Finite formation time effects in quasielastic (e,e') scattering on nuclear targets

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(Received 28 April 2000; published 11 August 2000)

The problem of the final state interaction, in quasielastic (e,e') scattering at large Q^2, is investigated by exploiting the idea that the ejected nucleon needs a finite amount of time to assume its asymptotic form. It is shown that when the dependence of the scattering amplitude of the ejected nucleon on its virtuality is taken into account, the final state interaction is decreased. The developed approach is simpler to implement than the one based on the color transparency description of the damping of the final state interaction, and is essentially equivalent to the latter in the case of the single rescattering term. The (e,e') process on the deuteron is numerically investigated and it is shown that, at $x=1$, appreciable finite formation time effects at Q^2 of the order of 10 $(\mathrm{GeV}/c)^2$ are expected.

PACS number(s): 25.30.−c, 13.60.−r, 24.10.−i

I. INTRODUCTION

Quasielastic (QE) (e,e') scattering on nuclei is considered to be a suitable process to look for color transparency (CT) effects in QCD [1,2]. The original idea was that at high $Q^2=\mathbf{q}^2-\nu^2$ ($\mathbf{q}$ and ν being the three-momentum and energy transfers, respectively) the state which emerges after the interaction (''the ejectile state'') is dominated by configurations of small size $\rho\sim\sqrt{1/Q^2}$. Since the color charge is supposed to be neutralized at small distances, the final state interaction (FSI) of the ejectile should vanish at high Q^2, providing, by this way, a clear signature of the underlying production mechanism. Unfortunately, a more detailed analysis reveals that the situation is not that simple, for it can be trivially shown that the transverse dimension of the ejectile is exactly equal to the one of the initial struck nucleon; this fact however is not, in principle, in contradiction with the vanishing of FSI at large Q^2 [3,4].

The latter effect, if operative, is a consequence of the cancellation between the various contributions of the different intermediate states produced after the initial interaction, in particular, of those with a large mass. In order to have a detailed theoretical description of CT, one should then be able to describe the propagation through the nucleus of all possible states of the ejectile, including the ones with very high masses, with the vanishing of FSI resulting from the destructive interference of all these different contributions. The practical implementation of such a program looks therefore to be a rather difficult task. Summing over all excited ejectile states seems to be technically feasible only in $3q$ or quark-diquark oscillator models, for which CT does not occur. The authors using these models are in fact forced to impose artificially CT by introducing a transverse form factor $\sim\exp(-\rho^2Q^2)$ [5,6]. Moreover, the high-mass states of the ejectile cannot certainly be described in terms of such models and require the inclusion of gluons and sea quarks.

In this paper we discuss a different approach to the problem. We want in fact to take into account the finite formation time (FFT) of the finally observed proton, which is an alternative and possibly more convenient way to represent the vanishing of FSI in QE (e,e') process at high Q^2. Let us first of all recall some relevant features of the theoretical description of high-energy hadron-nucleus interaction. It has long been known that after a particle has undergone an interaction, it should elapse a certain amount of time before it becomes capable of a new one. Such a phenomenon is formally due to the vanishing of the contribution of all planar diagrams at high energies. By studying the planar diagrams, one can ascribe the mechanism responsible for such an effect to the cancellation of the contributions to the absorptive part coming from different intermediate states. From a more formal point of view, it results from the dependence on the virtualities of the off-shell amplitudes appearing in the diagrams with several interactions. In the FFT approach, multiple interactions correspond to the following picture: the ejectile splits into its components (partons) which then interact with the target in parallel. From the diagrammatic point of view, this contribution is represented by nonplanar diagrams which substitute the planar ones as energy increases. Note that in the dispersion approach by Gribov [7] this substitution is not felt at all. One just closes the contour of integration over cumulative momentum transfers around the right-hand singularities. By doing so, one however assumes that there are also some nonzero left-hand singularities: otherwise the result would have been zero. This is precisely what happens if one takes only planar diagrams into account: they have no left-hand singularities. Then, although closing the contour around the right-hand singularities one seemingly obtains some nonzero contributions (specifically from the pole corresponding to the propagating particle itself), the sum of all contributions is finally equal to zero.

In QE (e,e') scattering on nuclei the situation is rather different, for the space-time point of the creation of the hadronic state, after the interaction of the photon with a nucleon as a whole, is fixed and it is inside the nucleus. Thus, unlike the case of hadron-nucleus interaction, where the projectile interacts while being in its asymptotic state, the hit nucleon will become capable of a new interaction only after the formation time which grows with its velocity. At large Q^2 it will then be able to interact only outside the nucleus and all FSI will vanish. In the diagrammatic language, the rescattering of the ejectile always includes a planar diagram. In the dispersion approach this means that there are no left singularities in the cumulative momentum transfers. Therefore one can expect that the FSI in QE (e,e') scattering will die out at high Q^2.

