Why is the three-nucleon force so odd?

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(Received 10 April 2000; published 20 September 2000)

By considering a class of diagrams which has been overlooked also in the most recent literature on three-body forces, we extract a new contribution to the three-nucleon interaction which specifically acts on the triplet odd states of the two-nucleon subsystem. In the static approximation, this $3N$-force contribution is fixed by the underlying $2N$ interaction, so in principle there are no free parameters to adjust. The $2N$ amplitude however enters in the $3NF$ diagram in a form which cannot be directly accessed or constrained by NN phase-shift analysis. We conclude that this new $3N$-force contribution provides a mechanism which implies that the presence of the third nucleon modifies the p-wave (and possibly the f-wave) components of the $2N$ subsystem in the triplet-isotriplet channels.

PACS number(s): 21.30.Cb, 25.10.+s, 25.40.Dn, 13.75.Cs

I. INTRODUCTION

The three-nucleon system represents an ideal testing ground for the nucleon-nucleon interaction [1]. This testing represented a challenge which required a great deal of efforts amongst various research groups active in this area in order to provide a comparison between extremely reliable calculations and precise measurements. In this field there have been four main areas of research which proved to be crucial for the progress in our understanding of the three-nucleon problem. These four areas are (1) NN phase-shift analysis, (2) $2N$ potentials, (3) $3N$ calculations, (4) experiments on few-nucleon systems. Recent advancements on these topics can be found in Ref. [2].

The combined result of all these studies and efforts has revealed that the presence of the third nucleon modifies the interaction between the remaining two. A serious difficulty must be faced at this point: namely, how can we conveniently describe the modifications of the NN force operated by the third nucleon? Most likely, we cannot avoid ambiguities in the description of the interaction modifications by the third nucleon, since the two-nucleon amplitudes enter off-the-energy-shell in the three-body quantum mechanical equations. And it is known [3], that it is possible to introduce (maybe, unrealistic) modifications in the NN off-shell structure that mimics the interaction effects due to the presence of the third nucleon. The lesson we must learn from this is that it is not sufficient to generate a phenomenological NN potential that perfectly reproduces the most up to date phase-shift analysis. One must also generate a NN potential by using theoretical insight as much as possible in order to constrain the off-shell properties of the NN amplitude. If this is not the case, a NN potential which fits precisely the experimental data but with the erroneous off-shell behavior would not provide reliable results for the $3N$ system, nor can be used to test for the presence of $3N$ forces.

On the other hand any realistic $2N$ potential underbinds the triton and this is the first signal that three-body nuclear forces do play a role, and that the off-shell ambiguities are not so dramatic [if the $2N$ potential is theoretically constrained in its one-pion-exchange (OPE) term and one assumes the requirements of minimal momentum dependence and/or nonlocality]. The off-shell ambiguities can be seen by the fact that one must adjust separately with each $2N$ potential the parameters of the $3N$ force in order to reproduce exactly the experimental binding energy of the triton [4], and this suggests that $2N$ forces and $3N$ forces must be generated consistently, within the same theoretical scheme.

Aside for the problem with the triton underbinding, other signals for possible evidences of $3NF$ effects must be sought in the $3N$ continuum. One signal arrived recently from the study of the unpolarized differential cross section for nd elastic scattering between 60 and 200 MeV. Continuum calculations with $2NF$ underestimate the minimum by a 30% effect (the Sagara discrepancy). Rigorous Faddeev calculations with the inclusion of $3NF$ completely solve the discrepancy between 60 and 140 MeV [5]. A coupled-channel calculation with the explicit treatment of the Δ-isobar excitation which leads to effective $3NF$'s in the three-nucleon subsystem reduces the Sagara discrepancy by a considerably large fraction [6]. Useful insights about the importance of possible $3NF$ diagrams should also be obtained from the study of pion production/absorption mechanisms in the $3N$ system, since progress has been recently made in the theoretical treatment of these reactions at the Δ resonance [7,8] and in the threshold region [9].

At the present stage, however, the existing $3NF$ models achieved only a limited success in explaining the discrepancies between theory and experiments, and it is possible that the spin-isospin structure of the full three-nucleon force is not well understood, yet. Indeed, as is well known [1], the comparison between theory and experiments reveals that existing $3N$ forces do not provide the correct structure of the vector analyzing powers, both for the proton A_{y0} and for the deuteron case iT_{11}, while the deuteron tensor polarization observables are described reasonably well. This has been evidenced also for pd scattering below breakup threshold [10], where variational techniques based on the pair correlated hyperspherical harmonic method allowed to incorporate the effects of the Coulomb interaction. The situation is still very much the same in these days as has been pointed out also in the most recent conference on few-body problems, held in Taipei in March 2000 (FBXVI).

This puzzling situation about the vector analyzing powers has been carefully analyzed in two recent publications [11,12]. In Ref. [11] it was concluded that, unless $3NF$ of new structure could be envisaged, the NN interaction in the ^3P_J states has to be modified in an energy dependent way (i.e., only for energies lower than 20 MeV). The study implicitly assumes that in this energy region the results from modern phase-shift analysis [13] could be possibly corrected in the triplet p waves without affecting appreciably the NN data. In Ref. [12] all the possibilities for solving the problem at the level of the two-body interaction (especially introducing modifications in the ^3P_J channels) have been investigated and then ruled out, with the conclusion that the only viable solution to the puzzle of the vector analyzing powers must come from a new $3NF$ contribution which has not yet been taken into account. The authors of Ref. [12] suggest a $3NF$ of the spin-orbit type, as a possible candidate.

