Separation of isoscalar, isovector, orbital, and spin contributions in $M1$ transitions in mirror nuclei

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Experimental data for $M1$ matrix elements in the $T=1/2$ mirror nuclei ^{27}Al and ^{27}Si are used to determine the isoscalar and isovector $M1$ components. The obtained isovector $M1$ matrix elements are compared to the Gamow-Teller matrix elements deduced from β decay and the ^{27}Al(^{3}He,t)^{27}Si charge exchange reaction to determine the contributions of the isovector spin and orbital components. The exchange current effects are taken into account. The results are compared with shell model calculations.

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I. INTRODUCTION

The magnetic dipole ($M1$) operator for $M1$ γ transitions and magnetic moments is dominated by the isovector (IV) spin (σ) term, but also contains IV orbital (l), isoscalar (IS) spin, and IS orbital components. On the other hand, the Gamow-Teller (GT) operator for GT β decays contains only the IV spin term [1,2]. These electromagnetic $M1$ and weak GT interactions also have different mesonic-exchange current (MEC) contributions [3,4]. It is well known that the diagonal matrix elements of these various terms of the $M1$ operator can be isolated by comparing the magnetic moments of mirror ground states and the GT β-decay matrix element which connects them [5]. Such separation into different components has been useful for the deeper understanding of the nuclear structure and the wave functions of the ground states. Equivalent information can be obtained for the off-diagonal matrix elements by comparing the $M1$-transition matrix elements of mirror nuclei to the analogous GT matrix elements which connect them, although it has been difficult to find cases where the analogous relationship between the $M1$ and GT transitions can be well utilized [6–8].

Previous studies on the contributions of the off-diagonal (transition) matrix elements have concentrated on the $\Delta T=1$ $M1$ transitions starting from the ground state (g.s.) of $T=0$ even-even target nucleus. The IV orbital contribution for the ^{32}S target was studied by comparing (p,n) and (e,e') reactions [9], and for the ^{28}Si target by comparing (^{3}He,t) and (e,e') reactions [10,11]. Results of the studies on $T=0$ targets showed that the ratios $B(M1)/B(\mathrm{GT})$ were not constant, but rather dependent on final state, suggesting that the IV orbital contribution to the $M1$ transition can be large depending on the configuration of the states [10,11]. The contribution of meson-exchange currents was also deduced from the study of analogous transitions [10–14].

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In this paper we use the analogous $M1$ γ decay data in $T=1/2$ mirror nuclei ^{27}Al and ^{27}Si together with GT matrix elements deduced from β decay and recent ^{27}Al(^{3}He,t)^{27}Si charge-exchange (CE) reaction data to extract the individual spin and orbital components for the off-diagonal $M1$ matrix elements. These off-diagonal matrix elements connect the ground states to the low-lying excited states of the $A=27$, $T=1/2$ system (see Fig. 1).

Recently the mirror-symmetry nature of the nuclear structure was established for the pair of nuclei ^{27}Al-^{27}Si up to $E_x\approx 9$ MeV based on the similarity of transition energies and strengths of analogous $M1$ and GT transitions in these nuclei [15]. In the analysis, the $M1$ γ transitions from excited states to the g.s. of ^{27}Al and the GT transitions deduced from the (^{3}He,t) reaction from the g.s. of ^{27}Al to the corre-

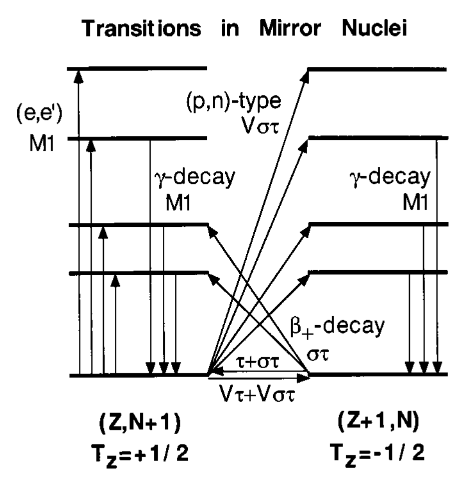

FIG. 1. $M1$ and GT transitions between $T=1/2$ states in $T=1/2$ mirror nuclei. Analogous $M1$ and GT transitions connecting the ground state of each nucleus with the excited states in the same nucleus and those in the conjugate nucleus, respectively, are indicated. In p-shell and sd-shell regions, $T_z=+1/2$ nucleus is stable, while $T_z=-1/2$ nucleus is β unstable. The type of reaction or decay and the relevant interactions causing each transition are shown with the arrows indicating the transitions.

sponding (analogous) excited states in ^{27}Si were compared. In spite of the overall similarity of $B(M1)$ and $B(\mathrm{GT})$ distributions, noticeable differences were observed for strengths of individual analogous transitions; the differences were larger in weaker transitions with $B(M1)\uparrow < 0.1\mu_N^2$ [15], suggesting that the IS and the IV orbital contributions are important.

This work and the others noted above use $B(\mathrm{GT})$ transition strengths obtained from CE reactions. This is based upon the fact that the 0° cross sections obtained from CE reactions, such as (p,n) or $(^3\mathrm{He},t)$ reactions, at intermediate energies are proportional to the $B(\mathrm{GT})$ values from β decays if the transition is not very weak [15,16]. The CE reactions have allowed a study of the GT transitions to a wider energy range, breaking the ''decay window restriction'' inherent to β decay measurements.

In Sec. II we review the formalism for the $M1$ and GT operators and matrix elements and show how they are used to determine the various spin and orbital contributions in $M1$ transitions. In Sec. III we analyze the data for the $A=27$, $T=1/2$ system. The results are discussed in Sec. IV and compared to shell-model calculations within the full sd-shell basis.

