fragments drastically at peripheral collisions [2]. Therefore, the present symmetric reaction will be also helpful to understand more about the above discrepancy.

As discussed by us and others, the density of the system plays a crucial role in deciding the fate of a reaction. We define the average density of reaction as

$$
\langle \rho \rangle = \left\langle \frac{1}{A_T + A_P} \sum_{i=1}^{A_T + A_P} \sum_{j=1}^{A_T + A_P} \frac{1}{(2 \pi L)^{3/2}} e^{-(\mathbf{x}_i - \mathbf{x}_j)^2 / 2L} \right\rangle, \quad (5)
$$

with \mathbf{x}_i and \mathbf{x}_j being the coordinates of *i*th and *j*th nucleons, respectively. In Fig. 1, we display the density for the system of $O+Br$ and $O+Ag$ at the incident energy of 50 and 200 MeV/nucleon, respectively. Due to larger compression, a higher density is reached at 200 MeV nucleon compared to 50 MeV/nucleon. Whereas, due to a large number of collisions at 200 MeV/nucleon, the final-state density is much

FIG. 1. The average density as a function of time. The left part is for $O+Br$ whereas the right part is for $O+Ag$ reactions. The upper and lower parts are at 50 and 200 MeV/nucleon, respectively. We display the results of simulations with soft and SMD equations of state with the Cugnon cross section and a constant cross section of 55 mb. The results with soft+Cug, SMD+Cug, soft+55 mb, and $SMD+55$ mb are displayed, respectively, with short dashed, solid, long dashed, and dash-double-dotted lines.

smaller than at 50 MeV/nucleon. Interestingly enough, the densities with Cugnon and 55 mb cross sections at 50 MeV/ nucleon are nearly the same. The inclusion of momentumdependent forces leads to reduction in the density. Naturally, due to the Pauli principle, almost all attempted collisions at 50 MeV/nucleon are blocked and thus a larger cross section does not make any difference. In contrast, due to the frequent nucleon-nucleon collisions at 200 MeV/nucleon, a larger cross section shatters the nuclear matter more and thus leads to lesser final-state density. Therefore, the maximal compression is decided by the nature of equation of state (i.e., by the equation of state with or without momentum-dependent interaction) rather than by the magnitude of nucleon-nucleon cross section. If one correlates the final-state density with the formation of fragments, one would expect that the soft interactions (with and without larger σ) should give different results compared to SMD (with and without collision) at 50 MeV/nucleon, whereas some effects of larger nucleonnucleon cross section on fragmentation should be visible at higher energies.

In Figs. 2 and 3, the time evolution is displayed for the collisions of $O+Br$ and $O+Ag$, respectively, at 50 and 200 MeV/nucleon. The displayed quantities are the heaviest fragment A^{max} , the light mass fragments (LMF's) $2 \le A \le 4$ and intermediate mass fragments $(MF's)$ which is defined as 5 $\leq A \leq 48$ for O+Br and $5 \leq A \leq 62$ for O+Ag reactions. We have also checked the time evolution of the mass distribution (not shown here) for the collisions of $O+Br$ and $O+Ag$, respectively, at three times, i.e., 60, 100 and 200 fm/*c*. The 200 fm/*c* is taken as freeze-out time in view of the fact the nuclei, generated in molecular-dynamical model, are no longer stable after 200 fm/*c*. In some cases, the multiplicity of the fragments continues to change even at 200 fm/*c*. In these cases, the reaction takes longer time. We in fact do not

FIG. 2. The time evolution of A^{max} , light mass fragments LMF's $(2 \leq A \leq 4)$ and heavy mass fragments IMF's ($5 \leq A \leq 48$) produced in the O+Br reaction at 50 MeV/nucleon (left part) and 200 MeV/ nucleon (right part) using soft and SMD with Cug and 40 mb cross sections, respectively. The results with $SMD+55$ mb are also displayed for intermediate mass fragments.

know whether the contribution to the mass yield after 200 fm/*c* is a real one or just spurious yield due to the destabilization of fragments. Therefore, we stop the reaction at 200 fm/*c*. Our calculations of the mass yield predicts two component shape with static equation of state (and larger cross section). This shape reduces to a greater extent once the momentum-dependent interactions are taken into picture. From Figs. 2 and 3, one can clearly see two distinguished patterns which emerges at 50 MeV/nucleon (i) with the static equation of state and (ii) with momentum-dependent equation of state (EOS). As expected, the role of nucleon-nucleon collisions is negligible at 50 MeV/nucleon. On the average, less than one collision per nucleon takes place at 50 MeV/ nucleon. Naturally, the static equation of state preserve the correlations between nucleons and therefore, the heavier *A*max is obtained in static EOS compared to SMD. As a result, lesser emission of LMF's and IMF's follows in soft EOS compared to SMD. One cannot neglect the importance of momentum-dependent interactions at low incident energies. An entirely different scenario can be seen once we shift to higher incident energies $(E=200 \text{ MeV/nucleon})$ where nucleon-nucleon collisions are quite important and the magnitude of different nucleon-nucleon cross sections plays a vital role. Now the average collision experienced by each nucleon is about 2 and the effect of the larger cross section is more prominant.