In this paper we discuss the dynamical mechanism which generate a new component to the three-body force. This mechanism has been obtained starting from a formalism [14] developed for the treatment of the pion dynamics in the $3N$ system. By projecting out the pion degrees of freedom from that formalism one obtains a $3N$ dynamical equation of Alt-Grassberger-Sandhas (AGS) type [15], which incorporates the spin-off of the pion dynamics beyond that already considered in the $2N$ interaction. It has been shown [16] that the new pionic terms in the $3N$ equation can be interpreted as irreducible diagrams, contributing to the construction of a $3NF$. Herein, we are mainly interested in one specific $3NF$ mechanism, which implies an intermediate $2N$-cluster formation while a pion is ''in flight.'' Since 2π mesonic retardation effects do play a role here, we analyzed this role, and found that they merely act as counter terms, to be subtracted because of the presence of a known cancellation effect [17,18]. This cancellation effect is correctly taken into account in the construction of all modern $3NF$'s [19]. The novelty of the approach presented herein is that we sized the effect of this cancellation more precisely, by allowing the mechanism to adjust to the effects of the nuclear medium. This was possible only because we started from an explicit treatment of the pion degree of freedom, while it would have not been possible to see the effect within the more common ''instantaneous'' approaches to the $3N$ force. The nature of these dispersive effects is well known in approaches devoted to the explicit treatment of the Δ degrees of freedom [20], however a discussion of the same effects in the presence of an explicit treatment of the pion dynamics can hardly be found in the literature.

From this new $3NF$ contribution we have extracted in a very natural way a specific component which acts only in the (triplet) odd waves of the $2N$ subsystem. We provide also the first derivation of the partial-wave expansion of this piece of $3N$ force, which can be used in current $3N$ calculations. The spin-isospin structure of this $3N$ force implies that at low energies the presence of the third nucleon modifies the $2N$ subsystem in the ^3P_J channels, and we suggest that this might be another possible candidate for explaining the inconsistencies registered between theory and experiments at low energies, at least in those cases most sensitive to the triplet p waves, such as A_{y0} and iT_{11}.

II. IRREDUCIBLE 3*N*-FORCE DIAGRAMS

The diagrammatic analysis discussed in this section is based on the systematic method developed in Ref. [14] to take into account the pion dynamics into the $3N$ system. The method has been originally designed for the treatment of the $3N$ dynamics above the pion threshold and represents an appropriate, connected-kernel, generalization of the standard Faddeev-AGS three-body equations [15,21] for the explicit inclusion of one meson degree of freedom. In the π-$3N$ space, the equations are labeled in terms of modified Yakubovskiǐ-type chains of partitions. In the $3N$ (no-pion) sector, the labelling structure leads to Faddeev-type components. Specific rules are provided on how the pion-nucleon vertex interaction couples the two sectors. By recursive application of the quasiparticle/separable-expansion method, a modified $3N$ AGS-quasiparticle equation is obtained where the explicit pion dynamics is built-in. This approach has been subsequently considered in Ref. [16] where an approximated, practical scheme for the solution of these equations has been designed. To the lowest order, the approximation scheme consists simply in replacing the inelastic components of the NN subamplitudes with their leading terms, represented by suitable combinations of the pion-nucleon vertex interaction. By setting to zero also these leading terms for the pion inelasticities in the NN subamplitudes, the NN subamplitudes become totally elastic and in this limit [14] one re-obtains the standard $3N$ Faddeev-AGS equation with $2N$ interactions, plus a completely disjoint standard Yakubovskiǐ-GS equation [22,23] for the π-$3N$ sector. In this limit, the input for the two separated three- and four-body equations are the fully elastic NN and πN t matrices.

In Ref. [16] it is discussed how to project out the pion degree of freedom from the treatment of Ref. [14]. The result of this procedure, i.e., cooling down the pion from the theory, can be recast in a very appealing way if one uses a finite-rank expansion of the elastic NN t matrix. There are methods to generate these expansions, such as the Ernst-Shakin-Thaler method [24], and with these methods very reliable and accurate separable expansions have been generated and tested [25–27].

The new $3N$ equation incorporating the pion dynamics has the standard AGS form [15]

$$X_{ab}=Z_{ab}+\sum_c Z_{ac}\tau_c X_{cb}, \tag{2.1}$$

where a, b, and c run over the three Faddeev components of the $3N$ system, and the only modification refers to the driving term, which can be separated into the following structure:

$$Z_{ab}=Z_{ab}^{\mathrm{AGS}}+Z_{ab}^{3N}. \tag{2.2}$$

The first contribution is, literally, the standard AGS driving term, which many groups have been calculating for years,

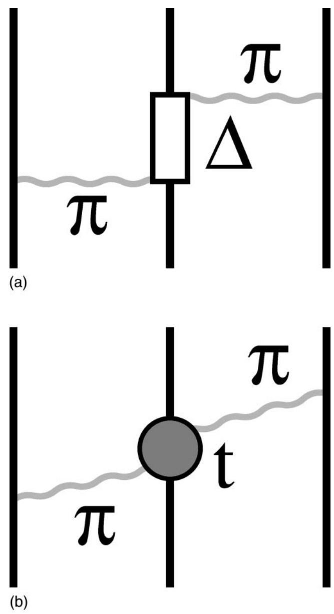

FIG. 1. Examples of well-known irreducible $3NF$ diagrams. The Fujita-Miyazawa diagram (top) and the nonpolar πN rescattering diagram (bottom).

while the second term represents the spin-off of the pion dynamics beyond that already contained in the $2N$ interaction. Following Ref. [16], it is possible to analyze all the pion-exchange diagrams contained in Z_{ab}^{3N} and it is found that they all correspond to irreducible $3N$ diagrams, and this establishes a link between this π-$3N$ approach and those formalisms considering irreducible $3N$ diagrams as generators of $3N$ interactions.

The diagrams that emerged from the analysis of Z_{ab}^{3N} can be classified according to their topological structures, and we can easily recognize irreducible diagrams that are well known. But we obtain also other diagrams which have been overlooked, to the best of our knowledge.

