Density fluctuations and multifragmentation of nuclear matter

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The density fluctuations of nuclear matter are studied within a mean-field model in which fluctuations are generated by an external stochastic field. The constraints imposed on the random force by the fluctuation-dissipation theorem are analyzed. It is shown that in the proximity of the borders of the spinodal region the assumption of a white-noise stochastic field can be reliably used. The domain distribution of the liquid phase in the spinodal decomposition of nuclear matter is derived. The related distribution of fragment sizes compares favorably with the experimental fragment distribution observed in heavy ion collisions.

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I. INTRODUCTION

Semiclassical kinetic equations for the one-body phase-space density provide a powerful tool for studying the dynamics of complex processes occurring in heavy ion collisions [1–3]. However, these equations in their original version give a deterministic description for the evolution of the one-body phase-space density and their solution represents the mean value of this density at each time. Thus, they are not able to account for phenomena such as the nuclear multifragmentation observed in heavy ion collisions. In this process fluctuations about the mean phase-space density are believed to play an essential role (for a review of nuclear multifragmentaion see, e.g., Ref. [4]). In the last decade, to remedy this drawback, an extension of the transport theory has been proposed [5–7]. This approach, which incorporates a fluctuating stochastic term into the kinetic equation, is usually known as the Boltzmann-Langevin equation, and was originally applied to the treatment of hydrodynamic fluctuations in the theory of classical fluids [8]. In Refs. [5–7] collisions between nucleons in the nuclear medium are regarded as random processes and the diffusion coefficient of the Langevin (fluctuating) term is ultimately related to the amplitude of the nucleon-nucleon scattering. The last step is a particular case of the fluctuation-dissipation theorem. More recently, a new method to take into account fluctuations has also been proposed [9]. In the approach of Ref. [9] the statistical fluctuations of the one-body phase-space density are directly introduced by assuming local thermodynamic equilibrium. The white-noise nature of the stochastic term is the basic assumption, generally shared by all works on this subject. The authors of Ref. [10], instead, have introduced an extension of the Boltzmann-Langevin theory by including a colored-noise term in the stochastic force; the occurrence of such a term has been ascribed to the finite nucleon-nucleon collision time. Actual applications to nuclear problems of this interesting approach still have to be made.

In the present paper, we study the density fluctuations and their time evolution by introducing a self-consistent stochastic field acting on the constituents of the system. The self-consistency condition is provided by the fluctuation-dissipation theorem. The evolution of the fluctuations is treated within a linear approximation for the stochastic field. For simplicity we consider an infinite homogeneous system. First, without introducing any particular assumption, we prove that a white-noise stochastic field cannot satisfy the self-consistency condition in general. Then, with reference to infinite nuclear matter and within a collisionless mean-field approximation, we specify the particular conditions under which the white-noise assumption for the stochastic field can be retained. These conditions are fulfilled for values of density and temperature lying in the proximity of the boundary of the spinodal region in the phase diagram, both inside and outside this region. Thus we consider nuclear matter in this physical situation and are able to solve the stochastic equation for the density fluctuations in a closed form.

With respect to the previous works on this subject [5–7], here we consider a different source of fluctuations: Landau damping. This source is present even when collisions have a negligible role in the evolution of the system.

In Sec. II we propose a procedure to determine the structure of domains formed within the system during a spinodal decomposition. If the fragmentation phenomenon observed in heavy ion collisions can be ascribed to a spinodal decomposition of the bulk of nuclei [11], we are allowed to identify the pattern according to which nuclear matter is decomposing, with the fragmentation pattern, and can compare the results of our nuclear matter calculations with the fragment distribution observed in heavy ion collisions. This comparison is made in Sec. III. Finally in Sec. IV a brief summary and conclusions are given.

Many papers, both theoretical and experimental, have been devoted to the multifragmentation problem. Here we mention only a few theoretical ones representing different approaches. In the statistical models of Refs. [12,13], a complete statistical equilibrium of all degrees of freedom is assumed in a freeze-out volume and the various exit channels are sorted according to their statistical weight in the microcanonical ensemble. In Ref. [14] instead nuclear multifragmentation has been described in terms of ''reducibility'' and ''thermal scaling.'' This means that fragments are emitted practically independently of each other and the one-fragment probability is given by a Boltzmann factor. In the dynamical models of Refs. [5–7] clusters are constructed from the one-body phase-space density governed by the Boltzmann-Langevin equation [15–17]. In order to take into account the

quantal nature of the system and the requirement of antisymmetrization, the quantum molecular dynamics model [18] and the more sophisticated fermionic (antisymmetrized) molecular dynamics model [19,20] have been developed. In addition, percolation [21] and lattice-gas models [22,23] have also been introduced. These models are particularly suitable to deal with the critical phenomena which can be expected to occur in multifragmantation. To conclude this nonexhaustive survey, we mention the calculations of Ref. [24], which are based on classical nucleation theory.

Even if it should eventually turn out that multifragmentation must be ascribed to very complicated processes, we think that in any case our present approach could give some insight into the underlying mechanism.

II. FORMALISM

A. White-noise assumption

The mean-field approximation allows us to obtain a self-consistent equation for the time evolution of the one-body density. We assume that the time scale of the terms neglected in the mean-field approximation is shorter than the characteristic times of the mean-field dynamics. In order to take into account thermodynamic fluctuations, quantum effects, and short-range correlations, we add to the mean field a stochastic term similar to the random force in the Langevin equation. We assume that this additional field is a Gaussian white noise with vanishing mean. In this case the time evolution of the density is a Markovian process. We will determine the conditions in which the white-noise assumption can be considered reasonable.

The additional stochastic mean field will induce density fluctuations with respect to the mean density. To be more specific, we assume that at the time $t=0$ in the system is present a density fluctuation $\delta\varrho(\mathbf{r},t=0)$, with $\delta\varrho(\mathbf{r},t)=\varrho(\mathbf{r},t)-\varrho_0$ (ϱ_0 is the density of the reference homogeneous state, i.e., the state towards which the system relaxes). Within a linear approximation for the stochastic mean field the Fourier coefficients of $\delta\varrho(\mathbf{r},t)$ for $t>0$ are given by (see, for example, Ref. [25], Sec. 15 I)

$$\delta\varrho_{\mathbf{k}}(t)=\delta\varrho_{\mathbf{k}}(t=0)-\frac{\delta\varrho_{\mathbf{k}}(t=0)}{D_k(\omega=0)}\int_0^t D_k(t-t')\,dt' + \int_0^t D_k(t-t')B_{\mathbf{k}}(t')dW_{\mathbf{k}}(t'), \tag{2.1}$$

where $D_k(t-t')$ is the response function of the nuclear medium and $D_k(\omega)$ its time Fourier transform. For symmetry reasons $D_k(t-t')$ and its Fourier transform depend only on the magnitude of the wave vector. In the second integral $B_{\mathbf{k}}(t')dW_{\mathbf{k}}(t')$ gives the contribution of the stochastic field in the interval dt'. The real and imaginary parts of the Fourier coefficients $W_{\mathbf{k}}(t')$ are indipendent components of a multivariate Wiener process [26]. The fact that the stochastic field is real requires $B^*_{\mathbf{k}}(t)=B_{-\mathbf{k}}(t)$ and $W^*_{\mathbf{k}}(t)=W_{-\mathbf{k}}(t)$.