II. ANALOGOUS *M*1 AND GT MATRIX ELEMENTS IN MIRROR NUCLEI

A. The *M*1 operator and matrix elements

The operator $\boldsymbol{\mu}$ for $M1$ transitions and magnetic moments consists of an orbital part $g_l \boldsymbol{l}$ and a spin part $g_s \boldsymbol{s}[=(1/2)g_s\boldsymbol{\sigma}]$. It can be rewritten as the sum of IS and IV terms (for example, see Refs. [2,8]) as

$$\boldsymbol{\mu}=\left\{\sum_{j=1}^{A}(g_l^{\mathrm{IS}}\boldsymbol{l}_j+g_s^{\mathrm{IS}}\boldsymbol{s}_j)-\sum_{j=1}^{A}(g_l^{\mathrm{IV}}\boldsymbol{l}_j+g_s^{\mathrm{IV}}\boldsymbol{s}_j)\tau_{zj}\right\}\mu_N, \tag{2.1}$$

where μ_N is the nuclear magneton. The coefficients g^{IS} and g^{IV} are the IS and IV combinations of gyromagnetic factors (g factors): $g_l^{\mathrm{IS}}=\frac{1}{2}(g_l^{\pi}+g_l^{\nu})$, $g_s^{\mathrm{IS}}=\frac{1}{2}(g_s^{\pi}+g_s^{\nu})$, $g_l^{\mathrm{IV}}=\frac{1}{2}(g_l^{\pi}-g_l^{\nu})$, and $g_s^{\mathrm{IV}}=\frac{1}{2}(g_s^{\pi}-g_s^{\nu})$. For bare protons and neutrons, the orbital and spin g factors are $g_l^{\pi}=1$ and $g_l^{\nu}=0$, and $g_s^{\pi}=5.586$ and $g_s^{\nu}=-3.826$, respectively. The z component of the isospin operator $\tau_{zj}=1$ for neutrons and -1 for protons.

Starting from the reduced matrix elements in spin, and following the convention of Edmonds [17], the $M1$ transition strength $B(M1)$ for the transition from the initial state with spin J_i and isospin T_i to the final state with J_f and T_f can be written [18]

$$B(M1)=\frac{1}{2J_i+1}\,\frac{3}{4\pi}\,|\langle J_f T_f T_{zf}\|\boldsymbol{\mu}\|J_i T_i T_{zi}\rangle|^2. \tag{2.2}$$

By applying the Wigner-Eckart theorem in the isospin space, we get

$$B(M1)=\frac{1}{2J_i+1}\,\frac{3}{4\pi}\,\mu_N^2\Bigg|\langle J_f T_f|||\sum_{j=1}^{A}\left(g_l^{\mathrm{IS}}\boldsymbol{l}_j+g_s^{\mathrm{IS}}\frac{1}{2}\boldsymbol{\sigma}_j\right) - \frac{C_{M1}}{\sqrt{2T_f+1}}\sum_{j=1}^{A}\left(g_l^{\mathrm{IV}}\boldsymbol{l}_j+g_s^{\mathrm{IV}}\frac{1}{2}\boldsymbol{\sigma}_j\right)\boldsymbol{\tau}_j|||J_i T_i\rangle\Bigg|^2 \tag{2.3}$$

$$=\frac{1}{2J_i+1}\,\frac{3}{4\pi}\,\mu_N^2\left[\left(g_l^{\mathrm{IS}}M_{M1}(l)+g_s^{\mathrm{IS}}\frac{1}{2}M_{M1}(\sigma)\right) - \frac{C_{M1}}{\sqrt{2T_f+1}}\left(g_l^{\mathrm{IV}}M_{M1}(l\tau)+g_s^{\mathrm{IV}}\frac{1}{2}M_{M1}(\sigma\tau)\right)\right]^2 \tag{2.4}$$

$$=\frac{1}{2J_i+1}\,\frac{3}{4\pi}\,\mu_N^2\left[M_{M1}^{\mathrm{IS}}-\frac{C_{M1}}{\sqrt{2T_f+1}}M_{M1}^{\mathrm{IV}}\right]^2. \tag{2.5}$$

The isospin Clebsch-Gordan (CG) coefficient $C_{M1}=(T_i T_{zi} 1 0|T_f T_{zf})$ comes out explicitly by the use of reduced matrix elements, where $T_z=T_{zf}=T_{zi}$ for $M1$ transitions. The $M_{M1}(l)$, $M_{M1}(\sigma)$, $M_{M1}(l\tau)$, and $M_{M1}(\sigma\tau)$ are the matrix elements defined by $\langle J_f T_f|||\Sigma_{j=1}^{A}\boldsymbol{l}_j|||J_i T_i\rangle$, $\langle J_f T_f|||\Sigma_{j=1}^{A}\boldsymbol{\sigma}_j|||J_i T_i\rangle$, $\langle J_f T_f|||\Sigma_{j=1}^{A}\boldsymbol{l}_j\boldsymbol{\tau}_j|||J_i T_i\rangle$, and $\langle J_f T_f|||\Sigma_{j=1}^{A}\boldsymbol{\sigma}_j\boldsymbol{\tau}_j|||J_i T_i\rangle$, respectively. The coefficient for the IV spin term, and therefore the transition matrix element $M_{M1}(\sigma\tau)$, is the largest in a usual case [1,19]. The M_{M1}^{IS} and M_{M1}^{IV} are the IS and IV terms of the $M1$ matrix element, respectively defined by

$$M_{M1}^{\mathrm{IS}}=g_l^{\mathrm{IS}}M_{M1}(l)+\frac{1}{2}g_s^{\mathrm{IS}}M_{M1}(\sigma) \tag{2.6}$$

and

$$M_{M1}^{\mathrm{IV}}=g_l^{\mathrm{IV}}\;M_{M1}(l\tau)+\frac{1}{2}g_s^{\mathrm{IV}}M_{M1}(\sigma\tau), \tag{2.7}$$

where the IV term is usually larger than the IS term due to the large value of the g_s^{IV} coefficient in the IV spin term. The IS term, therefore, may interfere destructively or constructively with the IV term. In addition, the orbital term may interfere constructively or destructively with the spin term. These interference effects are dependent on the configurations of the initial and final states.