The first class of diagram is represented by the diagrams reported in Fig. 1, which describes a pion rescattering by a third nucleon while being exchanged between the other two. As a matter of fact, such a 2π-exchange structure of the $3N$ potential has been the only one considered in all modern calculations. This has been pointed out also very recently by Friar *et al.* in Refs. [28,29]. On the upper part of the figure it is shown the Fujita-Miyazawa Δ-mediated interaction [30], which represents the prototype of this topological structure, and accounts for an important fraction of the generic pion rescattering process. In the lower part of Fig. 1 the more general pion-rescattering process is exhibited. The ''blob'' represents the πN amplitude, where one must subtract its polar part (corresponding to a nucleon propagating in the forward direction) to avoid double counting with the nucleonic multiple-scattering contributions, since these last are summed up to all orders in the dynamical $3N$ equations. The term which must be subtracted is shown in Fig. 2 and is called sometimes the (reducible) Born term. The existing $3N$ potentials differ mainly in the model calculation of the πN t matrix, whether it is constrained by current algebra and PCAC [31], or inspired by an effective meson-baryon Lagrangian constrained by chiral symmetry and current algebra [32], or determined by effective field-theoretic methods involving light-meson dynamics [33], or by the more systematic method of χPT [34], or finally by the form envisaged by the Fujita-Miyazawa term [35]. Since such contributions (and their short-range corrections) have been the subject of very extensive studies, we really have nothing to say in addition and will skip to the next classes.

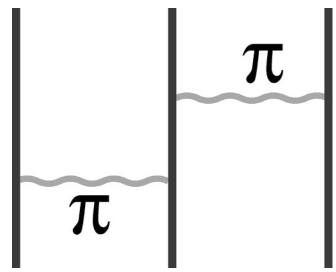

FIG. 2. The reducible Born diagram. This must be subtracted.

The structure of the second class of diagrams obtained by cooling down the pion from the π-$3N$ approach has been overlooked in all modern force calculations: it corresponds to the graph represented in Fig. 3. This describes a complete correlation between one of the two nucleons exchanging the meson and the third one while the pion is ''in flight.'' These diagrams should not be confused with those originated by pure mesonic retardation effects, analyzed and evaluated many years ago in a sequence of papers, in Refs. [36–38].

As has been shown later in Refs. [17,18], those diagrams contributing to the $3NF$ with pure mesonic retardation effects must be excluded because they cancel out against the corresponding contributions of pionic retardation effects arising from the Born term. This cancellation is correctly taken into account in the construction of modern $3NF$ contributions [29,19,39] and occurs also when considering $3NF$ retardation effects coming from the exchange of heavier mesons [33,17]. To take into account the effects of this cancellation we subtract from the diagram in Fig. 3 the second (irreducible) Born diagram shown in Fig. 4.

The reason for this subtraction can be understood in the following way: The diagram in Fig. 3 considers an irreduc-

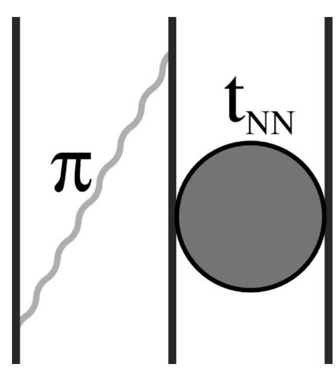

FIG. 3. Another irreducible $3NF$ diagram. The $2N$-correlation diagram.

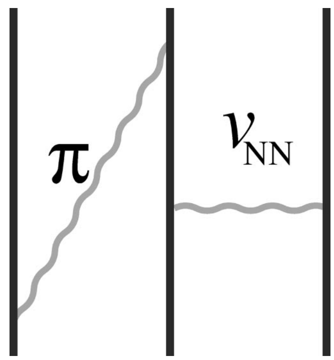

FIG. 4. The irreducible Born diagram. This also must be subtracted.

ible contribution wherein the pion propagates while a $2N$ subsystem *clusterizes*. Obviously, such an effect does happen above the pion threshold, since the reactions $Nd \to Nd\pi$ are observed; the problem is to determine up to what extent this mechanism is relevant at lower energies, where the $N+(NN)+\pi$ channel is asymptotically closed but may still be important as an intermediate state. The $3NF$ mechanism of Fig. 3 is dynamically more complete than the one shown in Fig. 4 which has been demonstrated to cancel out against the mesonic retardation corrections of the iterated OPEP term of Fig. 2 (see Refs. [17,33] for details). Therefore, it is the difference between the two diagrams in Figs. 3 and 4 that survives from the mesonic retardation effects and generates a new $3NF$ contribution. Had we replaced the full two-body t-matrix in this diagram by the input potential (this corresponds to an ''instantaneous,'' Born-type approximation) then the cancellation would be matched exactly and this $3NF$ effect would disappear. Hence, because of this incomplete cancellation, the effect *must* be entirely attributed to the energy dispersion of the intermediate $2N$ correlation, combined with a ''long leg pion'' diagram.

Finally, it is possible to generate from the study of Z_{ab}^{3N} other, more complicated, diagrams. In Fig. 5 it is shown just one example. One class embraces all possible *connected* $3N$ correlations while the exchanged pion is ''in flight.'' On the contrary, the diagrams of Fig. 3 represents all possible *disconnected* $2N$ correlations while the pion is ''in flight.''

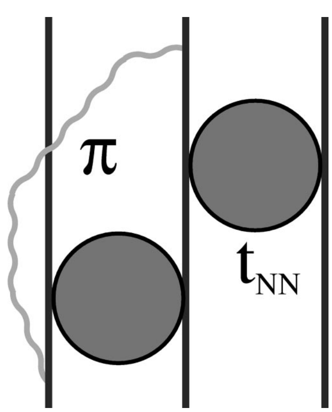

FIG. 5. Yet another irreducible $3NF$ diagram. The connected-correlation diagram.

We summarize this section with some comments. First, the analysis performed in this section was based on a new approach developed for the description of the pion dynamics within a nonrelativistic multinucleon context.

