

## Charge symmetry violation effects in pion scattering off the deuteron

V. V. Baru,<sup>\*</sup> A. E. Kudryavtsev,<sup>†</sup> and V. E. Tarasov<sup>‡</sup>

*Institute of Theoretical and Experimental Physics, 25 B. Chermushkinskaya Street, Moscow 117259, Russia*

W. J. Briscoe,<sup>§</sup> K. S. Dhuga,<sup>||</sup> and I. I. Strakovsky<sup>¶</sup>

*Center for Nuclear Studies and Department of Physics, The George Washington University, Washington, D.C. 20052*

(Received 5 January 2000; published 15 September 2000)

We discuss the theoretical and experimental situations for charge symmetry violation (CSV) effects in the elastic scattering of  $\pi^+$  and  $\pi^-$  on deuterium (D) and  ${}^3\text{He}/{}^3\text{H}$ . Accurate comparison of data for both types of targets provides evidence for the presence of CSV effects. While there are indications of a CSV effect in deuterium, it is much more pronounced in the case of  ${}^3\text{He}/{}^3\text{H}$ . We provide a description of the CSV effect on the deuteron in terms of single- and double-scattering amplitudes. The  $\Delta$ -mass splitting is taken into account. Theoretical predictions are compared with existing experimental data for  $\pi$ - $d$  scattering; a future article will speak to the  $\pi$ -three nucleon case.

PACS number(s): 25.80.Dj, 24.80.+y, 25.10.+s, 25.45.De

### I. INTRODUCTION

The study of CSV in the interaction of pions with nuclei in the Delta resonance region has been of considerable interest for the last two decades. The interaction of pions with light nuclei such as  ${}^2\text{H}$  [1–9],  ${}^3\text{He}/{}^3\text{H}$  [9–13], and  ${}^4\text{He}$  [14] has attracted particular attention. However, we note that quite a large data set also exists for scattering of  $\pi^+$  and  $\pi^-$  on  ${}^{12}\text{C}$ ,  ${}^{16}\text{O}$ , and  ${}^{40}\text{Ca}$  as well [15].

From the point of view of theory, the advantage of searching for CSV in the scattering of pions from light nuclei is that one can describe pion scattering in these systems in a relatively straight-forward manner. With this in mind, we limit ourselves to the consideration of the scattering of pions from deuterium,  ${}^3\text{He}$ , and  ${}^3\text{H}$ . Moreover, we anticipate that CSV effects are considerably diminished in the case of pion scattering from heavier nuclei because of the importance of processes such as absorption.

First, in order to evaluate the scale of CSV effect, we focus our theoretical efforts primarily on  $\pi d$  elastic scattering. In a following article, we will develop the formalism further to investigate CSV in the three-nucleon system.

A detailed analysis of the experimental situation will be given in the next section. Here, we want only to point out that in order to make a comparison between experimental data related to different projectile or target, we must deal with the same experimental measurables. Historically, the CSV experimental data were given in terms of asymmetry,  $A_\pi$  for the deuteron

$$A_\pi = \frac{d\sigma/d\Omega(\pi^-d) - d\sigma/d\Omega(\pi^+d)}{d\sigma/d\Omega(\pi^-d) + d\sigma/d\Omega(\pi^+d)}, \quad (1)$$

and in terms of ratios  $r_1$  and  $r_2$ , and superratio  $R$  for the  ${}^3\text{He}/{}^3\text{H}$  case

$$r_1 = \frac{d\sigma/d\Omega(\pi^+{}^3\text{H})}{d\sigma/d\Omega(\pi^-{}^3\text{He})},$$

$$r_2 = \frac{d\sigma/d\Omega(\pi^-{}^3\text{H})}{d\sigma/d\Omega(\pi^+{}^3\text{He})},$$

$$R = r_1 r_2. \quad (2)$$

Both interactions  $\pi^+{}^3\text{H}$  and  $\pi^-{}^3\text{He}$  for the ratio  $r_1$ , and  $\pi^-{}^3\text{H}$  and  $\pi^+{}^3\text{He}$  for the ratio  $r_2$  are isomirror interactions. Therefore, if charge symmetry is strictly observed, both  $r_1$  and  $r_2$  would be equal to 1.0. Of course, the Coulomb interaction is not charge symmetric and would have to be taken into account. The superratio  $R$  is the product  $r_1$  and  $r_2$ . So, if charge symmetry is universally true,  $R$  is also equal to 1.0.

The experimental data suggests evidence for a small effect in  $A_\pi$  for the deuteron (e.g.,  $A_\pi \simeq 2\%$  at 143 MeV [3]) with some indication of structure at scattering angles around  $90^\circ$  in c.m. frame. At the same time, a sizable effect is clearly seen in the  ${}^3\text{He}/{}^3\text{H}$  case. For example,  $r_2 = 0.7 \pm 0.1$  for  $T_\pi = 256$  MeV and  $\theta = 82^\circ$  [12]. Theoretical predictions for the asymmetry  $A_\pi$  in the deuteron case were given in Ref. [3]. To describe the asymmetry, authors of Ref. [3] used a single-scattering approximation with allowance for differently charged  $\Delta$ 's (1232). In this approximation, the CSV effect proved to be independent of the scattering angle with typical value proportional to  $\delta m_\Delta/\Gamma_\Delta$ . Approximately the same approach was used in the  ${}^3\text{He}/{}^3\text{H}$  case in Ref. [9].

A different approach for the  ${}^3\text{He}/{}^3\text{H}$  case was suggested in Ref. [16]. Authors of this paper used an optical potential to describe the pionic  ${}^3\text{He}/{}^3\text{H}$  amplitudes. The radial dependence of  $\pi A$  potentials was determined in terms of matter and spin densities for  ${}^3\text{He}$  and  ${}^3\text{H}$ . The Coulomb-nuclei interference effect in the vicinity of minima in differential cross sections was reported as the main reason for the CSV effect in the approach of Ref. [16]. However, this in-

<sup>\*</sup>Electronic address: baru@vxitep.itep.ru

<sup>†</sup>Electronic address: kudryavtsev@vitep5.itep.ru

<sup>‡</sup>Electronic address: tarasov@vitep5.itep.ru

<sup>§</sup>Electronic address: briscoe@gwu.edu

<sup>||</sup>Electronic address: dhuga@gwu.edu

<sup>¶</sup>Electronic address: igor@gwu.edu

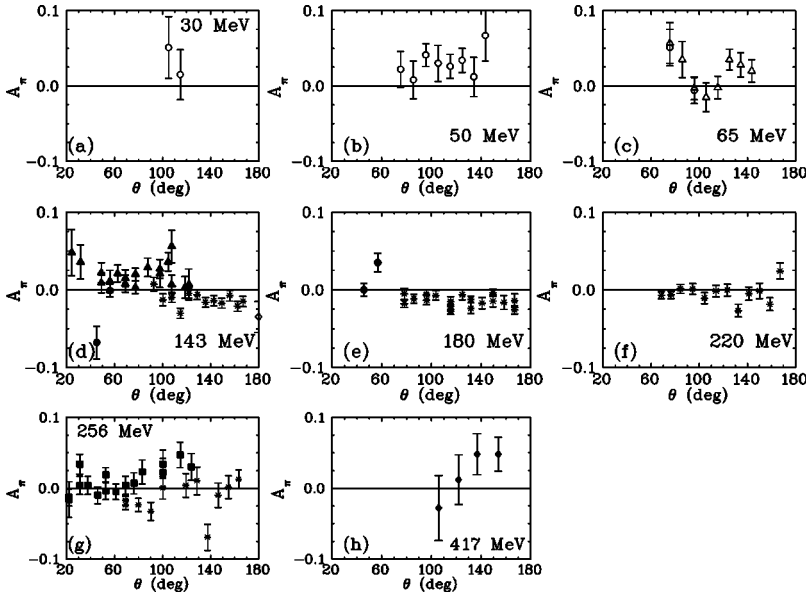


FIG. 1. Asymmetry  $A_\pi$  at different energies. (a) 30 MeV, (b) 50 MeV, (c) 65 MeV, (d) 143 MeV, (e) 180 MeV, (f) 220 MeV, (g) 256 MeV, and (h) 417 MeV for  $\pi d$  elastic scattering. Experimental data are from Ref. [7] (open circles), [6] (open triangles), [3] (filled triangles), [9] (filled circles), [2] (open diamonds), [5] (stars), [4] (filled squares), and [8] (filled diamonds).

terpretation was disputed by Briscoe and Silverman [17] because the authors of Ref. [16] obtained structure only near the  $90^\circ$  in  $r_2$  but could not at all explain the overall behavior of the experimental data.

