# In-medium meson propagators in relativistic nonlinear models

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Using the generating functional method, the  $\sigma$  and  $\omega$  meson propagators in nuclear matter have been determined in relativistic nonlinear models. The scalar and vector collective modes as well as the density dependence of the  $\omega$  meson mass have been considered.

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### I. INTRODUCTION

Since the original Walecka model [1], relativistic mean field models have been quite successful in describing nuclear matter and ground state properties of finite nuclei [1–4]. In these models, the nucleons are generally treated as Dirac particles interacting via scalar and vector meson fields in the mean field approximation through Yukawa or Dirac couplings. In addition, the most recent models (the so-called nonlinear models) [5–8] include couplings between the mesonic fields in the Lagrangian.

First, the models NL1 [5] and NL-SH [6], which contain  $\sigma$  self-coupling terms, were applied to the description of finite nuclei with great success. The next step has been to introduce an  $\omega$  self-coupling term in the Lagrangian in order to reduce the resulting strong scalar and vector potentials, closer to those obtained in the relativistic Brueckner-Hartree-Fock theory with realistic nucleon-nucleon interactions [9]. The parameter sets obtained [7], TM1 (for medium and heavy nuclei) and TM2 (for light nuclei) lead to results which compare extremely well with the existing data for both stable and unstable nuclei. More recently, an effective field theory that maintains the symmetry of quantum chromodynamics (QCD), based on an expansion of the Lagrangian in powers of the fields and their derivatives, has been proposed for finite nuclei calculations [8]. Naive dimensional analysis and the naturalness assumption have been used to truncate the Lagrangian at some finite order. Keeping terms up to fourth order in the Lagrangian, two parameter sets G1 and G2 are obtained [8]. In the same work [8], another parameter set, Q2, retaining only the same terms as TM1 and TM2 and providing a description of nuclear matter and finite nuclei almost as good as those provided by G1 and G2 is also given.

All of these models lead to a rather good description of the nuclear properties. The question that now arises is how do the mesons propagate in nuclear matter in these models? In this work, we have determined the in-medium propagators of the  $\sigma$  and  $\omega$  mesons in the seven nonlinear models we have previously listed. These models are namely NL1 and NL-SH with only scalar meson self-couplings, TM1, TM2, and Q2 with scalar and vector meson self-couplings, and, G1 and G2 which allow all scalar-vector couplings up to fourth order.

We develop the formalism in Sec. II and present our results in Sec. III. Finally, the conclusions are drawn in Sec. IV.

#### **II. FORMALISM**

For all of the models considered, the part of the Lagrangian density which will contribute to the scalar and vector meson propagations in symmetric nuclear matter reads

$$\mathcal{L} = \overline{\psi} [\gamma_{\mu} (i\partial^{\mu} - g_{\omega 1}V^{\mu}) - (M_{N} - g_{\sigma 1}\phi)]\psi + \frac{1}{2} (\partial_{\mu}\phi\partial^{\mu}\phi - m_{\sigma}^{2}\phi^{2}) + \frac{1}{2}m_{\omega}^{2}V_{\mu}V^{\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{3}g_{\sigma 3}\phi^{3} + \frac{1}{3}g_{\sigma\omega 3}\phi V_{\mu}V^{\mu} - \frac{1}{4}g_{\sigma 4}\phi^{4} + \frac{1}{4}g_{\omega 4}(V_{\mu}V^{\mu})^{2} + \frac{1}{4}g_{\sigma\omega 4}\phi^{2}V_{\mu}V^{\mu}, \qquad (1)$$

where  $M_N$  is the nucleon mass,  $m_\sigma$  and  $m_\omega$  the scalar and vector meson masses, and as usual

$$F_{\mu\nu} = \partial_{\mu} V_{\nu} - \partial_{\nu} V_{\mu} \,. \tag{2}$$

Lagrange equations yield

$$(\partial_{\mu}\partial^{\mu} + m_{\sigma}^{2} + g_{\sigma3}\phi + g_{\sigma4}\phi^{2} - \frac{1}{2}g_{\sigma\omega4}V_{\mu}V^{\mu})\phi$$
$$= g_{\sigma1}\bar{\psi}\psi + \frac{1}{3}g_{\sigma\omega3}V_{\mu}V^{\mu}, \qquad (3)$$

$$\partial_{\mu}F^{\mu\nu} + (m_{\omega}^{2} + \frac{2}{3}g_{\sigma\omega3}\phi + g_{\omega4}V^{\mu}V_{\mu} + \frac{1}{2}g_{\sigma\omega4}\phi^{2})V^{\nu}$$
$$= g_{\omega1}\overline{\psi}\gamma^{\nu}\psi, \qquad (4)$$

$$[\gamma_{\mu}(i\partial^{\mu}-g_{\omega 1}V^{\mu})-(M_{N}-g_{\sigma 1}\phi)]\psi=0.$$
(5)

