R-matrix fits involving levels of ^{8}Be

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R-matrix formulas are used to derive information about levels of ^{8}Be and reactions involving ^{8}Be. Data from the ^{4}He$(\alpha,p)^7$Li, ^{4}He$(\alpha,n)^7$Be, and ^{7}Li$(p,\alpha)^4$He reactions and from $\alpha+\alpha$ elastic scattering are fitted, and a value obtained for the zero-energy S factor for ^{7}Li$(p,\alpha)^4$He: $S(0)=58$ keV b. Recent measurements and analyses of the ^{7}Li$(d,\alpha\alpha)n$ and ^{7}Li$(p,\gamma_0)^8$Be reactions, and calculations concerned with low-lying intruder states in ^{8}Be, are discussed with reference to *R*-matrix fits.

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I. INTRODUCTION

In a recent paper, Spitaleri *et al.* [1] used the Trojan-horse method to extract the zero-energy S factor for the ^{7}Li$(p,\alpha)^4$He reaction from measurements of the ^{7}Li$(d,\alpha\alpha)n$ cross section. The value derived by Spitaleri *et al.* differs considerably from that obtained in an *R*-matrix fit to earlier ^{7}Li$(p,\alpha)^4$He data [2].

R-matrix formulas have been used to fit data for a variety of reactions involving levels of ^{8}Be. In addition to this fit to the low-energy ^{7}Li$(p,\alpha)^4$He data [2], measured values of the ^{7}Li$(p,\alpha)^4$He total cross section, angular distribution, and analyzing power at higher proton energies have been fitted [3], involving ^{8}Be levels in the excitation-energy region from about 16 to 25 MeV. Other ^{8}Be levels in this energy range contributed to *R*-matrix fits to ^{7}Li$(p,\gamma_0)^8$Be data [4,5]. Lower-lying ^{8}Be levels were involved in fitting data from $\alpha+\alpha$ elastic scattering and from reactions such as ^{9}Be$(p,d)^8$Be and ^{8}Li and ^{8}B β decay [6–8]; in addition to the well-known 0^+ ground state, 2^+ first-excited state at 3 MeV, and pair of isospin-mixed 2^+ levels at 16.6 and 16.9 MeV, also very broad 0^+ and 2^+ intruder states at about 10 MeV were required.

Some of these *R*-matrix fits [2,3] are repeated and modified here, because of the availability of additional data. Other fits [4–8] are discussed in the light of new data and new calculations.

The *R*-matrix fit [2] found $S(0)=65$ keV b for the ^{7}Li$(p,\alpha)^4$He zero-energy S factor. Later Rolfs and Kavanagh [9] gave $S(0)=52\pm8$ keV b from their direct measurement of the ^{7}Li$(p,\alpha)^4$He cross section. Spitaleri *et al.* [1] obtained $S(0)=36\pm7$ keV b. In their analysis of the ^{7}Li$(d,\alpha\alpha)n$ data using the Trojan-horse method, Spitaleri *et al.* made two approximations that seem to be incorrect; they used an $l=0$ transmission coefficient for the ^{7}Li$+p$ Coulomb barrier, and they excluded contributions from the 16.6 and 16.9 MeV levels of ^{8}Be. Their approach is discussed in the next section.

The early *R*-matrix fit [2] to the low-energy ^{7}Li$(p,\alpha)^4$He data of Spinka *et al.* [10] gave only moderate agreement with the later measurement by Rolfs and Kavanagh [9]. This fit [2] used some parameter values obtained in the *R*-matrix fit to data over an extended energy range [3]; the latter fit was subsequently criticized for predicting incorrectly $\alpha+\alpha$ elastic-scattering phase shifts [11] and the ^{4}He$(\alpha,p)^7$Li(478 keV) cross section [12]. In Sec. III, we repeat and extend this fit [3] to include the more-recent data [11,12], and then repeat the low-energy fit [2] (with some modifications) to include the Rolfs and Kavanagh [9] and other relevant data, in order to get a better value of $S(0)$.

In Sec. IV, we discuss the *R*-matrix fits to low-energy ^{7}Li$(p,\gamma_0)^8$Be data [4,5] in relation to recent measurements and calculations [13]. Section V discusses calculations [14] that question the existence of the low-lying intruder states in ^{8}Be that are required by the *R*-matrix fits to data [6–8].

II. INFORMATION FROM THE ^{7}Li$(d,\alpha\alpha)n$ REACTION

From their ^{7}Li$(d,\alpha\alpha)n$ measurements, Spitaleri *et al.* [1] extracted values of the nuclear part of the ^{7}Li$(p,\alpha)^4$He cross section by using the Trojan-horse method [15]. The usual two-body cross section was then obtained by multiplying by a Coulomb-barrier transmission coefficient, and normalizing to the measured cross section [9] at the ^{7}Li$+p$ c.m. energy $E\approx300$ keV. For the transmission coefficient, they used the ^{7}Li$+p$ penetration factor for relative orbital angular momentum $l=0$. Decay to two α particles is possible, however, only from positive-parity states of the ^{8}Be system, so that only odd values of l can contribute to the ^{7}Li$(p,\alpha)^4$He cross section, and at low energies one should use the $l=1$ penetration factor (as was used in an earlier work [16] by the same group). The penetration factor increases with energy more rapidly for $l=1$ than for $l=0$, consequently use of the $l=1$ factor would lead to an even smaller value of $S(0)$.