As expected, the heaviest fragment is detected in a static

FIG. 3. Same as Fig. 2, but for the $O + Ag$ reaction. Note that now IMF's are defined with masses $5 \leq A \leq 62$.

soft equation of state with Cugnon cross section which is followed by the soft+40 mb/SMD+Cug. The lightest A^{max} is obtained with $SMD+40$ mb. The difference in A^{max} with static EOS and all other combinations is larger compared to the later one. The above effect is also reflected in the formation of IMF's which follows the same trend like *A*max. The soft EOS generates much less IMF's compared to SMD $SMD+40$ mb/soft+40 mb which yield nearly the same numbers of IMF's close to 2.5 in the case of $O+Br$ and 3 in the case of the $O+Ag$ reaction. An additional extra large nucleon nucleon cross section $(=55 \text{ mb})$ does not alter the results appreciably. As reported in Ref. $[11]$, the measured multiplicity of IMF's in $O + Br/Ag$ is 5.6. The use of MDI+large cross section at least reduces the disagreement with experimental data to a larger extent (the MDI $+55$ mb generates 2.5 and 3.5 IMF's, respectively, in $O+Br$ and $O+Ag$ reaction which is much larger to the IMF's produced with the static+Cug. option which yields 1.6 and 1.2 IMF's in $O+Br$ and $O+Ag$ reactions, respectively). The above picture is not valid for LMF's where $SMD+Cug$. yields different LMF's compared to $SMD+40$ mb. The former yield two LMF's less than the latter one. One has to keep in mind that the LMF's are produced during the collision whereas IMF's are the remnants of the target-spectator matter. Therefore, the different magnitude of the nucleon-nucleon cross section will yield different light mass fragment production. This argument is also evident from the saturation time of IMF's and LMF's. The saturation time for IMF's is about 100 fm/*c* which is less compared to LMF's which is about 150 fm/*c*. At least for the production of IMF's, the role of momentumdependent interactions is unique at both incident energies. The inclusion of MDI leads to enhancement in the production of IMF's. One does not expect the different forms of *NN* cross section to influence the results at low incident energies. At higher incident energies, the role of the nucleon-nucleon cross section depends on the equation of state. If one has the static equation of state, a larger effect of different *NN* cross sections can be seen on the multiplicity of intermediate mass fragments whereas if the momentum-dependent interactions are taken into account, the above effects are either negligible or their role is reduced. The apparent cause seems to be that both the momentum-dependent interactions and the larger nucleon-nucleon cross section act in the same direction, i.e., they tend to destroy the initial correlations among nucleons which are very important for multifragmentation. Once the MDI has broken the system into IMF's and has destroyed the initial correlations, the additional *NN* cross section does not yield much different results. It is important to note that this picture is valid when the system is mildly excited.

In the absence of nucleon-nucleon collisions (in a Vlasov mode), even the head on collision ($b=0$ fm) yields two nuclei passing each other, because all initial correlations are preserved in a Vlasov mode. The inclusion of MDI deflect the particles in the transverse direction during the initial phase of the reaction yielding a large flow. In a similar manner, a larger cross section yields more collisions and more destruction of the initial correlations, thus, yields also larger flow. In recent studies on disappearance of flow, the balance energy (the energy at which attractive and repulsive interactions balance each other and flow disappears) was found to be sensitive towards both the momentum-dependent interactions and towards larger cross section $[17]$. On the other hand, the use of static EOS leads to different spectrum once a larger cross section is taken into account. Here the inclusion of larger cross section yields the same effects as the inclusion of momentum-dependent interactions. The picture is different for LMF's which are produced during the collision where any additional effect in terms of cross section or MDI yields 20% different results. We have also checked the average binding energy of LMF's and IMF's produced with soft, soft+40 mb, SMD and SMD+40 mb. In all cases, the LMF's have a typical binding energy of -4 MeV/nucleon whereas IMF's have binding energy of the order of -6 to -7 MeV/nucleon indicating that the fragments are properly bound.