It should be pointed out that the mechanism which makes the FSI vanish due to CT or FFT is the same: the cancellation of the contributions from various propagating states. Many authors believe that in QCD the FFT is a direct consequence of color transparency: a colorless quark system created at a point needs a finite time to reach its asymptotic configuration with the corresponding cross section [8–10]. However the FFT and color transparency, although interrelated in QCD, are in fact different properties. The FFT effects, as mentioned, are related to the vanishing of the planar diagrams, which can be demonstrated to occur at high energies for the so-called soft theories, of the $\lambda\phi^3$ type or in theories in 1+1 dimension, without color transparency properties. On the other hand, one can choose physical processes in which no FFT effects seem to be operative but color transparency effects are obvious. An example of such a process is the diffractive dissociation of a fast pion into a pair of jets with large transverse momenta, where a small size pion configuration appears to be created from the start [11–13]. Thus the reader should not get an erroneous impression that the FFT can be considered as a sole and universal mechanism responsible for diminishing the FSI in all conceivable physical processes.

In the particular case of quasielastic (e,e') scattering, the FFT approach allows a description of the interaction where all effects of FSI vanish explicitly at high Q^2, so that all the necessary cancellations are automatically implemented. In the present paper we discuss the possible modifications of the standard Glauber formula, allowing the amplitudes to depend on their virtualities in a way compatible with the standard analytical properties so as to guarantee the vanishing of the amplitudes at high virtualities, which is precisely the property which leads to all FFT effects. It should be noted that our approach is not very much more restrictive than the currently used multichannel Glauber approach. In fact, a virtuality dependent nucleon-nucleon amplitude simulates, to a large extent, the propagation and the interaction of excited intermediate states. Thus a model for the propagation of a $3q$ state through the nucleus can be (approximately) translated into the propagation of the nucleon with a particular dependence of its interaction amplitude on the virtuality. The case of the single rescattering will be worked out in detail, showing that in this case the two approaches are equivalent. In the FFT approach, model building means specifying the dependence of the amplitude on the virtuality, which phenomenologically seems to be a simpler task than constructing a model for the propagation and the interaction of excited $3q$ states, as required by the conventional description of color transparency. The present approach can be properly generalized so as to take into account also the excited nucleon states. We do not try to discuss such a more elaborate feature in the present paper, and we assume, moreover, the simplest possible factorizable form of dependence of the amplitudes on their virtualities. Our final result is that final state interactions vanish at high Q^2 as mM^2R_A/Q^2, where R_A is the radius of the nucleus, m the nucleon mass and M^2 its average excitation mass squared. The case of FSI with a deuteron target will be considered in detail (an interesting approach to the problem has also been discussed in Ref. [14]). Although in this case one does not expect appreciable FSI effects, the two-body system has the advantage that its structure is well known, and, moreover, only the single rescattering term has to be considered, in which case, as it will be shown, different theoretical approaches to FSI converge to the same result.

Our paper is organized as follows. In Sec. II the general formalism for treating the FSI in the Glauber approach when the amplitudes depend on the virtuality of the external lines is presented. In Sec. III the high Q^2 limit of the approach is investigated. Section IV is devoted to the numerical application to the deuteron target; finally, the conclusions are drawn in Sec. V.

II. FORMALISM

A straightforward way to incorporate FFT effects, generated by the dependence of the scattering amplitudes on the virtualities of the colliding particles, is through the Feynman diagrams formalism. The amplitude describing n consecutive rescattering of the ejectile emerging from the interaction of the struck nucleon with the incoming virtual photon, is depicted in Fig. 1. It corresponds to the usual Glauber approximation of the scattering amplitude [15]. Our notations are as follows: (i) The four-momentum of the target nucleus is denoted Ap and we work in rest system of the nucleus, so that $\mathbf{p}=0$; (ii) the momenta of the nucleons before (after) all interactions are denoted k_i (k_i'); (iii) the spectators correspond to $i=n+2,\dots,A$, for which $k_i=k_i'$; (iv) the active nucleon is labeled 1, and nucleons from 2 to $n+1$ are the ones on which the active nucleon 1 rescatters. Correspondingly, the number of rescatterings goes from 2 to $n+1$; (v) the momentum transferred in the ith rescattering is q_i, so that $k_i'=k_i+q_i$ for $i=2,\dots,n+1$; (vi) the momentum of the active nucleon after interaction with the photon is $k_1^{(1)}=k_1+q$, and after the ith rescattering $k_1^{(i)}=k_1+q-\Sigma_{j=1}^{i}q_j$. The derivation of the amplitude corresponding to Fig. 1 is a standard one, so that, in the following, only those points which are related to the FFT effects, generated by the dependence of the amplitudes on the virtuality of the nucle-