Following the standard procedure (see for example Ref. [1]), we first introduce the action *S* written as the integral of the Lagrangian density. This integral is then evaluated approximately, keeping only linear and quadratic variations about the classical fields. We denote  $\phi_0$  and  $V_0^{\mu}$  as the  $\sigma$  and  $\omega$  classical fields that satisfy the Lagrange equations [Eqs. (3) and (4)], and,  $\sigma$  and  $\omega^{\mu}$  as the quantum fluctuations about these fields. Note that the linear terms in  $\sigma$  and  $\omega^{\mu}$  in *S* have disappeared since  $\phi_0$  and  $V_0^{\mu}$  satisfy the Lagrange equations. We now introduce the generating functional for the Lagrangian density of Eq. (1)

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$$\widetilde{W}(\overline{\zeta},\zeta,J_{\mu},J) = \frac{W(\overline{\zeta},\zeta,J_{\mu},J)}{[W(\overline{\zeta},\zeta,J_{\mu},J)]_{\overline{\zeta}=\zeta=J_{\mu}=J=0}},$$
(6)

with

$$W(\overline{\zeta},\zeta,J_{\mu},J) = \int D(\overline{\psi})D(\psi)D(\omega^{\mu})D(\sigma) \\ \times \exp\left\{i\int d^{4}x[\mathcal{L}+\overline{\zeta}\psi+\overline{\psi}\zeta+J_{\mu}\omega^{\mu}+J\sigma]\right\}.$$
(7)

In this expression, the integrals run over all values of the fields  $\bar{\psi}(x)$ ,  $\psi(x)$ ,  $\omega^{\mu}(x)$ , and  $\sigma(x)$  at each point *x* in space time and  $\zeta(x)$ ,  $\bar{\zeta}(x)$ ,  $J^{\mu}(x)$ , and J(x) are the corresponding source functions. In order to limit ourselves to tractable expressions, we have expanded here the exponential in Eq. (7) only up to first order in  $g_{\sigma 3}$ ,  $g_{\sigma 4}$ ,  $g_{\omega 4}$ ,  $g_{\sigma \omega 3}$ ,  $g_{\sigma \omega 4}$  and second order in  $g_{\sigma 1}$ ,  $g_{\omega 1}$ .

By performing variational derivatives of  $\tilde{W}$  with respect to the source functions, we can generate all the Green functions of the theory. The  $\sigma$  and  $\omega$  meson propagators may then be calculated by the following functional derivatives:

$$iG_{\sigma}^{(2)}(x_1 - x_2) = \left\{ \left[ -i\frac{\delta}{\delta J(x_1)} \right] \left[ -i\frac{\delta}{\delta J(x_2)} \right] \times \widetilde{W}(\overline{\zeta}, \zeta, J_{\mu}, J) \right\}_{\overline{\zeta} = \zeta = J_{\mu} = J = 0}, \quad (8)$$
$$iG_{\sigma,\mu\nu}^{(2)}(x_1 - x_2) = \left\{ \left[ -i\frac{\delta}{\delta J(x_1)} \right] \left[ -i\frac{\delta}{\delta J(x_1)} \right] \right\}_{\overline{\zeta} = \zeta = J_{\mu} = J = 0}, \quad (8)$$

$$\sum_{\omega \ \mu\nu} (x_1 \ x_2) \left[ \left[ \sum_{\nu} \delta J^{\mu}(x_1) \right] \left[ \sum_{\nu} \delta J^{\nu}(x_2) \right] \right] \\ \times \widetilde{W}(\overline{\zeta}, \zeta, J_{\mu}, J) \left\{ \sum_{\overline{\zeta} = \zeta = J_{\mu} = J = 0}^{-1} \right].$$
(9)