The stochastic part of the mean field is completely determined once the coefficients $B_{\mathbf{k}}(t)$ are known. In order to gain information about these coefficients we concentrate on the correlations of density fluctuations at equilibrium. Because of the independence of the components of the multivariate Wiener process $W_{\mathbf{k}}(t)$, only the terms with $\mathbf{k}'=-\mathbf{k}$ survive. Within a linear approximation, these correlations can be expressed by means of the same quantities that appear in Eq. (2.1). This does, in a sense, correspond to the Onsanger hypothesis about the decay of deviations from equilibrium [25]. The equation for the equilibrium fluctuations is obtained from Eq. (2.1) by moving the initial time to $-\infty$, without including any particular condition at finite times. Then the correlations are given by the equation (the angular brackets denote ensemble averaging)

$$\langle \delta\varrho_{\mathbf{k}}(t)\,\delta\varrho_{-\mathbf{k}}(t')\rangle = \int_{-\infty}^{min(t,t')} dt_1\, D_k(t-t_1)D_k(t'-t_1) \times B_{\mathbf{k}}(t_1)B_{-\mathbf{k}}(t_1). \tag{2.2}$$

The time-translation invariance of the left-hand side of Eq. (2.2) requires that the coefficients $B_{\mathbf{k}}(t)$ must be constant. Taking the Fourier transform, Eq. (2.2) gives

$$\langle \delta\varrho_{\mathbf{k}}(\omega)\,\delta\varrho_{-\mathbf{k}}(\omega')\rangle = 2\pi\delta(\omega+\omega')\,\langle(\delta\varrho_{\mathbf{k}}\delta\varrho_{-\mathbf{k}})(\omega)\rangle = 2\pi\delta(\omega+\omega')\,D_k(\omega)D_k(\omega')\,|B_{\mathbf{k}}|^2. \tag{2.3}$$

By exploiting the fluctuation-dissipation theorem

$$\langle(\delta\varrho_{\mathbf{k}}\delta\varrho_{-\mathbf{k}})(\omega)\rangle = -\frac{2}{1-e^{-\beta\omega}}\mathrm{Im}\,D_k(\omega), \tag{2.4}$$

where $\beta=1/T$ is the inverse temperature (we use units such that $\hbar=c=k_B=1$), we obtain for the coefficients $B_{\mathbf{k}}$ the equation

$$|B_{\mathbf{k}}|^2 = -\frac{2}{1-e^{-\beta\omega}}\frac{\mathrm{Im}\,D_k(\omega)}{|D_k(\omega)|^2}. \tag{2.5}$$

We have used the relation $D_k(-\omega)=D^*_k(\omega)$. Equation (2.5) can be satisfied only if the right-hand side does not depend on ω. This can occur only in particular situations; thus the original white-noise assumption about the stochastic mean field is not correct in general. This result is quite general, so we can conclude that for a perturbed system approaching an equilibrium state, fluctuations about the average trajectory cannot usually be accounted for by a white-noise stochastic force.

Now, with reference to symmetric nuclear matter, we discuss particular physical situations in which Eq. (2.5) can have a solution. Only in such conditions is the assumption of a white-noise stochastic field valid. The relevant quantity is the linear-response function $D_k(\omega)$. We evaluate $D_k(\omega)$ within a self-consistent mean-field approximation. In order to derive compact analytical expressions, here we use the linearized Vlasov equation for calculating the response function. This equation can be regarded as a semiclassical approximation to the random phase approximation, valid in the long-wavelength limit. We also use a Skyrme-like form of

the nucleon-nucleon effective interaction. Our self-consistent mean-field potential is given by

$$U = a\frac{\varrho}{\varrho_{eq}} + b\left(\frac{\varrho}{\varrho_{eq}}\right)^{\alpha+1} - d\,\nabla^2\varrho, \tag{2.6}$$

where ϱ_{eq} is the saturation density of nuclear matter. For the parameters in Eq. (2.6) we take the values

$$a = -356.8 \text{ MeV}, \quad b = 303.9 \text{ MeV}, \quad \alpha = \frac{1}{6},$$

$$d = 130 \text{ MeV fm}^5.$$

The values of a, b, and α reproduce the binding energy (15.75 MeV) of nuclear matter at saturation ($\varrho_{eq} = 0.16$ fm^{-3}) and give an incompressibility modulus of 201 MeV. For the values of d we follow the prescriptions of Ref. [27].

The response function is given by

$$D_k(\omega) = \frac{D_k^{(0)}(\omega)}{1 - \mathcal{A}_k D_k^{(0)}(\omega)}, \tag{2.7}$$

where $D_k^{(0)}(\omega)$ is the noninteracting particle-hole propagator, and

$$\mathcal{A}_k = a\frac{1}{\varrho_{eq}} + \frac{b}{\alpha+1}\frac{1}{\varrho_{eq}^{\alpha+1}}\varrho_0^{\alpha} + d\,k^2 \tag{2.8}$$

are the Fourier coefficients of the effective interaction. Here ϱ_0 is the density of the reference homogeneous state.

By substituting the expression (2.7) for $D_k(\omega)$ into Eq. (2.5), we obtain

$$|B_{\mathbf{k}}|^2 = -\frac{2}{1-e^{-\beta\omega}}\frac{\mathrm{Im}\,D_k^{(0)}(\omega)}{|D_k^{(0)}(\omega)|^2}. \tag{2.9}$$

This equation shows that the coefficients $B_{\mathbf{k}}$ do not explicitely depend on the nucleon-nucleon effective interaction. However, we remark that $D_k^{(0)}(\omega)$ is the propagator of independent particles that are moving in the mean field of the reference homogeneous state,

$$U_0 = a\frac{\varrho_0}{\varrho_{eq}} + b\left(\frac{\varrho_0}{\varrho_{eq}}\right)^{\alpha+1}, \tag{2.10}$$

and thus the interaction between constituents does enter, although not explicitly, into the expression of $B_{\mathbf{k}}$.

