Causality in an excluded volume model

Nirvikar Prasad,[1] K. K. Singh,[2] and C. P. Singh[1]
[1]*Department of Physics, Banaras Hindu University, Varanasi 221 005, India*
[2]*Department of Physics, Birla Institute of Techology, Pilani, India*
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We have earlier proposed a thermodynamically consistent equation of state (EOS) incorporating the Van der Waal's type of excluded volume effect for a hot and dense hadron gas consisting of many species of finite-size baryons. We demonstrate here that our excluded volume approach does not violate causality.

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Search of an equation of state (EOS) for a hot and dense hadron gas (HG) is still continuing because it describes the behavior of the matter under extreme conditions of temperature and density. However, until the advent of relativistic heavy-ion collisions, there is almost no direct experimental evidence on which to base this type of EOS, although these kinds of situations existed in the early Universe and/or in the core of a neutron star. A great deal of theoretical work had also been done in this direction. The precise nature of the quark-hadron phase transition will become clear if we know the correct EOS for both the phases separately [1–3]. Many phenomenological models have recently been proposed [4–15] for obtaining an EOS of a hot and dense HG. This, however, gives rise to widely varying conclusions and ensuing controversies. The results for the same physical quantities from different models are found to be much different.

If one assumes hadrons as pointlike and noninteracting particles in a dense and hot hadron gas, the phase transition to a quark-gluon plasma (QGP) will not materialize because at a large density or temperature, a large number of resonances will be produced. Consequently the pressure of HG will always be larger than that of the QGP. Therefore, one will not get a transition from a multicomponent HG to a QGP if one uses an EOS of an ideal HG. Moreover, it is expected that the hadronic interactions get significant when hadrons are closely packed in a dense and hot hadron gas. The important feature of the hadronic interactions at high densities is a short range repulsive force between a pair of baryons. This force has been considered in the literature either by giving a hard-core geometrical volume to each baryon [4–11] or by the mean-field approach formulated in the language of the thermodynamics of the extended objects [12–14]. One commonly used method for incorporating the hard-core repulsion in the EOS of the HG, is to consider the excluded volume effect [4–11] in the form of Van der Waal's correction in molecular physics. However, these models suffer from two main and severe deficiencies. First the EOS employed in these models is thermodynamically inconsistent because variables such as the baryon density n_B cannot be derived from a thermodynamical potential Ω (i.e., $n_B \neq \partial\Omega/\partial\mu$, where μ is the baryon chemical potential). Second and more crucial deficiency lies in the violation of causality in these models, i.e., sound velocity v_S is larger than the velocity of light c in the hot and dense HG medium. In other words, $v_S > 1$ in the unit of $c=1$ and the information travels at a speed larger than c in the medium. Several proposals appeared in the literature for correcting the first inconsistency [9–11,15]. But most of them still suffer from the second one. Recently we have presented a new model which incorporated the excluded-volume correction in a thermodynamically consistent way [15]. The purpose of this paper is to show that our model does not violate causality for a dense and hot hadron gas.

A thermodynamically consistent EOS for HG has been obtained [15] by directly evaluating the partition function of the grand canonical ensemble for a dense, hot HG consisting of baryons of kind i with finite-size volume V_i as

$$\ln Z_i^{\rm ex} = \frac{g_i}{6n^2 T}\int_{v_i}^{V-\Sigma N_j V_j} dV \int_0^\infty dk\, \frac{k^4}{(k^2+m_i^2)^{1/2}} \times[\lambda_i^{-1}\exp(E_i/T)+1]^{-1}, \tag{1}$$

where $\Sigma N_j V_j$ is the total excluded volume for j kinds of baryons, g_i is the spin-isospin degeneracy factor for the ith baryon. Using the Boltzmann approximation, we get

$$\ln Z_i^{\rm ex} = V(1-\Sigma n_j V_j) I_i \lambda_i \tag{2}$$

in the thermodynamic limit (i.e., $V\rightarrow$large). Here n_j is the baryon number density for jth baryon obtained after excluded volume correction, λ_i is the fugacity of ith hadron $\lambda_i = \exp(\mu_i/T)$, and I_i is the momentum space integral

$$I_i = \frac{g_i}{2n^2} m_i^2 T K_2(m_i/T), \tag{3}$$

where K_2 is the modified Bessel function of second kind. From Eq. (2), one gets

$$n_i = (1-\mathbf{R}) I_i \lambda_i - I_i \lambda_i^2 \frac{\partial \mathbf{R}}{\partial \lambda_i}, \tag{4}$$

where $\mathbf{R} = \Sigma n_j V_j$ is the ratio of the occupied volume to the total volume V.