In our investigation, we study the role of double-scattering on CSV because of mass splitting of  $\Delta$  isobars. It is widely known that the single-scattering approximation reproduces a differential cross section fairly well in the forward hemisphere. But for scattering angles beyond  $90^\circ$ , the double-scattering term is important and should be included. The influence of multiple scattering terms on differential cross section for deuteron case was studied long ago in Refs. [18–20]. But the influence of double and multiple scattering on CSV effects was never studied in detail.

In Sec. III, we explain how the basic ingredients of the scattering amplitude and constraints such as single and double scattering, and the Coulomb interaction are combined for  $\pi d$  elastic scattering. These results and the prospect for improvement are summarized in Sec. IV. The  ${}^3\text{He}/{}^3\text{H}$  case is considered in forthcoming paper.

## II. ANALYSIS OF EXPERIMENTAL SITUATION

The CSV effect was first observed in the difference of total  $\pi^\pm d$  cross sections in PSI and reported in Ref. [1]. This has been widely discussed, see, e.g., the book by Ericson and Weise [21]. There have been several measurements for both  $\pi^+ d$  and  $\pi^- d$ . The first systematic study of the CSV effect in the differential  $\pi^\pm d$  cross sections was done at LAMPF and presented in Ref. [22]. Soon after, the asymmetry  $A_\pi$  for  $T_\pi=143$  MeV was presented for the range of laboratory scattering angles between  $20^\circ$  and  $115^\circ$  [3]. The experiment was repeated for approximately the same range of scattering angles at  $T_\pi=256$  MeV [4]. We note that the structure in the asymmetry seen in Ref. [3] was not seen in the TRIUMF measurements of Ref. [5]. In the meantime, some indications for CSV effects were also obtained at low energies 30, 50, and 65 MeV at TRIUMF [6,7]. We also mention the high-

energy Gatchina data at  $T_\pi=417$  MeV [8] which also shows some indications on CSV.

We recall that the asymmetry (1), and ratios (2), are two different measures of CSV effects. As in the  ${}^3\text{He}/{}^3\text{H}$  case, we denote the ratio  $r=r_1=r_2$

$$r = \frac{d\sigma/d\Omega(\pi^- d)}{d\sigma/d\Omega(\pi^+ d)} = 1 + \epsilon.$$

Then, in the case of small magnitudes of CSV, we get

$$A_\pi \approx \epsilon/2.$$

Clearly, this tiny effect would require high-quality data.

Smith *et al.* [5] reported a  $-1.5\%$  asymmetry in the  $\pi d$  cross sections at back angles, with uncertainties of  $0.6\%$  at the different angles. The energy dependence of the asymmetry between 30 and 417 MeV is shown in Fig. 1.

## III. THEORETICAL CONSIDERATION OF CSV EFFECT IN DEUTERON

We see two possible ways to interpret the experimental situation. The first way is that one may conclude that there is really no effect in deuterium in accordance with statement [5] and that the effect in the  ${}^3\text{He}/{}^3\text{H}$  case is influenced correspondingly by specific three-body configurations of  ${}^3\text{He}$  and  ${}^3\text{H}$ . By this, we mean the possible influence of three-body, CSV forces which are absent in the  ${}^2\text{H}$  case and/or differences in the description of  ${}^3\text{He}$  and  ${}^3\text{H}$  wave functions (WF) as a consequence of an additional Coulomb repulsion between two protons in the  ${}^3\text{He}$  case (see in this connection Ref. [23]).

The second scenario is to suggest that the effect may be seen in both cases  ${}^2\text{H}$  and  ${}^3\text{He}/{}^3\text{H}$ , but in deuterium, the effect is small in comparison with  ${}^3\text{He}/{}^3\text{H}$ . There should still be some angular dependence for the CSV effect in deuterium. However, Masterson *et al.* [3] have shown that within

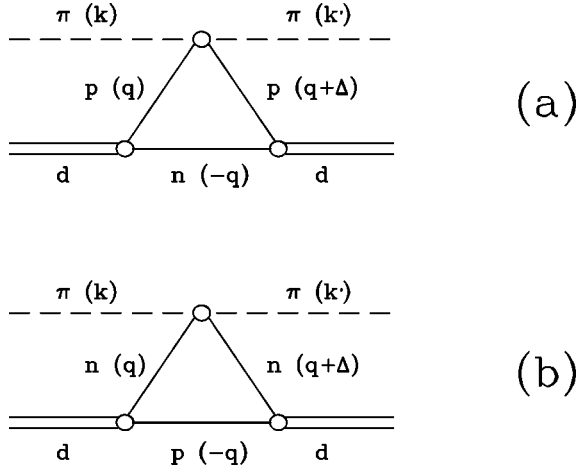


FIG. 2. Single-scattering amplitudes for  $\pi^+d$  on the proton (a) and the neutron (b).

the impulse single-scattering approximation the angular dependence for CSV is absent when only scattering via the  $P_{33}$  is considered. The inclusion of others  $S$  and  $P$  waves does not change the situation dramatically as all the phases except  $P_{33}$  are small in the region of interest. So, we need to look beyond the single-scattering approximation and to consider multiple scattering of pions.

(1) *Single-scattering approximation.* Everywhere below, we shall use the following notations:

$$k_{\text{c.m.}} = \frac{m}{m + \omega} k, \quad w = m + \omega - \frac{k^2}{2(m + \omega)},$$

where  $\omega$  is the pion energy,  $w_i$  are the masses of isobars, and here and below indices 1–4 in the notations of amplitudes, masses and widths mean the corresponding isobar isospin state:

$$i = 1, 2, 3, 4$$

for

$$\Delta^{++}, \Delta^+, \Delta^0, \Delta^-.$$

We suppose  $\Gamma_{\text{el}} = \Gamma_{\text{tot}} = \Gamma_0 = 120$  MeV. The values  $w_i$  ( $i = 1, 2, 3, 4$ ), we calculate according to the formula from Ref. [21] [p. 124, Eq. (4.16)]:

$$w_i = a - bI_i + cI_i^2,$$

where  $I_i$  is the third component of isospin for the  $i$ th term from the  $\Delta$  multiplet. Using the average resonance value from the PDG [24]  $w_0 = 1232$  MeV, we get  $a = 1231.8$  MeV,  $b = 1.38$  MeV, and  $c = 0.13$  MeV. In this approximation, the  $\pi d$  amplitude is the sum of the two Feynman diagrams shown in Fig. 2.