We shall now show that in the classical limit $\omega/T \ll 1$, the right-hand side of Eq. (2.9) does not depend on ω; thus the assumption of a white-noise stochastic field can be considered valid in that limit.

In the actual physical situations considered in this paper the values of the temperature are small enough with respect to the Fermi temperature so that the Pauli principle is still operating. Therefore the strength of the particle-hole excitations having energies much higher than kv_F (v_F is the Fermi velocity) can be considered negligible. Moreover, the relevant values of the wave vector k turn out to be such that the quantity kv_F is of the same order of magnitude as T. Thus the limit $\omega/T \ll 1$ also implies $\omega/kv_F \ll 1$.

The noninteracting particle-hole propagator $D_k^{(0)}(\omega)$ acquires a very simple form in the long-wavelength (Vlasov) limit. The imaginary part is

$$\mathrm{Im}\,D_k^{(0)}(\omega) = \frac{1}{4\pi}m^2\frac{\omega}{k}\int d\varepsilon_p \frac{\partial n_p}{\partial \varepsilon_p}\theta\left(1-\frac{\omega}{kv}\right), \tag{2.11}$$

where

$$n_p = \frac{4}{e^{\beta(\epsilon_p - \tilde{\mu})}+1}$$

is the mean occupation number of nucleons with kinetic energy $\epsilon_p = p^2/2m$ and $v = p/m$. The effective chemical potential $\tilde{\mu}$ is measured with respect to the uniform mean field U_0.

For $\omega/kv_F \ll 1$ the imaginary part of $D_k^{(0)}(\omega)$ is given by

$$\mathrm{Im}\,D_k^{(0)}(\omega) = -\frac{1}{\pi}m^2\frac{\omega}{k}\frac{1}{e^{-\beta\tilde{\mu}}+1} + O\left(\left(\frac{\omega}{kv_F}\right)^3\right), \tag{2.12}$$

while the real part of $D_k^{(0)}(\omega)$ in the long-wavelength limit takes the form

$$\mathrm{Re}D_k^{(0)}(\omega) = -\frac{1}{2\pi^2}\int dp\, p^2 \frac{\partial n_p}{\partial \varepsilon_p} \times\left(-1+\frac{1}{2}\frac{\omega}{kv}\ln\frac{1+\omega/kv}{|1-\omega/kv|}\right). \tag{2.13}$$

For T sufficiently low with respect to $\tilde{\mu}$, the most important contribution to the integral in Eq. (2.13) comes from a small domain of ε_p around $\tilde{\mu}$. So we can take $\omega/kv \ll 1$ in evaluating the integral, and obtain

$$\mathrm{Re}D_k^{(0)}(\omega) = -\frac{\partial \varrho_0}{\partial \tilde{\mu}} + O\left(\left(\frac{\omega}{kv_F}\right)^2\right). \tag{2.14}$$

With $D_k^{(0)}(\omega)$ given by Eqs. (2.12) and (2.14), the right-hand side of Eq. (2.9) is independent of ω to the lowest significant order in ω/T. Thus, for $\omega/T \ll 1$ the magnitude of the coefficients $B_{\mathbf{k}}$ is given by

$$|B_{\mathbf{k}}|^2 = \frac{2}{\pi}m^2\left(\frac{\partial\tilde{\mu}}{\partial\varrho_0}\right)^2\frac{T}{e^{-\beta\tilde{\mu}}+1}\frac{1}{k}. \tag{2.15}$$

The phases of $B_{\mathbf{k}}$, instead, remain unknown. However, we will see that only the quantities $|B_{\mathbf{k}}|^2$ are needed to determine the probability distribution of density fluctuations. Finally, we remark that $|B_{\mathbf{k}}|$ for a given $\mathbf{k}$ is determined solely by the density and temperature of nuclear matter.

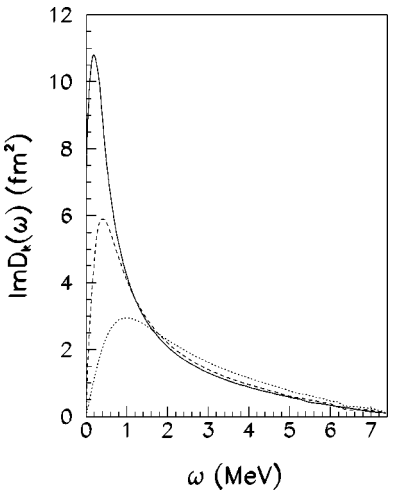

FIG. 1. Imaginary part of the response function $D_k(\omega)$ of Eq. (2.7) for hot nuclear matter ($T=5$ MeV) at different densities approaching the critical value $\varrho_c=0.617\,\varrho_{eq}$ (dotted line, $\varrho_0=0.70\,\varrho_{eq}$; dashed line, $\varrho_0=0.65\,\varrho_{eq}$; solid line, $\varrho_0=0.63\,\varrho_{eq}$). The value of k is $0.1\,k_F$.

The white-noise assumption is justified if the excitation strength is concentrated in a narrow range of energy close to zero. This condition requires that $\mathrm{Im}\,D_k(\omega)$ be a sharply peaked function in the proximity of $\omega=0$ and be negligible elsewhere. The imaginary part of $D_k(\omega)$ displays this feature for values of temperature and density near the borders of the spinodal region, since the pole of $D_k(\omega)$ lying on the imaginary axis moves towards $\omega=0$ as the system approaches the mechanical instability. This is shown in Fig. 1, where we report $\mathrm{Im}\,D_k(\omega)$ calculated with Eq. (2.7) using the complete expression of $D_k^{(0)}(\omega)$. With our effective interaction, for $T=5$ MeV the spinodal region starts at $\varrho_c=0.617\varrho_{eq}$. The values of ϱ_0 used in Fig. 1 are close to this critical value.

