Comment on ''Nucleon form factors and a nonpointlike diquark''

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Bloch *et al.* [Phys. Rev. C **60**, 062201 (1999)] presented a calculation of the electromagnetic form factors of the nucleon using an ansatz for the quark-diquark solution of the relativistic three-quark Faddeev equation. In this Comment it is pointed out that the calculation of these form factors stems from a three-quark bound state current that contains overcounted contributions. The corrected expression for the three-quark bound state current is derived.

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The proper way to include an external photon into a few-body system of strongly interacting particles described by integral equations has recently been discussed in detail [1,2]. In particular, it has been shown how to avoid the overcounting problems that tend to plague four-dimensional approaches [1]. The purpose of this Comment is to point out that just this type of overcounting is present in the work of Bloch *et al.* [3] who calculated the electromagnetic form factors of the nucleon using an ansatz for the quark-diquark solution of the four-dimensional Faddeev equation for three quarks.

In Ref. [2] we showed that the bound state electromagnetic current of three identical particles is given by

$$j^{\mu}=\frac{1}{6}\sum_{i=1}^{3}\bar{\Psi}\left(\Gamma_i^{\mu}D_{0i}^{-1}+\frac{1}{2}v_i^{\mu}d_i^{-1}-\frac{1}{2}v_i\Gamma_i^{\mu}\right)\Psi, \qquad (1)$$

where Ψ ($\bar{\Psi}$) is the wave function of the initial (final) three-body bound state, Γ_i^{μ} is the electromagnetic vertex function of the ith particle, d_i is the propagator of particle i, v_i is the two-body potential between particles j and k (ijk is a cyclic permutation of 123), v_i^{μ} is the five-point function resulting from the gauging of v_i, and $D_{0i}\equiv d_j d_k$ is the free propagator of particles j and k. Because Ψ is fully antisymmetric, the sum over i in Eq. (1) can be replaced by three times the $i=3$ contribution. In this way the second term on the right hand side (RHS) of Eq. (1) defines the two-body interaction current contribution

$$j^{\mu}_{\text{two-body}}=\frac{1}{4}\bar{\Psi}v_3^{\mu}d_3^{-1}\Psi, \qquad (2)$$

while the first and third terms together make up the one-body current contribution to the bound state current. As discussed in Ref. [1], the first term on the RHS of Eq. (1) defines an electromagnetic current

$$j^{\mu}_{\text{overcount}}=\frac{1}{2}\bar{\Psi}\Gamma_3^{\mu}D_{03}^{-1}\Psi, \qquad (3)$$

which overcounts the one-body current contributions, while the third term defines a current

$$j^{\mu}_{\text{subtract}}=\frac{1}{4}\bar{\Psi}v_3\Gamma_3^{\mu}\Psi, \qquad (4)$$

which plays the role of a subtraction term in that it removes the overcounted contributions. Here we shall not be concerned with the two-body interaction current, but rather, endeavor to examine the cancellations taking place between the first (''overcount'') and last (''subtract'') terms in detail. Thus we stress that the correct one-body contribution to the current, also known as the impulse approximation, is given by

$$j^{\mu}_{\text{impulse}}=j^{\mu}_{\text{overcount}}-j^{\mu}_{\text{subtract}}. \qquad (5)$$

To reveal these cancellations one writes the bound state wave function in terms of its Faddeev components $\Psi=\Psi_1+\Psi_2+\Psi_3$, where

$$\Psi_i=\frac{1}{2}D_{0i}v_i\Psi. \qquad (6)$$

These components obey the Faddeev equations

$$\Psi_i=D_{0i}t_i\Psi_j=D_{0i}t_i\Psi_k, \qquad (7)$$

where t_i is the t matrix for the j-k system, and for identical fermions possess the symmetry properties [2]

$$P_{12}\Psi_1=-\Psi_2,\quad P_{13}\Psi_1=-\Psi_3,\quad P_{23}\Psi_1=-\Psi_1,\ \text{etc.}, \qquad (8)$$

where P_{ij} is the operator interchanging particles i and j. Writing Eq. (3) in terms of these components and using Eqs. (8), $j^{\mu}_{\text{overcount}}$ becomes a sum of five terms

$$j^{\mu}_{\text{overcount}}=\frac{1}{2}\bar{\Psi}_3\Gamma_3^{\mu}D_{03}^{-1}\Psi_3+\bar{\Psi}_3\Gamma_1^{\mu}D_{01}^{-1}\Psi_3+\bar{\Psi}_2\Gamma_1^{\mu}D_{01}^{-1}\Psi_3$$
$$+\bar{\Psi}_2\Gamma_2^{\mu}D_{02}^{-1}\Psi_3+\bar{\Psi}_2\Gamma_3^{\mu}D_{03}^{-1}\Psi_3. \qquad (9)$$

The diquark model used in Ref. [3] is equivalent to invoking the separable approximation for the two-body t matrix $t_i=h_i\tau_i\bar{h}_i$, with τ_i playing the role of the diquark propagator and h_i describing the quark-quark-diquark vertex. For separable interactions, it is usual to define the spectator-quasiparticle (quark-diquark) amplitude X_i through the equation [1]