The more practical approach in Ref. [16] is sufficiently systematic to generate three topologically different structures of irreducible diagrams for the $3N$ force. The more general approach of Ref. [14] will generate additional classes of irreducible (and reducible) $3NF$ diagrams. The first diagrammatic structure that emerged is well known and is practically the only one explored in modern few-nucleon calculations. Then there are irreducible diagrams whose topological structure was not known. Most interesting is the class of diagrams considering a $2N$ correlation while the pion is being exchanged. These diagrams should not be confused with the mesonic retardation effects, which produce a net cancellation amongst themselves. To our knowledge, this contribution to the $3NF$ has never been considered before and will be analyzed in the next two sections of this paper. Finally, we have also revealed the presence of another class based on connected $3N$ correlations while the pion is being exchanged. From the connected-kernel approach considered in Ref. [14], other more complicated classes could be produced. Their basic building blocks, however, are always the $2N$ t-matrices and the πN amplitudes, and therefore it is clear that the two fundamental ingredients for the construction of the $3NF$ are the ones shown in Figs. 1 and 3.

III. THE ''ODD'' CONTRIBUTION TO THE 3N FORCE

In this section, we will first derive a $3N$ interaction from the irreducible $3N$-force diagram shown in Fig. 3. Moreover, we will discuss how this irreducible $3NF$ produces a contribution acting in the triplet odd-states for the $2N$ subsystem.

To derive a $3N$ interaction from the diagram in Fig. 3, we sum all the possible diagrams corresponding to a correlation between nucleons ''1'' and ''2'' while the pion is ''in flight.'' There are four of such diagrams and their sum provides the contribution for an irreducible $3N$ force in one single Faddeev component; namely the component where nucleon 3 represents the spectator. (This scheme of diagrammatic resummation is a very natural and automatic consequence of the formalism discussed in Ref. [14].) We denote the resulting component of the $3N$ force as V_3^{3N}. The complete $3N$ interaction obviously will result from the sum over all three Faddeev components, or equivalently from all diagrams resulting from the cyclic permutations of the nucleons in the four diagrams mentioned above. Hence the total $3NF$ contribution will result from $V^{3N}=V_1^{3N}+V_2^{3N}+V_3^{3N}$.

The component V_3^{3N} must be calculated according to the expression

$$V_3^{3N}=f_1G_0^{(4)}\tilde{t}_{12}G_0^{(4)}f_3^\dagger+f_2G_0^{(4)}\tilde{t}_{12}G_0^{(4)}f_3^\dagger+f_3G_0^{(4)}\tilde{t}_{12}G_0^{(4)}f_1^\dagger +f_3G_0^{(4)}\tilde{t}_{12}G_0^{(4)}f_2^\dagger, \tag{3.1}$$

where f_1 ($f_1^\dagger$) represents the πNN vertex interaction for exchanged pion production (absorption) on nucleon 1, $G_0^{(4)}$ denotes the intermediate propagation of the three nucleons plus

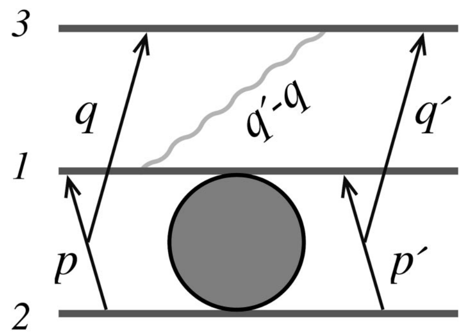

FIG. 6. Notation on the $3N$ Jacobi momenta used in the text.

the exchanged pion, $\tilde{t}_{12}$ represents the subtracted $2N$ t matrix, describing the correlation between nucleons 1 and 2 while the pion is ''in flight.'' One must observe right from the start that one cannot identify this amplitude with the on-shell $2N$ t matrix. Indeed this subtracted amplitude enters off-shell in the diagram, and with an energy shift. And below pion production threshold, $\tilde{t}_{12}$ must be real because the free Green's function $G_0^{(4)}$ is not singular in this region of the real axis.

If one now considers the first of these four diagrams with the Jacobi coordinates depicted as in Fig. 6, using the static approximation and assuming the process in the c.m. (center of mass) of the system, one obtains that this diagram corresponds to the following contribution:

$$D_1=\sum_\alpha \frac{f_{\pi NN}}{m_\pi}\frac{\boldsymbol{\sigma_1}\cdot(\mathbf{q}-\mathbf{q}')}{\sqrt{(2\pi)^3 2\omega}}\tau_1^\alpha G_0^{(4)} \times\tilde{t}_{12}\left(\mathbf{p},\mathbf{p}';E-\frac{q^2}{2\nu}-\omega_\pi\right)G_0^{(4)}\frac{f_{\pi NN}}{m_\pi}\frac{\boldsymbol{\sigma_3}\cdot(\mathbf{q}-\mathbf{q}')}{\sqrt{(2\pi)^3 2\omega}}\tau_3^{-\alpha}. \tag{3.2}$$

With the sum over α it is intended that all three isospin components of the pion field are summed up (in pseudospherical representation), while ν is the reduced mass of the spectator nucleon with respect to the c.m. of the pair.

To derive the final expression of this diagram, we will use the static approximation. Originally, the Green's function on the left should read

$$G_0^{(4)}=\frac{1}{E-p^2/2\mu-q^2/2\nu-\omega_\pi}, \tag{3.3}$$

and similarly the one on the right should be

$$G_0^{(4)}=\frac{1}{E-p'^2/2\mu-q^2/2\nu-\omega_\pi}. \tag{3.4}$$

Using the static approximation, we assume that

$$E\simeq\frac{p^2}{2\mu}+\frac{q^2}{2\nu}\simeq\frac{p'^2}{2\mu}+\frac{q^2}{2\nu}. \tag{3.5}$$

In this case, both Green's functions on the left and right of $\tilde{t}_{12}$ can be approximated by the same expression, namely,

$$G_0^{(4)}\simeq-\frac{1}{\omega_\pi}. \tag{3.6}$$

We introduce also $\mathbf{Q}$ as the momentum transferred by the pion, $\mathbf{Q}=\mathbf{q}'-\mathbf{q}$, hence $\omega_\pi=\sqrt{m_\pi^2+Q^2}$.