B. Distribution of fluctuations

We now derive from Eq. (2.1) the probability distribution for $\delta\varrho_{\mathbf{k}}(t)$ in the limit $\omega/T\ll 1$, and for values of temperature and density in the proximity of the spinodal region. The response function $D_k(\omega)$ has a pole in the lower part of the imaginary ω axis, at a position given by

$$i\Gamma_k = i\frac{\pi}{m^2}(1+e^{-\beta\tilde{\mu}})\frac{\partial\varrho_0}{\partial\tilde{\mu}}\frac{\left(\left.\frac{\partial^2 f}{\partial\varrho_0^2}\right|_T + dk^2\right)}{\mathcal{A}_k}k. \quad (2.16)$$

We have used the relation

$$\left.\frac{\partial\tilde{\mu}}{\partial\varrho_0}\right|_T = \left.\frac{\partial^2 f}{\partial\varrho_0^2}\right|_T - \mathcal{A}_0, \quad (2.17)$$

where f is the free-energy density and $\mathcal{A}_0=\mathcal{A}_{k=0}$.

In Eq. (2.16) the relevant quantity is the isothermal stiffness $\partial^2 f/\partial\varrho_0^2|_T$, which vanishes on the boundary of the spinodal region. Since we limit our calculations to the proximity of the spinodal region, we neglect $\partial^2 f/\partial\varrho_0^2|_T$ with respect to $\mathcal{A}_0$ in evaluating $D_k(t)$. Furthermore, in actual calculations the typical values of k which come into play are such that the term $d\,k^2$ is smaller than $|\mathcal{A}_0|$; thus we also neglect this term with respect to $|\mathcal{A}_0|$. This approximation is consistent with the long-wavelength limit adopted in the calculation of $D_k(\omega)$.

Substituting into Eq. (2.1) the response function $D_k(t-t')$ calculated with these approximations, the equation for the fluctuations $\delta\varrho_{\mathbf{k}}(t)$ becomes

$$\delta\varrho_{\mathbf{k}}(t)=\delta\varrho_{\mathbf{k}}(t=0)e^{\Gamma_k t}+\tilde{B}_k\int_0^t e^{\Gamma_k(t-t')}\,dW_k(t'), \quad (2.18)$$

where

$$|\tilde{B}_k|=\frac{1}{|\mathcal{A}_0|}\sqrt{\frac{2\pi T}{m^2}(1+e^{-\beta\tilde{\mu}})\,k}, \quad (2.19)$$

and Γ_k is given by Eq. (2.16), neglecting the term $d\,k^2$ in $\mathcal{A}_k$. We recall that Γ_k is negative, so that $|\Gamma_k|$ represents the damping rate of fluctuations, which vanishes for long wavelengths when $\partial^2 f/\partial\varrho_0^2|_T\to 0$.

Equation (2.18) represents an Ornstein-Uhlenbeck process [26] with $|\Gamma_k|$ as drift coefficient and $\tilde{B}_k$ as diffusion coefficient. The corresponding Fokker-Planck equation for the probability distribution $P[\delta\varrho_{\mathbf{k}}(t)]$ reads

$$\frac{\partial}{\partial t}P[\delta\varrho_{\mathbf{k}}(t)]=|\Gamma_k|\frac{\partial}{\partial\delta\varrho_{\mathbf{k}}(t)}\delta\varrho_{\mathbf{k}}(t)P[\delta\varrho_{\mathbf{k}}(t)] + \frac{1}{2}|\tilde{B}_k|^2\frac{\partial^2}{\partial\delta\varrho_{\mathbf{k}}^2(t)}P[\delta\varrho_{\mathbf{k}}(t)]. \quad (2.20)$$

For simplicity we assume the state of the system at $t=0$ to be homogeneous on average ($\langle\delta\varrho_{\mathbf{k}}(t=0)\rangle=0$ for $k\neq 0$). Equation (2.18) says that this property holds during time evolution. In this case the solution of Eq. (2.20) is a Gaussian distribution with zero mean value. Whenever it is necessary, a nonvanishing mean value can easily be introduced. The explicit expression of the distribution $P[\delta\varrho_{\mathbf{k}}(t)]$ is

$$P[\delta\varrho_{\mathbf{k}}(t)]=N_1\exp\left(-\frac{1}{2}\sum_{\mathbf{k}}\delta\varrho_{\mathbf{k}}^*(t)\frac{1}{\sigma_k^2(t)}\delta\varrho_{\mathbf{k}}(t)\right), \quad (2.21)$$

with the variance $\sigma_k^2(t)$ given by

$$\sigma_k^2(t)=\sigma_k^2(t=0)\,e^{2\Gamma_k t}+\frac{T}{f''+d\,k^2}(1-e^{2\Gamma_k t}). \quad (2.22)$$

Here the constant N_1 is a normalization factor and we have introduced the abbreviation

$$f'' = \left.\frac{\partial^2 f}{\partial \varrho_0^2}\right|_T .$$

For $t \to \infty$, Eq. (2.21) reproduces the usual Gaussian approximation with variance

$$\sigma_k^2(t=\infty) = \frac{T}{f'' + d\,k^2} \tag{2.23}$$

for the equilibrium thermodynamical fluctuations [28]. We furthermore remark that Eq. (2.22) for the time evolution of the variance is similar to that obtained with different approaches in previous works on this subject [7,29].

For later purposes we report also the distribution of the fluctuations in ordinary space:

$$P[\delta\varrho(\mathbf{r},t)] = N_2 \exp\left(-\frac{1}{2}\int d\mathbf{r}\, d\mathbf{r}'\, \delta\varrho(\mathbf{r},t) \times M(\mathbf{r},\mathbf{r}',t)\,\delta\varrho(\mathbf{r}'t) \right), \tag{2.24}$$

where

$$M(\mathbf{r},\mathbf{r}',t) = \frac{1}{V}\sum_{\mathbf{k}} e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')}\frac{1}{\sigma_k^2(t)},$$

and N_2 is an appropriate normalization factor.

The diffusion coefficients of Eq. (2.19) are derived by means of the fluctuation-dissipation theorem, which concerns only fluctuations about equilibrium. In Ref. [30] a way has been suggested to extend the treatment of stable cases to processes where instabilities can develop. Following that suggestion we include in our approach the case of nuclear matter merged in the spinodal region. In practice, we still assume the validity of Eq. (2.21) for the probability distribution of the density fluctuations, with the variance $\sigma_k^2(t)$ of Eq. (2.22) calculated with the values of temperature and density of the new situation. This amounts to treating the diffusion coefficients for the unstable case as an analytic continuation of the stable-case coefficients in the (ϱ, T) plane. The reliability of such a procedure lies in the fact that both the growing rate Γ_k and the diffusion coefficient $\tilde{B}_k$ change smoothly when the system crosses the stability boundary and enters the spinodal region. The pole of $D_k(\omega)$, in turn, continuously moves along the imaginary axis from the lower part to the upper part of the complex ω plane [see Eq. (2.16)]. In order to preserve causality, the integral for calculating the Fourier antitransform $D_k(t)$ must be performed along a path which cuts the imaginary axis above the pole.