The magnetic moment μ of a state with spin J and isospin T is defined by

$$\mu=\langle JJTT_z|\mu_z|JJTT_z\rangle. \tag{2.8}$$

By applying the Wigner-Eckart theorem in the spin space,

$$\mu=\frac{(JJ10|JJ)}{\sqrt{(2J+1)}}\langle JTT_z\|\boldsymbol{\mu}\|JTT_z\rangle, \tag{2.9}$$

where the CG coefficient $(JJ10|JJ)=\sqrt{J/J+1}$. By further applying the Wigner-Eckart theorem in the isospin space, and by putting Eq. (2.1), we get

$$\mu=\frac{\sqrt{J}}{\sqrt{(J+1)(2J+1)}}\mu_N\left[M_{M1}^{\rm IS}-\frac{C_{M1}}{\sqrt{2T+1}}M_{M1}^{\rm IV}\right], \tag{2.10}$$

where the IS and IV matrix elements are obtained by making the initial and the final states to be the same in the definitions given by Eqs. (2.6) and (2.7), respectively. The CG coefficient is $C_{M1}=(TT_z10|TT_z)$.

B. Analogous *M*1 matrix elements in mirror nuclei

In a $\Delta T=0$ $M1$ transition between states with the same $T(\neq 0)$, both IS and IV contributions are expected. If an $M1$ transition is more than average strength, it is usual that the IV contribution is about one order of magnitude larger than the IS contribution [1,19]. For analogous transitions in $T_z=\pm T$ mirror nuclei, the isospin CG coefficients C_{M1} change sign, and we can rewrite Eq. (2.5) as

$$B(M1)_\pm=\frac{1}{2J_i+1}\frac{3}{4\pi}\mu_N^2\left[M_{M1}^{\rm IS}\mp\frac{|C_{M1}|}{\sqrt{2T_f+1}}M_{M1}^{\rm IV}\right]^2, \tag{2.11}$$

where $|C_{M1}|=\sqrt{1/3}$ for our $T=1/2$ case. Here we can choose real IS and IV matrix elements for the $M1$ transitions between particle-bound states.

Under the assumption that the IV term is larger than the IS term, we can separately extract the IS and the IV transition strengths $B_{\rm IS}(M1)$ and $B_{\rm IV}(M1)$ in the $\Delta T=0$ $M1$ transition by solving Eqs. (2.11) as simultaneous equations

$$B_{\rm IS}(M1)\equiv\frac{1}{2J_i+1}\frac{3}{4\pi}\mu_N^2[M_{M1}^{\rm IS}]^2 \tag{2.12}$$

$$=\frac{1}{4}[\sqrt{B(M1)_+}-\sqrt{B(M1)_-}]^2, \tag{2.13}$$

and

$$B_{\rm IV}(M1)\equiv\frac{1}{2J_i+1}\frac{3}{4\pi}\mu_N^2\frac{C_{M1}^2}{2T_f+1}[M_{M1}^{\rm IV}]^2 \tag{2.14}$$

$$=\frac{1}{4}[\sqrt{B(M1)_+}+\sqrt{B(M1)_-}]^2. \tag{2.15}$$

Similarly from the moments of the $T_z=\pm T$ nuclei $\mu_\pm$, we can relate the IS and IV moments $\mu_{\rm IS}=\frac{1}{2}(\mu_++\mu_-)$ and $\mu_{\rm IV}=\frac{1}{2}(\mu_+-\mu_-)$, respectively, to the IS and IV $M1$ matrix elements:

$$\mu_{\rm IS}=\frac{\sqrt{J}}{\sqrt{(J+1)(2J+1)}}\mu_N M_{M1}^{\rm IS} \tag{2.16}$$

and

$$\mu_{\rm IV}=\frac{\sqrt{J}}{\sqrt{(J+1)(2J+1)}}\mu_N\frac{-|C_{M1}|}{\sqrt{2T+1}}M_{M1}^{\rm IV}. \tag{2.17}$$

C. Isoscalar contribution to the *M*1 operator

In order to judge whether the IS term is making a constructive or a destructive contribution in each $M1$ transition, we introduce the following ratio:

$$R_{\rm IS}=\frac{B(M1)}{B_{\rm IV}(M1)}. \tag{2.18}$$

If the IS contribution is constructive (destructive) in a specific $M1$ transition, then the ratio is larger (smaller) than unity. In the analogous $M1$ transitions of $T_z=\pm T$ mirror nuclei, the smaller IS matrix element couples with the larger IV one in a reverse way depending on the T_z value, as seen from Eqs. (2.11). It is expected that the ratios $R_{\rm IS}$ for the analogous $M1$ transitions show the relationship like a ''seesaw.'' Similarly, for magnetic moments we can introduce the following ratio:

$$R_{\rm IS}=\left(\frac{\mu}{\mu_{\rm IV}}\right)^2. \tag{2.19}$$

D. GT transitions

As is well known, the GT β decay is related to the simple $\boldsymbol{\sigma\tau}$ operator. The situation is almost the same in hadron CE reactions at intermediate energies at 0° (i.e., in the limit of $q=0$), because the IV spin part of the effective interaction is dominant there [16,20].