The elementary  $\pi N$  amplitude in terms of  $\delta_{33}(k)$  phase looks similar to the following:

$$\hat{f}_{\pi N} = \frac{1}{2ik} (e^{2i\delta_{33}(k)} - 1) \frac{2 + \vec{t} \cdot \vec{\tau}}{3} \{2\hat{k} \cdot \hat{k}' + i\vec{\sigma} \cdot [\hat{k} \times \hat{k}']\}, \quad (3)$$

where  $\vec{\sigma}$  and  $\vec{\tau}$  are Pauli matrices and  $\hat{f}_{\pi N}$  is the operator in spin and isospin space of the  $\pi N$  system. The deuteron wave function in  $S$ -wave approximation is

$$\frac{1}{\sqrt{2}} \psi_d(p) w_2^+ (\vec{\epsilon} \cdot \vec{\sigma}) \sigma_2 w_1^*$$

(here  $w_1$  and  $w_2$  are the nucleon spinors and  $\vec{\epsilon}$  is the polarization vector of deuteron), and the expression for amplitude  $f_1$ , which correspond to Fig. 2(a), has the form

$$f_{\pi d}^{(1)} = \frac{2}{E_{\text{c.m.}}^{\pi d}} \int \frac{d\vec{p}}{(2\pi)^3} E_{\text{c.m.}}^{\pi N} f_{33}(k_{\text{c.m.}}) \psi_d(\vec{p}) \psi_d\left(\vec{p} - \frac{\vec{\Delta}}{2}\right) \times \{2(\vec{\epsilon} \cdot \vec{\epsilon}')(\hat{k} \cdot \hat{k}') - [\vec{\epsilon} \times \vec{\epsilon}'] \cdot [\hat{k} \times \hat{k}']\}. \quad (4)$$

Here  $\vec{\Delta} = \vec{k} - \vec{k}'$  is the three-dimension momentum transfer;  $f_{33}(k) = 1/2ik (e^{2i\delta_{33}(k)} - 1)$ ;  $\vec{\epsilon}(\vec{\epsilon}')$  is the polarization vector of initial (final) deuteron;  $\hat{k} = \vec{k}_{\text{c.m.}}/k_{\text{c.m.}}$  and  $\hat{k}' = \vec{k}'_{\text{c.m.}}/k_{\text{c.m.}}$  are the units vectors, where  $\vec{k}_{\text{c.m.}}(\vec{k}'_{\text{c.m.}})$  is the momentum of initial (final) pion in the rest frame of subprocess  $\pi N \rightarrow \pi N$ .

At this stage, we make some simplifications. We shall neglect Fermi motion of the nucleon and consider (for a while) the expression (4) in the static limit, i.e.,  $\omega/m \rightarrow 0$ . Then,  $2E_{\text{c.m.}}^{\pi N}/E_{\text{c.m.}}^{\pi d} \rightarrow 1$ ,  $k_{\text{c.m.}} \rightarrow k$ . So, we get

$$\hat{f}_{\pi d}^{(1)} = \frac{4}{3} f_{33}(k) \{2(\vec{\epsilon} \cdot \vec{\epsilon}')(\hat{k} \cdot \hat{k}') - [\vec{\epsilon} \times \vec{\epsilon}'] \cdot [\hat{k} \times \hat{k}']\} \times \int \Psi_D^2(r) e^{i(\vec{\Delta}/2) \cdot \vec{r}} d\vec{r}. \quad (5)$$

For this amplitude, the differential cross section with the unpolarized initial deuteron has the following form:

$$\frac{d\sigma_{\pi d}^{(1)}}{d\Omega} = \frac{32}{27} (6 \cos^2 \theta + \sin^2 \theta) |f_{33}(k)|^2 F_D^2(\Delta), \quad (6)$$

where  $F_D(\Delta) = \int \Psi_D^2(r) e^{i\vec{\Delta} \cdot \vec{r}/2} d\vec{r}$ . This expression agrees with that given in Ref. [3]. The ratio 6:1 between the terms proportional to  $\cos^2 \theta$  and  $\sin^2 \theta$  reflects the ratio of non-spin-flip to spin-flip amplitudes in this approximation.

(2) *Charge symmetry breaking effect.* First consider the elementary  $\pi^+ p$  amplitude in terms of a  $\Delta(1232)$  pole. The amplitude looks similar to a standard Breit-Wigner amplitude

$$f_{\pi^+ p} = -\frac{1}{2k} \frac{\Gamma_1}{w - w_1 + i\Gamma_1/2}, \quad (7)$$

where  $w_1$  and  $\Gamma_1$  are the mass and the full width, respectively, of the  $\Delta^{++}$  resonance. Making a linear expansion of

this amplitude around the mean value of the mass  $w_0$  and the width  $\Gamma_0$  for the  $\Delta$  resonance, we get

$$f_{\pi^+p} = -\frac{1}{2k} \frac{\Gamma_0}{w-w_0+i\Gamma_0/2} \left( 1 + \frac{\delta\Gamma_1}{\Gamma_0} + \frac{\delta w_1 - i\delta\Gamma_1/2}{w-w_0+i\Gamma_0/2} \right), \quad (8)$$

where  $\delta\Gamma_1 = \Gamma_1 - \Gamma_0$  and  $\delta w_1 = w_1 - w_0$ . So, using Eq. (8), we get that the charge asymmetry in  $\pi^\pm d$  scattering in this approximation is

$$A_\pi = \frac{3}{4} \frac{C_\Gamma(w-w_0)^2 + (w-w_0)C_M\Gamma_0}{\Gamma_0[(w-w_0)^2 + \Gamma_0^2/4]}, \quad (9)$$

where the parameters  $C_M$  and  $C_\Gamma$  are expressed in terms of  $\Delta$  mass and width splitting:

$$C_M = \delta w_4 + \frac{1}{3} \delta w_3 - \frac{1}{3} \delta m_2 - \delta m_1 \approx 4.6 \text{ MeV},$$

$$C_\Gamma = \delta\Gamma_4 + \frac{1}{3} \delta\Gamma_3 - \frac{1}{3} \delta\Gamma_2 - \delta\Gamma_1 \approx 1.7 \text{ MeV}.$$

These values are taken from the Masterson *et al.* paper [3] and are in agreement with the most recent data [24]. The leading correction in Eq. (9) comes from the factor  $C_M$  and later on when looking for CSV effects, we will take into account this factor only.

Notice that in the approximation considered above, the quantity  $A_\pi$ , according to Eq. (8), does not depend on scattering angle  $\theta$ . This is the consequence of the simplification we used. Namely, we took into account the impulse approximation with the  $\pi N$  scattering in the  $P_{33}$  wave. As was demonstrated in Ref. [3], the inclusion of other  $S$  and  $P$  waves does not change the picture dramatically but leads to a smooth dependence of  $A_\pi$  versus scattering angle  $\theta$ . (Note, the deviation from calculated constant value is much smaller than the experimental data.) Nevertheless, as was shown in Ref. [3], the inclusion of the CSV effect in the form (8) already raises the possibility of describing the observed CSV on the deuteron at 143 MeV for scattering angles  $\theta \leq 80^\circ$ .

(3) *Double-scattering approximation.* The  $\pi d$  differential cross section in the approximation (6) has a minimum at the scattering angle around  $90^\circ$ , where the non-spin-flip amplitude vanishes. For this reason, the contribution from the double-scattering term may be essential in this region of scattering angles. There are three diagrams for the double-scattering process which are depicted in Fig. 3. The sum of these amplitudes is proportional to the combination

$$\frac{1}{3} [f_{33}(k)]^2 + \frac{1}{3} [f_{33}(k)]^2 - \frac{2}{9} [f_{33}(k)]^2, \quad (10)$$

where the last term comes from the diagram with the virtual charge exchange [Fig. 3(c)]. To estimate the contribution of diagrams of Fig. 3, let us use the so-called fixed-centers approximation. This method for  $\pi d$  scattering was first used by Brueckner [25] (see also Ref. [18]). Its accuracy was later estimated by Kolybasov and Kudryavtsev [19,20].