After performing a Fourier transform, we obtain the inmedium  $\sigma$  and  $\omega$  meson propagators in momentum space

$$iG_{\sigma}^{(2)}(q) = iG_{\sigma}^{0}(q) - (ig_{\sigma 1})^{2}iG_{\sigma}^{0}(q) \\ \times \int \frac{d^{4}k}{(2\pi)^{4}} \operatorname{Tr}[iG_{N}^{0}(k)iG_{N}^{0}(k+q)] iG_{\sigma}^{0}(q) \\ + iG_{\sigma}^{0}(q) i\left[-2g_{\sigma 3}\phi_{0} - 3g_{\sigma 4}\phi_{0}^{2} + \frac{1}{2}g_{\sigma \omega 4}V_{0}^{\lambda}V_{0\lambda}\right] \\ \times iG_{\sigma}^{0}(q), \qquad (10)$$

$$iG^{(2)}_{\omega \mu\nu}(q) = iG^{0}_{\omega \mu\nu}(q) - (ig_{\omega 1})^{2} iG^{0}_{\omega \mu\alpha}(q)$$

$$\times \int \frac{d^{4}k}{(2\pi)^{4}} \operatorname{Tr}[i\gamma^{\alpha}G^{0}_{N}(k)i\gamma^{\beta}G^{0}_{N}(k+q)]$$

$$\times iG^{0}_{\omega \beta\nu}(q) + iG^{0}_{\omega \mu\alpha}(q)$$

$$\times i\left[g^{\alpha\beta}\left(\frac{2}{3}g_{\sigma\omega3}\phi_{0} + g_{\omega4}V^{\lambda}_{0}V_{0\lambda} + \frac{1}{2}g_{\sigma\omega4}\phi^{2}_{0}\right) + 2g_{\omega4}V^{\alpha}_{0}V^{\beta}_{0}\right]iG^{0}_{\omega \beta\nu}(q).$$
(11)

In Eqs. (10) and (11), we have neglected the contributions quadratic in  $g_{\sigma3}$ ,  $g_{\sigma4}$ ,  $g_{\omega4}$ ,  $g_{\sigma\omega3}$ , and  $g_{\sigma\omega4}$  since in realistic effective models in which the naturalness assumption is required, these contributions should be much smaller than the ones taken into account in this work. The second order  $\sigma\omega$ -meson propagator (which mixes the  $\sigma$  and  $\omega$  mesons) is obtained similarly:

$$iG_{\sigma\omega\mu}^{(2)}(q) = g_{\sigma 1}g_{\omega 1} iG_{\sigma}^{0}(q) \int \frac{d^{4}k}{(2\pi)^{4}} \operatorname{Tr}[iG_{N}^{0}(k)i\gamma^{\alpha} \\ \times G_{N}^{0}(k+q)]iG_{\omega\alpha\mu}^{0}(q) + iG_{\sigma}^{0}(q) \\ \times i\left[\left(\frac{2}{3}g_{\sigma\omega3} + g_{\sigma\omega4}\phi_{0}\right)V_{0}^{\alpha}\right]iG_{\omega\alpha\mu}^{0}(q).$$

$$(12)$$

In these expressions,  $G^0_{\sigma}(q)$ ,  $G^0_{\omega \mu\nu}(q)$ , and  $G^0_N(k)$  are, respectively, the  $\sigma$ ,  $\omega$ , and nucleon propagators in free space.

Following the procedure already developed in Ref. [10], the polarization operators in the  $\sigma$ ,  $\omega$ , and  $\sigma\omega$  channels become

$$\Pi_{\sigma\sigma}(q) = \Pi^{W}_{\sigma\sigma}(q) + \Pi^{\rm nl}_{\sigma\sigma}, \qquad (13)$$

$$\Pi^{\mu\nu}_{\omega\omega}(q) = \Pi^{W\,\mu\nu}_{\omega\omega}(q) + \Pi^{\mathrm{nl}\,\mu\nu}_{\omega\omega}, \qquad (14)$$

$$\Pi^{\mu}_{\sigma\omega}(q) = \Pi^{W\,\mu}_{\sigma\omega}(q) + \Pi^{\mathrm{nl}\,\mu}_{\sigma\omega}, \qquad (15)$$