Spitaleri *et al.* [1] assumed that only the quasifree breakup part of the ^{7}Li$(d,\alpha\alpha)n$ yield should be included in their calculation of the ^{7}Li$(p,\alpha)^4$He cross section. They also assumed that sequential decay through the 16.6 and 16.9 MeV levels of ^{8}Be ‘‘represents an undesired physical background which has to be subtracted . . . ,’’ but this is open to question. Spitaleri *et al.* represented the quasifree breakup and the sequential decay through states of ^{8}Be in their Figs. 1(a) and 1(b). We present these in slightly different form in Fig. 1, where it is assumed that the ^{7}Li(d,n) reaction proceeds by stripping; in both parts of Fig. 1, the neutron can be considered as a spectator, so that both represent quasifree

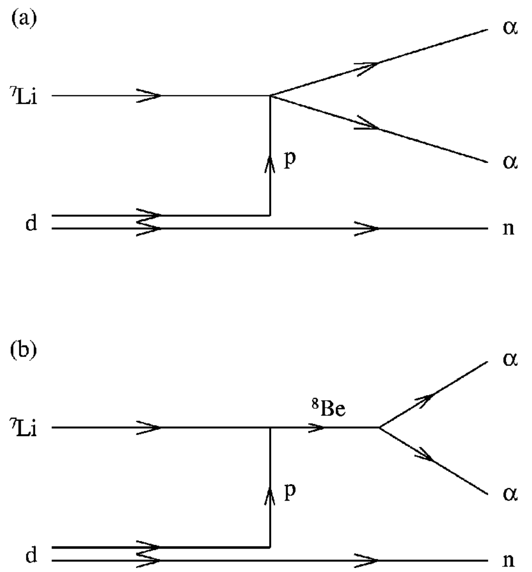

FIG. 1. Representations of possible reaction mechanisms for the reaction ^{7}Li$(d,\alpha\alpha)n$: (a) direct three-body breakup, (b) sequential decay through states of ^{8}Be.

processes. Sequential decay through the 16.6 and 16.9 MeV states of ^{8}Be should not be excluded on the grounds that it cannot be quasifree.

As an alternative to the Trojan-horse method, the ^{7}Li$(d,\alpha\alpha)n$ data could be analyzed by using *R*-matrix formulas. The contribution to the cross section for the two-stage reaction ^{7}Li$(p,\alpha)^4$He due to N levels of ^{8}Be of given J^π can be written [17]

$$\sigma_{p\alpha}=(\pi g/k_p^2)\sum_{sl}\left|\sum_{\lambda,\mu=1}^{N}\Gamma_{\lambda psl}^{1/2}\Gamma_{\mu\alpha}^{1/2}A_{\lambda\mu}\right|^2 \tag{1}$$

with

$$(\mathbf{A}^{-1})_{\lambda\mu}=(E_\lambda-E)\delta_{\lambda\mu}-\sum_c \gamma_{\lambda c}\gamma_{\mu c}L_c^0 \tag{2}$$

and

$$\Gamma_{\lambda psl}^{1/2}=(2P_l)^{1/2}\gamma_{\lambda psl}, \quad \Gamma_{\mu\alpha}^{1/2}=(2P_{\alpha J})^{1/2}\gamma_{\mu\alpha}, \tag{3}$$

where s is the ^{7}Li$+p$ channel spin. The summations over λ and μ in Eq. (1) cover both bound and unbound levels in the ^{7}Li$+p$ channel. The summation over c in Eq. (2) covers all decay channels, including psl and α. The yield of the ^{7}Li$(d,\alpha\alpha)n$ reaction due to the same N levels can be written, as a function of E [18],

$$\sigma_{dn,\alpha}\propto\sum_x\left|\sum_{\lambda,\mu=1}^{N}G_{\lambda x}^{1/2}\Gamma_{\mu\alpha}^{1/2}A_{\lambda\mu}\right|^2, \tag{4}$$

where $G_{\lambda x}^{1/2}$ is a feeding amplitude and x labels the quantum numbers for the formation process that give incoherent contributions. For sufficiently high deuteron energies, $G_{\lambda x}$ is a slowly-varying function of E. If the ^{7}Li(d,n) reaction proceeds by stripping, then one has

$$G_{\lambda x}^{1/2}\propto\gamma_{\lambda psl}. \tag{5}$$

The formula (4) applies for energies E above and below the ^{7}Li$+p$ threshold. For energies above the threshold, if only one l-value contributes significantly, one has from Eqs. (1), (3), (4), and (5)

$$\sigma_{p\alpha}\propto(1/E)\,P_l\,\sigma_{dn,\alpha}. \tag{6}$$

It may be noted that $\sigma_{dn,\alpha}$ contains contributions from ^{8}Be levels above and below the threshold, and the same applies to $\sigma_{p\alpha}$. It seems that Spitaleri *et al.* [1] were not justified in excluding contributions from the subthreshold 16.6 and 16.9 MeV levels.

III. *R*-MATRIX FIT TO ^{7}Li$(p,\alpha)^4$He AND RELATED DATA

In the previous *R*-matrix fit to low-energy ^{7}Li$(p,\alpha)^4$He data [2], a three-level approximation was used, and the data consisted of values of the total cross section and angular-distribution coefficients for $E_p\lesssim 600$ keV. Parameter values for one of the levels were taken from an earlier fit to more-extensive data, including analyzing-power coefficients, for $E_p\lesssim 7$ MeV [3].

This earlier fit [3] involved two 0^+ levels and four 2^+ levels, with contributions from proton and neutron channels corresponding to the ground and first-excited states of ^{7}Li and ^{7}Be, as well as the $\alpha+\alpha$ channel. The formulas were modified to take account of very broad background levels, which contributed to the general trend of the real part of the $\alpha+\alpha$ phase shifts [6,7]. The imaginary parts of the phase shifts and sharp changes in the real parts were not included in the fitted data, also no data directly involving the ^{7}Li excited state or either state of ^{7}Be were included. Hence it is not surprising that the predictions of this fit showed discrepancies with later measured values of the phase shifts [11] and of cross sections for the ^{4}He$(\alpha,p)^7$Li(478 keV) reaction [12].