The expression of the double-scattering diagrams without elementary  $\pi N$  spin-orbit forces in this fixed centers approximation has the form [20]

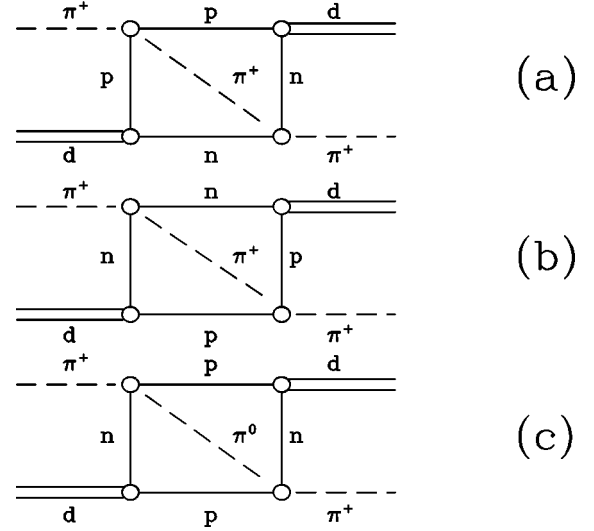


FIG. 3. Double-scattering amplitudes for  $\pi^+ d$ : elastic (a) and (b), and with virtual charge exchange (c).

$$\begin{aligned} f_{\pi d}^{(2)} &= \frac{4}{3} f_{33}(k) 2F_2(\theta, k) \\ &= \frac{4}{3} f_{33}(k) 2 \left( 1 - \frac{1}{3} \right) f_{33}(k) \hat{k}_i \cdot \hat{k}'_j \\ &\quad \times \int \Psi_D^2(r) e^{i(\vec{k} + \vec{k}')/2 \cdot \vec{r}} [h_1(r) \hat{r}_i \cdot \hat{r}_j + h_2(r) \delta_{ij}] d\vec{r}, \end{aligned} \quad (11)$$

where the functions  $h_1(r)$  and  $h_2(r)$  are

$$h_1(r) = \frac{e^{ikr}}{r} - \frac{3e^{ikr}}{k^2 r^3} + \frac{3}{k^2 r^3} + \frac{3ie^{ikr}}{kr^2}, \quad (12)$$

$$h_2(r) = \frac{e^{ikr}}{k^2 r^3} - \frac{1}{k^2 r^3} - \frac{ie^{ikr}}{kr^2}, \quad (13)$$

and the factor  $(1 - \frac{1}{3})$  in the right-hand side of Eq. (11) is specially introduced to clear up the relation between relative contributions of the elastic double-scattering term (it is proportional to 1) and the virtual charge-exchange diagram (it is  $\propto -\frac{1}{3}$ ).

This form of the functions  $h_1(r)$  and  $h_2(r)$  corresponds to a certain choice for the off-shell dependence for  $f_{\pi N}$  amplitudes. For more details, see Ref. [20]. In expression (11),  $\hat{k}$  and  $\hat{r}$  are the units vectors,  $\hat{k} = \vec{k}/k$ ,  $\hat{r} = \vec{r}/r$ , and  $\hat{k}_i$  is the  $i$ -component of this vector.

The sum of the single- and double-scattering diagrams in this approximation<sup>1</sup> is

<sup>1</sup>We omit temporarily the spin-flip amplitudes taking into account only the non-spin-flip amplitudes. The inclusion of spin-flip will be done later.

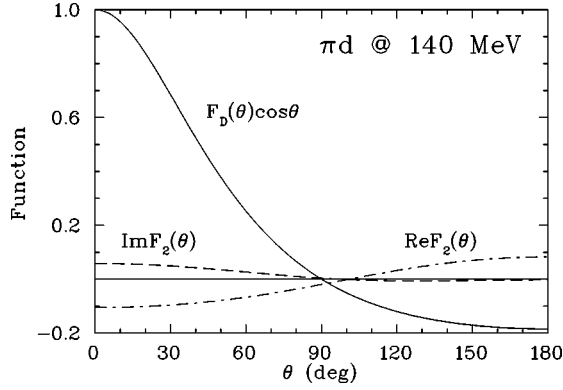


FIG. 4. Amplitudes for  $\pi d$  elastic scattering without spin-flip at 140 MeV. Solid curve gives  $F_D(\theta)\cos\theta$ . The real (imaginary) parts of amplitude  $F_2(\theta)$  is plotted with dash-dotted (dashed) lines.

$$f_{\pi d}^{(1+2)} = \frac{4}{3} f_{33}(k) 2 [F_D(\theta)\cos\theta + \text{Re} F_2(\theta) + i \text{Im} F_2(\theta)]. \quad (14)$$

The functions  $F_D(\theta)\cos\theta$ ,  $\text{Re} F_2(\theta)$ , and  $\text{Im} F_2(\theta)$  are shown in Fig. 4. We see from this figure that the amplitude of double-scattering is strongly suppressed at forward angles versus single scattering. But at larger than  $90^\circ$  angles, the contributions of single and double scattering are comparable. Clearly, the inclusion of the interference effects at this angular range will be essential.

(4) *Spin-flip amplitude.* Now, we take into account both the non-spin-flip and spin-flip parts of the elementary  $\pi N$  amplitude (3). As in our previous discussion, we will take into account the single- and double-scattering terms without any recoil effects (i.e., in the fixed-center approximation). The double-scattering term of the  $\pi d$ -scattering amplitude is

$$f_{\pi d}^{(2)} = \frac{8\pi(E_{\text{c.m.}}^{\pi N})^2}{mE_{\text{c.m.}}^{\pi d}} N f_{33}^2(k_{\text{c.m.}}) \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{d^3\vec{q}_1}{(2\pi)^3} \psi_d(\vec{q}) \times \psi_d\left(\vec{q}_1 - \frac{\vec{\Delta}}{2}\right) \frac{U}{s^2 - k^2 - i0}. \quad (15)$$

Here  $N$  is the isotopic factor, which has been already used in Eq. (10), for  $\pi^\pm d$ -scattering  $N = 4/9 = 1/3 + 1/3 - 2/9$ .

The denominator  $s^2 - k^2 - i0$  comes from the pion propagator, where  $\vec{s} = \vec{k}_1 + \vec{q} - \vec{q}_1$  is the virtual pion three-momenta in the lab system.  $U$  stands for the expression which includes the spin effects,

$$U = \text{Tr}\{O' S_2 O S_1\}, \quad O = \frac{1}{\sqrt{2}} \vec{\epsilon} \cdot \vec{\sigma},$$

$$S_2 = 2\hat{s} \cdot \hat{k}' + i\vec{\sigma} \cdot [\hat{s} \times \hat{k}'], \quad (16)$$

$$S_1 = 2\hat{k} \cdot \hat{s} + i\vec{\sigma} \cdot [\hat{s} \times \hat{k}].$$

Here  $O$  is spin operator in the S-wave part of the initial deuteron wave function, and  $O' = (1/\sqrt{2})\vec{\epsilon}' \cdot \vec{\sigma}$  is the same for

the final deuteron;  $S_{1,2}$  are spin parts of the  $\pi N$  amplitudes<sup>2</sup>;  $\hat{s} = \vec{s}/s$  is the unit vector. Let us represent  $U$  as

$$U = \hat{s}_i \cdot \hat{s}_j Q_{ij} \quad (17)$$

and define the integral

$$I_{ij} = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{d^3\vec{q}_1}{(2\pi)^3} \psi_d(\vec{q}) \psi_d\left(\vec{q}_1 - \frac{\vec{\Delta}}{2}\right) \frac{\hat{s}_i \cdot \hat{s}_j}{s^2 - k^2 - i0}. \quad (18)$$

The tensor  $O_{ij}$  in Eq. (17) can be obtained from Eqs. (16). The integral (18) may be rewritten in the form

$$I_{ij} = J_1 \hat{\kappa}_i \cdot \hat{\kappa}_j + J_2 \delta_{ij}, \quad \text{where } \hat{\kappa} = \vec{\kappa}/\kappa,$$

$$\vec{\kappa} = (\vec{k} + \vec{k}')/2. \quad (19)$$

Here the quantities  $J_1$  and  $J_2$  are complex functions, which depend on  $k$  and  $\theta$ . They depend on the deuteron WF as well, and are given in the Appendix.