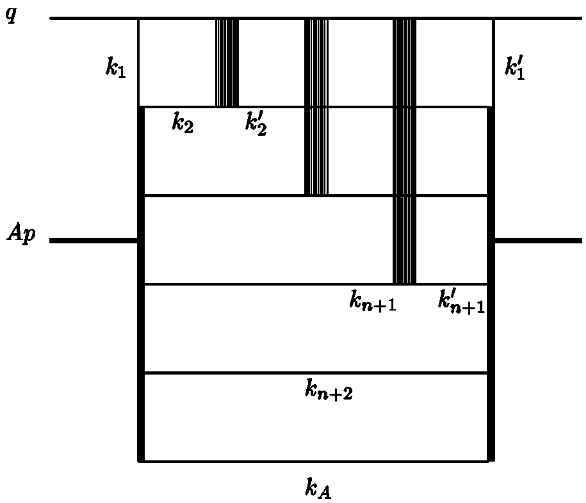

FIG. 1. The forward scattering amplitude.

ons, are stressed; more details on the derivation of the amplitude are given in the Appendix.

The expression for the amplitude with n rescatterings, corresponding to Fig. 1 reads

$$i\mathcal{A}^{(n)}=(1/2)xQ^2(4m\nu)^n\frac{A!}{(A-n-1)!}\times\int\prod_{j=2}^{n+1}\frac{d^4k_j}{(2\pi)^4}\frac{d^4k'_j}{(2\pi)^4}P(k_j)P(k'_j)\times\prod_{j=n+2}^{A}\frac{d^4k_j}{(2\pi)^4}P(k_j)P(k_1)P(k'_1)P(k_1^{(1)})\times\prod_{j=2}^{n+1}P(k_1^{(j)})if_ji\gamma(k_1,q)i\gamma(k'_1,q)\times i\Phi(k_i)i\Phi(k'_i). \quad (1)$$

In Eq. (1), $P(k)$ denotes the propagator of the nucleon with momentum k, and f_j, $j=2,\ldots,n+1$ the corresponding rescattering amplitudes, which are assumed to depend, in addition to the energy and momentum transfer, also upon the virtuality of the fast nucleon which has been struck by the photon. On the mass shell, they are normalized according to

$$2\,\mathrm{Im}\,f=\sigma^{(\mathrm{tot})} \quad (2)$$

[the factor $(4m\nu)^n$ originates from this normalization]. The vertex Φ describes the transition of the nucleus into A nucleons and the vertex γ is the form factor of the on shell nucleon (only the electric form factor appears in our spinless model). The factor $(1/2)xQ^2$ originates from the initial photon interaction, and $x=Q^2/(2k_1\cdot q)$ is the usual Bjorken scaling variable. With our normalization, twice the imaginary part of the amplitude gives the corresponding contribution to the structure function F_2.

The integrations over the energies and the transverse components of all momenta, as well as over the longitudinal momentum components of the spectators, are performed in the standard way. As a result, the amplitude is expressed as an integral over the transverse coordinate b of the struck nucleon with respect to the direction of the virtual photon and the longitudinal interaction points z_i, $i=1,2,\ldots,n+2$, with the appropriate nuclear density matrices. One is left with the nontrivial momentum integrations over the $n+1z$ components of k_1 and q_i, $i=2,3,\ldots,n+1$, through which the virtualities of the active nucleon are expressed as

$$v_i=(k_1^{(i)})^2-m^2=Q^2\left(\frac{1}{x}-1\right)+\frac{Q^2}{xm}\left(\sum_{j=2}^{i}q_{jz}-k_{1z}\right). \quad (3)$$

For the sake of simplicity we make the simplest possible assumption on the dependence of the amplitude f_j on the two virtualities v_j and v_{j-1}, namely we assume the factorized form

$$f_j=F(v_{j-1})F(v_j)f, \quad (4)$$

where f is the on-shell amplitude and $F(v)$ a form factor exhibiting the dependence of f on the virtuality of the external lines, normalized according to $F(0)=1$ and decreasing with v. In the same manner we introduce also the dependence of the off-mass-shell electric form factor on the virtuality of the nucleon

$$\gamma(k_1,q)=F(v_1)\gamma(Q^2);\quad \gamma(k'_1,q)=F(v_{n+1})\gamma(Q^2). \quad (5)$$

The whole v dependence of the integrand is then given by the factorized expression

$$\prod_{i=1}^{n+1}\frac{F^2(v_i)}{-v_i-i0}$$

multiplied by the exponential

$$\exp[i\Delta(z_{n+2}-z_1)],$$

which does not depend on v_i, and the exponential

$$\exp\left(i\frac{xm}{Q^2}\sum_{j=2}^{n+1}v_j(z_j-z_{j+1})\right)$$

which is symmetric in all v_j's. In these expressions we have used $\Delta=m(1-x)$ and have denoted $z_{n+2}\equiv z'_1$ (the last interaction point in the longitudinal space). All integrations on the virtualities v lead therefore to the same function

$$iJ(-z)=\int\frac{dv}{2\pi}\frac{F^2(v)}{-v-i0}\exp\left(i\frac{xm}{Q^2}vz\right). \quad (6)$$