We further assume

$$\frac{\partial \mathbf{R}}{\partial \lambda_i} = \frac{\partial \Sigma n_j V_j}{\partial \lambda_i} = \frac{\partial n_i}{\partial \lambda_i} V_i . \tag{5}$$

We thus make a simplifying assumption that the number density of the ith baryon n_i depends only on its fugacity λ_i. Thus it is not a function of the fugacities of other kinds of baryons in a multicomponent hadron gas. We have examined the consequences of this assumption in the earlier paper [15]. We, therefore, get finally

$$n_i = \frac{Q_i(1-\Sigma_{j\neq i} n_j V_j)}{\lambda_i V_i} e^{1/I_i V_i \lambda_i}, \tag{6}$$

where

$$Q_i = \int_0^{\lambda_i} \exp(-1/I_i V_i \lambda_i) d\lambda_i . \tag{7}$$

It is difficult to obtain n_i from Eq. (6) because it depends on all n_j. Defining X as the ratio of the total occupied volume by baryons and the available volume, we get

$$X = \frac{\Sigma_i n_i V_i}{1-\Sigma n_i V_i} = \sum_i \frac{Q_i}{\lambda_i \exp(-1/\lambda_i I_i V_i) - Q_i}. \tag{8}$$

Here R is related to X as follows:

$$R = \frac{X}{X+1}. \tag{9}$$

Thus we get the final solution from Eq. (6)

$$n_i = \frac{Q_i(1-R)}{\lambda_i V_i} \frac{\exp(1/I_i V_i \lambda_i)}{1-(Q_i/\lambda_i)\exp(1/I_i V_i \lambda_i)}. \tag{10}$$

It is easier to evaluate this equation numerically. Once we know baryon densities and R, we can easily calculate pressure as follows:

$$P^{\mathrm{ex}}(T,\mu) = (1-R)P^0(T,\mu) + P_M . \tag{11}$$

Here $P^0(T,\mu)$ is the total pressure arising from the summation of the partial pressures of all kinds of pointlike baryons and P_M is the pressure due to mesons. One can now obtain the energy density $P = TS_i + \Sigma \mu_i n_i - P^{\mathrm{ex}}$ and the entropy density by differentiating P^{ex} with respect to T. Similarly total number of particles are $n = \Sigma n_i + n_M$ where n_M is the total meson number density. It is difficult to get an analytic expression for velocity of sound v_S from the above. But one can obtain v_S from the slope of the curve between pressure and energy density at a constant S/n.

We have considered here a HG with seven baryonic components viz. N, Λ (1236), Λ, Σ, Ξ, Σ^* (1385), and Λ^* (1405) and their antiparticles. Similarly we have taken π, k, η, K^*, ρ, ω, ϕ mesons in the meson spectrum. In order to conserve strangeness, we have used the criterion of net zero strangeness in the hadron gas. We have further simplified our calculation by using the same volume for each baryon component as $V = (4/3)\pi r^3$ with a hard-core radius $r = 0.8$ fm.

It should be emphasized that we have used the classical (Boltzmann) limit in our model. We, therefore, cannot use our model for a large value of μ or baryon density. However, our aim in this paper is to show the difference between our calculation and the model of Rishcke *et al.* [9] for smaller values of μ.

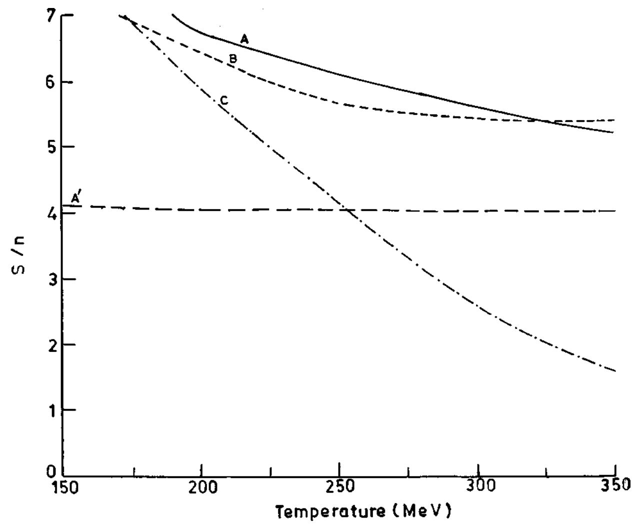

FIG. 1. Variations of entropy per particle vs temperature at $\mu = 100$ MeV. The solid line curve A shows the results in our model, the dashed-dotted curve C shows the results of Rischke model, and those in Kuono-Takagi model are shown by dashed curve B. The curve A′ represents the variations of S/n in our model for massless hadrons (at $\mu = 100$ MeV).

In Fig. 1, we have shown the variations of the specific entropy, i.e., total entropy per particle (S/n) with temperature T of the hadron gas at $\mu = 100$ MeV. S/n decreases for HG as temperature increases and entropy per particle at asymptotically large T takes a value slightly larger than 5.