Using Eqs. (17) and (18), we obtain for  $f_{\pi d}^{(2)}$  the expression of the type  $f_{\pi d}^{(2)} \sim I_{ij} Q_{ij}$ . Let us rewrite the amplitudes  $f_{\pi d}^{(1)}$  and  $f_{\pi d}^{(2)}$  in the form

$$f_{\pi d}^{(1)} = A_1 \epsilon_i \epsilon_j' T_{ij}^{(1)}, \quad f_{\pi d}^{(2)} = A_2 \epsilon_i \epsilon_j' T_{ij}^{(2)}, \quad (20)$$

where the tensor  $T_{ij}^{(1)}$  can be obtained from Eq. (5), and  $T_{ij}^{(2)}$ —from the relation  $I_{ij} Q_{ij} = \epsilon_i \epsilon_j' T_{ij}^{(2)}$ . Finally, we get

$$T_{ij}^{(1)} = 2z \delta_{ij} + \hat{k}_i' \hat{k}_j - \hat{k}_j' \hat{k}_i, \quad T_{ij}^{(2)} = a_{ij} J_1 + b_{ij} J_2,$$

and

$$a_{ij} = \frac{1}{2} (5 + 3z) \delta_{ij} - 2\hat{\kappa}_i \hat{\kappa}_j + 3\hat{k}_i' \hat{k}_j - \hat{k}_j' \hat{k}_i, \quad (21)$$

$$b_{ij} = 4z \delta_{ij} + 5\hat{k}_i' \hat{k}_j - 3\hat{k}_j' \hat{k}_i,$$

where  $z = (\hat{k} \cdot \hat{k}')$ . The values  $A_1$  and  $A_2$  in Eqs. (20) for the case of  $\pi^+ d$  scattering are

$$A_1 = \frac{2(m + \omega)}{2m + \omega} \left( f_1 + \frac{1}{3} f_2 \right) F_D(\theta), \quad (22)$$

$$A_2 = \frac{8\pi(m + \omega)^2}{m(2m + \omega)} \frac{2}{3} f_2 \left( f_1 - \frac{1}{3} f_2 \right) \left( f_i = \frac{1}{k_{\text{c.m.}}} \frac{\Gamma/2}{w_i - w - i\Gamma/2} \right)$$

[here we use more accurate values  $E_{\text{c.m.}}^{\pi N} = m + \omega$  and  $E_{\text{c.m.}}^{\pi d} = 2m + \omega$  than in the simplified version used in Eq. (5)]. In the case of the  $\pi^- d$  elastic scattering, one should substitute  $f_1 \rightarrow f_4$  and  $f_2 \rightarrow f_3$  in expressions (22). If  $\Delta$ -mass splitting is absent, then Eqs. (22) are reduced to

<sup>2</sup>The technique we used is discussed in more details in our recent paper [26].

$$A_1^{(0)} = \frac{2(m+\omega)}{2m+\omega} \frac{4}{3} f_0 F_D(\theta), \quad (23)$$

$$A_2^{(0)} = \frac{8\pi(m+\omega)^2}{m(2m+\omega)} \frac{4}{9} f_0^2 \left( f_0 = \frac{1}{k_{\text{c.m.}}} \frac{\Gamma_0/2}{w_0 - w - i\Gamma_0/2} \right).$$

After averaging over initial and summation over final polarization of deuteron, we can write the final result for the cross section  $\sigma(\theta) \equiv d\sigma/d\Omega$  as the sum of three terms:

$$\sigma(\theta) = \sigma_{11}(\theta) + \sigma_{12}(\theta) + \sigma_{22}(\theta), \quad (24)$$

where  $\sigma_{11}$  and  $\sigma_{22}$  are the contributions from the single and double scattering, respectively, and  $\sigma_{12}$  is the single-double interference term. The expressions for these cross sections are given below:

$$\sigma_{11}(\theta) = \frac{1}{3} |A_1|^2 T_{ij}^{(1)*} T_{ij}^{(1)} = \frac{2}{3} |A_1|^2 (1 + 5z^2), \quad (25)$$

$$\begin{aligned} \sigma_{12}(\theta) &= \frac{2}{3} \text{Re}[A_1^* A_2 T_{ij}^{(1)*} T_{ij}^{(2)}] \\ &= \frac{2}{3} \text{Re}[A_1^* A_2 [(4 + 11z + 9z^2)J_1 + (8 + 20z^2)J_2]], \end{aligned} \quad (26)$$

$$\begin{aligned} \sigma_{22}(\theta) &= \frac{1}{3} |A_2|^2 T_{ij}^{(2)*} T_{ij}^{(2)} \\ &= \frac{1}{3} |A_2|^2 \left[ \frac{1}{4} (75 + 90z + 27z^2) |J_1|^2 \right. \\ &\quad \left. + (16 + 25z + 15z^2) (J_1 J_2^* + J_1^* J_2) \right. \\ &\quad \left. + (34 + 34z^2) |J_2|^2 \right]. \end{aligned} \quad (27)$$

Taking into account that the leading CSV correction comes from the mass splitting and this splitting is small, it would be useful to represent the formula for the cross section in a linearized in  $\delta m_\Delta$  form. In this limit, the expression for asymmetry has the form

$$\begin{aligned} A_\pi &= -\frac{C_M}{2\sigma^{(0)}\Gamma} \left\{ 3(B_0 + B_0^*) \left[ \frac{1}{2} \sigma_{11}^{(0)}(\theta) + \sigma_{22}^{(0)}(\theta) \right] \right. \\ &\quad \left. + 2 \text{Re} \left[ A_1^* A_2 \left( B_0 + \frac{1}{2} B_0^* \right) [(4 + 11z + 9z^2)J_1 \right. \right. \right. \\ &\quad \left. \left. \left. + (8 + 20z^2)J_2 \right] \right] \right\}, \end{aligned} \quad (28)$$

and correspondingly ratio  $r = 1 + 2 A_\pi$ . Here  $B_0 = \Gamma_0/2/(w_0 - w - i\Gamma_0/2)$ ; the values  $\sigma^{(0)}$ ,  $\sigma_{11}^{(0)}$ , and  $\sigma_{22}^{(0)}$  are defined by Eqs. (24), (25), and (27), respectively, after substitutions  $A_1 \rightarrow A_1^{(0)}$  and  $A_2 \rightarrow A_2^{(0)}$  from Eqs. (23).

Hence all the CSV corrections depend on the same linear combination of masses, as in the single-scattering term, i.e., on the parameter  $C_M \approx 4.6$  MeV. Note that the inclusion of the double-scattering introduces no new parameters, i.e., the effect is still primarily dominated by  $C_M$ .