The subtracted t matrix is estimated according to the expression

$$\tilde{t}_{12}\left(\mathbf{p},\mathbf{p}';E-\frac{q^2}{2\nu}-\omega_\pi(Q)\right)=t_{12}\left(\mathbf{p},\mathbf{p}';E-\frac{q^2}{2\nu}-\omega_\pi(Q)\right) -v_{12}(\mathbf{p},\mathbf{p}'), \tag{3.7}$$

where the potential-like term $v_{12}(\mathbf{p},\mathbf{p}')$ contains only OPE/OBE-type diagrams. The quantity $\tilde{t}_{12}$ depends on the Jacobi momenta $\mathbf{p}$, $\mathbf{p}'$, $\mathbf{q}$, and $\mathbf{q}'$ in a complicated way. The important feature, however, is that the energy of the $2N$ subsystem is shifted to negative values by the spectator kinetic energy *and* by the mesonic term ω_π. In the case of heavier mesons ($\omega_x\gg\omega_\pi$) the $2N$ energy becomes so negative (large in absolute value) that t_{12} is close to v_{12}, and hence the two quantities almost cancel each others. As we will see further on, with the pion the results are quite different because this meson—the Goldstone boson of the underlying chiral theory—is so light.

One has to repeat the same derivation also for the other three diagrams and sum over all the contributions. As a result, the third Faddeev component of the irreducible $3N$ force generated by the four terms given by Eq. (3.1), can be expressed as

$$V_3^{3N}=\frac{f_{\pi NN}^2}{m_\pi^2}\frac{1}{(2\pi)^3} \times\left[\frac{(\boldsymbol{\sigma_1}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})(\boldsymbol{\tau_1}\cdot\boldsymbol{\tau_3})+(\boldsymbol{\sigma_2}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})(\boldsymbol{\tau_2}\cdot\boldsymbol{\tau_3})}{\omega_\pi^2}\right] \times\frac{\tilde{t}_{12}(\mathbf{p},\mathbf{p}';E-q^2/2\nu-\omega_\pi)}{2\omega_\pi} +\frac{f_{\pi NN}^2}{m_\pi^2}\frac{1}{(2\pi)^3}\frac{\tilde{t}_{12}(\mathbf{p},\mathbf{p}';E-q'^2/2\nu-\omega_\pi)}{2\omega_\pi} \times\left[\frac{(\boldsymbol{\sigma_1}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})(\boldsymbol{\tau_1}\cdot\boldsymbol{\tau_3})+(\boldsymbol{\sigma_2}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})(\boldsymbol{\tau_2}\cdot\boldsymbol{\tau_3})}{\omega_\pi^2}\right] \tag{3.8}$$

and the total contribution to the $3N$ force will be given by summing up this contribution together with those obtained from this by cyclic permutations of the three nucleons.

This formula implies several aspects on which we would like to comment. (i) We have used the nonrelativistic reduction of the πNN vertex. (ii) We have neglected nucleon recoil effects on the basis that the pion mass m_π is much smaller than the nucleon mass, M. (iii) We have made use of the static approximation, implying that the pion ''in flight'' exchange momentum but not energy with the nucleon 3. (iv) The form of the force obtained above is made symmetrical by the combined sum of the four diagrams, and below pion threshold the resulting expression is Hermitian, being $\tilde{t}_{12}$ real.

It may be convenient to transform V_3^{3N} introducing the spin and isospin operators for the $2N$ subsystem, $\mathbf{S}_{12}=\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2$, and $\mathbf{T}_{12}=\boldsymbol{\tau}_1+\boldsymbol{\tau}_2$, respectively. Then it is a matter of simple algebraic manipulations to rewrite Eq. (3.8) in the following form:

$$
\begin{aligned}
V_3^{3N}=&\frac{f_{\pi NN}^2}{2m_\pi^3}\frac{1}{(2\pi)^3}\frac{(\mathbf{S_{12}}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})}{m_\pi^2+Q^2}(\mathbf{T_{12}}\cdot\boldsymbol{\tau_3})\tilde{t}_{12}\left(\mathbf{p},\mathbf{p}';E-\frac{q^2}{2\nu}-m_\pi\right)+\frac{f_{\pi NN}^2}{2m_\pi^3}\frac{1}{(2\pi)^3}\tilde{t}_{12}\left(\mathbf{p},\mathbf{p}';E-\frac{q'^2}{2\nu}-m_\pi\right)\\
&\times\frac{(\mathbf{S_{12}}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})}{m_\pi^2+Q^2}(\mathbf{T_{12}}\cdot\boldsymbol{\tau_3})-\frac{f_{\pi NN}^2}{2m_\pi^3}\frac{1}{(2\pi)^3}\frac{(\boldsymbol{\sigma_1}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})}{m_\pi^2+Q^2}(\boldsymbol{\tau_2}\cdot\boldsymbol{\tau_3})\tilde{t}_{12}\left(\mathbf{p},\mathbf{p}';E-\frac{q^2}{2\nu}-m_\pi\right)\\
&-\frac{f_{\pi NN}^2}{2m_\pi^3}\frac{1}{(2\pi)^3}\tilde{t}_{12}\left(\mathbf{p},\mathbf{p}';E-\frac{q'^2}{2\nu}-m_\pi\right)\frac{(\boldsymbol{\sigma_1}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})}{m_\pi^2+Q^2}(\boldsymbol{\tau_2}\cdot\boldsymbol{\tau_3})\\
&-\frac{f_{\pi NN}^2}{2m_\pi^3}\frac{1}{(2\pi)^3}\frac{(\boldsymbol{\sigma_2}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})}{m_\pi^2+Q^2}(\boldsymbol{\tau_1}\cdot\boldsymbol{\tau_3})\tilde{t}_{12}\left(\mathbf{p},\mathbf{p}';E-\frac{q^2}{2\nu}-m_\pi\right)\\
&-\frac{f_{\pi NN}^2}{2m_\pi^3}\frac{1}{(2\pi)^3}\tilde{t}_{12}\left(\mathbf{p},\mathbf{p}';E-\frac{q'^2}{2\nu}-m_\pi\right)\frac{(\boldsymbol{\sigma_2}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})}{m_\pi^2+Q^2}(\boldsymbol{\tau_1}\cdot\boldsymbol{\tau_3}).
\end{aligned}
\tag{3.9}
$$

Here, we have also approximated the two normalization factors of the pion field, i.e., the two square roots in the denominators, as

$$\frac{1}{\sqrt{2\omega}\sqrt{2\omega}}\simeq\frac{1}{2m_\pi},\tag{3.10}$$

and we have shifted the $2N$ amplitudes by the pion mass m_π, in place of ω_π. These approximations are really not essential, but they simplify considerably the formulas of the partial wave expansions, without really altering the physics. It is clear, however, that a more consistent calculation requires the employment of the exact expressions.