Here the calculations of Ref. [3] are modified, with the fitted data including the ^{4}He$(\alpha,p)^7$Li total cross sections and angular distribution coefficients for both states of ^{7}Li [12], the ^{4}He$(\alpha,n)^7$Be total cross section (summed over the two ^{7}Be states) [12], the $\alpha+\alpha$ elastic-scattering complex phase shifts [11], as well as the analyzing-power data used in Ref. [3]. These data are shown in Figs. 2–6, together with the *R*-matrix best fit. The ^{4}He(α,p) and ^{4}He(α,n) data cover the full range given in Ref. [12] (for E_α values from about 39 MeV to 49 MeV). In addition, because we are here particularly interested in the low-energy region, we include values of the ^{4}He$(\alpha,p)^7$Li(g.s.) total cross section and angular-distribution coefficients for $E_\alpha\approx 35-38$ MeV obtained from the ^{7}Li$(p,\alpha)^4$He data used in Ref. [3]. The analyzing-power measurements cover the range $E_\alpha=36-47$ MeV. These sets of data include values of the experimental uncertainties. The real parts of the phase shifts δ_J and the inelasticities η_J are given in Ref. [11] at closely spaced energies for $E_\alpha=30-70$ MeV, without uncertainties. We use the values of δ_J and η_J $(J=0,2,4)$ for E_α between 35

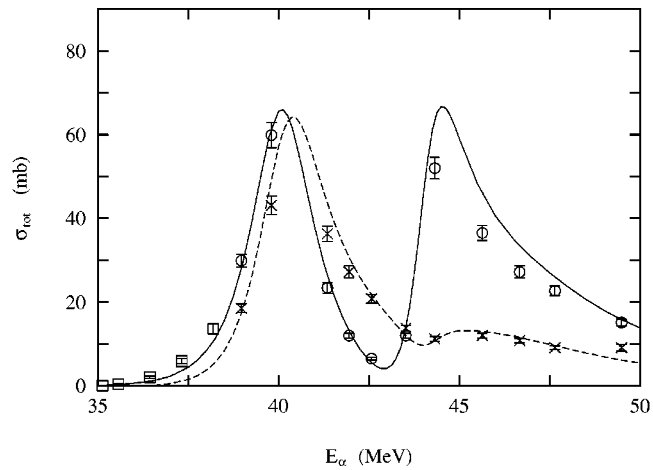

FIG. 2. Total cross section for the ^{4}He$(\alpha,p)^7$Li reaction as a function of α-particle energy. The experimental points [12] are for production of the ^{7}Li ground state (circles) and first-excited state (crosses). The ground-state points are extended to lower energies by using ^{7}Li$(p,\alpha)^4$He data [3] (squares). The curves are *R*-matrix best fits to the data shown in Figs. 2–6; ground state (solid line), excited state (dashed line).

and 49 MeV, at intervals of 0.5 or 1.0 MeV, measured from Fig. 1 of Ref. [11], with uncertainties assigned rather arbitrarily ($\pm 2°$, $\pm 10°$, and $\pm 5°$ for δ_0, δ_2, and δ_4, respectively, and ± 0.02 for each η_J).

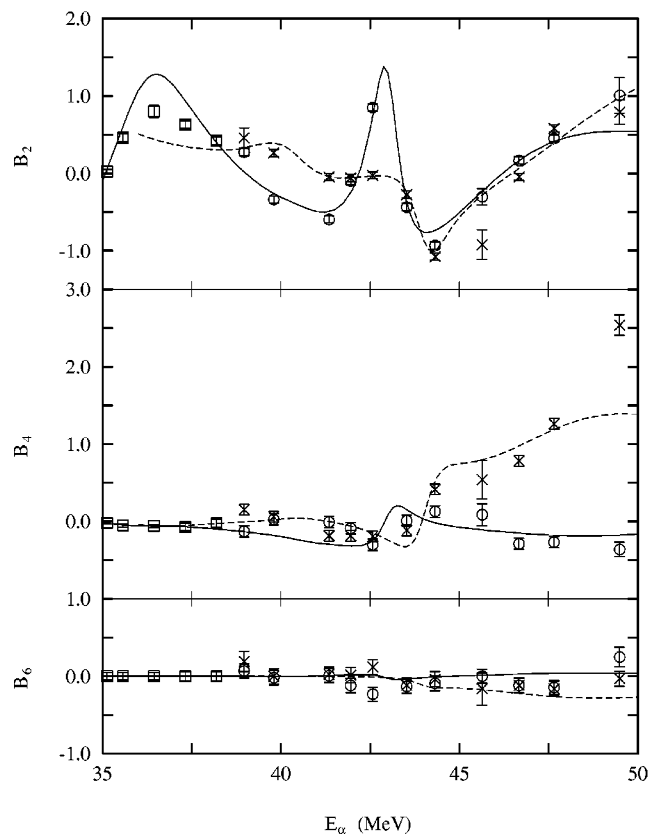

FIG. 3. Angular-distribution coefficients B_L for the ^{4}He$(\alpha,p)^7$Li reaction as functions of α-particle energy. The points and curves have the same meaning as in Fig. 2.

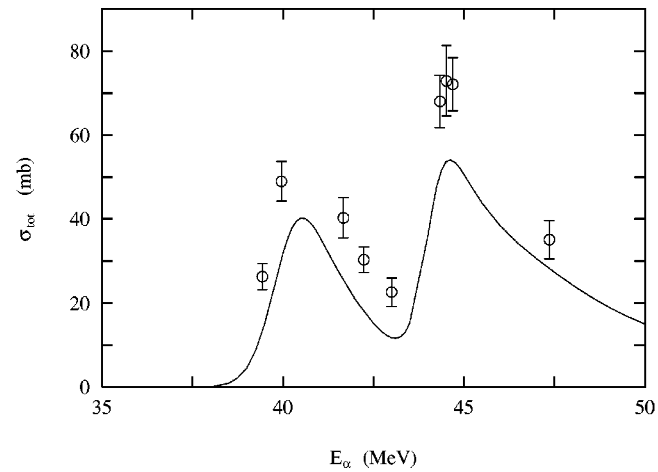

FIG. 4. Total cross section for the ^{4}He$(\alpha,n)^7$Be reaction (sum of ground-state and excited-state contributions), as a function of α-particle energy. The experimental points are from Ref. [12], and the curve is for the *R*-matrix best fit as in Fig. 2.