where

$$\Pi_{\sigma\sigma}^{\rm nl} = 2g_{\sigma3}\phi_0 + 3g_{\sigma4}\phi_0^2 - \frac{1}{2}g_{\sigma\omega4}V_0^{\lambda}V_{0\lambda}, \qquad (16)$$

$$\Pi^{\mathrm{nl}\;\mu\nu}_{\omega\omega} = -g^{\mu\nu} (\frac{2}{3}g_{\sigma\omega3}\phi_0 + g_{\omega4}V_0^{\lambda}V_{0\lambda} + \frac{1}{2}g_{\sigma\omega4}\phi_0^2) - 2g_{\omega4}V_0^{\mu}V_0^{\nu}, \qquad (17)$$

$$\Pi_{\sigma\omega}^{nl\ \mu} = -\left(\frac{2}{3}g_{\sigma\omega3} + g_{\sigma\omega4}\phi_0\right)V_0^{\mu},\tag{18}$$

are the polarizations yielded by nonlinear meson coupling terms. The polarizations  $\Pi_{\sigma\sigma}^{W}(q)$ ,  $\Pi_{\omega\omega}^{W\,\mu\nu}(q)$ , and  $\Pi_{\sigma\omega}^{W\,\mu}(q)$ , obtained from the original Walecka model (particle-hole excitations) are calculated with an interacting nucleon propagator taking a form analogous to the noninteracting one but with  $M_N^* = M_N + \Sigma^{s*}$ , where  $\Sigma^{s*}$  is the scalar component of the nucleon self-energy. These contributions can be found in Ref. [11]. Note that we have omitted the contribution arising from antinucleons.

In symmetric nuclear matter, for a static and uniform system in mean-field theory, in the rest frame of nuclear matter, the classical fields  $\phi_0$  and  $V_{0\mu}$  can be obtained from the Lagrange equations, Eqs. (3) and (4)

$$\phi_0 = \frac{g_{\sigma 1} \rho_s + \frac{1}{3} g_{\sigma \omega 3} V_0^2}{m_{\sigma}^2 + g_{\sigma 3} \phi_0 + g_{\sigma 4} \phi_0^2 - \frac{1}{2} g_{\sigma \omega 4} V_0^2},$$
 (19)

$$V_{0\mu} = \delta_{\mu 0} V_0 = \delta_{\mu 0} \frac{g_{\omega 1} \rho_B}{m_{\omega}^2 + \frac{2}{3} g_{\sigma \omega 3} \phi_0 + g_{\omega 4} V_0^2 + \frac{1}{2} g_{\sigma \omega 4} \phi_0^2},$$
(20)

where the scalar and baryonic nuclear densities,  $\rho_s$  and  $\rho_B$ , are defined as usual as

$$\rho_s = \langle \bar{\psi}\psi \rangle, \tag{21}$$

$$\rho_B = \langle \psi^{\dagger} \psi \rangle. \tag{22}$$

The equation for the nucleon field then becomes

$$[i\gamma_{\mu}\partial^{\mu} - g_{\omega 1}\gamma^{0}V_{0} - M_{N}^{*}]\psi = 0, \qquad (23)$$

with

$$M_N^* = M_N - g_{\sigma 1} \phi_0. \tag{24}$$

The  $\omega$  meson propagator as well as the polarization  $\prod_{\omega\omega}^{\mu\nu}$ are symmetric second rank tensors and can be expanded in terms of independent symmetric tensors (see Appendix B of Ref. [12]). As usual, we have dropped the terms proportional to  $q^{\mu}q^{\nu}$  since they vanish when coupled to a conserved current. Thus, the polarization in the  $\omega$  and  $\sigma\omega$  channel can be written as

$$\Pi^{\mu\nu}_{\omega\omega} = -\hat{g}^{\mu\nu}\Pi^{q}_{\omega} + \frac{\hat{\eta}^{\mu}\hat{\eta}^{\nu}}{\hat{\eta}^{2}}\Pi^{\eta}_{\omega} + \frac{\hat{\eta}^{\mu}q^{\nu} + q^{\mu}\hat{\eta}^{\nu}}{\sqrt{2}(q^{2}\hat{\eta}^{2})^{1/2}}\Pi^{q\eta}_{\omega}, \quad (25)$$