(5) *Coulomb interaction.* Now, we consider the fact that the charged pions interact with the deuteron by the Coulomb force. The elementary  $\pi N$  amplitude, which corresponds to

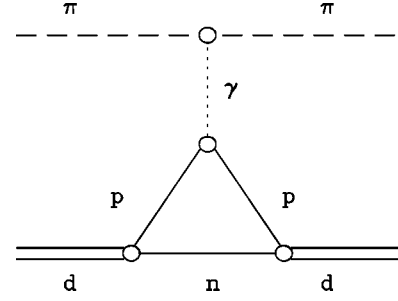


FIG. 5. Feynman diagrams for the Coulomb  $\pi p$  and  $\pi d$  amplitudes.

the interaction of a pion with a proton via  $\gamma$  exchange, is drawn in Fig. 5. In terms of bispinors, the expression for this diagram is

$$M_{\pi p}^{(\gamma)} = \frac{4\pi e^2}{t} \bar{u}_2(k_1 + k_2)_\mu \gamma^\mu u_1.$$

Neglecting the magnetic interaction and adding the Coulomb phase, we finally get for the Coulomb amplitude

$$\begin{aligned} f^\gamma &= \frac{M_{\pi p}^{(\gamma)}}{8\pi(m+\omega)} \\ &= -\frac{e^2}{2k_{\text{c.m.}}^2 \sin^2(\theta/2)} \frac{\omega m}{(m+\omega)} \\ &\quad \times \exp \left[ -\frac{2ie^2}{k_{\text{c.m.}}} \frac{\omega m}{(m+\omega)} \ln \left( \sin \frac{\theta}{2} \right) \right], \end{aligned} \quad (29)$$

where  $e^2 = \frac{1}{137}$ . Below we use the amplitude  $f^\gamma$  (29) convoluted with the proton density of deuteron as a crude approximation to the Coulomb pion-deuteron scattering amplitude  $f_{\pi d}^{(\gamma)}$ . We took into account the square of this amplitude as well as its interference with single- and double-scattering terms. Technically, it is more suitable to introduce in addition to the values  $A_1$  and  $A_2$  the new one  $A_C$ :

$$A_C = \frac{2(m+\omega)}{2m+\omega} \left( f_1 + f^\gamma + \frac{1}{3} f_2 \right) F_D(\theta). \quad (30)$$

In terms of these  $A_1$ ,  $A_2$ , and  $A_C$ , the cross sections  $\sigma_{11}$  and  $\sigma_{12}$  now have the form

$$\sigma_{11}(\theta) = \frac{2}{3} [6z^2 |A_C|^2 + (1 - z^2) |A_1|^2], \quad (31)$$

$$\begin{aligned} \sigma_{12}(\theta) &= \frac{2}{3} \text{Re} \{ A_C^* A_2 [(11z + 13z^2)J_1 + 28z^2 J_2] \\ &\quad + A_1^* A_2 [(4 - 4z^2)J_1 + (8 - 8z^2)J_2] \}, \end{aligned} \quad (32)$$

and the expression for  $\sigma_{22}$  is given by expression (27).

Note, that a fairly thorough study of the Coulomb effects on pion-deuteron scattering and CSV effects were performed in Ref. [27], see also Refs. [3,4]. As we are mainly interested in looking for CSV effects, which come from the double-scattering term and  $\Delta$ -isobars mass splitting, we limit

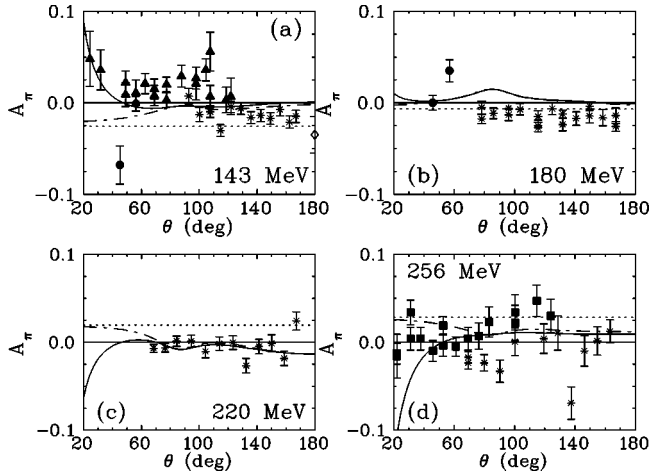


FIG. 6. Asymmetry for  $\pi d$  elastic scattering with the Coulomb interaction taken into account. (a) 143 MeV, (b) 180 MeV, (c) 220 MeV, and (d) 256 MeV. Experimental data are from Refs. [2–4], [6–9]. Notation is the same as in Fig. 1. Solid curves give the total amplitude. Single (and double) scattering without Coulomb corrections is shown by dashed (dash-dotted) curves.

ourselves to the Coulomb amplitude in crude approximations (29). Note also, that another source of CSV effects in the  $\pi d$  elastic scattering may come from the direct isospin breaking effect in the strong  $\pi N$  amplitudes, see in this connection Ref. [28]. We do not consider the influence of this possible interaction on the value of  $A_\pi$  in this paper.

The curves for asymmetry  $A_\pi$  with the Coulomb interaction taken into account are given in Figs. 6. If we consider the  $\pi^- d$  scattering instead of  $\pi^+ d$ , we should substitute in expressions (22) and (30):  $f_1 \rightarrow f_4$ ,  $f_2 \rightarrow f_3$ , and  $f^\gamma \rightarrow -f^\gamma$ . From Fig. 6, we see that single scattering does not depend on the scattering angle but a change of sign of the asymmetry does occur between 180 and 220 MeV according to the expression, given by Eq. (9).

#### IV. CONCLUSION AND FUTURE PROSPECTS

In this paper, we calculate the charge asymmetry,  $A_\pi$ , (due to  $\Delta$ -isobar mass splitting) in the elastic scattering of the  $\pi d$  system. We explicitly combine the contributions of single and double scattering to form the scattering amplitude. In each rescattering, the resonance interaction in the pion-nucleon  $P_{33}$  wave is taken into account. The analytic expression for double scattering was obtained in the framework of spin and isospin variables. In addition, the Coulomb interaction between the charged pion and the proton of the deuteron has been taken into account. The scattering amplitude is calculated within a fixed-centers approximation, i.e., the Fermi motion of the nucleons is not included. Likewise, the interference of nonresonant amplitudes in single and double scattering are not taken into account. This follows from the work of Masterson *et al.* [3], where it was shown that  $S$ -wave single scattering has only a small effect on  $A_\pi$  when compared to the effect of the  $P_{33}$  interaction. We also show the sensitivity of the angular distribution of  $A_\pi$  to double scattering relative to the single scattering approximation.

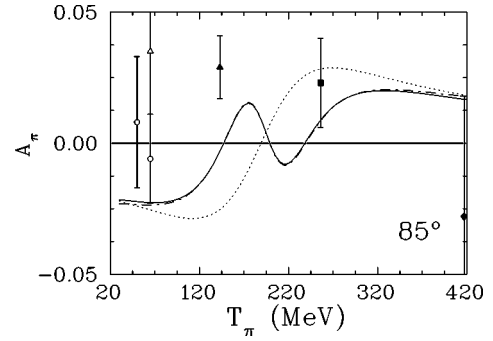


FIG. 7.  $85^\circ$  energy dependence of asymmetry  $A_\pi$  for  $\pi d$  elastic scattering with the Coulomb interaction taken into account. Notation is the same as is in Fig. 6.