We include two 0^+ levels, four 2^+ levels, and one 4^+ level. As in Ref. [3], each level is assumed to be $T=0$. The lower 0^+ level at about 20 MeV and the three lowest 2^+ levels are closely related to those used in Ref. [3]. The lowest 2^+ level represents the known 2^+ levels at 16.6 and 16.9

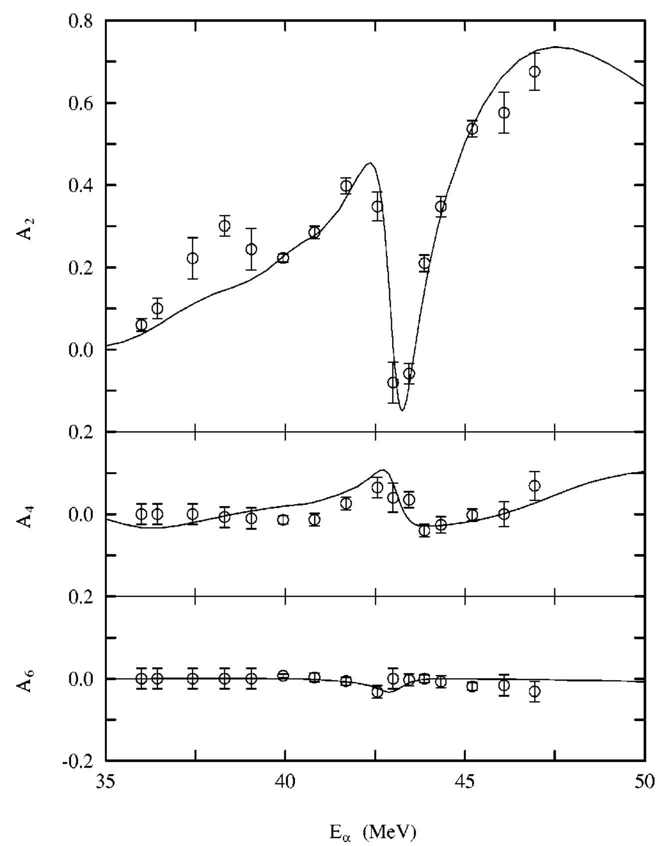

FIG. 5. Analyzing-power coefficients A_L for the ^{7}Li$(p,\alpha)^4$He reaction, as functions of the equivalent α-particle energy for the inverse reaction. The experimental points are from Ref. [3], and the curves are for the *R*-matrix best fit of Fig. 2.

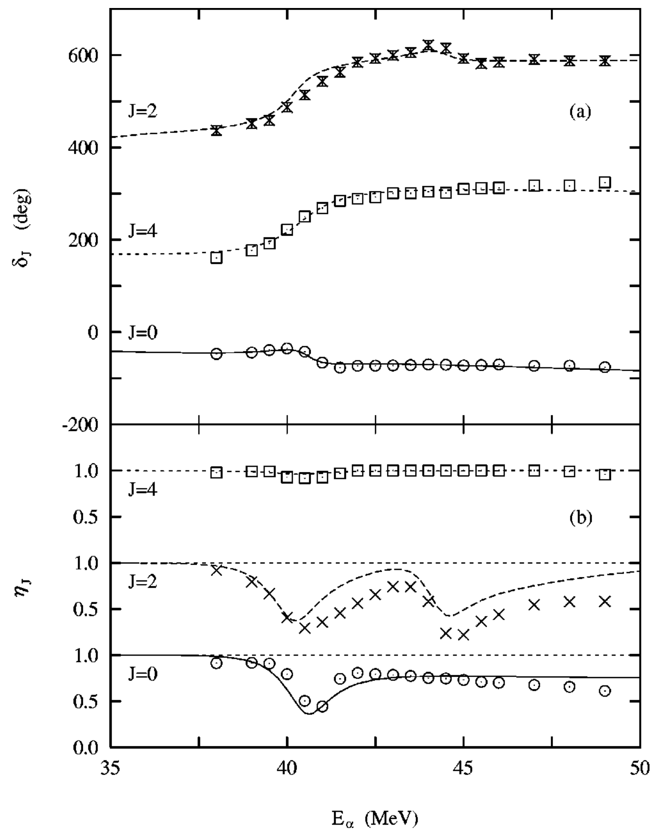

FIG. 6. (a) Real parts δ_J of the $\alpha+\alpha$ scattering phase shifts and (b) inelasticities η_J as functions of α-particle energy. The points are from Ref. [11], with the assigned uncertainties not shown when less than the symbol size.

MeV, which lie below the ^{7}Li$+p$ threshold, and the other two are at about 20 and 22 MeV. A feature of the present data not apparent in the data available in Ref. [3] is that each of the inelasticities η_0 and η_2 shows a steady decline from its low-energy value of unity as the energy increases, with rapid fluctuations due to narrow levels superimposed. These trends lead us to introduce broad background 0^+ and 2^+ levels, which we locate at $E_x=30$ MeV ($E_\alpha\approx 60$ MeV). A 4^+ level near 20 MeV is needed to fit the δ_4 and η_4 data. As in Ref. [3], trends in the real phase shifts δ_J are accounted for by factors $Q_{\alpha J}$ ($J=0,2,4$) attributed to additional broad levels of ^{8}Be with reduced widths large for the α channel and zero for all other channels [6,7]. Also, as in Ref. [3], *p*- and *f*-wave nucleons are included ($l=1,3$), we use B_c equal to $-l$ for nucleon channels and zero for α channels, and the channel radii are taken as 4.22 fm for nucleon channels and 6.0 fm for α channels. This value of the α channel radius was chosen to be reasonably near to the values obtained from fitting scattering and reaction data [6,7] (see Sec. V).

TABLE II. Background-parameter values for *R*-matrix fits to ^{4}He$(\alpha,p)^7$Li and other data.

J^π	E_0 (MeV)	α_J (deg)	β_J (deg MeV^{-1})
0^+	20.5	35.0	-4.50
2^+	20.5	424.0	-2.00
4^+	20.5	152.0	-4.00

The best-fit level-parameter values are given in Table I. The notation is as in Ref. [3]; the level parameters are the eigenenergy E_λ of level λ for each J value, and reduced-width amplitudes $\gamma_{\lambda psl}$ for the ^{7}Li$+p$ ground-state channel, $\gamma_{\lambda pl}$ for the excited-state channel (with $s=1$), and $\gamma_{\lambda\alpha}$ for the $\alpha+\alpha$ channel. In all, 35 parameters were varied, not all at the same time. Also slight adjustments were made to the background phase shifts $\overline{\delta_J}$, which are represented by [19]

$$\overline{\delta_J}=\alpha_J+\beta_J(E-E_0), \tag{7}$$

where E is here the c.m. energy in the $\alpha+\alpha$ channel; the best-fit values of α_J and β_J are shown in Table II. It is possible that better fits could be obtained with parameter values different from those in Tables I and II, but the present set seems to reproduce the data sufficiently well. The biggest systematic discrepancy, for the ^{4}He$(\alpha,n)^7$Be cross section shown in Fig. 4, is presumably due mainly to the assumption of $T=0$ for all levels.