If nuclear correlations are disregarded, the vertex functions Φ can be expressed through the factorized nuclear density matrix

$$\rho(bz_1,bz_2,\ldots,bz_{n+1}|bz_{n+2},bz_2,\ldots,bz_{n+1})=\rho(bz_1|bz_{n+2})\prod_{j=2}^{n+1}\rho(bz_j), \quad (7)$$

where (see Fig. 1) the above quantity is non diagonal in the coordinate 1, diagonal in coordinates $2\cdots(n+1)$, and it is integrated over the coordinates $(n+2)\cdots A$. As a result we obtain the amplitude with n rescatterings in the following form:

$$\mathcal{A}^{(n)}=\gamma^2(Q^2)f^n\frac{x^2m}{2}\frac{A!}{(A-n-1)!}\times\int d^2b\,dz_1dz_{n+2}\prod_{j=2}^{n+1}dz_j\rho(bz_j)\times\prod_{j=1}^{n+1}iJ(z_{j+1}-z_j)\times\exp[i\Delta(z_{n+2}-z_1)]\rho(bz_1|bz_{n+1}). \quad (8)$$

With the dependence of the amplitudes on the virtualities turned on, all effective propagators $J(z)$ go to zero in the large Q^2 limit. Thus, for large Q^2, the bulk of the rescattering contribution comes from the single rescattering term

$$\mathcal{A}^{(1)}=\gamma^2(Q^2)f\frac{x^2m}{2}A(A-1)\times\left\langle\int dz\rho(b_1,z)iJ(z_1'-z)iJ(z-z_1)\right\rangle_1, \quad (9)$$

where the notation $\langle\cdots\rangle_1$ means that the quantity in brackets, i.e., $O(b_1,z_1',z_1)\equiv\int dz\rho(b_1,z)iJ(z_1'-z)iJ(z-z_1)$, has to be averaged over the coordinates of the active nucleon, according to

$$\langle O(b_1,z_1',z_1)\rangle_1\equiv\int d^2b_1dz_1dz_1'\rho(b_1z_1|b_1z_1')\times\exp[i\Delta(z_1'-z_1)]O(b_1,z_1',z_1). \quad (10)$$

If the dependence of the f's on their virtualities is neglected and one puts $F(v)=1$, then $J(z)=\theta(z)$ so that, after summing over n in Eq. (8), the standard Glauber result is obtained:

$$\mathcal{A}=iA(1/2)mx^2\gamma^2\langle\{[1+ifT(b_1,z_1',z_1)]^{A-1}-1\}\rangle_1, \quad (11)$$

where

$$T(b_1,z_1',z_1)=\int_{z_1}^{z_1'}dz\rho(b_1,z). \quad (12)$$

The introduction of the dependence of the amplitudes on the virtualities, by means of the factorized approximation (4), is effectively equivalent to the replacement of the usual $\theta(z)$ in the nucleon propagator with the function $J(z)$, which depends on the virtuality through the form factor $F(v)$.

This simple rule suggests a possible different derivation of the scattering amplitude, which is equivalent to the one just considered, when the dependence of the amplitude on the virtuality of the external lines is turned off. Instead of choosing a particular set of Feynman diagrams, one may assume that the scattering matrix on the nucleus factorizes into the product of scattering matrices on individual nucleons. In our case we assume that the scattering matrix of the first (active) nucleon on the other $A-1$ nucleons is given by

$$S(r_2,r_3,\ldots,r_A|r_1)=\prod_{j=2}^{A}s(r_j), \quad (13)$$

where $r_j=(b_j,z_j)$, and $s(r_j)$ is the scattering matrix on the nucleon j. In accordance with the results obtained from the consecutive rescattering diagram we take

$$s(r_j)=1-J(z_1'-z_j)J(z_j-z_1)\Gamma(b_j-b_1), \quad (14)$$

where b_1 is the impact parameter of the active nucleon, z_1 and z_1' its longitudinal coordinates before and after the interaction, and Γ the nucleon-nucleon profile as a function of the relative transverse distance of the two interacting nucleons, normalized as $\int d^2b\Gamma(b)=-if$. The two functions J, which describe the propagation of the active nucleon to and from the collision point z_j, are replaced by θ functions in the standard Glauber approach. By averaging Eq. (13) over the positions of the nucleons from 2 to A, by making use of the approximation given by Eq. (7) and by disregarding the dimensions of the nucleon as compared with the dimensions of the nucleus, we obtain

$$\langle S(r_2,r_3,\ldots,r_A|r_1)\rangle_{A-1}\equiv\int\prod_{j=2}^{A}d^3r_j\rho(b_jz_j)S(r_2,r_3,\ldots,r_A|r_1)=\left(1+if\int dzJ(z_1'-z)J(z-z_1)\rho(b_1,z)\right)^{A-1}. \quad (15)$$

The final amplitude is obtained by averaging Eq. (15) over the coordinates of the first nucleon and by multiplying the result with the appropriate factor, as in Eq. (11).