Both expressions (3.8),(3.9) are symmetrical at sight. The latter one has the advantage that it is possible to isolate from the rest the contribution given by the first two terms, which we rewrite as

$$
\begin{aligned}
V_3^{*}=&\frac{f_{\pi NN}^2}{2m_\pi^3}\frac{1}{(2\pi)^3}\frac{(\mathbf{S_{12}}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})}{m_\pi^2+Q^2}(\mathbf{T_{12}}\cdot\boldsymbol{\tau_3})\\
&\times\tilde{t}_{12}\left(\mathbf{p},\mathbf{p}';E-\frac{q^2}{2\nu}-m_\pi\right)+\text{H.c.}
\end{aligned}
\tag{3.11}
$$

The remaining part of V_3^{3N}, i.e., the sum over the last four terms, mixes together the spin components of nucleon 1 with the isospin components of nucleon 2. We will not discuss further this contribution and leave it for future studies. The last four terms in Eq. (3.9) is just one of the many irreducible contributions which should be added up to build the full spin-isospin structure of the $3N$ interaction.

In the following, we will focus the attention on V_3^*, which has an interesting spin-isospin structure since it depends only on the spectator and pair coordinates. Because of the presence of the spin-isospin operators $\mathbf{S_{12}}$ and $\mathbf{T_{12}}$, such term vanishes unless the $2N$ pair is in a triplet state for *both* spin and isospin coordinates. And since the nucleon pair must be in an antisymmetric state because of the generalized Pauli principle, then the allowed orbital momentum of the pair can be only odd. This means that this contribution to the irreducible $3N$ force acts only in triplet odd states (^3P_J waves, ^3F_J waves, etc.) of the two-nucleon subsystem. In other words, the third nucleon, by means of this contribution modifies selectively the triplet odd states of the $2N$ subsystem with respect to a free, isolated nucleon-nucleon pair. We believe that this mechanism can possibly modify those observables particularly sensitive to the triplet p and f waves and might therefore affect also the nucleon-deuteron vector analyzing powers.

IV. PARTIAL WAVE DECOMPOSITION

We provide the partial-wave decomposition of V_3^*. We will work in the so-called channel spin coupling since this is the most natural scheme for nucleon-deuteron scattering.

The channel-spin coupling is defined according to the following notation:

$$w=\{[(ls)j\sigma]KL\}\Gamma\Gamma_z, \tag{4.1}$$

where l, s, and j represent, respectively, the orbital momentum, spin, and total spin-angular momentum of the pair (hence, s represents the quantum number associated to the operator $\mathbf{S_{12}}$). The total spin of the pair j is then coupled with the intrinsic spin of the spectator $\sigma=1/2$ to provide the so-called channel spin K. And finally, this is coupled to the orbital angular momentum of the third nucleon L, to give the total angular momentum of the $3N$ system Γ and its azimuthal component Γ_z.

We observe that V_3^* has an interesting structure in the pair-spectator coordinate system. The structure is that of a OPE contribution, but in the spectator coordinates, multiplied by a full $2N$ interaction depending on the internal coordinates of the pair. Given this structure, it is not so difficult to perform the partial wave decomposition of V_3^*. Indeed, one can consider the spectator coordinates and separate the $3N$ potential into a spin-spin component and a tensor one, following the standard procedure for the OPE term (see, e.g., Refs. [40,41]),

$$\frac{(\mathbf{S_{12}}\cdot\mathbf{Q})(\boldsymbol{\sigma_3}\cdot\mathbf{Q})}{m_\pi^2+Q^2}=\frac{1}{3}\left[-\frac{m_\pi^2}{m_\pi^2+Q^2}(\mathbf{S_{12}}\cdot\boldsymbol{\sigma_3})+\frac{Q^2}{m_\pi^2+Q^2}\boldsymbol{\Sigma}_{\mathbf{12}}(\hat{\mathbf{Q}})\right], \tag{4.2}$$

where $(\hat{\mathbf{Q}})$ is the angular part of the spectator momentum, and $\boldsymbol{\Sigma}_{\mathbf{12}}(\hat{\mathbf{Q}})=3(\mathbf{S_{12}}\cdot\hat{\mathbf{Q}})(\boldsymbol{\sigma_3}\cdot\hat{\mathbf{Q}})-(\mathbf{S_{12}}\cdot\boldsymbol{\sigma_3})$ is the tensor operator.

In the above equation, we have neglected the contact term, on the grounds that its contribution will be unavoidably smeared out when taking into account the extended structure of the sources of the meson field. The extended nature of the sources have to be included in V_3^* by means of phenomenological πNN form factors. In a fully consistent calculation these form factors should be the same as those used in the standard $2N$ OPE contribution. Then V_3^* is completely fixed by the full expression of the $2N$ potential.