Using the reduced matrix element in spin as well as in isospin and following the convention of Edmonds [17], the GT transition strength $B(\mathrm{GT})$ is expressed [15,18] as

$$B(\mathrm{GT})=\frac{1}{2J_i+1}\frac{1}{2}\frac{C_{\rm GT}^2}{2T_f+1}\left|\langle J_fT_f|||\sum_{j=1}^{A}(\boldsymbol{\sigma}_j\boldsymbol{\tau}_j)|||J_iT_i\rangle\right|^2 \tag{2.20}$$

$$=\frac{1}{2J_i+1}\frac{1}{2}\frac{C_{\rm GT}^2}{2T_f+1}[M_{\rm GT}(\sigma\tau)]^2, \tag{2.21}$$

where $C_{\rm GT}$ is the isospin CG coefficient $(T_iT_{zi}1\pm1|T_fT_{zf})$, and the $M_{\rm GT}(\sigma\tau)$ is the IV spin-type GT matrix element.

E. Contributions of MECs

It is known that the MEC contributions which come mainly from one-pion exchange are different for the $M1$ (vector) and GT (axial-vector) operators [21,22]. In particular for the IV spin operator, there is an MEC enhancement in $M1$ transitions [10–14]. The enhancement can be expressed by the ratio of squared matrix elements of the IV spin terms of the analogous $M1$ and GT transitions as

$$R_{\rm MEC}=\frac{[M_{M1}(\sigma\tau)]^2}{[M_{\rm GT}(\sigma\tau)]^2}. \tag{2.22}$$

In the analyses using shell-model wave functions obtained by the use of the Wildenthal's USD interaction [23] for initial and final states, it was found that experimental $M1$-transition strengths are better described by reducing the g factors of the $M1$ operator by about $15-20\,\%$ than those of free-nucleon values [24–26]. In the $A=27$ region, the best fit has been achieved by taking $g_s^{\rm eff}$ to be $0.85\times g_s^{\rm free}$ [26]. On the other hand, in order to reproduce GT transition strengths, the average renormalization factor of 0.76 ± 0.03 was needed for the effective GT operator [24]. Taking the squared ratio of these renormalization factors, we get the average $R_{\rm MEC}=1.25$ for the middle of the sd shell. This is the value we will use in the following analysis. Recent experiments [10–15] which compare (e,e') and/or γ-decay strengths with β decay and/or CE-reaction strengths in the sd shell deduce values of $R_{\rm MEC}$ in the range of 1.15 to 1.5, whose average is consistent with 1.25 but whose variation shows some possible state dependence for this quantity.

F. Other corrections

In addition to the MEC corrections discussed in the previous section, there are two smaller effects which we need to address. They are the effects of isospin mixing, which is know to be important in the interpretation of mirror GT and $M1$ transition strengths in the p-shell nuclei, and of the ''tensor'' contribution to the effective operators.

In the formalism above the matrix elements of the individual spin and orbital operators are identical in the corresponding $M1$ transitions of mirror nuclei. Also the GT transitions between mirror analogous states are the same. It is well known in the p-shell nuclei that the strengths of mirror GT transitions differ by up to 20%. Towner [27] has interpreted this based on the different binding energies of the proton and neutron in mirror nuclei making the β_+ decay and the β_- decay, respectively. The effect of the Coulomb interaction is dependent on the overlaps of radial wave functions, which is a form of isospin mixing. The known mirror GT decays in the sd shell, however, do not show any significant mirror asymmetry [24]. Our estimates of the radial overlaps do not show a significant mirror dependence, which is probably due to the overall tighter binding and also to the fact that the d orbit having a larger centrifugal barrier dominates the GT decay.

Another source of mirror asymmetry is in the change in the valence wave functions due to the Coulomb interaction with the sd shell. In Sec. IV, we will show that the effect on the individual matrix elements is rather small by comparing the results of shell-model calculations using the isospin conserving and isospin nonconserving interactions. The effect of the ''tensor'' term will also be evaluated through shell-model calculations in Sec. IV.

G. Orbital contribution to the *M*1 operator

Both IS and IV matrix elements $M_{M1}^{\rm IS}$ and $M_{M1}^{\rm IV}$ given by Eqs. (2.6) and (2.7), respectively, consist of orbital and spin terms. In the IS term, the operator $\boldsymbol{l}_j$ can be eliminated by using the relationship of operators $\boldsymbol{l}_j=\boldsymbol{j}_j-\boldsymbol{s}_j$. The summation of $\boldsymbol{j}_j$ operator, $\boldsymbol{J}=\Sigma_{j=1}^{A}\boldsymbol{j}_j$, is the total angular momentum operator. Since the matrix element of $\boldsymbol{J}$ between states that are internally orthogonal is zero, only the contribution from $\boldsymbol{s}_j$ remains [19]. Similar elimination of the operator $\boldsymbol{l}_j$ in the IV term is not possible, because the summation $\Sigma_{j=1}^{A}\boldsymbol{j}_j\boldsymbol{\tau}_j$ is not an external operator.

Our main interest here is to deduce the orbital and spin contributions in the IV matrix element. If the spin term is larger than the orbital term, the spin and orbital contributions are obtained separately by combining the values from Eqs. (2.7), (2.14), (2.15), (2.21), and (2.22). In reality, however, this is not always guaranteed, and the possible duality of the solution is not excluded. For example, a complete cancellation of spin and orbital contributions was observed for an IV $M1$ transition in ^{28}Si [10,11], suggesting that the absolute values of spin and orbital terms can be of equal quantity in a transition which is not very strong.

In order to determine whether the spin and orbital terms make a constructive or a destructive interference in each IV part of the $M1$ transition, we find from Eq. (2.7) that it is useful to introduce the following ratio:

$$R_{\rm OC}=\frac{[M_{M1}^{\rm IV}]^2}{[(1/2)g_s^{\rm IV}M_{M1}(\sigma\tau)]^2}. \tag{2.23}$$

Here, an $R_{\rm OC}$ value larger than unity shows that the orbital term enhances the transition, and vice versa, except in a rather weak transition in which (a) the signs of spin and orbital terms are different and (b) the spin term is hindered and is much smaller than the orbital one $(|\frac{1}{2}g_s^{\rm IV}M_{M1}(\sigma\tau)|\ll|g_l^{\rm IV}M_{M1}(l\tau)|)$.