The previous fit [2] to the low-energy ^{7}Li$(p,\alpha)^4$He data assumed that contributions came only from three 2^+ levels of ^{8}Be, at 16.6, 16.9, and about 20 MeV, and that *f*-wave nucleon channels could be neglected. The parameter values for the 20 MeV level were taken from the earlier fit [3] to the higher-energy data, after allowance for a different choice of B_c values. For the 16.6 and 16.9 MeV levels, the nucleon

TABLE I. Level-parameter values for *R*-matrix fits to ^{4}He$(\alpha,p)^7$Li and other data. $a_l=4.22$ fm, $a_\alpha=6.0$ fm, $B_l=-l$, $B_\alpha=0.0$. Eigenenergies are given in MeV, reduced-width amplitudes in MeV$^{1/2}$.

J^π	λ	E_λ	$\gamma_{\lambda p11}$	$\gamma_{\lambda p21}$	$\gamma_{\lambda p13}$	$\gamma_{\lambda p23}$	$\gamma_{\lambda p1}$	$\gamma_{\lambda p3}$	$\gamma_{\lambda\alpha}$
0^+	1	21.01	0.033	0.0	0.0	0.0	-0.707	0.0	0.321
	2	30.0	2.047	0.0	0.0	0.0	0.314	0.0	-0.525
2^+	1	15.85	0.091	0.388	-0.332	-1.101	0.006	-0.472	0.225
	2	20.17	-0.170	0.270	-0.297	0.123	0.322	0.048	0.226
	3	22.29	-0.028	-0.397	-0.089	0.034	-0.080	0.115	-0.142
	4	30.0	0.834	0.172	0.638	0.064	0.468	1.506	-0.188
4^+	1	20.42	0.0	0.0	-0.049	-0.008	0.0	0.072	0.305

TABLE III. Level-parameter values for 2^+ states of ^{8}Be. $a_1=4.22$ fm, $a_\alpha=6.0$ fm, $B_1=-1.60$, $B_\alpha=0.0$. Eigenenergies are given in MeV, reduced-width amplitudes in MeV$^{1/2}$.

E_λ	$\gamma_{\lambda p11}$	$\gamma_{\lambda p21}$	$\gamma_{\lambda p1}$	$\gamma_{\lambda n11}$	$\gamma_{\lambda n21}$	$\gamma_{\lambda n1}$	$\gamma_{\lambda\alpha}$
16.76	0.605	1.137	0.093	0.264	0.378	0.041	0.103
16.85	−0.264	−0.378	−0.041	−0.605	−1.137	−0.093	0.081
20.75	−0.228	0.320	0.318	0.228	−0.320	−0.318	0.209
31.01	0.836	0.183	0.469	−0.836	−0.183	−0.469	−0.184

reduced-width amplitudes were assumed to be related by the two-state isospin-mixing model, with approximately maximal mixing, and values of small amplitudes were taken from shell-model calculations. The energies and α-particle reduced-width amplitudes of these levels were determined by fitting the positions and widths that had been obtained earlier [7]. Only three parameters were varied in fitting the $^7\mathrm{Li}(p,\alpha)^4\mathrm{He}$ data, which included the total cross section measured by Spinka *et al.* [10] at four energies with $E_p=130-561$ keV.

We now modify this fit in several respects. Later measurements of the $^7\mathrm{Li}(p,\alpha)^4\mathrm{He}$ S factor by Rolfs and Kavanagh [9] covered the range $E_{\mathrm{c.m.}}=24.6-873$ keV ($E_p\approx30-1000$ keV), with angular distributions measured for $E_p=50-900$ keV. Other measurements are available at these and even lower energies. Harmon [20] gave $S(0)=49\pm2$ keV b from his cross section measurements for $E_p=20-250$ keV, but his results are normalized to the S factor for the $^6\mathrm{Li}(p,\alpha)^3\mathrm{He}$ cross section, which he apparently assumed to have the same angular distribution as $^7\mathrm{Li}(p,\alpha)^4\mathrm{He}$. Engstler *et al.* [21] gave the S factor for $E_{\mathrm{c.m.}}=12.7-1000$ keV and the angular distribution for $E_{\mathrm{c.m.}}=26-1000$ keV; for $E_{\mathrm{c.m.}}\lesssim50$ keV, the S factor is enhanced considerably by electron screening. We fit the Rolfs and Kavanagh data [9], in addition to those of Spinka *et al.* [10] (including their values of the angular-distribution coefficient as given by Rolfs and Kavanagh). The positions and widths of the 16.6 and 16.9 MeV levels are now given very precisely by the $\alpha+\alpha$ elastic-scattering measurements of Hinterberger *et al.* [19]. They used a simplified two-level *R*-matrix formula to fit their data, and gave the resultant values of the level parameters. As we use *R*-matrix formulas that have energy-dependent penetration and shift factors, and include contributions from nucleon channels, we cannot make direct use of their parameter values. Instead we construct the $\alpha+\alpha$ d-wave phase shift for $E_\alpha=32-36$ MeV ($E_{\mathrm{c.m.}}<654$ keV) from the parameter values given in their Tables I and IV, and then include these phase shift values, with assigned uncertainties of $\pm2°$, in the fitted data.

In addition to the three 2^+ levels at 16.6, 16.9, and about 20 MeV that were used before [2], we include the broad background 2^+ level at about 30 MeV. The factors $Q_{\alpha2}$ are included as before. Because of the additional data, we allow variations in more parameters than the three allowed before, but still relate neutron reduced-width amplitudes to proton reduced-width amplitudes in the same way.