For the spin-spin part of V_3^*, we obtain the following partial-wave decomposition:

$$\begin{aligned}\langle p,q,w|V_3^*(\text{spin}-\text{spin})|p',q',w'\rangle&=\delta_{ss'}\delta_{s1}\delta_{LL'}\delta_{KK'}\delta_{\Gamma\Gamma'}\delta_{\Gamma_z\Gamma_z'}\hat{j}\hat{j}'12\frac{2}{\pi}I_L(q,q')\\&\times(-)^{l+j+K+j'+1/2}\begin{Bmatrix}1&j&j'\\l&1&1\end{Bmatrix}\begin{Bmatrix}1&j&j'\\K&\frac{1}{2}&\frac{1}{2}\end{Bmatrix}\\&\times\left[\tilde{t}_{12}\left(p,p';E-\frac{q^2}{2\nu}-m_\pi\right)\right]_{ll'}^{j's'}\frac{1}{2m_\pi}+\text{H.c.}\end{aligned} \tag{4.3}$$

For the tensor-spin component the decomposition in partial waves is slightly more complicated, being

$$\begin{aligned}\langle p,q,w|V_3^*(\text{tensor})|p',q',w'\rangle&=\delta_{ss'}\delta_{s1}\delta_{\Gamma\Gamma'}\delta_{\Gamma_z\Gamma_z'}\hat{j}\hat{j}'\hat{K}\hat{K}'\hat{L}\hat{L}'12\frac{2}{\pi}\sqrt{30}I_{LL'}(q,q')\\&\times(-)^{l+j+\Gamma+K'}\begin{pmatrix}L&2&L'\\0&0&0\end{pmatrix}\begin{Bmatrix}1&j&j'\\l&1&1\end{Bmatrix}\\&\times\begin{Bmatrix}K&2&K'\\L'&\Gamma&L\end{Bmatrix}\begin{Bmatrix}1&j&j'\\1&\frac{1}{2}&\frac{1}{2}\\2&K&K'\end{Bmatrix}\\&\times\left[\tilde{t}_{12}\left(p,p';E-\frac{q^2}{2\nu}-m_\pi\right)\right]_{ll'}^{j's'}\frac{1}{2m_\pi}+\text{H.c.}\end{aligned} \tag{4.4}$$

The two quantities $I_L(q,q')$ and $I_{L,L'}(q,q')$ represent well known Fourier-Bessel transforms of Yukawa-type functions

$$I_L(q,q')=-\frac{f_{\pi NN}^2}{12\pi}\int_0^\infty j_L(qR)\,j_L(q'R)\left(\frac{e^{-m_\pi R}}{R}\right)R^2dR \tag{4.5}$$

and

$$I_{LL'}(q,q')=-\frac{f_{\pi NN}^2}{12\pi}\int_0^\infty j_L(qR)\,j_{L'}(q'R)\times\left[\frac{e^{-m_\pi R}}{R}\left(1+\frac{3}{m_\pi R}+\frac{3}{(m_\pi R)^2}\right)\right]R^2dR. \tag{4.6}$$

The resulting analytical expressions for these integrals are well known [42].

Finally, the potential matrix elements for the spin and tensor parts must be multiplied by the isotopic component, which is the same in both cases:

$$\langle(t\tau)TT_z|(\mathbf{T_{12}}\cdot\boldsymbol{\tau_3})|(t'\tau)T'T_z'\rangle=\delta_{tt'}\delta_{t1}\delta_{TT'}\delta_{T_zT_z'}12(-)^{3/2+T}\begin{Bmatrix}T&\frac{1}{2}&1\\1&1&\frac{1}{2}\end{Bmatrix}. \tag{4.7}$$

Here $\tau=1/2$ is the isospin of the spectator nucleon, t is the isotopic spin of the $2N$ pair, and T is the total isospin of the $3N$ system. As ought to be expected, only for isovector pairs is this matrix element nonzero.

Equation (3.11) represents an OPE potential in the spectator-pair coordinates, times a subtracted $2N$ t matrix for the internal coordinates of the pair. This $2N$ amplitude is quite off shell, because of the presence of a pion-exchange term, in addition to the standard shift due to the spectator nucleon. In the limit of a heavy meson exchange ($m_x\rightarrow 1$ GeV) the energy of the $2N$ subsystem will be large and negative, and t_{12} will be dominated by its Born OBE term v_{12}. Hence the effect of this $3NF$ is suppressed because approximately $\tilde{t}_{12}\simeq 0$. On the contrary, as shown in Fig. 7,