Our calculations indicate some enhancement of  $A_\pi$  in the region of angles around  $90^\circ$ . For example, at  $T_\pi = 180$  MeV (in a range of maximum effect of the delta resonance) there is evidence for the growth of  $A_\pi$  from 0.002 at  $50^\circ$  to 0.015 at  $85^\circ$ . [We can expect some enhancement at  $85^\circ$  due to the behavior of  $F_D(\theta)\cos\theta$ ,  $\text{Re} F_2(\theta)$ , and  $\text{Im} F_2(\theta)$  shown in Fig. 4.] Clearly, the magnitude of  $A_\pi$  is not large. However, it is consistent with the magnitude of the experimental data and their uncertainties. Of some concern is the apparent discrepancy between the sign of the calculated  $A_\pi$  and that of the data. The source of this discrepancy is not yet known and continues to be investigated. Perhaps a more revealing picture (Fig. 7) is the one showing the energy behavior of  $A_\pi$  at  $85^\circ$ . Again, the calculated  $A_\pi$  indicates a change of sign, this time as a function of energy, but the data error bars are far too large to allow any conclusive statements. We conclude that to confirm these predictions for the asymmetry on the deuteron, one needs to have data that are approximately 2–3 times better in precision than currently available.

The results of our study indicate that the contribution of the double scattering as a source of charge asymmetry within the  $\Delta$  mass and width splitting has the same order of magnitude as the experimental data. This means that double scattering is an important element of CSV, and should not be ignored. Moreover, because the number of multiple scattering diagrams increases with increasing number of nucleons, it is our contention that pion multiple-scattering effects should be quite important in the study of CSV in the interaction of pions with nuclei heavier than the deuteron, in particular,  $^3\text{He}$ ,  $^3\text{H}$ , and  $^4\text{He}$ .

We note that there are other mechanisms that might play an important role in CSV for the deuteron and other light nuclei, but these are not discussed here. In the case of  $^3\text{He}/^3\text{H}$ , one clear example would be the difference in the  $^3\text{He}$  and  $^3\text{H}$  wave functions. The interested reader is directed to Ref. [23], where this case is discussed in detail. In fact, a complete classification of possible CSV sources is given in Ref. [29].

#### ACKNOWLEDGMENTS

The authors acknowledge useful communications with B. L. Berman, J. Friar, and S. Kamalov. One of us (A.K.) ac-

knowledges the hospitality extended by the Center for Nuclear Studies of The George Washington University. This work was supported in part by the U. S. Department of Energy Grants No. DE-FG02-99ER41110 and DE-FG02-95ER40901 with the Russian grant for Basic Research No. 98-02-17618. I.S. gratefully acknowledges a contract from Jefferson Lab under which this work was done. The Thomas Jefferson National Accelerator Facility (Jefferson Lab) is operated by the Southeastern Universities Research Association (SURA) under U.S. DOE Contract No. DE-AC05-85-84ER40150.

## APPENDIX

Here we give the expressions for the integrals  $J_1$  and  $J_2$ :

$$J_1 = \frac{1}{4} \int dr r^2 \psi^2(r) [(3E_2 - E_0)h_1(r)],$$

$$J_2 = \frac{1}{4} \int dr r^2 \psi^2(r) [(E_0 - E_2)h_1(r) + 2E_0h_2(r)].$$
(A1)

Here  $E_n = \int_{-1}^{+1} e^{i\kappa r z} z^n dz$ ,  $\kappa = k \cos(\theta/2) = k \sqrt{(1+z)/2}$  and functions  $h_1(r)$  and  $h_2(r)$  are given in the main text, see Eqs. (12) and (13).

Let us calculate the integral  $J_1$ . For this purpose, it is suitable to use the following representation for under integral function:

$$(3E_2 - E_0)h_1(r) = \sum_{m=1}^{16} a_m \frac{e^{ib_m r}}{r^{n_m}}.$$
(A2)

Here  $n_m = 2, 3, 4, 5, 6, 4, 5,$  and  $6$  for  $m = 1, 2, 3, 4, 5, 6, 7,$  and  $8$ , respectively;  $n_m = n_{m-8}$  for  $9 \leq m \leq 16$ , and

$$a_1 = -2ik^{-1}x^{-1},$$

$$a_2 = 6k^{-2}x^{-1}(1+x^{-1}),$$

$$a_3 = 6ik^{-3}x^{-1}(1+3x^{-1}+x^{-2}),$$

$$a_4 = -18k^{-4}x^{-2}(1+x^{-1}),$$

$$a_5 = -18ik^{-5}x^{-3},$$

$$a_6 = -6ik^{-3}x^{-1},$$

$$a_7 = 18k^{-4}x^{-2},$$

$$a_8 = 18ik^{-5}x^{-3},$$

$$b_1 = b_2 = b_3 = b_4 = b_5 = (1+x)k,$$

$$b_6 = b_7 = b_8 = xk,$$
(A3)

where  $x = \cos(\theta/2)$ . These equations (28) after the replacement  $x \rightarrow -x$  define the values  $a_m$  and  $b_m$  for  $9 \leq m \leq 16$  as  $a_m = a_{m-8}$  and  $b_m = b_{m-8}$ .

In calculations, we use a realistic deuteron wave function (in  $S$ -wave approximation) of the Bonn potential [30], parametrized as  $\psi(r) = \sum_i c_i (e^{-\alpha_i r}/r)$ , where  $\alpha_i > 0$ . With this form of  $\psi(r)$ , we get

$$J_1 = \frac{1}{4} \int \sum_{ijm} c_i c_j a_m e^{(ib_m - \alpha_i - \alpha_j)r} \frac{dr}{r^{n_m}}.$$
(A4)

To evaluate this integral, one may use a general relation

$$\int_0^\infty \sum_i c_i e^{a_i x} \frac{dx}{x^{n_i}} = \sum_i c_i \frac{a_i^{n_i-1}}{(n_i-1)!} [S_{n_i-1} - \ln a_i],$$
(A5)

where  $S_n = \sum_{k=1}^n 1/k$  and  $S_0 = 0$ . The formula (27) is derived for the case  $n_i \geq 1$  and is valid if this integral converges (i.e.,  $\text{Re } a_i < 0$  and the under integral function is finite at  $x \rightarrow 0$ ). These conditions are satisfied for the integral (26), and we finally get

$$J_1 = \frac{1}{4} \sum_{ijm} c_i c_j a_m \frac{(ib_m - \alpha_i - \alpha_j)^{n_m-1}}{(n_m-1)!} \times \left( S_{n_m-1} - \ln \sqrt{(\alpha_i + \alpha_j)^2 + b_m^2} + i \text{atan} \frac{b_m}{\alpha_i + \alpha_j} \right).$$
(A6)

To obtain the expression for  $J_2$ , one may use the analogous representation

$$(E_0 - E_2)h_1(r) + 2E_0h_2(r) = \sum_{m=1}^{14} a_m \frac{e^{ib_m r}}{r^{n_m}}.$$
(A7)

Here  $n_m = 3, 4, 5, 6, 4, 5,$  and  $6$  for  $m = 1, 2, 3, 4, 5, 6,$  and  $7$ , respectively;  $n_m = n_{m-7}$  for  $8 \leq m \leq 14$  and

$$a_1 = -2k^{-2}x^{-1}(1+x^{-1}),$$

$$a_2 = -2ik^{-3}x^{-1}(1+3x^{-1}+x^{-2}),$$

$$a_3 = 6k^{-4}x^{-2}(1+x^{-1}),$$

$$a_4 = 6ik^{-5}x^{-3},$$

$$a_5 = 2ik^{-3}x^{-1},$$

$$a_6 = -6k^{-4}x^{-2},$$

$$a_7 = -6ik^{-5}x^{-3},$$

$$b_1 = b_2 = b_3 = b_4 = (1+x)k,$$

$$b_5 = b_6 = b_7 = xk,$$
(A8)

where  $x = \cos(\theta/2)$ . These equations (A1) after the replacement  $x \rightarrow -x$  define the values  $a_m$  and  $b_m$  for  $8 \leq m \leq 14$  as  $a_m = a_{m-7}$  and  $b_m = b_{m-7}$ . Thus, for the integral  $J_2$  we get the similar equations (28) in which the values  $n_m$ ,  $a_m$ , and  $b_m$  are defined by Eqs. (29) and (30).