TABLE IV. Background-parameter values for 2^+ states of ^{8}Be.

E_0 (MeV)	α_2 (deg)	β_2 (deg MeV^{-1})
16.84	77.61	−5.33

The best-fit values of the parameters are given in Tables III and IV, and the corresponding fits to the data are shown in Figs. 7–9. It is interesting that the angular-distribution coefficient B_2 shown in Fig. 8 becomes negative for $E_{\mathrm{c.m.}}\lesssim100$ keV, although Rolfs and Kavanagh apparently assumed $B_2\to0$ as $E_{\mathrm{c.m.}}\to0$. The change of sign is in agreement with the measurements of Engstler *et al.* [21], which were not included in the fit. From our *R*-matrix fit we obtain $S(0)=58$ keV b. This agrees with the value 58.7 keV b given by Engstler *et al.* from a polynomial fit to previous data (essentially from Rolfs and Kavanagh) for $E_{\mathrm{c.m.}}\geqslant100$ keV. It seems that the value of $S(0)$ obtained by Spitaleri *et al.* [1] is much too low.

IV. THE $^7\mathrm{Li}(p,\gamma_0)^8\mathrm{Be}$ S FACTOR

In a recent measurement, Spraker *et al.* [13] found a negative slope for the $^7\mathrm{Li}(p,\gamma_0)^8\mathrm{Be}$ S factor for proton energies between 40 and 100 keV. This appears to be not inconsistent with the measurement of Zahnow *et al.* [22] for $E_p=100-1500$ keV; their S factor is approximately constant up to about 250 keV. Cecil *et al.* [23] found a constant S factor for $E_p=40-170$ keV.

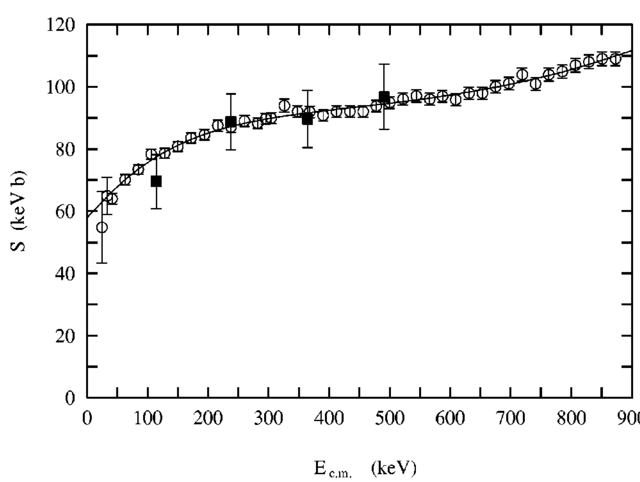

FIG. 7. S factor for the $^7\mathrm{Li}(p,\alpha)^4\mathrm{He}$ reaction as a function of the $^7\mathrm{Li}+p$ c.m. energy. The experimental points are from Ref. [9] (open circles) and Ref. [10] (filled squares). The curve is the *R*-matrix best fit to the data shown in Figs. 7–9.

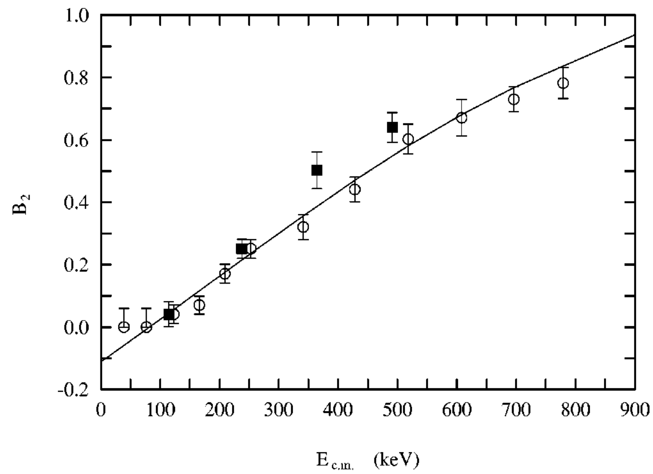

FIG. 8. Angular-distribution coefficient B_2 (denoted by a_2 or A_2 in Ref. [9]) as a function of the ^{7}Li+p c.m. energy. The points and curve have the same meaning as in Fig. 7.

An R-matrix fit to ^{7}Li$(p,\gamma_0)^8$Be data, including the S factor, angular distribution, and analyzing power, over a wide energy range ($E_p \lesssim 1500$ keV), found a positive slope of the low-energy S factor [5]. This fit included $M1$ contributions from two 1^+ levels of ^{8}Be (the 17.64 and 18.15 MeV levels), and $E1$ contributions from two 1^- levels (the GDR and a lower-lying $T=1$ level). All of these levels lie above the region of low proton energies, and this is essentially the reason for the positive slope of the S factor.

Spraker *et al.* [13] seek to explain their observed negative slope by including an $E2$ contribution from the 2^+, 16.6 MeV level of ^{8}Be, which lies below the ^{7}Li+p threshold. They say ''The influence of this state has not been included in previous work.'' In Ref. [4], however, possible contributions from the 16.6 and 16.9 MeV levels were considered, and estimated to be less than 1% of the observed S factor. This upper limit is reduced by using more-recent experimental values for the $E2$ ground-state γ widths of the 16.6 and 16.9 MeV levels [24].

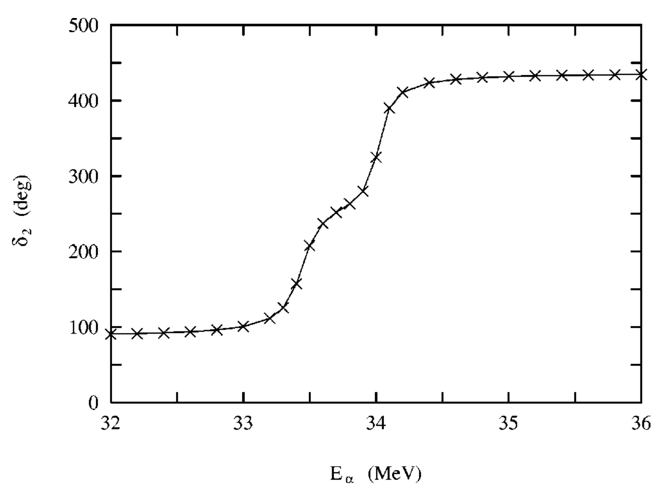

FIG. 9. Real part δ_2 of the $\alpha+\alpha$ d-wave scattering phase shift as a function of α-particle energy. The points are obtained from the parameter values in Ref. [19]. The curve has the same meaning as in Fig. 7.