It can be readily seen that, in absence of any dependence upon the virtualities, that is, when $J(z)$ is replaced by $\theta(z)$, the standard Glauber result (11) is recovered. However, it is instructive to notice that, with the dependence on virtualities turned on, the expression (15) is generally different from Eq. (8) obtained from the consecutive scattering diagram. Only the single rescattering contribution is identical in Eqs. (8) and (15). Thus the assumption of the factorizability of the nuclear scattering matrix (13) generally has a physical meaning different from selecting the consecutive scattering diagrams. The factorizability (13) seems to be a more fundamental assumption, applicable also to high-energy scattering. As we shall see, the production amplitudes derived on its basis reproduce correctly the Glauber result in absence of any dependence on the virtualities, and thus satisfy the AGK rules [16], in contrast to the amplitudes obtained from the discontinuities of Eq. (8) (see the Appendix). The consecutive scattering diagram approach to FSI, with the factorization hypothesis (4), gives moreover problems with unitarity, which are avoided when FSI is expressed as in Eq. (15). In the following we will therefore work in the scheme where the S matrix is factorized as a product of scattering matrices on individual nucleons. The two approaches give in any case the same result for the deuteron, which will be considered in Sec. IV.

We are interested in QE (e,e') scattering on nuclear targets, i.e., in the process involving the production of a proton in the final state. In absence of any FSI, the contribution of this process to the inclusive structure function $F_2^{N/A}$, to be denoted $F_2^{N/A,(0)}$, is proportional to the square modulus of the nucleon photoproduction amplitude and it is given by

$$F_2^{N/A,(0)}(x,Q^2)=(1/2)A\gamma^2(Q^2)x^2m\langle 1\rangle_1. \tag{16}$$

The FSI is obtained by multiplying each production amplitude by the S matrix (13) taken between the initial and final nuclear states. Since the final active nucleon is physical, its virtuality is zero and, accordingly, one has $J(z_1'-z_j)=1$. The scattering matrix on a nucleon should then contain only one function J:

$$s(r_j)=1-J(z_j-z_1)\Gamma(b_j-b_1). \tag{17}$$

After summing over the final nuclear states one obtains

$$\langle S(r_2,r_3,\ldots,r_A|r_1)S^*(r_2,r_3,\ldots,r_A|r_1')\rangle_{A-1} \tag{18}$$

which replaces $\langle 1\rangle_1$ in the average of Eq. (16). Equation (18), which can be evaluated in a straightforward way, differs from the Glauber result [17] only by the substitution of all functions $\theta(z)$ with the functions $J(z)$. One thus obtains

$$\begin{aligned}\langle S(r_2,r_3,\ldots,r_A|r_1)&S^*(r_2,r_3,\ldots,r_A|r_1')\rangle_{A-1}\\ &=\left(1+if\int dz J(z-z_1)\rho(b_1,z)\right.\\ &\quad -if^*\int dz J^*(z-z_1')\rho(b_1,z)\\ &\quad \left.+\sigma^{el}\int dz J(z-z_1)J^*(z-z_1')\rho(b_1,z)\right)^{A-1}.\end{aligned} \tag{19}$$

From Eq. (19) one obtains the single rescattering contribution to the inclusive structure function in the following form:

$$F_2^{N/A,(1)}=(1/2)A(A-1)\gamma^2(Q^2)x^2m\left\langle\int dz\rho(b_1,z)[ifJ(z-z_1)-if^*J^*(z-z_1')+\sigma^{el}J(z-z_1)J^*(z-z_1')]\right\rangle_1. \tag{20}$$

The expression above can also be obtained from the discontinuities of the amplitude (8), corresponding to the consecutive rescattering diagram of Fig. 1 (see the Appendix).

III. UNITARITY AND HIGH Q^2 BEHAVIOR

At high Q^2 the dependence of the amplitudes on their virtualities becomes of primary importance. On rather general grounds we may express the form factor squared as

$$F^2(v)=\int_0^{+\infty}\frac{dv'v'\tau(v')}{v'-v-i0} \tag{21}$$

with the normalization

$$\int_0^{+\infty}dv\,\tau(v)=1.$$

Note that $\tau(v)$ needs not be real. From Eq. (6) we find

$$J(z)=\theta(z)\int_0^{+\infty}dv\,\tau(v)\left[1-\exp\left(-i\frac{xmvz}{Q^2}\right)\right]. \tag{22}$$