In the unstable case, the time behavior of the variance $\sigma_k^2(t)$ in Eq. (2.22) is similar to that predicted by linear theories of the spinodal decomposition of alloys and fluids (for an extensive review on this subject see Ref. [31]). The variance grows exponentially for the fluctuations with wave number

$$k < k_c = \sqrt{\frac{|f''|}{d}}, \tag{2.25}$$

while it tends to the aymptotic value $\sigma_k(t=\infty)$ of Eq. (2.23) for $k > k_c$. In particular the growth rate Γ_k presents a maximum for $k = k_M = k_c/\sqrt{3}$. This means that the pattern of the regions which contain coherently correlated fluctuations is asymptotically characterized by the wavelength $\lambda_M = 2\pi/k_M$. These features for the growth rate of unstable modes are analogous to those obtained in Ref. [16] within a different scheme.

C. Size of fragments

Starting from the probability distribution for density fluctuations given by Eq. (2.24), we can determine the corresponding distribution for the size of the correlation domains. It has already been recalled that the stable and unstable cases can be treated within the same scheme. Thus we shall invesigate two different situations which could be explored by nuclear matter during a nucleus-nucleus collision: in one case the system is in the metastable region and relaxes towards a local minimum of the free energy, while in the other case the system is merged in the spinodal zone and develops density fluctuations which grow with time and will eventually lead to decomposition. According to our approximations, we limit our analysis in both cases to values of temperature and density in the proximity of the borders of the spinodal region. Moreover, we consider homogenous nuclear matter in both cases, and still assume that $\langle \delta\varrho_{\mathbf{k}}(t=0)\rangle = 0$ for $k \neq 0$.

Before performing explicit calculations, we make a few remarks. It is known that linear theories are unable to describe the late stages of the spinodal decomposition of alloys and fluids (see Ref. [31] and references quoted therein). In particular, they predict a limiting value for the length scale that characterizes the pattern of the correlation domains. This value is given by the wavelength λ_M for which the growth rate of fluctuations has a maximum. Instead, Monte Carlo simulations and experimental results [32] show a continuous coarsening of the domains with increasing time. However, it has been argued in Ref. [33] that the early-time Monte Carlo results are consistent with a linear theory, provided that a stochastic force is included.

In the physical situations considered in the present paper (heavy ion collisions), the value of the characteristic wavelength λ_M is larger than 10 fm, beyond the size of the nuclear system involved. Moreover, the corresponding growth time $1/\Gamma_{k_M}$ is of the same order of magnitude as the characteristic times of the nucleus-nucleus collisions in the energy range considered here. Thus the fluctuations with wave number k_M are still far from being the predominant ones in this time interval. This means that the processes that we are investigating correspond to an early stage of the spinodal decomposition. Then we can expect reliable results from our approach, at least at a qualitative level.

From Eq. (2.24) we obtain the usual expression for the equilibrium correlation function:

$$G(|\mathbf{r}-\mathbf{r}'|)=\frac{1}{4\pi}\frac{T}{d}\frac{e^{-|\mathbf{r}-\mathbf{r}'|/\xi}}{|\mathbf{r}-\mathbf{r}'|}, \tag{2.26}$$

where

$$\xi=\sqrt{\frac{d}{f''}}$$

is the correlation length. This quantity, which represents the average extension of the correlation domains, can be obtained by an appropriately weighted integral of the correlation function:

$$\xi=\int d\mathbf{r}d\mathbf{r}'F(\mathbf{r},\mathbf{r}')G(|\mathbf{r}-\mathbf{r}'|). \tag{2.27}$$

The function $F(\mathbf{r},\mathbf{r}')$ is a suitable weight function. Here we extend this relation between averaged quantities to fluctuating quantities, for systems both at equilibrium and out of equilibrium. We then assume that the size of correlation domains at time t is given by a quadratic functional of the fluctuations $\delta\varrho(\mathbf{r},t)$:

$$b=L(t)\frac{\int d\mathbf{r}d\mathbf{r}'\,\delta\varrho(\mathbf{r},t)F(\mathbf{r},\mathbf{r}')\delta\varrho(\mathbf{r}',t)}{\int d\mathbf{r}d\mathbf{r}'F(\mathbf{r},\mathbf{r}')G(|\mathbf{r}-\mathbf{r}'|,t)}, \tag{2.28}$$

where $L(t)=\langle b\rangle$ is the length scale that characterizes the pattern of the domains, and $G(|\mathbf{r}-\mathbf{r}'|,t)$ is the correlation function for systems out of equilibrium. The latter quantity is the space Fourier transform of the variance of Eq. (2.22).

In order to simplify calculations, we further choose for $F(\mathbf{r},\mathbf{r}')$ a separable form. The requirement that b should be positive for any function $\delta\varrho(\mathbf{r},t)$ enforces a symmetric form

$$F(\mathbf{r},\mathbf{r}')=f(\mathbf{r})f(\mathbf{r}') \tag{2.29}$$

of the weight function. This form allows us to obtain a closed expression for the probability distribution of b. In addition, with this choice the final results are entirely independent of the function $f(\mathbf{r})$.

Now we derive the probability distribution for b as a function of the length scale $L(t)$. Later we shall give a procedure for determining $L(t)$.

For a given probability distribution $P[\delta\varrho(\mathbf{r},t)]$, the related probability distribution for b, at a given time t, can be obtained by means of the functional integral

$$P(b,t)=\int d[\delta\varrho(\mathbf{r},t)]\,\delta\Bigg(b-\frac{L(t)}{C}\int d\mathbf{r}\,\delta\varrho(\mathbf{r},t)\times F(\mathbf{r},0)\,\delta\varrho(0,t)\Bigg)P[\delta\varrho(\mathbf{r},t)], \tag{2.30}$$

where we have put

$$C=\int d\mathbf{r}d\mathbf{r}'f(\mathbf{r})G(|\mathbf{r}-\mathbf{r}'|,t)f(\mathbf{r}') \tag{2.31}$$

in order to simplify the notation.