Nevertheless, Spraker *et al.* claim that they can obtain agreement with their observed negative slope, in a model that contains contributions only from direct $E1$ capture and from the 16.6 MeV level; however, this model and extensions of it do not describe other observed quantities such as the angular distribution and analyzing power for ^{7}Li$(p,\gamma_0)^8$Be [13].

Spraker *et al.* assumed a linear function of energy to fit their low-energy S factor

$$S(E_{\rm c.m.}) = S_0 + S_1 E_{\rm c.m.}. \tag{8}$$

With their data normalized to those of Zahnow *et al.* [22] at $E_p = 98.3$ keV, they obtained $S_0 = 0.50 \pm 0.07$ keV b and $S_1 = (-9.5 \pm 3.2) \times 10^{-4}$ b, giving $S_1/S_0 = -1.9$ MeV^{-1}. [For ^{7}Li$(p,\gamma_1)^8$Be, they obtained $S_1/S_0 = -1.3$ MeV^{-1}.] The R-matrix fit [5] gave $S_1/S_0 = 0.5$ MeV^{-1}.

In support of their negative slope, Spraker *et al.* gave their results for the S factor for ^{7}Li$(p,\alpha)^4$He, which they measured at the same time—with normalization to the results of Rolfs and Kavanagh [9], they found $S_0 = 49 \pm 4.4$ keV b and $S_1 = 0.036 \pm 0.003$ b, or $S_1/S_0 = 0.7$ MeV^{-1}. They say that their results are in excellent agreement with those of Rolfs and Kavanagh, who gave $S_0 = 52 \pm 8$ keV b, but did not give a value of S_1. The R-matrix fit to the Rolfs and Kavanagh data shown in Fig. 7 gives $S_1/S_0 = 2.9$ MeV^{-1} for the energy range $E_p = 40$–100 keV. Similarly the polynomial fit of Engstler *et al.* [21] gives $S_1/S_0 = 3.2$ MeV^{-1}. The discrepancy for ^{7}Li$(p,\alpha)^4$He between the S_1/S_0 values of Spraker *et al.* (0.7) and the fits to the Rolfs and Kavanagh data (2.9 and 3.2) is about the same as that for ^{7}Li$(p,\gamma_0)^8$Be between the S_1/S_0 values of Spraker *et al.* (–1.9) and the R-matrix calculation [5] (0.5). This might lead one to question the claim by Spraker *et al.* that ''there is not a systematic problem in this technique which produces negative slopes.''

Spraker *et al.* [13] also measured the ^{7}Li$(p,\gamma_0)^8$Be analyzing power at 90°. From their Fig. 5, this decreases from 0.42 to 0.12 as E_p decreases from 80 to 40 keV (although the text says the decrease is from 0.4 to about 0.25). From the formulas in Ref. [5], $A_y(90°) = b_1/(1-\frac{1}{2}a_2) \approx b_1$ (since a_2 is very small), and the R-matrix fit [5] to higher-energy data predicts a decrease from 0.38 to 0.35 as E_p decreases from 80 to 40 keV.

V. USE OF *R*-MATRIX FORMULAS IN CALCULATIONS FOR ^{8}Be BELOW THE ^{7}Li+p THRESHOLD

The R-matrix fits of Secs. III and IV depend explicitly on properties of ^{8}Be levels in the energy region near and above the ^{7}Li+p threshold. The fits of Sec. III also involve properties of ^{8}Be levels below this threshold; the factors $Q_{\alpha J}$ are attributed [3] to broad levels of ^{8}Be that were required in earlier R-matrix fits [6,7]. These fits were made to $\alpha+\alpha$ elastic-scattering phase shifts and to data from reactions in which ^{8}Be is an unstable product nucleus, decaying to two α particles, e.g., ^{9}Be$(p,d)^8$Be$(\alpha)^4$He, and from ^{8}Li and ^{8}B β decay. In order to obtain consistent fits to the scattering and reaction data, the $\alpha+\alpha$ channel radius a_α needed to be large, about 6.5 or 7.0 fm, and this large value implied the existence of excited 0^+ and 2^+ states of ^{8}Be at about 10 MeV excitation energy. These states were very broad, and had to be interpreted as intruder states, as they could not belong to the lowest $(1s^4 1p^4)$ shell-model configuration. Warburton [25] sought to avoid such low-lying intruder states by using a smaller channel radius (4.5 fm), but he could not then consistently fit the scattering and reaction data with the same values of the *R*-matrix parameters.

Recently, Fayache *et al.* [14] have queried the existence of low-lying intruder states in ^{8}Be. They carried out shell-model calculations including higher configurations $[(0+2+4)\hbar\omega]$ and also made deformed-oscillator model calculations; they did not find any low-lying intruder states in ^{8}Be, although they did in the neighboring nuclei ^{10}Be and ^{12}C. In a Comment [26] on this work, it was argued that their models were not sufficiently realistic for a decision to be possible on whether or not such intruder states exist in ^{8}Be, but a Reply [27] to the Comment argued that the calculations were relevant. In particular, the Reply says ''We therefore feel that we made . . . a very solid case to the effect that there are no low-lying intruders in ^{8}Be.'' If such states do not exist, the justification for the calculations of Sec. III would be considerably weakened. It seems, however, that several of the arguments used by Fayache *et al.* [27] to support their conclusion are open to question.