- [1] E. Pedroni, K. Gabathuler, J. J. Domingo, W. Hirt, P. Schwaller, J. Arvieux, C. H. Q. Ingram, P. Gretillat, J. Piffaretti, N. W. Tanner, and C. Wilkin, *Nucl. Phys.* **A300**, 321 (1978).
- [2] R. J. Holt, J. R. Specht, E. J. Stephenson, B. Zeidman, R. L. Burman, J. S. Frank, M. J. Leitch, J. D. Moses, M. A. Yates-Williams, R. M. Laszewski, and R. P. Redwine, *Phys. Rev. Lett.* **43**, 1229 (1979).
- [3] T. G. Masterson, J. J. Kraushaar, R. J. Peterson, R. S. Raymond, R. A. Ristinen, R. L. Boudrie, E. F. Gibson, and A. W. Thomas, *Phys. Rev. C* **26**, 2091 (1982).
- [4] T. G. Masterson, J. J. Kraushaar, R. J. Peterson, R. S. Raymond, R. A. Ristinen, J. L. Ullmann, R. L. Boudrie, D. R. Gill, E. F. Gibson, and A. W. Thomas, *Phys. Rev. C* **30**, 2010 (1984).
- [5] G. R. Smith, D. R. Gill, D. Ottewell, G. D. Wait, P. Walden, R. R. Johnson, R. Olszewski, R. Rui, M. E. Sevier, R. P. Trelle, J. Brack, J. J. Kraushaar, R. A. Ristinen, H. Chase, E. L. Mathie, V. Pafilis, R. B. Schubank, N. R. Stevenson, A. Reinat, and Y. Alexander, *Phys. Rev. C* **38**, 240 (1988).
- [6] M. D. Kohler, J. T. Brack, B. Clausen, J. J. Kraushaar, B. J. Kriss, R. A. Ristinen, K. Vaziri, G. R. Smith, D. F. Ottewell, M. E. Sevier, R. P. Trelle, and N. R. Stevenson, *Phys. Rev. C* **44**, 15 (1991).
- [7] M. D. Kohler, R. A. Ristinen, J. J. Kraushaar, B. J. Kriss, E. F. Gibson, G. R. Smith, D. F. Ottewell, J. T. Brack, M. Kermani, J. Jaki, and M. Metzler, *Phys. Rev. C* **48**, 1884 (1993).
- [8] A. B. Gridnev, M. B. Ishmuhametov, N. G. Kozlenko, S. P. Kruglov, A. Yu. Majorov, A. Yu. Marevsky, and V. V. Sumachev, *Phys. Part. Nuclei* **58**, 410 (1995).
- [9] B. M. K. Nefkens, W. J. Briscoe, A. D. Eichon, D. H. Fitzgerald, A. Mokhtari, and J. A. Wightman, *Phys. Rev. C* **41**, 2770 (1990).
- [10] C. Pillai, D. B. Barlow, B. L. Berman, W. J. Briscoe, A. Mokhtari, B. M. K. Nefkens, and M. E. Sadler, *Phys. Rev. C* **43**, 1838 (1991).
- [11] S. K. Mattheews, W. J. Briscoe, C. Bennhold, B. L. Berman, R. W. Caress, K. S. Dhuga, S. N. Dragic, S. S. Kamalov, N. J. Nicholas, M. F. Taragin, L. Tiator, S. J. Greene, D. B. Barlow, B. M. K. Nefkens, C. Pillai, J. W. Price, L. D. Isenhower, M. E. Sadler, I. Slaus, and I. Supek, *Phys. Rev. C* **51**, 2534 (1995).
- [12] K. S. Dhuga, B. L. Berman, W. J. Briscoe, R. W. Caress, S. K. Mattheews, D. B. Barlow, B. M. K. Nefkens, C. Pillai, J. W. Price, S. J. Greene, I. Slaus, and I. Supek, *Phys. Rev. C* **54**, 2823 (1996).
- [13] W. J. Briscoe *et al.* (in preparation).
- [14] B. Brinkmüller, C. L. Blilie, D. Denhard, M. K. Jones, G. M. Martinez, S. K. Nanda, S. M. Sterbenz, Y.-F. Yen, L. G. Atencio, S. J. Greene, C. L. Morris, S. J. Seestrom, G. R. Burleson, K. S. Dhuga, J. A. Faucett, R. W. Garnett, K. Maeda, C. F. Moore, S. Mordechai, A. Williams, S. H. Yoo, and L. C. Bland, *Phys. Rev. C* **44**, 2031 (1991).
- [15] K. S. Dhuga *et al.* (in preparation).
- [16] Kr. T. Kim, Y. E. Kim, and R. H. Landau, *Phys. Rev. C* **36**, 2155 (1987).
- [17] W. J. Briscoe and B. H. Silverman, *Phys. Rev. C* **39**, 282 (1989).
- [18] W. R. Gibbs, *Phys. Rev. C* **3**, 1127 (1971); W. R. Gibbs, A. T. Hess, and W. B. Kaufmann, *ibid.* **13**, 1982 (1976).
- [19] A. E. Kudryavtsev, *JETP* **34**, 490 (1971); V. M. Kolybasov and A. E. Kudryavtsev, *Nucl. Phys.* **B41**, 510 (1972).
- [20] V. M. Kolybasov and A. E. Kudryavtsev, *Phys. Part. Nuclei* **17**, 42 (1973); A. E. Kudryavtsev, in *Proceedings of the International Seminar, 1973, Moscow, ITEP, Atomizdat, Moscow, 1974* (unpublished), Vol. 3, p. 30.
- [21] T. Ericson and W. Weise, *Pions and Nuclei* (Clarendon, Oxford, 1988).
- [22] T. G. Masterson, E. F. Gibson, J. J. Kraushaar, R. J. Peterson, R. S. Raymond, R. A. Ristinen, and R. L. Boudrie, *Phys. Rev. Lett.* **47**, 220 (1981).
- [23] W. R. Gibbs and B. F. Gibson, *Phys. Rev. C* **43**, 1012 (1991).
- [24] Particle Data Group, C. Caso *et al.*, *Eur. Phys. J. C* **3**, 1 (1998).
- [25] K. A. Brueckner, *Phys. Rev.* **89**, 834 (1953); **90**, 717 (1953).
- [26] V. E. Tarasov, V. V. Baru, and A. E. Kudryavtsev, *Phys. At. Nucl.* (to be published).
- [27] J. Fröhlich, B. Saghai, C. Fayard, and G. H. Lamot, *Nucl. Phys.* **A435**, 738 (1985).
- [28] W. R. Gibbs, Li Ai, and W. B. Kaufmann, *Phys. Rev. Lett.* **74**, 3740 (1995).
- [29] W. B. Kaufmann and W. R. Gibbs, *Ann. Phys. (N.Y.)* **214**, 84 (1992).
- [30] R. Machleidt, K. Holinde, and Ch. Elster, *Phys. Rep.* **149**, 1 (1987).