The above discussion is further strengthened by taking the peripheral collision of $Xe+Sn$ at 400 MeV/nucleon. In Fig. 4, we display the *A*max, free nucleons along with light, medium and intermediate mass fragments. We see a similar trend like the one obtained for the case of the asymmetric reaction. In contrast to $O + Br$ (which is less asymmetric compared to $O+Ag$, here we see the appreciable effect of the large cross section in the presence of both the static equation of state and the momentum-dependent interactions. One of the causes for this appreciable effect seems to be the larger density in $Xe+Sn$ collisions compared to the $O + Br/Ag$ reaction. We have also examined these effects in central collisions of $Xe+Sn$ and similar trends were obtained. As soon as the target and projectile overlap, the nucleons from these nuclei feels a large repulsion which de-

FIG. 4. The same as Fig. 2, but for the reaction of $Xe+Sn$ at $b=8$ fm and at 400 MeV/nucleon.

flects them in the transverse direction. One sees from the figure that with MDI, there is early emission of fragments compared to soft. Naturally, collisions start playing a role during the same time.

To illustrate the relative role of the momentum-dependent interactions and nucleon-nucleon cross section, we rather define a relative probability

$$
\text{Rel}_{\text{prob}}^{\text{Frag}}\% = \left| \frac{(\text{Mult})^{\sigma^L} - (\text{Mult})^{\sigma^{\text{Cug.}}}}{(\text{Mult})^{\sigma^{\text{Cug.}}}} \right| X \quad 100.
$$

Here Mult stands for the multiplicity of the same kind of particles with Cugnon or with larger cross section of 55 mb using the same kind of equation of state. In other words, this relative probability defines the change in the multiplicity of the same kind of fragments in the presence of larger cross sections. We here take the static soft equation of state in one case and soft with momentum-dependent equation of state in other case. The relative probability percentage is depicted in Fig. 5 for the central reaction of $O + Br/Ag$ at 200 MeV/ nucleon and for the peripheral collisions of $Xe+Sn$ at 400 MeV/nucleon. The displayed results are between 60 and 200 fm/*c*. The left-hand side deals with IMF's whereas the right-hand side represents the LMF's. Very interestingly, one sees quite a different response of the larger cross section in the presence of the simple static equation of state and momentum-dependent interactions. The maximum final effect (of the large cross section) in the presence of momentum-dependent interaction is about 50% compared to about 600% with the static soft equation of state. Further one

FIG. 5. The $\text{Rel}_{\text{Prob}}^{\text{Frag}}$ % as a function of time. The left and right parts are for IMF's and LMF's, respectively. The $O+Ag$ and $O+Br$ reactions are at 200 MeV/nucleon whereas the $Xe+Sn$ reaction is at 400 MeV/nucleon.

also notices that the maximum effect in most of the cases occurs between 60 and 90 fm/ c which (while looking at Fig. 1) is the time when the reaction has finished and the matter starts breaking into fragments due to the density fluctuations. Once the cracks and/or breaks are complete (at $100 \text{ fm}/c$) the relative enhancement in the yields seems to be saturating. To understand this (behavior of) relative probability, we have to keep in the mind that the clusterization is performed within spatial correlation method which takes care of the spatial coordinates only and no reference is made about the relative momentum of nucleons. As a result, the nucleons with large relative momentum can be a part of the same fragment. Further, as is evident from Figs. 2–4, the MST method will a give single fragment at the times of high density which seems to decay afterwards. In other words, the spatial correlation method is not suited for the cases where the density is quite large and excitation energy is low. In all present three cases, the system is mildly excited. In other words, the destruction in the initial nucleon-nucleon correlations seems to be very small (the major parts of the remnant seem to be propagating with the same initial velocity). A close look at the $Xe+Sn$ reaction (Fig. 4) depicts that the heaviest fragments (detected in MST) with soft equation of state are bigger than the mass range defining the intermediate mass fragment. Once larger cross section is incorporated into the soft equation of state, the nucleon-nucleon collision rate increases which cracks the spectator matter into fragments. As a result, we see a huge enhancement in the production of IMF's. From Figs. 2–4, one sees that the A^{max} (at 200 fm/*c*) in the soft equation of state is about 37, 62, and 85, respectively, for the O+Br, O+Ag, and $Xe+Sn$ reactions. On the other hand, the inclusion of larger cross sections in soft EOS leads to a decrease in the *A*^{max} and now it reads 15, 30, and 56 which is well below the upper limit of IMF's. In addition, the third largest fragments with soft EOS are below the lower limit of the IMF's. Once the MDI is included, it leads to strong repulsion for any pair of nucleons from the target and projectile (due to large relative momentum) and therefore, it again forbids the formation of heavy fragments $(e.g.,)$ the A^{max} with MDI (using the normal cross section) reads 24, 40, and 60, respectively, for $O+Br$, $O+Ag$, and $Xe+Sn$. Under this physical picture, a further use of larger nucleonnucleon cross sections may destroy the remaining nucleonnucleon correlations. This will not make a difference for heavy fragments as they are already counted in the IMF windows, but can have a strong effect for light mass fragments. For example, in the $Xe+Sn$ reaction (see Fig. 4), the MDI and $MDI+55$ mb give nearly the same number of heavy fragments in the mass window between 20 and 65. Whereas the light mass fragments $(5-9$ mass window) register enhancement from 1.5 (with MDI) to 3.3 (with MDI+55 mb) which explain the major part of the enhancement in the production of IMF's with σ =55 in MDI (it enhances from 3.8) to 6.3). In other words, the effect of larger cross sections in the presence of MDI is confined to the further splitting of the light medium mass fragments and its effect is least for heavy fragments. It seems that the momentum-dependent interactions (which act during the initial phase of the reaction) along with the normal cross section are able to separate the matter in the spectator and participant by pushing the spectator matter. In this case, the use of larger cross sections increases the probability of nucleon-nucleon collision and hence plays a role in the participant zone and thus breaking the light medium fragments (formed in the reaction) into still lighter fragments. In view of this, it will be difficult to extract knowledge about the magnitude of the nucleon-nucleon cross section from fragmentation. As noted above, there is a strong effect of larger cross sections in the presence of static equations of state, but this effects reduces to a very small level once the momentum dependence of the equation of state is used.