$$\Pi^{\mu}_{\sigma\omega} = \frac{\hat{\eta}^{\mu}}{\hat{\eta}^2} \Pi^{\eta}_{\sigma\omega} + \frac{q^{\mu}}{q^2} \Pi^{q}_{\sigma\omega}, \qquad (26)$$

where

$$\Pi_{\omega}^{q} = \Pi_{\omega}^{qW} + \frac{2}{3}g_{\sigma\omega3}\phi_{0} + g_{\omega4}V_{0}^{2} + \frac{1}{2}g_{\sigma\omega4}\phi_{0}^{2}, \qquad (27)$$

$$\Pi_{\omega}^{\eta} = \Pi_{\omega}^{\eta W} - 2\,\hat{\eta}^2 g_{\omega 4} V_0^2, \qquad (28)$$

$$\Pi_{\omega}^{q\,\eta} = -2\,\sqrt{2}g_{\,\omega4}V_0^2(q\,.\,\eta) \left(\frac{\hat{\eta}^2}{q^2}\right)^{1/2},\tag{29}$$

$$\Pi^{\eta}_{\sigma\omega} = \Pi^{W}_{\sigma\omega} - \hat{\eta}^2 (\frac{2}{3}g_{\sigma\omega3} + g_{\sigma\omega4}\phi_0) V_0, \qquad (30)$$

$$\Pi^{q}_{\sigma\omega} = -(q,\eta)(\frac{2}{3}g_{\sigma\omega3} + g_{\sigma\omega4}\phi_0)V_0, \qquad (31)$$

with

$$\hat{g}^{\mu\nu} = g^{\mu\nu} - \frac{q^{\mu}q^{\nu}}{q^2}, \qquad (32)$$

$$\hat{\eta}^{\mu} = \eta^{\mu} - \frac{(q \cdot \eta)q^{\mu}}{q^2}, \qquad (33)$$



FIG. 1. The  $\sigma$  and  $\omega$  collective modes at nuclear matter saturation density obtained with models NL1 (upper part) and NL-SH (lower part). For each model, the two upper solid curves are the  $\sigma$ and longitudinal  $\omega$  mesonic branch modes and the lower solid curve is the zero-sound mode. The dashed curve represents the transverse  $\omega$  mesonic branch.

and

$$q^2 = \nu^2 - \mathbf{q}^2. \tag{34}$$

Here  $\eta^{\mu}$  describes the uniform motion of the medium, and is such that, in the nuclear matter rest frame we have  $\eta^{\mu} = (1,0,0,0)$ . Then, as in Ref. [10], Dyson equations taking into account the effects of summing polarization insertions up to all orders are built for the  $\sigma$  and  $\omega$  meson propagators (including the  $\sigma$ - $\omega$  mixing term)

$$\widetilde{G}_{\sigma}(q) = \overline{G}_{\sigma}(q) + \overline{G}_{\sigma}(q) \Pi^{\mu}_{\sigma\omega}(q) \overline{G}_{\omega \ \mu\nu}(q) \Pi^{\nu}_{\sigma\omega} \widetilde{G}_{\sigma}(q),$$
(35)

$$\widetilde{G}_{\omega \ \mu\nu}(q) = \overline{G}_{\omega \ \mu\nu}(q) + \overline{G}_{\omega \ \mu\alpha}(q) \Pi^{\alpha}_{\sigma\omega}(q) \times \overline{G}_{\sigma}(q) \Pi^{\beta}_{\sigma\omega}(q) \widetilde{G}_{\omega \ \beta\nu}(q), \qquad (36)$$

where

$$\bar{G}_{\sigma}(q) = G^{0}_{\sigma}(q) + G^{0}_{\sigma}(q)\Pi_{\sigma\sigma}(q)\bar{G}_{\sigma}(q), \qquad (37)$$

and

$$\bar{G}_{\omega\ \mu\nu}(q) = G^0_{\omega\ \mu\nu}(q) + G^0_{\omega\ \mu\alpha}(q)\Pi^{\alpha\beta}_{\omega\omega}(q)\bar{G}_{\omega\ \beta\nu}(q)$$
(38)

are the meson propagators when the  $\sigma$ - $\omega$  mixing is not taken into account. These equations, which can be solved formally, yield the following expressions for the  $\sigma$  and  $\omega$  meson propagators:

$$\tilde{G}_{\sigma} = \frac{q^2 - m_{\omega}^2 - \Pi_{\omega}^q * + \Pi_{\omega}^{\eta} *}{(q^2 - m_{\sigma}^2 - \Pi_{\sigma\sigma}^*)(q^2 - m_{\omega}^2 - \Pi_{\omega}^q * + \Pi_{\omega}^{\eta} *) + \Pi_{\sigma\omega}^{*2}},$$
(39)

$$\begin{split} \tilde{G}^{\mu\nu}_{\omega} &= \left( -\hat{g}^{\mu\nu} + \frac{\hat{\eta}^{\mu} \hat{\eta}^{\nu}}{\hat{\eta}^{2}} \right) \frac{1}{q^{2} - m_{\omega}^{2} - \Pi_{\omega}^{q}} - \frac{\hat{\eta}^{\mu} \hat{\eta}^{\nu}}{\hat{\eta}^{2}} \\ &\times \frac{q^{2} - m_{\sigma}^{2} - \Pi_{\sigma\sigma}^{q}}{(q^{2} - m_{\sigma}^{2} - \Pi_{\sigma\sigma}^{*})(q^{2} - m_{\omega}^{2} - \Pi_{\omega}^{q} * + \Pi_{\omega}^{\eta} *) + \Pi_{\sigma\omega}^{*2}}, \end{split}$$

$$(40)$$

provided that

$$\Pi_{\sigma\sigma}^{*} = \Pi_{\sigma\sigma}^{W} + \Pi_{\sigma\sigma}^{nl} + \frac{\Pi_{\sigma\omega}^{q^{2}}}{a^{2}m_{\omega}^{2}}, \qquad (41)$$

$$\Pi^q_{\omega} * = \Pi^q_{\omega}, \qquad (42)$$

$$\Pi_{\omega}^{\eta} * = \Pi_{\omega}^{\eta} + \frac{\Pi_{\omega}^{q \eta 2}}{2m_{\omega}^{2}}, \qquad (43)$$

$$\Pi_{\sigma\omega}^{*2} = \frac{\Pi_{\sigma\omega}^{\eta 2}}{\hat{\eta}^{2}} + \frac{\Pi_{\sigma\omega}^{q}\Pi_{\omega}^{q\eta}}{2m_{\omega}^{2}} \left( \frac{2\sqrt{2}\Pi_{\sigma\omega}^{\eta}}{(q^{2}\hat{\eta}^{2})^{1/2}} + \frac{\Pi_{\sigma\omega}^{q}\Pi_{\omega}^{q\eta}}{q^{2}m_{\omega}^{2}} \right).$$
(44)

A similar procedure for the  $\sigma$ - $\omega$  propagator (see Ref. [12]) leads to

$$\tilde{G}^{\mu}_{\sigma\omega} = -\frac{\hat{\eta}^{\mu}}{(\hat{\eta}^2)^{1/2}} \frac{\Pi^*_{\sigma\omega}}{(q^2 - m^2_{\sigma} - \Pi^*_{\sigma\sigma})(q^2 - m^2_{\omega} - \Pi^q_{\omega} * + \Pi^\eta_{\omega} *) + \Pi^{*2}_{\sigma\omega}},\tag{45}$$

In Eq. (40), the  $\omega$  propagator has been split into its transverse (first term) and longitudinal (second term) parts and we have dropped all terms proportional to  $q^{\mu}q^{\nu}$  since they vanish when coupled to a conserved current. Obviously, when the polarization vanishes, the  $\sigma$  and  $\omega$  propagators in free space are recovered.

#### **III. RESULTS**

The preceding formalism has been used for the determination of the scalar and vector collective modes and of the density dependence of the  $\omega$  meson mass in symmetric nuclear matter. These results are displayed for the seven nonlinear models: NL1, NL-SH, TM1, TM2, Q2, G1, and G2. The values of the coupling constants can be extracted from Refs. [5–8].

In Figs. 1, 2, and 3, we show the  $\sigma$  and  $\omega$  collective modes at nuclear matter saturation density. For each model used, the two upper solid curves (in the timelike region) are the  $\sigma$  and longitudinal  $\omega$  mesonic branch modes and the lower solid curve (in the spacelike region) is the zero-sound mode. The dashed curve represents the transverse  $\omega$  mesonic branch. We can remark that the shape of the mesonic branches is only weakly dependent on the model used. On the other hand, the zero-sound mode depends strongly on the model since, for example, we can see that no zero sound exists at saturation density for the model G1.