The term $[M_{M1}(\sigma\tau)]^2$ in Eq. (2.23) can be replaced by the corresponding term of the analogous GT transition $[M_{\rm GT}(\sigma\tau)]^2$ by using Eq. (2.22), which is further known from the $B(\rm GT)$ value of the GT transition [see Eq. (2.21)]. Then the ratio $R_{\rm OC}$ is expressed by using $B_{\rm IV}(M1), R_{\rm MEC}$, and $B(\rm GT)$ as

$$R_{\rm OC}=\frac{8\pi}{3\mu_N^2(g_s^{\rm IV})^2}\frac{C_{\rm GT}^2}{C_{M1}^2}\frac{1}{R_{\rm MEC}}\frac{B_{\rm IV}(M1)}{B(\rm GT)}, \tag{2.24}$$

where the directions of the $M1$ and the corresponding GT transitions are so selected that the initial and the final states are respectively common or analogous to keep the consistency of the spin factor $2J+1$ (see Sec. III). The ratio of squared CG coefficients becomes 2 for the transitions between $T=1/2$ states in mirror nuclei.

The ratio $R_{\rm OC}$ can also be obtained for the ground states of $T_z=\pm1/2$ nuclei by using the $\mu_\pm$ and the $B(\rm GT)$ between these states. Putting $M_{M1}^{\rm IV}$ known from Eq. (2.17) into Eq. (2.23), we get

TABLE I. Corresponding low-lying states in ^{27}Al and ^{27}Si and the strengths of $M1$ (μ) and GT transitions from the g.s. of each nucleus. For details of transition strengths, see text. The excitation energies are given in units of MeV. The $B(M1)\uparrow(\mu)$ values are given in units of μ_N^2 (μ_N).

States in ^{27}Al			States in ^{27}Si		
E_x [a]	$2\cdot J^{\pi}$ [a]	$B(M1)\uparrow$ [b]	E_x [a]	$B(M1)\uparrow$ [b]	B(GT) [c]
0.0	5^+	3.642 [d]	0.0	-0.865 [e]	0.307 ± 0.044
1.014	3^+	0.015 ± 0.001	0.957	0.019 ± 0.001	$[(2.2\pm0.3)\times10^{-4}]$ [f]
2.211	7^+	0.150 ± 0.004	2.164	0.102 ± 0.010	0.079 ± 0.006
2.735	5^+	0.046 ± 0.007	2.648	0.022 ± 0.005	0.039 ± 0.004
2.982	3^+	0.245 ± 0.013	2.866		0.173 ± 0.012
3.957	3^+	0.145 ± 0.012	3.804		0.079 ± 0.007
4.410	5^+	0.226 ± 0.028	4.289	0.075 ± 0.037	0.097 ± 0.009

[a]From Ref. [29].
[b]Calculated using data from Ref. [28,29].
[c]From Ref. [15], see text.
[d]Magnetic moment μ from Ref. [30], where given error is small.
[e]Magnetic moment μ from Ref. [31], where given error is small.
[f]Transition contains little GT strength.

$$R_{\rm OC}=\frac{2(J+1)}{J\mu_N^2(g_s^{\rm IV})^2}\frac{C_{\rm GT}^2}{C_{M1}^2}\frac{1}{R_{\rm MEC}}\frac{\mu_{\rm IV}^2}{B({\rm GT})}. \qquad (2.25)$$

III. DATA EVALUATION FOR $A=27$, $T=1/2$ MIRROR NUCLEI

A. $B(M1)$ and B(GT) values

The J^{π} values of the g.s. are $5/2^+$ for the ^{27}Al-^{27}Si mirror nuclei. The $M1$ operator connects the $5/2^+$ g.s. to the excited states with J^{π} values $3/2^+$, $5/2^+$, and $7/2^+$. Mean lifetimes of levels in the mirror nuclei ^{27}Al-^{27}Si are available up to $E_x=4.5$ MeV from Ref. [28]. The $B(M1)$ values to the g.s. are obtained for each nucleus using data compiled by Endt [29]. For each excited state, $B(M1)$ (in units of μ_N^2) to the g.s. is calculated from the known mean lifetimes, γ-ray branching ratios to the g.s., $M1$ and $E2$ mixing ratios, and γ-ray energies (for example, see Refs. [2,15]).

In order to compare the $B(M1)$ values directly with the B(GT) values obtained in transitions starting from the g.s. of one of the mirror nuclei, we use the $B(M1)\uparrow$ values which would be obtainable in an (e,e')-type transition from the g.s. The $B(M1)\uparrow$ values are calculated from $B(M1)$ values

$$B(M1)\uparrow=\frac{2J_j+1}{2J_{\rm g.s.}+1}B(M1), \qquad (3.1)$$

where $J_{\rm g.s.}$ and J_j are the spin values of the g.s. and the jth excited state, respectively. The $B(M1)\uparrow$ values from the g.s. to the analogous states in ^{27}Al and ^{27}Si are listed in columns 3 and 5 of Table I. The values of g.s. magnetic moments are taken from Refs. [30,31].