We now compare the relative role of the cross section and equation of state by confronting the results with the experimental data of Jakobsson. In Figs. 6 and 7, we display our calculations with the experimental data of Jakobsson *et al.* [1]. We display in Fig. 6 our calculation of the soft equation of state with the constant cross section of 40 and 55 mb along with the energy-dependent cross section of Cugnon and in-medium *G*-matrix cross section. In Fig. 7, the Cugnon cross section is displayed along with a constant cross section of 40 and 55 mb. From Fig. 6, it is clear that the use of Cugnon or *G*-matrix cross sections gives nearly the same results indicating that the role of in-medium effects in the present context is only marginal. At lower energies, all cross sections (which yield the same results, as expected) fail to reproduce the experimental atomic charge identification.

FIG. 6. The atomic charge distribution *dN*/*dZ* as function of charge for the reaction of $O+Br$. The displayed calculations are using soft EOS with the cross sections from Cugnon, *G* matrix, and a constant cross section of 40 and 55 mb. We also display the result of the *G*-matrix cross section. The data (represented by a solid dot) is taken from Ref. $[1]$. The units of energy are MeV/nucleon.

Whereas at higher incident energies, the heavy fragment (observed with Cug/G -matrix cross sections) disappears if the simulations are carried with a larger cross section of 40 mb/55 mb. A larger cross section, though, does not help at low incident energies due to the forbidden space, it helps to a larger extent to get rid of the heavy fragment at higher incident energies. We have also checked the results with the hard equation of state which gives similar trends as obtained with soft equation of state.

In Fig. 8, we display the results of MDI with cross section of Cugnon and 55 mb along with the experimental measurements of Jakobsson *et al.* [1]. We see no difference at 50 MeV/nucleon with or without larger cross section. At 200 MeV/nucleon, the use of SMD with σ =55 mb improves the results a little bit. A larger cross section alone is not enough to explain the emulsion data at 200 MeV/nucleon. The inclusion of the momentum-dependent interactions are essential to explaining the data. Our conclusion is also supported by the earlier analysis of Leray *et al.* [10].

IV. SUMMARY

In the present paper, the relative importance of model ingredients was analyzed by studying both the symmetric and asymmetric reaction. Our analysis clearly indicates that both the MDI and the larger cross section help to improve

FIG. 7. Same as Fig. 6, but for the $O + Ag$ reaction and using cross sections from Cugnon and constant cross sections of 40 and 55 mb only. The data is taken from $[1]$.

the agreement with experimental data. As far as the relative importance of both ingredients is concerned, our study reveals that the role of MDI is essential and the use of larger cross sections is optional once the MDI are included in the Hamiltonian. With the inclusion of MDI, the sensitivity of the cross section is reduced. In the absence of MDI, the inclusion of larger σ yields entirely different results. Both

FIG. 8. The atomic charge distribution *dN*/*dZ* calculated using SMD with different cross sections taken from Cugnon and a constant cross section of 55 mb.

these effects depend strongly on the degree of excitation the system has. If the system is highly excited, the inclusion of MDI or a larger cross section does not play any role. In the case of mildly excited systems, both (the momentumdependent interaction and the larger cross section) give an entirely different picture. The effect of larger cross sections reduces if the momentum-dependent interactions are already included in the Hamiltonian which will make it difficult to

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extract the magnitude of the cross section from multifragmentation.

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