We have also calculated the  $\omega$  meson effective mass defined by the position of the mesonic branch at zero momentum, as a function of density (at zero momentum one cannot distinguish between transverse and longitudinal modes). Note that in the limit  $\mathbf{q}^2 \rightarrow 0$  the particle-hole excitations contribute nothing to the meson effective masses. Thus, when

nonlinear  $\omega$  meson self-couplings are not taken into account in the meson propagators (as for the NL1 and NL-SH models), the  $\omega$  mass remains unchanged (equal to the free  $\omega$ mass). The curves in Fig. 4 represent the  $\omega$  meson effective mass calculated with the models NL1, NL-SH, TM1, TM2, Q2, G1, and G2. As we can see, the behavior of the  $\omega$  meson effective mass when density increases, drastically depends on the model. For TM1, TM2, G1, and G2 an increase is observed, while for Q2 the mass decreases. These differences



FIG. 2. Same as Fig. 1 but for TM1 (upper part), TM2 (middle), and Q2 (lower part).



FIG. 3. Same as Fig. 1 but for G1 (upper part) and G2 (lower part).

are easily understood considering that, at zero momentum, both the particle-hole contributions and the terms of the polarization containing  $\hat{\eta}^2$  vanish and, consequently, the only modification of the mass comes from the last three terms in Eq. (27). Hence, since the mean-field values,  $\phi_0$  and  $V_0$ , do not vary very much at zero momentum from one model to another, the density dependence of the  $\omega$  effective mass is entirely determined by the values of the three coupling constants:  $g_{\sigma\omega3}$ ,  $g_{\omega4}$ , and  $g_{\sigma\omega4}$ . For the TM1, TM2, and Q2 models where  $g_{\sigma\omega3}$  and  $g_{\sigma\omega4}$  are equal to zero, the mass increases when  $g_{\omega 4}$  is positive (as in TM1 and TM2), and decreases when  $g_{\omega 4}$  is negative (as in Q2). In the case of G1 and G2, these three coupling constants are, respectively, (680.6 MeV, 86.41, -64.95) and (6663 MeV, 71.71, 8.385). For the G2 model, the three coupling constants are positive and, thus, the mass increases strongly, while for the G1 model, a partial cancellation of the last two terms of Eq. (27)  $(g_{\omega 4} \text{ and } g_{\sigma \omega 4} \text{ are of the same order of magnitude and of}$ opposite sign) and a rather weak positive value for  $g_{\sigma\omega3}$  lead to a weak increase of the mass. Let us mention that an increase of the  $\omega$  mass contradicts what is obtained in many other models. Indeed, many authors using either the Nambu-Jona-Lasinio model [13], or QCD sum rules [14,15], or dilatational symmetry of the chiral Lagrangian [16] obtain a





decrease of the  $\omega$  meson mass when the density increases.

## **IV. CONCLUSION**

In this work, using the generating functional method, we have determined the in-medium propagators of the  $\sigma$  and  $\omega$ mesons in the seven relativistic mean-field nonlinear models NL1, NL-SH, TM1, TM2, Q2, G1, and G2. This study was motivated by the fact that, although the nonlinear models are known to provide a rather good description of nuclear properties, nothing was known on how the mesons propagate in nuclear matter with these models. As an application of this formalism, we have considered the scalar and vector collective modes and the density dependence of the  $\omega$  meson mass. The shape of the mesonic branches found are only weakly dependent on the model used, in contrast with the zero-sound mode which depends strongly on the model. Concerning the  $\omega$  meson effective mass, the behavior when density increases is strongly model dependent. For NL1 and NL-SH the mass is constant, for TM1, TM2, and G1 a weak increase is obtained, for G2 the increase is stronger, while for Q2 the mass decreases weakly when density increases as expected from "QCD inspired" models. Considering these discrepancies and since, as it is well known (see for example the discussion in Ref. [8]) that the determination of so many coupling constants by a minimization procedure faces serious difficulties, one can reasonably question the reliability of the values obtained for high order coupling constants such as  $g_{\sigma\omega3}$ ,  $g_{\omega4}$ , and  $g_{\sigma\omega4}$ , and thus for the  $\omega$  meson effective mass in these models.

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