The simplest choice of $\tau(v)$ is evidently $\tau(v)=\delta(v-M^2)$ and in this case

$$J(z)=\theta(z)\left[1-\exp\left(-i\frac{z}{l(Q^2)}\right)\right], \tag{23}$$

where

$$l(Q^2)=\frac{Q^2}{xmM^2} \tag{24}$$

has the obvious meaning of a formation length growing linearly with Q^2. In this case, at the single rescattering level, our model for the FFT coincides with the standard two-channel Glauber model for the propagating nucleon and its excited state of mass squared $m^{*2}=m^2+M^2$, provided that the amplitudes and the production vertices are constrained in a definite way. Indeed, using Eq. (23) in Eq. (9), we obtain

$$\begin{aligned}\mathcal{A}^{(1)}=&-\gamma^2(Q^2)f\frac{x^2m}{2}A(A-1)\\ &\times\left\langle\int dz\rho(b_1,z)\theta(z_1'-z)\theta(z-z_1)\right.\\ &\left.\times(1-e^{-i(z_1'-z)/l}-e^{-i(z-z_1)/l}+e^{-i(z_1'-z_1)/l})\right\rangle_1.\end{aligned} \tag{25}$$

On the other hand, the two-channel Glauber model with two ejectile states 1 (the nucleon) and 2 (its excited state) leads to the single rescattering contribution

$$\mathcal{A}^{(1)}=-\frac{x^2m}{2}A(A-1)\Big\langle\int dz\rho(b_1,z)\,\theta(z_1'-z)\,\theta(z-z_1)\times(\gamma_1^2f_{11}+\gamma_1\gamma_2\,f_{21}e^{-i(z_1'-z)/l}+\gamma_1\gamma_2\,f_{12}e^{-i(z-z_1)/l}+\gamma_2^2\,f_{22}e^{-i(z_1'-z_1)/l})\Big\rangle_1, \quad (26)$$

where $f_{ik}=f_{ki}$, $i,k=1,2$ are the forward scattering amplitudes for transitions $i\rightarrow k$ and γ_i, $i=1,2$ are vertices for the production of the two ejectile states. One immediately observes that Eqs. (25) and (26) coincide if

$$f_{11}\gamma_1+f_{12}\gamma_2=0, \quad f_{21}\gamma_1+f_{22}\gamma_2=0 \quad (27)$$

and, moreover, if $\gamma_{11}^2f_{11}$ in Eq. (26) is identified with γ^2f in Eq. (25). The meaning of the sum rules (27) is that when applying the matrix f_{ik} to the vector γ_i one obtains zero, which is the condition for propagating eigenstates of the forward scattering matrix with zero eigenvalue in nuclear medium [18]. As discussed in Ref. [19], it is precisely the condition for color transparency.

In fact in the case of two channels one may easily see that both unitarity, $2\,\mathrm{Im}\,f_{il}=\Sigma_{j=1,2}f_{ij}f_{jl}^*$, and the transparency conditions are satisfied by

$$f_{12}=f_{21}=-\xi f_{11}, \quad f_{22}=\xi^2f_{11}, \quad (28)$$

where ξ is the (real) ratio of the form factors γ_1 and γ_2, whose value is obtained by $\xi^2=|f_{12}|^2/|f_{11}|^2=\sigma_{\mathrm{inel}}/\sigma_{\mathrm{el}}$. All parameters are then fixed by the value of the total and the elastic nucleon-nucleon cross sections, namely, by the imaginary part and by the modulus of f_{11}. The resulting expression of the single rescattering correction to the forward amplitude is then given by Eq. (25) with $f=f_{11}$ and $\gamma=\gamma_1$.

With a larger number of rescatterings, our model with the choice (23) generates amplitudes which are different and essentially simpler as compared to the two-state Glauber model. This raises the problem of unitarity in our approach.

A simple way to satisfy unitarity to all orders of rescattering in our model, is to ensure that unitarity is fulfilled for the individual scattering matrices (17). Namely, one has to enforce

$$2\,\mathrm{Re}[J(z)\Gamma(b)]\geqslant|J(z)\Gamma(b)|^2 \quad (29)$$

at all values of z and b. At first sight this condition is not so easy to fulfill, since it involves the real part of the nucleon-nucleon scattering amplitude on the left-hand side, which may have different signs at different energies. However we can satisfy Eq. (29) if we assume that $\tau(v)$ is itself an analytic function in the lower half plane. Rotating the contour in Eq. (22) to pass along the negative imaginary axis, we can rewrite Eq. (22) as

$$J(z)=\theta(z)\int_0^{+\infty}dv\,\tau_1(v)\left[1-\exp\left(-\frac{xmvz}{Q^2}\right)\right], \quad (30)$$

where $\tau_1(v)=-i\tau(-iv)$. Now it is sufficient to require that τ_1 is positive to have $0\leqslant J(z)\leqslant 1$, in which case Eq. (29) is satisfied provided the amplitude Γ is itself unitary.