From the β decay study of the ^{27}Si g.s., B(GT) values are known for several excited states of ^{27}Al up to $E_x=2.98$ MeV [29]. B(GT) values in units where $B({\rm GT})=3$ for the β decay of the free neutron are listed in the last column of Table I. Assuming the charge symmetry of the nuclear force, we expect the same B(GT) values for the analogous transitions from the ^{27}Al g.s. to the excited states of ^{27}Si. The B(GT) values of 3.8 and 4.3 MeV states are available from a good resolution ^{27}Al(^{3}He,t) reaction performed at an incident energy 150 MeV/nucleon [15]. The proportionality between the B(GT) values obtained in β decays and the 0° cross sections is well known for (p,n) reactions at $E_p=100$ MeV and higher [16]. A similar proportionality has been reported for transitions with B(GT) values larger than 0.04 in the ^{27}Al(^{3}He,t) reaction [15].

B. IS and orbital contributions

$B(M1)$ values could be calculated for four pairs of analogous $M1$ transitions in the ^{27}Al-^{27}Si mirror system as shown in Table I. The values for $B_{\rm IS}(M1)$ and $B_{\rm IV}(M1)$ are derived using Eqs. (2.13) and (2.15), and are listed in columns 4 and 5 of Table II. The values of $B_{\rm IS}(M1)$ are smaller than the values of $B_{\rm IV}(M1)$ by more than one order. The constructive or destructive contributions of the IS term, respectively, are known from $R_{\rm IS}>1$ or <1 calculated using Eqs. (2.18) and (2.19). They are given in columns 6 and 7 of Table II for ^{27}Al and ^{27}Si, respectively. Since the constructive and destructive contributions of the IS term are reversed in the $T_z=\pm1/2$ nuclei, the $R_{\rm IS}$ values for ^{27}Al ($T_z=+1/2$) and ^{27}Si ($T_z=-1/2$) show a seesaw-like relationship.

The orbital contributions in the IV term $R_{\rm OC}$, which are common in the analogous transitions and also in the g.s. magnetic moments of the mirror nuclei pair, are calculated by using Eqs. (2.24) and (2.25). Based on the discussion given in Sec. II E, the value $R_{\rm MEC}=1.25$ together with the value $g_s^{\rm IV}=4.706$ are used. The results are listed in the last column of Table II.

IV. DISCUSSION

The experimental $B(M1)\uparrow$ values available for the four pairs of analogous transitions in the ^{27}Al-^{27}Si mirror nuclei

TABLE II. Experimental $B_{IS}(M1)(\mu_{IS})$, $B_{IV}(M1)$ (μ_{IV}), R_{IS}, and R_{OC} values for the corresponding states in ^{27}Al and ^{27}Si. For the definition of these values, see text. The excitation energies are given in units of MeV. The values of $B_{IS}(M1)(\mu_{IS})$ and $B_{IV}(M1)(\mu_{IV})$ are given in units of $\mu_N^2(\mu_N)$.

E_x of states in ^{27}Al	in ^{27}Si	$2\cdot J^\pi$	$B_{IS}(M1)$	$B_{IV}(M1)$	R_{IS} in ^{27}Al	in ^{27}Si	R_{OC}
0.0	0.0	5^+	1.388[a]	2.254[b]	2.61	0.15	3.3 ± 0.5
1.014	0.957	3^+	$(6\pm5)\times10^{-5}$	0.017 ± 0.001	0.89 ± 0.07	1.12 ± 0.09	(51.3 ± 8.0)[c]
2.211	2.164	7^+	$(1.2\pm0.5)\times10^{-3}$	0.125 ± 0.006	1.20 ± 0.06	0.82 ± 0.08	0.96 ± 0.12
2.735	2.648	5^+	$(1.1\pm0.8)\times10^{-3}$	0.033 ± 0.004	1.40 ± 0.28	0.67 ± 0.18	0.51 ± 0.10
2.982	2.866	3^+					
3.957	3.804	3^+					
4.410	4.289	5^+	0.010 ± 0.007	0.140 ± 0.028	1.61 ± 0.37	0.53 ± 0.28	0.88 ± 0.19

[a]IS magnetic moment μ_{IS}.
[b]IV magnetic moment μ_{IV}.
[c]Not reliable, because of small IV spin term.

ranged from 0.015 to 0.23 μ_N^2. The $B_{IS}(M1)$ values, the strengths related to the IS term of the $M1$ operator, are more than one order of magnitude smaller than the $B_{IV}(M1)$ values for all transitions, as seen from columns 4 and 5 of Table II. The cross term of the IS and IV matrix elements, however, can be large, and the effective contribution of the IS term can be significant in the $M1$ transition. The ratio R_{IS} is defined to show how the pure IV transition is modified by the contribution of the IS term. The constructive and destructive IS contributions are distinguished by larger and smaller values than unity, respectively. As we see, R_{IS} varies from 0.5 to 1.6, showing that the $B(M1)$ can be modified by about $\pm50\%$ through the interference of IV and IS terms, although the $B_{IS}(M1)$ itself is small. This shows that R_{IS} is a sensitive signature for the IS contribution in the $M1$ transition.

The same is true for the orbital contribution in the IV term, i.e., the contribution of the orbital term is well represented by the ratio R_{OC} even if the term itself is small. The reliable R_{OC} values obtained for the three transitions ranged from 0.51 to 0.96, showing slightly destructive contributions of the IV orbital term against the IV spin term. The large value of $R_{OC}=3.3$ obtained for the g.s. moments indicates that the orbital contribution is large. The larger orbital contribution in magnetic moments than in $M1$ transitions is explained from the different expectation values of the $\boldsymbol{l}$ operator evaluated between single-particle states; in the magnetic moments, the expectation values have the order of the orbital quantum number L of the state, while in the $M1$ transition, it has the order of 1 (for example, see Ref. [4]). For the $J^\pi=(5/2)^+$ g.s. with some $d_{5/2}$ single-particle nature, enhancement of orbital contribution is expected for the magnetic moments due to the L value of 2.