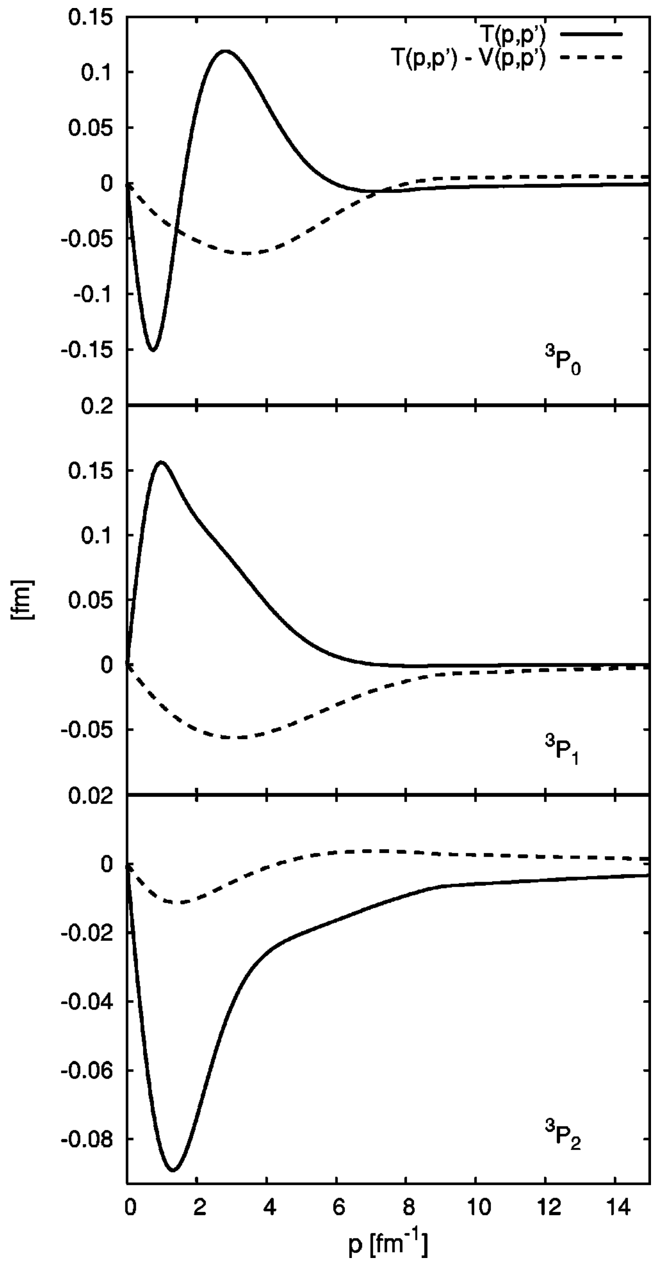

FIG. 7. Effect of the subtraction in the triplet p waves. The full line represents the unsubtracted $2N$ t matrix for $E=-150$ MeV and $p'=0.89$ fm^{-1} while the pion is ''in flight.'' The dashed line shows the same amplitude when mesonic retardations (discussed in the text) are subtracted. A 15–30 % effect survives from the cancellation. This generates the new $3NF$ component.

for the lightest meson, a 15–30 % effect (at least) survives from the cancellation and this generates the ''odd'' contribution to the $3NF$. The figure shows the comparison between the unsubtracted (solid line) and the subtracted (dashed line) t matrices, for a $2N$ energy of -150 MeV, consistent with the calculation of the diagram shown in Fig. 3. The t matrices have been calculated in the relevant triplet p states, with the Bonn B potential [41], which is of OBE type. Then the subtracted amplitude is given simply by Eq. (3.7). The lines show the t_{12} and $\tilde{t}_{12}$ amplitudes as a function of the momentum p, while p' was fixed at the value $p'=0.89$ fm^{-1}. With other values of p' we found the same effect. Also, in order to show that the cancellation could not produce an overall vanishing result we made a more stringent test, by assuming that when the mass of the particle ''in flight'' is of the order of $\Lambda \simeq 1$ GeV, then the diagram of Fig. 3 is canceled *exactly* against the mesonic retardation corrections of Fig. 4. In this case, the subtracted t matrix entering in the *pionic* diagram can be evaluated according to the expression

$$\tilde{t}_{12}\left(\mathbf{p},\mathbf{p}';E-\frac{q^2}{2\nu}-\omega_\pi(Q)\right) \approx t_{12}\left(\mathbf{p},\mathbf{p}';E-\frac{q^2}{2\nu}-\omega_\pi(Q)\right) - t_{12}(\mathbf{p},\mathbf{p}';-\Lambda), \qquad (4.8)$$

and this expression can be employed with all types of phenomenological NN potentials. We checked $\tilde{t}_{12}$ for an energy of the $2N$ subsystem around -150 MeV, for the Bonn B and for the Paris potential [43], and with both interactions we found that a 10–20 % effect was surviving after the subtraction, thus providing evidence that the cancellation cannot hold exactly and simultaneously in both cases of light- and heavy-meson exchanges. However, it is also possible to modify the behavior of the $2N$ amplitude in this energy region, so that to obtain a $3NF$ contribution of larger or smaller size, without obviously altering the constraints to the $2N$ amplitudes from comparison with phase-shift analyses.

V. SUMMARY AND CONCLUSIONS

As discussed in the Introduction, discrepancies between theoretical calculations and experimental measurements give a clear indication that $3NF$'s of new structure are badly needed. The existing $3NF$ models do not seem to provide in full the correct spin-isospin structure of the $3N$ force.

In this paper, we have suggested a new mechanism which generates an irreducible $3NF$ whose structure is topologically very different from those explored up to now. Therefore, the force generated by this mechanism should be considered as an additional contribution to the full $3N$ interaction. This mechanism is generated under the hypothesis that the standard few-nucleon dynamics and the pion-exchange processes are intertwined more strongly than what has been generally assumed up to now. In particular, it is the intermediate formation of a virtual $2N$ cluster during a pion-exchange process that gives rise to this new $3NF$ term. In an instantaneous approach, such a mechanism is 100% suppressed because of the presence of a well-known cancellation effect which involves the meson-retardation corrections of the reducible Born term of Fig. 2 as well as all possible irreducible $3NF$ diagrams obtained by subsuming the exchange of two pions in their variety of possible time orderings. It was known also that the same cancellation occurs when considering similar processes involving heavy-meson exchanges.

We have sized the effect of this cancellation more precisely by considering a complete $2N$ rescattering process, not just its Born term (of Fig. 4). This was possible by extrapolating the NN t matrix downwards to negative energy by a shift given by the spectator kinetic energy and by the relativistic energy of the meson. The result of this study reveals that a 15–30 % contribution survives from this cancellation in case of a cluster formation while a pion is ‘‘in flight.’’ Instead, the cancellation is more pronounced when considering the same process while a heavy meson in ‘‘in flight.’’ The reason for this difference can be entirely attributed to the fact that the mass of the pion is approximately comparable to the average momenta exchanged between nucleons in nuclear process, while the mass of the heavier mesons are larger.

We have studied the spin-isospin structure of the $3NF$ generated by this new, pion-induced mechanism and we have extracted an important contribution which selectively operates in the triplet-odd waves for the $2N$ subsystem. While we acknowledge that the way to fully understand the spin-isospin structure of the $3N$ interaction is still long and difficult, we conclude that this new term may possibly contribute to this structure, especially by affecting those spin observables most sensible to the ^3P_J waves, such as the Nd vector analyzing powers.

ACKNOWLEDGMENTS

The work of L.C. was performed under the Murst-PRIN Project ‘‘Fisica Teorica del Nucleo e dei Sistemi a Piú Corpi.’’ The work of W.S. was supported by the Natural Science and Engineering Research Council of Canada.

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