With the distribution of Eq. (2.24) and using the integral representation of the δ function,

$$\delta(x)=\frac{1}{2\pi}\int d\eta e^{i\eta x},$$

the equation for the distribution $P(b,t)$ takes the form

$$P(b,t)=\frac{N_2}{2\pi}\int d\eta e^{i\eta b}\int d[\delta\varrho(\mathbf{r},t)]\times\exp\Bigg[-\frac{1}{2}\int d\mathbf{r}\,d\mathbf{r}'\,\delta\varrho(\mathbf{r},t)\Bigg(M(\mathbf{r},\mathbf{r}',t)+2i\eta\frac{L(t)}{C}f(\mathbf{r})f(\mathbf{r}')\Bigg)\delta\varrho(\mathbf{r}',t)\Bigg]. \tag{2.32}$$

The functional integral is of Gaussian type and allows us to express the result of the integration in closed form:

$$P(b,t)=\frac{N_2}{2\pi}\int d\eta e^{i\eta b}\frac{1}{\left(\det\frac{1}{2\pi}\left[\hat{M}+2i\eta\frac{L(t)}{C}\hat{F}\right]\right)^{1/2}}. \tag{2.33}$$

The quantities $\hat{M}$ and $\hat{F}$ are infinite-dimensional operators, with matrix elements $M(\mathbf{r},\mathbf{r}',t)$ and $f(\mathbf{r})f(\mathbf{r}')$, respectively, in the coordinate representation.

The determinant in the last equation can be factorized as

$$\det\frac{1}{2\pi}\left[\hat{M}+2i\eta\frac{L(t)}{C}\hat{F}\right]=\det\left[\frac{\hat{M}}{2\pi}\right]\det\left[\mathbf{1}+2i\eta\frac{L(t)}{C}\hat{M}^{-1}\hat{F}\right], \tag{2.34}$$

where $\mathbf{1}$ is the unit matrix. The square root of the first factor on the right-hand side and the normalization constant N_2 of Eq. (2.33) coincide and cancel. What remains to be evaluated is the inverse of the square root of the second determinant. For this purpose we write the determinant in exponential form and expand the exponent in a power series. Thus, we obtain the following formal expression:

$$\begin{aligned}&\left(\det\left[\mathbf{1}+2i\eta\frac{L(t)}{C}\hat{M}^{-1}\hat{F}\right]\right)^{-1/2}\\&=\exp\left[-\frac{1}{2}\mathrm{Tr}\ln\left(\mathbf{1}+2i\eta\frac{L(t)}{C}\hat{M}^{-1}\hat{F}\right)\right]\\&=\exp\left[-\frac{1}{2}\sum\frac{1}{n}(-1)^{1+n}\left(2i\eta\frac{L(t)}{C}\right)^n\mathrm{Tr}(\hat{M}^{-1}\hat{F})^n\right].\end{aligned} \tag{2.35}$$

We recall that the matrix element $M^{-1}(\mathbf{r},\mathbf{r}',t)$ and the correlation function $G(|\mathbf{r}-\mathbf{r}'|,t)$ coincide. Because of the separable form chosen for the function $F(\mathbf{r},\mathbf{r}')$, Eq. (2.29), the trace operation on the generic n term of Eq. (2.35) simply

yields C^n. Thus the series can be resummed and gives $\ln[1+2i\eta L(t)]$. Then the probability distribution $P(b,t)$ acquires the form

$$P(b,t)=\frac{1}{2\pi}\int d\eta \frac{e^{i\eta b}}{[1+2i\eta L(t)]^{1/2}}.$$

A simple integration in the complex η plane gives the final result

$$P(b,t)=\frac{1}{\sqrt{\pi}}\frac{1}{\sqrt{2L(t)b}}e^{-b/[2L(t)]}. \tag{2.36}$$

From the probability distribution of the domain size we can derive the distribution of the number of nucleons A that are contained in a correlation domain, assumed to be spherical. For a homogeneous liquid the relation between A and the size b is $b=2r_0A^{1/3}$, where r_0 is determined by the actual density. With a simple transformation of variables we obtain, for the probability distribution of A, $P(A,t)$, the equation

$$P(A,t)=\frac{1}{3}\frac{1}{\sqrt{\pi}}\sqrt{\frac{r_0}{2L(t)}}A^{-5/6}e^{-[r_0/L(t)]A^{1/3}}. \tag{2.37}$$

Further, to take into account that A is a discrete variable we express the probability of finding a correlation domain containing A nucleons, $Y(A)$, through the integral

$$Y(A)=\int_{A-1}^{A} dA\, P(A,t). \tag{2.38}$$

For large A, $Y(A)$ tends to coincide with $P(A,t)$.

III. RESULTS

The distribution $P(A,t)$ and the probability $Y(A)$ are completely determined once the ratio between the length scale $L(t)$ and the mean interparticle spacing r_0 is fixed. The parameter $L(t)$ sets the scale for the decrease of the correlation function $G(r,t)$ with increasing r. We can obtain an estimate of $L(t)$ by analyzing the behavior of $G(r,t)$ as a function of r at a given t. The correlation function is initially determined by the variance $\sigma_k^2(t=0)$; then, in the stable case, it asymptotically assumes the form given in Eq. (2.26), with the appropriate correlation length $\xi=L(t=\infty)$, while in the unstable case, it acquires a damped oscillatory behavior characterized by the asymptotic wavelength λ_M. In order to illustrate the general features of the function $L(t)$, we simply assume that the initial fluctuations are negligible, $\sigma_k^2(t=0)\approx 0$. In this case the function $G(r,t)$ is completely determined by the density and temperature of nuclear matter. Here we consider two sample values for the density ($\varrho_0=0.65\varrho_{eq}$ and $\varrho_0=0.58\varrho_{eq}$) and a single value for the temperature ($T=5$ MeV). This temperature is in the range of values expected for the nuclear multifragmentation process [4]. The two corresponding points in the phase diagram (ϱ,T) lie in the metastable region and in the spinodal region, respectively, and are sufficiently close to the boundary of the spinodal zone to justify our assumption of a white-noise stochastic field. In Figs. 2 and 3 we show the behavior of $G(r,t)$ as a function of r at three different values of time, both in the stable and unstable situations. In the stable case of Fig. 2, a simple inspection of the behavior of $G(r,t)$ shows that it is reasonably well reproduced by a function like that on the right-hand side of Eq. (2.26) [obviously with ξ replaced by $L(t)$]. We adopt such a form for $G(r,t)$; then, by comparison with its true behavior shown in Fig. 2, we can determine $L(t)$. For the unstable case shown in Fig. 3, the situation is slightly more involved because the asymptotic regime is reached only after a very long time. For this case, we simply assume that $L(t)$ does coincide with the distance at which the value of $G(r,t)$ is reduced by 80% with respect to its value at $r=1$ fm (because of our approximations, we cannot expect the present approach to be reliable for distances shorter than 1 fm).