The experimentally extracted results are compared with shell-model calculations carried out in the full sd model space by using Wildenthal's USD interaction [23] and the computer code OXBASH [32]. The $B(M1)$ and $B(\mathrm{GT})$ values, respectively, are obtained by using effective $M1$ and GT operators [24–26]. The calculated values of E_x,J^π,$B(M1)\uparrow$ for ^{27}Al and ^{27}Si, and $B(\mathrm{GT})$ are presented in columns 1 through 5 of Table III for the states and transitions studied experimentally. Although the same E_x values are obtained for both of these nuclei due to the use of charge-symmetry

TABLE III. Results of shell-model calculations assuming pure isospin. Values are shown for $B(M1)\uparrow(\mu)$, $B_{IS}(M1)(\mu_{IS})$, $B_{IV}(M1)(\mu_{IV})$, R_{IS}, and R_{OC} obtained for the corresponding states in ^{27}Al and ^{27}Si. For details of the calculation, see text. The excitation energies are given in units of MeV. The values of $B(M1)\uparrow(\mu)$, $B_{IS}(M1)(\mu_{IS})$, and $B_{IV}(M1)(\mu_{IV})$ are given in units of $\mu_N^2(\mu_N)$.

E_x	$2\cdot J^\pi$	$B(M1)\uparrow$ in ^{27}Al	in ^{27}Si	$B(\mathrm{GT})$	$B_{IS}(M1)$	$B_{IV}(M1)$	R_{IS} in ^{27}Al	in ^{27}Si	R_{OC}
0.0	5^+	3.676[a]	-0.879[a]	0.347	1.399[b]	2.278[c]	2.61	0.15	3.02
1.264	3^+	5.7×10^{-3}	7.1×10^{-3}	3.1×10^{-3}	2.0×10^{-5}	6.4×10^{-3}	0.89	1.12	1.24
2.326	7^+	0.127	0.095	0.075	5.8×10^{-4}	0.111	1.15	0.86	0.89
2.709	5^+	0.040	0.026	0.040	3.6×10^{-4}	0.032	1.22	0.80	0.50
2.780	3^+	0.307	0.213	0.214	2.1×10^{-3}	0.258	1.19	0.83	0.73
4.027	3^+	0.103	0.077	0.055	4.6×10^{-4}	0.090	1.15	0.86	0.99
4.139	5^+	0.208	0.165	0.084	6.2×10^{-4}	0.186	1.12	0.89	1.33

[a]Magnetic moment μ.
[b]IS magnetic moment μ_{IS}.
[c]IV magnetic moment μ_{IV}.

TABLE IV. Results of shell-model calculations which are the same as Table III but include isospin mixing. Values are shown for $B(M1)\uparrow(\mu)$, $B_{\rm IS}(M1)(\mu_{\rm IS})$, $B_{\rm IV}(M1)(\mu_{\rm IV})$, $R_{\rm IS}$, and $R_{\rm OC}$ obtained for the corresponding states in ^{27}Al and ^{27}Si. For details of the calculation, see text.

		$B(M1)\uparrow$					$R_{\rm IS}$		
E_x	$2\cdot J^{\pi}$	in ^{27}Al	in ^{27}Si	B(GT)	$B_{\rm IS}(M1)$	$B_{\rm IV}(M1)$	in ^{27}Al	in ^{27}Si	$R_{\rm OC}$
0.0	5^+	3.666[a]	−0.869[a]	0.345	1.399[b]	2.268[c]	2.61	0.15	3.02
1.264	3^+	3.8×10^{-3}	7.5×10^{-3}	3.7×10^{-3}	1.6×10^{-4}	5.5×10^{-3}	0.69	1.37	0.90
2.326	7^+	0.124	0.095	0.075	4.8×10^{-4}	0.109	1.14	0.87	0.87
2.709	5^+	0.041	0.026	0.038	4.2×10^{-4}	0.033	1.24	0.79	0.53
2.780	3^+	0.305	0.212	0.214	2.1×10^{-3}	0.256	1.19	0.83	0.73
4.027	3^+	0.098	0.077	0.052	3.3×10^{-4}	0.087	1.13	0.88	1.02
4.139	5^+	0.218	0.167	0.086	8.7×10^{-4}	0.192	1.14	0.87	1.36

[a]Magnetic moment μ.
[b]IS magnetic moment $\mu_{\rm IS}$.
[c]IV magnetic moment $\mu_{\rm IV}$.

nuclear interaction, the experimental E_x values and the J^{π} values of the low-lying states up to 4.5 MeV are rather well reproduced. Also the $B(M1)\uparrow$ values larger than $2\times10^{-2}\mu_N^2$ are calculated within 10−20% errors for both nuclei, except the very weak transition to the theoretical $3/2^+$ state at 1.264 MeV. The only exception is the transition to the theoretical state at 4.139 MeV where the calculated strength in ^{27}Si is a factor of two larger than experiment. All B(GT) values larger than 4×10^{-2} are well reproduced, again except the very weak transition to the 1.264 MeV state.

The calculated $B_{\rm IS}(M1)$ and $B_{\rm IV}(M1)$ values are given in columns 6 and 7 of Table III. Both $B_{\rm IS}(M1)$ and $B_{\rm IV}(M1)$ values are in good agreement with the experimental values within the the experimental error. If we compare the results in terms of $R_{\rm IS}$, however, we see again a rather large difference for the g.s. →4.139 MeV transition.

The calculated $R_{\rm OC}$ values using $R_{\rm MEC}=1.25$ are given in the last column. For the g.s. magnetic moment, the g.s. →2.326 MeV, and g.s.→2.709 MeV transitions, good agreements are seen with the experimental results. For the g.s.→4.139 MeV transition, however, rather different result is obtained; the experimental $R_{\rm OC}$ value suggests the destructive orbital contribution, while the shell-model value suggests the constructive contribution.