For a hadron gas consisting of free, massless, point particles such as pions only, we get $S_\pi/n_\pi \simeq 4$. Since we do not assign a hard-core volume to mesons, the expressions for entropy density and number density for mesons do not change in the excluded volume approach. For a hadron gas of free and massive mesons treated as point particles, using relativistic Boltzmann distribution, one obtains

$$\frac{S}{n} = 4 + \frac{m}{T} \frac{K_1(m/T)}{K_2(m/T)}, \tag{12}$$

where K_1 and K_2 are modified Bessel functions of first and second kinds, respectively. Equation (12) clearly predicts the value for S/n larger than 4 and it decreases slowly as temperature increases. Equation (12) is modified for massive pointlike baryons of only one species and we get

$$\frac{S}{n} = 4 + \frac{m}{T} \frac{K_1(m/T)}{K_2(m/T)} - \frac{\mu}{T}. \tag{13}$$

We conclude that the entropy per particle for a hadron gas having a multicomponent baryons and mesons saturate at a larger value, i.e., $S/n \simeq 5$ in our excluded-volume approach. The value of S/n is almost independent of the temperature at large T and is only sensitive to the values of the parameters eg. masses of the baryons and mesons, etc.

In Fig. 1, we have also shown the ratio S/n calculated in other thermodynamically consistent approach, i.e., Rischke model [9]. For comparison, we have also demonstrated the ratio in the thermodynamically inconsistent approach, i.e., Kuono-Takagi model [8]. We notice that the result for S/n in

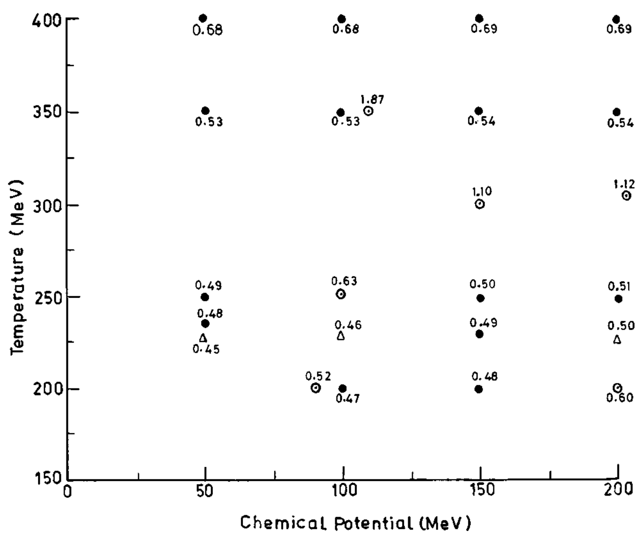

FIG. 2. Variations of the velocity of sound in the T and μ plane are shown in our model by closed circles, in the Kuono and Takagi model by open triangles, and in the Rischke model by open circles with a dot in the middle.

the Kuono-Takagi model shows a saturation at a slightly larger value $S/n \simeq 6$. However, the Rischke model shows a strikingly different type of behavior and S/n in this model decreases rapidly as temperature increases. This behavior is opposite to what we get in the other two models. Thus Fig. 1 demonstrates the differences in the predictions of various theoretical approaches being used to incorporate the excluded volume in the HG models at large T or μ. In Fig. 1 we have also shown the results of our model at $\mu = 100$ MeV for the HG with massless hadrons. We find that the curve for S/n converges to 4 at high temperatures. This limiting value for S/n at high temperatures and low μ indicates that the prediction of the excluded-volume approach is compatible with what we expect from Eqs. (12) and (13).

In Fig. 2, we have shown the variation of the velocity of sound in the T-μ plane. The velocity of sound is given by $v_S^2 = \partial P/\partial \epsilon$ at a constant S/n. We find that the maximum value of v_S one gets in our model is $v_S = 0.70$ when $T \simeq 400$ MeV and $\mu \simeq 200$ MeV. Thus our excluded volume model does not indicate any violation of causality. Moreover, we find that our values of v_S lie between $v_S = (1/3)^{1/2} (=0.58)$ which we get in an ideal gas of ultrarelativistic particles and $v_S = (2/3)^{1/2} (=0.8)$ which one expects for an ideal gas of nonrelativistic particles. The deviation mainly arises because the constituents of HG have a large interaction at large temperatures and chemical potentials. For the sake of comparison, we have also shown the values of v_S obtained in the Kuono-Takagi as well as Rischke models in the whole T, μ plane. Furthermore, we find that the value of $v_S^2 < 1$ in the case of the Kuono-Takagi model also. However, in the Rischke model, we find that v_S^2 is far larger than one at $T \geqslant 250$ MeV and hence violates causality. In conclusion, we have formulated a thermodynamically consistent EOS for a hot and dense HG after properly accounting for the geometrical volume of the baryons. We have demonstrated that our model does not violate causality as the sound velocity in the gas does not exceed one. Although we have not been able to take the case of a Fermi degenerate gas but we still find a large difference between various models existing in the literature at low values of μ. Thus a proper and realistic way of making excluded volume correction in the EOS of HG has been achieved in our model which does not suffer from either of the two main inconsistencies, i.e., thermodynamical inconsistency as well as the violation of causality.

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