Fayache *et al.* [27] claim that more-realistic shell-model calculations using the Arizona interaction [28] give results that completely agree with theirs. These calculations for ^{8}Be and similar ones for ^{10}Be [29] do give results that agree with those that Fayache *et al.* [14] obtained for their interaction (c), for which the lowest calculated intruder state was near 30 MeV for both ^{8}Be and ^{10}Be, but do not agree at all with the results for their interactions (a) and (b), on which Fayache *et al.* based their conclusion that finding low-lying intruder states in ^{10}Be does not prove that they are also present in ^{8}Be. As is said in the Comment [26], such shell-model calculations [14,28,29] using a harmonic-oscillator basis would not be expected to predict states of the type suggested by the *R*-matrix fits to ^{8}Be data [6,7], as they are very unbound.

Also Fayache *et al.* [27] suggest ambiguity in the *R*-matrix analysis [6,7], by saying that there is uncertainty in how the parameters should be chosen and referring to a recent analysis of *s*-wave $\alpha+\alpha$ scattering by Humblet *et al.* [30], which found no evidence for the existence of a resonance near 9 MeV. This analysis [30] adopted a channel radius of 6 fm and consequently the values of the *R*-matrix parameters were comparable with those in Ref. [6], including an *R*-matrix pole near 9 MeV. At this pole energy, the resonant phase shift $\beta=\delta-\phi$ increases through 90°. Humblet *et al.* [30] defined a physical resonance by requiring the total phase shift δ to increase through 90°, and it is obvious that there is no ''resonance'' near 9 MeV, but this does not imply any ambiguity in the *R*-matrix parameter values.

In seeking to understand why some of their calculations gave low-lying intruder states in ^{10}Be and ^{12}C but not in ^{8}Be, Fayache *et al.* [14] considered the Nilsson diagram. They said that intruder states in ^{10}Be and ^{12}C are formed by

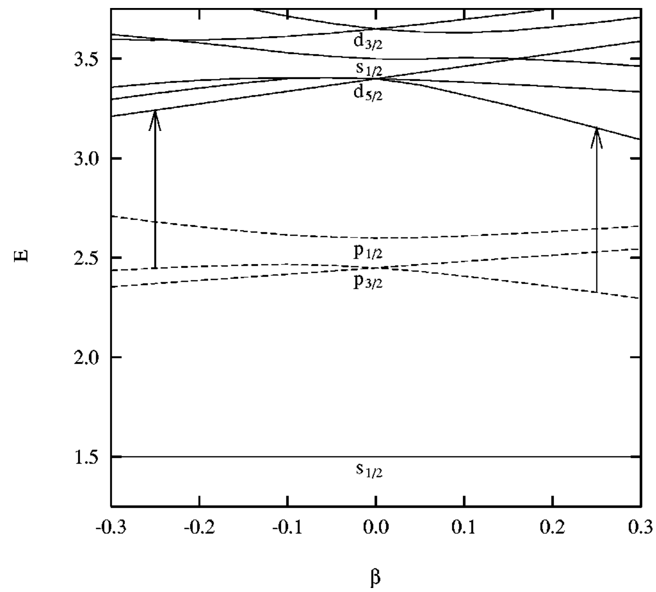

FIG. 10. Nilsson diagram (from Ref. [31]). Eigenenergies (in units of $\hbar\omega_0$) as functions of the deformation parameter β. The arrows indicate nucleon excitations involved in the formation of intruder states in ^{8}Be ($\beta>0$) and ^{12}C ($\beta<0$).

taking nucleons from upward-going lines, whereas for ^{8}Be the nucleons must be taken from a down-going line, which costs much more energy. To illustrate this they show, however, only the prolate side of the Nilsson diagram. Since their ^{10}Be ground state is triaxial and ^{12}C is oblate, it is not at all clear that their argument is valid, even though the Reply [27] insists that it is. For ^{12}C on the oblate side, the line from which the nucleons must come is also down-going (for increasing $|\beta|$); as illustrated in Fig. 10, the excitation energy required to form an intruder state in ^{12}C is about the same as that for ^{8}Be on the prolate side, for the same value of $|\beta|$.

Fayache *et al.* [27] say that an intruder state in ^{8}Be can be formed only by exciting one of the α particles. If, however, the ^{8}Be ground state is regarded as a $3s$ oscillation of two α particles, then a 0^+ intruder state can be formed by a $4s$ oscillation, without exciting either α particle.

It seems that model calculations of the type performed by Fayache *et al.* [14] and the supporting arguments that they give [14,27] are not adequate for reliable conclusions to be drawn on whether or not there are low-lying intruder states in ^{8}Be. The situation remains that consistent *R*-matrix fits to scattering and reaction data require such states.

VI. SUMMARY

R-matrix formulas have been used and can be used to fit data for various reactions involving ^{8}Be and to give information about levels of ^{8}Be.

The ^{7}Li$(d,\alpha\alpha)n$ measurements of Spitaleri *et al.* [1] could be fitted and the ^{7}Li$(p,\alpha)^4$He *S* factor derived using the *R*-matrix formulas of Sec. II, as an alternative to the Trojan-horse method that they adopted.

A large amount of data from the ^{4}He$(\alpha,p)^7$Li, ^{4}He$(\alpha,n)^7$Be, and ^{7}Li$(p,\alpha)^4$He reactions and from $\alpha+\alpha$ scattering is fitted in Sec. III, and a value obtained for the

zero-energy S factor for the ${}^7\mathrm{Li}(p,\alpha){}^4\mathrm{He}$ reaction $S(0)=58$ keV b. This is somewhat less than the value 65 keV b obtained in an earlier R-matrix fit [2], but much larger than the recent value 36 ± 7 keV b given by Spitaleri *et al.* [1].

The suggestions by Spraker *et al.* [13] that the slope of the low-energy S factor for the ${}^7\mathrm{Li}(p,\gamma_0){}^8\mathrm{Be}$ reaction is negative, in contradiction with the prediction of an R-matrix fit to higher-energy data [5], and that this can be attributed to a contribution from the subthreshold 16.6 MeV level of ${}^8\mathrm{Be}$, are questioned in Sec. IV.

It is suggested in Sec. V that arguments [14,27] against the existence of low-lying intruder states in ${}^8\mathrm{Be}$, which are required by consistent R-matrix fits to scattering and reaction data, are not convincing.

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