The largest disagreement between theory and experiment in the comparisons above appear for the third $5/2^+$ state, and they are all related to the factor of two difference (outside the rather large experimental error) between theory and experiment for the transition to the 4.289 MeV state in ^{27}Si. The good agreement between experiment and theory for the analogue $M1$ transition to the 4.410 MeV state in ^{27}Al indicates that the problem may be with the experimental lifetime for the 4.289 MeV state in ^{27}Si. The mean lifetime of $\tau_m=5\pm2$ psec is at the lower end for those measured in Ref. [28]. The calculated mean lifetime based on the USD interaction is 2.3 psec. It would be useful to confirm the experimental value.

It is expected that the change in the valence wave functions due to the Coulomb interaction with the sd shell can cause asymmetry of mirror $M1$ decays. We have evaluated this effect by using the Ormand-Brown isospin nonconserving interaction [33] together with the USD interaction. The results with no isospin mixing (Table III) can be compared to the results with isospin mixing (Table IV). In particular compare the $R_{\rm OC}$ values in Tables III and IV. Except for the very weak matrix element to the 1.264 MeV state, we see that isospin mixing (mirror asymmetry in the $M1$ matrix elements) has a very small effect on our results.

The other consideration is that the effective operator for $M1$ and GT has a form which goes beyond the free-nucleon forms of Eqs. (2.1) and (2.20) by the addition of a ''tensor'' term of the form $[Y^{(2)}\otimes\boldsymbol{\sigma}]^{(1)}$ [24]. This term arises from the core-polarization and MEC corrections to the model-space operators [21,22]. In order to evaluate this effect, we compare the results with the full effective operator for $M1$ and GT which includes the tensor term [24–26] (those in Table III) to another calculation which is the same except that the tensor operator is omitted (those in Table V). In particular compare the $R_{\rm OC}$ values in Tables III and V. The tensor effect is larger than isospin mixing, but it is on the same order as the experimental errors (Table II). If the experimental errors were much smaller, the tensor term in the operator would have an influence on the interpretation we attempt to make.

The method of decomposition discussed here can be applied to other mirror pairs such as ^{11}B-^{11}C, which can be used for solar neutrino detection [34]. Nuclear levels of ^{11}B and ^{11}C are excited by neutrinos through weak neutral and charged currents, respectively. However, for these lighter p-shell nuclei the effect of isospin mixing due to the Coulomb interaction is more important [27] and must be evaluated. The operators $\boldsymbol{\sigma\tau}$ and $\boldsymbol{\sigma}$, respectively, appear in the axial IV and axial IS neutral currents, where the latter, in addition to the former, is expected to contribute in neutrino inelastic scatterings. It is, therefore, important to know these contributions separately. Since the types of the relevant operators are the same as those of the spin terms of the $M1$ operator, the present study identifying the contribution of each individual term in ''electromagnetic'' $M1$ transitions can be used to obtain information on ''weak'' transitions by the neutral currents, which play important roles in the study

TABLE V. Results of shell-model calculations which are the same as Table III but exclude the tensor part of the effective operator. Values are shown for $B(M1)\uparrow(\mu)$, $B_{IS}(M1)(\mu_{IS})$, $B_{IV}(M1)(\mu_{IV})$, R_{IS}, and R_{OC} obtained for the corresponding states in ^{27}Al and ^{27}Si. For details of the calculation, see text.

		$B(M1)\uparrow$					R_{IS}		
E_x	$2\cdot J^\pi$	in ^{27}Al	in ^{27}Si	B(GT)	$B_{IS}(M1)$	$B_{IV}(M1)$	in ^{27}Al	in ^{27}Si	R_{OC}
0.0	5^+	3.538[a]	-0.759[a]	0.332	1.390[b]	2.149[c]	2.71	0.12	2.81
1.264	3^+	4.0×10^{-3}	5.5×10^{-3}	3.3×10^{-3}	0.3×10^{-4}	4.7×10^{-3}	0.85	1.17	0.87
2.326	7^+	0.112	0.084	0.074	5.0×10^{-4}	0.097	1.15	0.86	0.80
2.709	5^+	0.045	0.030	0.041	3.9×10^{-4}	0.037	1.22	0.80	0.55
2.780	3^+	0.349	0.245	0.220	2.3×10^{-3}	0.294	1.19	0.83	0.81
4.027	3^+	0.116	0.088	0.057	5.0×10^{-4}	0.101	1.15	0.86	1.09
4.139	5^+	0.225	0.179	0.086	6.7×10^{-4}	0.201	1.12	0.89	1.41

[a]Magnetic moment μ.
[b]IS magnetic moment μ_{IS}.
[c]IV magnetic moment μ_{IV}.

of solar as well as supernova neutrinos and also in the search of spin-coupled cold dark matter.

V. SUMMARY

The present work has demonstrated how the IS and IV as well as the spin and orbital contributions to $M1$ transitions can be evaluated by comparing experimental information on the $B(M1)_\pm$ values and B(GT) value of the analogous transitions in mirror nuclei. R_{IS} is defined to show how the IS term, although it is smaller than the IV term, contributes in a $M1$ transition. R_{OC} is defined to show the contribution of the IV orbital term in the IV term. The detailed analysis was carried out for the mirror nuclei ^{27}Al and ^{27}Si. The B(GT) values from β decay were supplemented by those obtained with the (^{3}He,t) CE reaction. The orbital contribution was also calculated for the g.s. magnetic moments. Through the comparison with the results of shell-model calculations, it is suggested that R_{IS} and R_{OC} are good signatures for the validity test of both experimental and shell-model results. The method described here can be applied to other mirror nuclei. Since the operators $\boldsymbol{\sigma\tau}$ and $\boldsymbol{\sigma}$ are common with the ones in the axial IV and axial IS neutral currents in weak transitions, the results of the decomposition can be used to obtain information on the matrix elements needed to calculate inelastic neutrino scattering.

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