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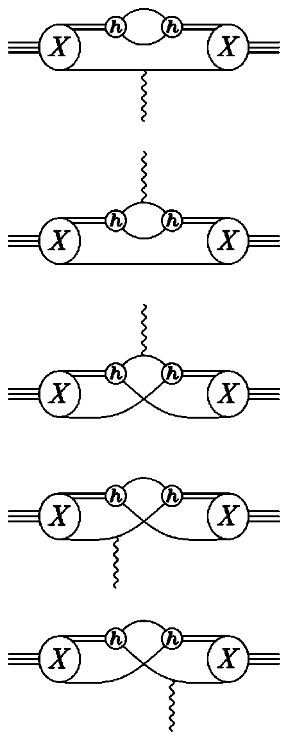

FIG. 1. Illustration of the five terms making up $j^{\mu}_{\rm overcount}$ in the case of separable interactions, Eq. (11). For a three-quark system, single, double, and triple lines correspond to a quark propagator d_i, a diquark propagator τ_i, and a three-quark bound state (the nucleon), respectively. The wiggly line indicates the single-quark electromagnetic current Γ_i^{μ}. The correct impulse approximation is obtained by removing the first and fourth of these diagrams (counting from the top).

$$\Psi_i = G_0 h_i \tau_i X_i, \tag{10}$$

where $G_0 = d_1 d_2 d_3$. In terms of these amplitudes the contribution of Eq. (9) becomes

$$\begin{aligned} j^{\mu}_{\rm overcount} = & \frac{1}{2}\bar{X}_3 d_3 \Gamma_3^{\mu} d_3 \tau_3 (\bar{h}_3 d_1 d_2 h_3)\tau_3 X_3 \\ & + \bar{X}_3 \tau_3 (\bar{h}_3 d_1 \Gamma_1^{\mu} d_1 d_2 h_3) d_3 \tau_3 X_3 \\ & + \bar{X}_2 \tau_2 d_2 \bar{h}_2 d_1 \Gamma_1^{\mu} d_1 h_3 d_3 \tau_3 X_3 \\ & + \bar{X}_2 \tau_2 d_2 \bar{h}_2 \Gamma_2^{\mu} d_2 d_1 h_3 d_3 \tau_3 X_3 \\ & + \bar{X}_2 \tau_2 d_2 \bar{h}_2 d_3 \Gamma_3^{\mu} d_1 h_3 d_3 \tau_3 X_3 . \end{aligned} \tag{11}$$

The five terms summed on the RHS of Eq. (11) are illustrated in Fig. 1. The last four terms are identical to the contributions $2\Lambda_{\mu}^{i}$ ($i=2,\ldots,5$) of Ref. [3], while the first term on the RHS of Eq. (11) differs from Λ_{μ}^{1} only in that our diquark propagator contains a dressing bubble. With or without this bubble, Eq. (11) does not give the correct impulse approximation.

With the help of Eq. (6), the subtraction term of Eq. (4) can be expressed as

$$j^{\mu}_{\rm subtract} = \frac{1}{2}\bar{\Psi}_3 D_{03}^{-1} \Gamma_3^{\mu} \Psi_3 + \bar{\Psi}_2 D_{02}^{-1} \Gamma_2^{\mu} \Psi_3 . \tag{12}$$

Comparison with Eq. (9) shows that the first and fourth terms of Eq. (9) are overcounted.[1] Thus the correct expression for the impulse approximation is

$$j^{\mu}_{\rm impulse} = \bar{\Psi}_3 \Gamma_1^{\mu} D_{01}^{-1} \Psi_3 + \bar{\Psi}_2 \Gamma_1^{\mu} D_{01}^{-1} \Psi_3 + \bar{\Psi}_2 \Gamma_3^{\mu} D_{03}^{-1} \Psi_3 . \tag{13}$$

For the work of Ref. [3], this means that the correct impulse approximation is given by the sum of their Λ_{μ}^{2}, Λ_{μ}^{3}, and Λ_{μ}^{5} only, and not, as claimed in their work, by the sum of all five Λ_{μ}^{i}'s. Diagrammatically this means that the correct impulse approximation to the nucleon current in the diquark model corresponds to the sum of the second, third, and fifth diagrams of Fig. 1.

A further comment regarding Ref. [3] concerns the numerical values obtained for the contributions Λ_{μ}^{1} and Λ_{μ}^{5}. For separable interactions Eq. (7) imply that the amplitudes X_i satisfy the equations $X_i = \bar{h}_i D_{0i} h_j \tau_j X_j$, where $i \neq j$. Using the time-reversed version of these equations one obtains $\bar{X}_3 = \bar{X}_2 \tau_2 \bar{h}_2 d_1 d_2 h_3$ which can be used to simplify the last term of Eq. (11):

$$\bar{X}_2 \tau_2 d_2 \bar{h}_2 d_3 \Gamma_3^{\mu} d_1 h_3 d_3 \tau_3 X_3 = \bar{X}_3 d_3 \Gamma_3^{\mu} d_3 \tau_3 X_3 . \tag{14}$$

The RHS of this equation is just $2\Lambda_{\mu}^{1}$ of Ref. [3] and we have therefore shown that $\Lambda_{\mu}^{1} = \Lambda_{\mu}^{5}$. This equality appears not to be reflected in the numerical results of Ref. [3] as is evident from their Table II. In turn this suggests that the ansatz used in Ref. [3] to parametrize the quark-diquark amplitude, their Eqs. (26) and (27), is inconsistent with a true solution of the relativistic Faddeev equation. Finally, we note that the errors of Ref. [3] have been perpetuated in a recent paper [4].

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[1] Actually the fourth and fifth terms of Eq. (9) are identical, as can easily be shown using Eqs. (7). Thus, it should be understood that overcounting is due to *either* the fourth or fifth terms.

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