Making again the simplest choice $\tau_1(v)=\delta(v-M^2)$, we obtain a purely real $J(z)$

$$J(z)=\theta(z)\left[1-\exp\left(-\frac{z}{l(Q^2)}\right)\right], \quad (31)$$

where the formation length $l(Q^2)$ is defined by Eq. (24). In the past the formation length was often introduced into the rescattering picture in a straightforward manner, essentially by changing the function $\theta(z)$ by $\theta(z-l)$ in the rescattering matrix. In our approach, with Eq. (29), we also find a real damping factor in the rescattering matrix, which, however, has a much softer behavior and vanishes at high Q^2 only as $\sim 1/Q^2$:

$$J(z)\simeq\theta(z)\frac{xmz}{Q^2}\int dv\,v\,\tau_1(v)=\theta(z)\frac{xmzM^2}{Q^2}, \quad Q^2\rightarrow\infty, \quad (32)$$

where $M^2=\langle v\rangle$ is the average excitation mass squared (with m^2 subtracted). Of course, Eq. (32) is true only if this average exists, that is if the integration over v in the first expression on the right-hand side converges. If not, the vanishing of $J(z)$ at large Q^2 is slower.

Assuming Eq. (32) we find that the propagation of the ejectile between any two points in the nucleus along the z axis, gives a small factor $\sim mM^2R_A/Q^2$. Equation (15) then tells us that the amplitude with n rescatterings behaves as $1/l^{2n}$, that is, as $1/Q^{4n}$. The leading rescattering correction will come from the single rescattering term and it is of order $1/Q^4$. It is interesting that the n-fold rescattering amplitude obtained from the consecutive rescattering diagram [Eq. (8)] generally has a slower decrease with Q^2, namely, it decreases as $1/Q^{2(n+1)}$ (with the exception of the single rescattering term, $n=1$, when both amplitudes coincide). This means that due to FFT the total absorptive corrections to the structure function, generated by the direct interaction of the incoming photon with a nucleon as a whole, is of order $1/Q^4$, as compared with the plane wave impulse approximation. It is remarkable that the contribution of the rescattering, although also vanishing at $Q^2\rightarrow\infty$, has a relative order of $1/Q^2$, and so it is substantially larger than the total absorptive corrections. This follows from our expression (20) for the discontinuity. Evidently in the limiting case $Q^2\rightarrow\infty$ only the two first terms survive, which correspond to the cut of the nucleon propagators (which does not correspond properly to a rescattering, but rather to an interference term). Using Eq. (31) we find in the limit $Q^2\rightarrow\infty$

$$\frac{F_2^{N/A,(1)}}{F_2^{N/A,(0)}}=-12(A-1)mxM^2\sigma^{\mathrm{tot}}\frac{M^2}{Q^2\langle 1\rangle_1}\times\langle(U(b,z_1)+U(b,z_1')-z_1T(b,z_1)-z_1'T(b,z_1'))\rangle_1, \quad (33)$$

where

$$U(b,z)=\int_z^{+\infty} dz' z' \rho(b,z') \tag{34}$$

and $\sigma^{\rm tot}=2\,{\rm Im}\,f$ is the total cross section for the NN interaction.

IV. FSI FOR THE DEUTERON TARGET

The deuteron structure function may be written as follows:

$$F_2^d=F_2^{N/d,(0)}+F_2^{N/d,(1)}+F_2^{N^*/d,(1)}, \tag{35}$$

where $F_2^{N/d,(0)}$ is the expression obtained in impulse approximation, $F_2^{N/d,(1)}$ the contribution to the deuteron structure function in presence of FSI and with a proton in the final state, while all other contributions to the structure function are represented by $F_2^{N^*/d,(1)}$. The structure function is obtained by working out the imaginary parts of the the forward virtual photon-deuteron amplitude. For a deuteron target the amplitude without rescattering is given by

$$\mathcal{A}^{(0)}=(1/2)i\gamma^2(Q^2)x^2m\int d^2b\,dz_1dz_1'\psi(b,z_1)\psi(b,z_1')\times\theta(z_1'-z_2)\exp[i\Delta(z_1'-z_1)], \tag{36}$$

where $\psi(b,z)$ is the deuteron wave function and, to keep into account virtual photon finite energy effects, $\Delta=Q^2(1-x)/(2q_zx)$. The discontinuity of this amplitude, corresponding to the cut active nucleon line, gives the contribution to the inclusive deuteron structure function generated by the production of a fast nucleon $F_2^{N/d}$. We find from Eq. (36)

$$F_2^{N/d,(0)}(x,Q^2)=\pi x^2m\gamma^2(Q^2)\int d^3k\,\phi^2(k)\times\delta[k_z-Q^2(1-x)/(2q_zx)], \tag{37}$$

$\phi(k)$ being the deuteron wave function in momentum space.