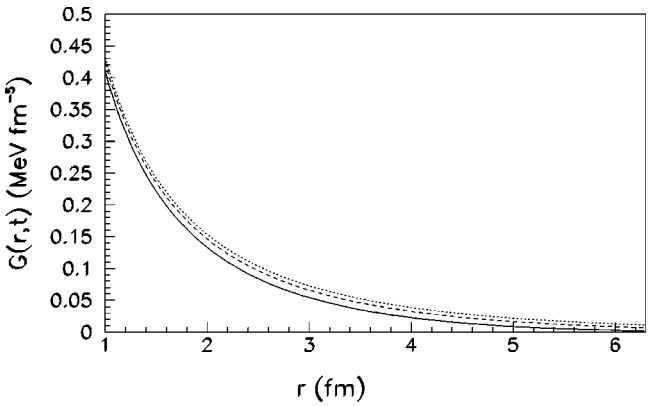

FIG. 2. Spatial behavior of the time-dependent correlation function $G(r,t)$ for nuclear matter on the stable side of the spinodal curve ($\varrho_0=0.65\,\varrho_{eq}$, $T=5$ MeV). The three curves correspond to different values of t (solid line, $t=50$ fm/c; dashed line, $t=100$ fm/c; dotted line, $t=200$ fm/c).

At a given time t the value of the length scale $L(t)$ depends strongly on the distance from the boundary of the spinodal zone; the shorter this distance, the larger is $L(t)$. In Fig. 4 the calculated length $L(t)$ is displayed as a function of t for the two chosen sets of parameters. The values of t are in

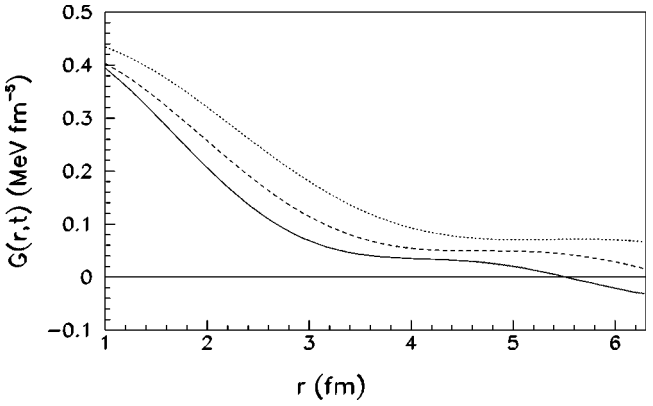

FIG. 3. Same as Fig. 2, but for the unstable case ($\varrho_0=0.58\,\varrho_{eq}$).

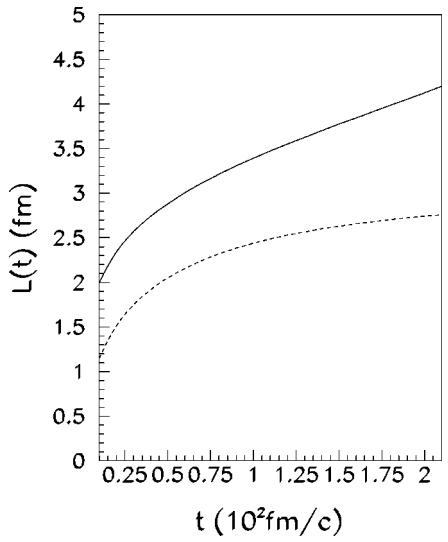

FIG. 4. Behavior of the length scale $L(t)$ within the spinodal region (solid line, $\varrho_0 = 0.58\,\varrho_{eq}$) and outside it (dashed line, $\varrho_0 = 0.65\,\varrho_{eq}$).

the range that is relevant for nuclear fragmentation [4]. Figure 4 shows that for $t \sim 200$ fm/c, $L(t)$ practically reaches its asymptotic value [$L(\infty) \approx 3.0$ fm] in the metastable situation, whereas in the unstable case $L(t)$ is still much smaller than $L(\infty) \approx 12$ fm.

In the two physical situations considered here, two different processes could drive nuclear matter towards a spinodal decomposition. In the metastable case, if the density fluctuations are large enough, the nuclear system can explore the unstable region for a time sufficiently long to move towards a phase separation. In the unstable case instead, fluctuations grow with time until they cause the decomposition of the nuclear system. In both cases we expect that the pattern of domains containing the liquid phase is determined by the probability distribution $P(b,t)$ or $P(A,t)$ of Eqs. (2.36) and (2.37).

In order to assess the degree of validity of our approach, we compare the results of our calculations with the corresponding experimental data by identifying the probability $Y(A)$ of Eq. (2.38) with the distribution of the fragment yield. Since experimentally the fragments are detected according to their charge, we have to transform $P(A,t)$ and $Y(A)$ into the corresponding functions of Z. We assume a homogeneous distribution also for the charge $Z = [(1 - \alpha)/2]A$, with $\alpha = (N-Z)/A$, and use $\alpha = 0.2$, which corresponds to the average asymmetry of the nuclear systems considered.

In Fig. 5 the probability $Y(Z)$ is displayed as a function of Z on a double logarithmic scale for three different values of the ratio $L(t)/r_0$. The range of values for $L(t)/r_0$ has been chosen in accordance with that of $L(t)$ in Fig. 4. Figure 5 shows that $Y(Z)$ can be fit with good accuracy by a power law $Y(Z) = Y_0 Z^{-\tau_{eff}}$. The values of the effective exponent τ_{eff} lie between 1.17 for $L(t)/r_0 = 4$ and 1.42 for $L(t)/r_0 = 2$.

The power-law behavior of the fragment yield and the determination of the exponent have been the subject of several experimental studies of multifragmentation (see, for example, the recent papers in [34,35]). The observed values of the exponent are in the interval $\sim 1.2 - 1.5$ for nuclear rections with beam energies lower than $\sim 40A$ MeV, whereas they exceed the value of 2 at higher energies [34,35]. A value of the exponent $\tau_{eff} \geqslant 2$ can be unlikely reproduced by our calculations because we would need an unreasonably low value for the ratio $L(t)/r_0$. However, in various papers [35–37] it has been remarked that the effects of collective motions, which have not been taken into account by our present approach, should become more important with increasing beam-energy.

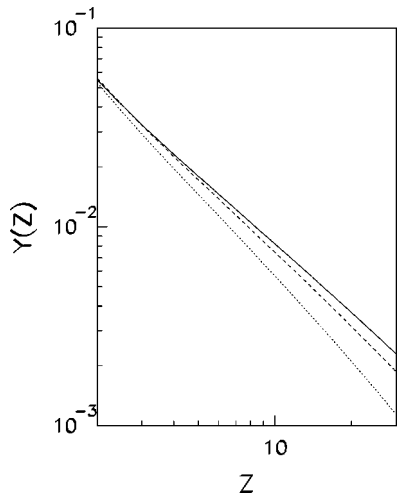

FIG. 5. Fragment distribution $Y(Z)$ calculated for different values of the ratio $L(t)/r_0$. Solid line, $L(t)/r_0 = 4$; dashed line, $L(t)/r_0 = 3$; dotted line, $L(t)/r_0 = 2$.

Figures 6 and 7 show a comparison between the charge distributions predicted by our approach, $Y(Z)$, and recent experimental data obtained by the Multics/Miniball Collaboration for Au+Au collisions at an incident energy of $E = 35A$ MeV [38] and by the INDRA Collaboration for ^{129}Xe+Sn and ^{155}Gd+^{238}U collisions at $E = 32A$ MeV and $E = 36A$ MeV, respectively [39]. The calculations have been

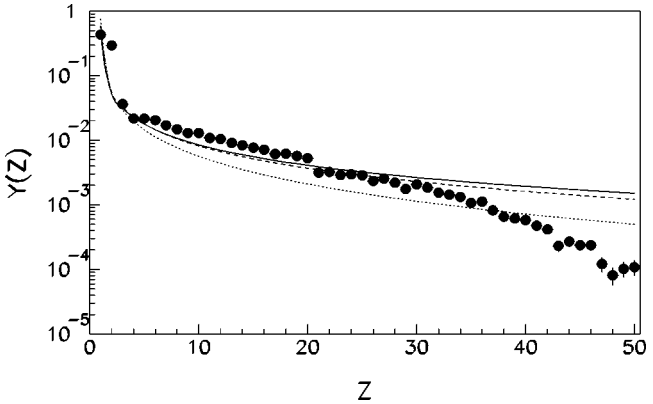

FIG. 6. Comparison of fragment distribution $Y(Z)$ calculated for $L(t)/r_0 = 6$ (solid line), 4 (dashed line), and 2 (dotted line) with experimental distribution for the reaction Au+Au at $35A$ MeV. Data from Ref. [38] have been normalized to 1.

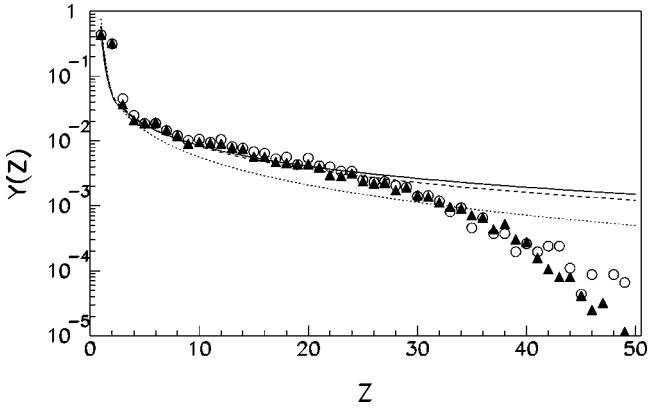

FIG. 7. Same as Fig. 6, but for the reactions ^{129}Xe+Sn at $E=32A$ MeV (triangles) and ^{155}Gd+^{238}U at $E=36A$ MeV (circles). Data from Ref. [39] have been normalized to 1.

performed for three values of the parameter $L(t)/r_0$. We have normalized the experimental distributions to one in order to perform the comparison on an absolute scale. We can see that the agreement between experimental data and the calculated charge distributions is quite satisfactory for $Z<30-35$ and that for the lighter fragments the experimental points are better reproduced with larger values of the ratio $L(t)/r_0$. For $Z>30-35$ the observed distribution presents a slope steeper than that predicted by our calculations. This faster decrease should be ascribed to finite-size effects [40] which have not been included in our nuclear matter treatment.

IV. SUMMARY AND CONCLUSIONS

We have studied the density fluctuations associated with a one-body treatment of nuclear dynamics. In our approach the fluctuations are generated by adding a stochastic term to the mean field. This additional random force is determined by a self-consistency condition required by the fluctuation-dissipation theorem. We have treated the effects of the stochastic field in linear approximation and this has allowed us to express the time evolution of the fluctuations in a closed form.

First we have analyzed the nature of the stochastic field and have shown that in general a white-noise assumption for the stochastic field is not consistent with the fluctuation-dissipation theorem. Then we have studied the particular physical conditions in which the white-noise nature of the stochastic term can be retained. These conditions include hot nuclear matter at a temperature $T\approx 5$ MeV, where the system can be still considered degenerate. We have found that for a Fermi system the treatment of density fluctuations by means of a white-noise stochastic term is justified when the limit $\omega/T\ll 1$ gives a reasonable approximation to the density-density response. This condition is better satisfied when the density and temperature of the system are close to the borders of the spinodal region in the (ϱ,T) plane. Thus, in the limit $\omega/T\ll 1$ the equilibrium fluctuations can be adequately described by means of thermodynamic functions and we can expect that in this limit the purely quantum fluctuations will play a negligible role also for systems not too far from equilibrium. We have extended the results obtained for the probability distribution of a metastable system to unstable situations. This has been achieved by extrapolating the relevant quantities across the boundary of the spinodal region. Because of the linear approximation used for evaluating the response of the system to the stochastic force, the fluctuations have a Gaussian probability distribution.

In the final part of this paper we have introduced a procedure to determine the size and mass distributions of the domains containing correlated density fluctuations; then we have compared the obtained mass distribution to the yield of light fragments observed in the multifragmentation of heavy nuclei. The procedure proposed here is quite general and can be applied to any Gaussian fluctuation distribution.

Our approach can account both for the observed power-law distribution and for the value of the effective exponent found experimentally, but for the exponent the agreement is limited to collisions with beam energies lower than $\sim 40A$ MeV. This discrepancy between our predictions and the observed values of the effective exponent in collisions of higher energies deserves further investigations. A detailed comparison with experiment has shown that our approach fairly reproduces the measured charge distributions for $Z<30-35$. Since we are dealing with infinite nuclear matter, we expect to overestimate the number of fragments having a large fraction of the mass of the emitting source.

Finally, we remark that the obtained mass distribution contains only one parameter, the ratio between the time-dependent length scale of domains $L(t)$ and the mean interparticle spacing r_0. This ratio can become large. A more detailed comparison of the present model with experimental data could also give an estimate of the time required by the system to break up.

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