The amplitude with a single rescattering is written as

$$\mathcal{A}^{(1)}=-(1/2)\gamma^2x^2m\int dz_1dz_1'd^2b\,\psi(b,z_1)i\Gamma(b)\psi(b,z_1')\times J(-z_1)J(z_1')\exp[i\Delta(z_1'-z_1)],$$

where b is the distance between the proton and the neutron in transverse space. More explicitly in the two-channel model one has

$$\mathcal{A}^{(1)}=(1/2)\gamma^2x^2m\int d^2b\,i\Gamma(b)[X(b,x,Q^2)]^2, \tag{38}$$

where

$$X(b,x,Q^2)=i\int dz\,\psi(b,z)J(-z)\exp(i\Delta z)=\int\frac{d^3k}{(2\pi)^{3/2}}\phi(k)e^{i\mathbf{k}_t\mathbf{b}}\left[\frac{1}{k_z-\Delta-i0}-\frac{1}{k_z-\Delta+1/l-i0}\right]. \tag{39}$$

The corresponding cross section to produce a fast nucleon has two different contributions: from the cut of the amplitude Γ and from the cut of the nucleon propagators. The sum of the two discontinuities from the cut nucleon propagators is given by

$${\rm Disc}_1\mathcal{A}^{(1)}=ix^2m\gamma^2\int d^2b\,Y(b,x){\rm Re}[i\Gamma(b)X(b,x,Q^2)], \tag{40}$$

where

$$Y(b,x)=\int\frac{d^3k}{(2\pi)^{3/2}}\phi(k)e^{i\mathbf{k}_t\mathbf{b}}2\pi\delta(k_z-\Delta) \tag{41}$$

and, in the Bjorken limit, it is a real function independent of Q^2.

As for the discontinuity corresponding to a cut across the rescattering blob Γ, since we are interested in the contribution of the scattered nucleon to the inclusive structure function, only the elastic part of the unitarity sum over the intermediate states has to be retained. We obtain

$${\rm Disc}_2\mathcal{A}^{(1)}=i(1/2)x^2m\gamma^2\int d^2b|\Gamma(b)|^2|X(b,x,Q^2)|^2. \tag{42}$$

The contribution to the inclusive deuteron structure function due to the nucleon rescattering in the final state, is given by the sum of the two discontinuities (40) and (42) divided by i

$$F_2^{N/d,(1)}=-i({\rm Disc}_1\mathcal{A}^{(1)}+{\rm Disc}_2\mathcal{A}^{(1)}). \tag{43}$$

Note that at low energy, when no elastic channels are open and $\sigma_{\rm tot}=\sigma_{\rm el}$, one has

$$F_2^{N/d,(1)}=x^2m\gamma^2\int d^2b\left\{-2\,{\rm Im}\,\Gamma(b)+\frac{|\Gamma(b)|^2}{2}\right\}\frac{[Y(b,x)]^2}{4}=x^2m\gamma^2\int d^2b\{-{\rm Im}\,\Gamma(b)\}\frac{[Y(b,x)]^2}{4}=2\,{\rm Im}\,\mathcal{A}^{(1)}. \tag{44}$$

The only contributions to the imaginary part of the forward amplitude is given, in this case, by the the two discontinuities (40) and (42) where only the elastic intermediate state is present. At higher energies the inelastic channels become more and more important. The effect is to add further contributions to the imaginary part of the forward amplitude. As it may be seen by looking at the behavior of X, Eq. (38), as a function of the formation length l, the additional contri-

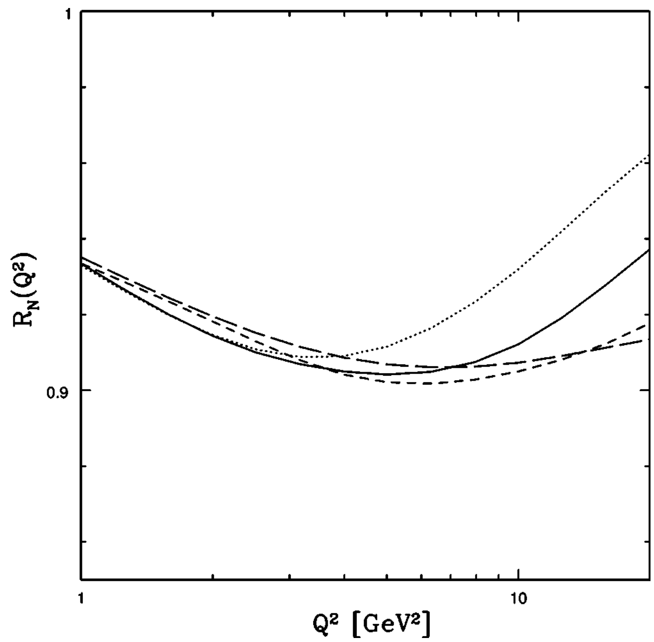

FIG. 2. Values of $R_N(Q^2)$ [Eq. (45)] at $x=1$ for the deuteron target with different choices of the excited nucleon mass: $m^*=1.44$ GeV dotted line, $m^*=1.8$ GeV continuous line, $m^*=2.4$ GeV short-dashed line. The long-dashed line corresponds to the standard Glauber result, where no dependence of the amplitude on the virtuality of the external lines is taken into account.

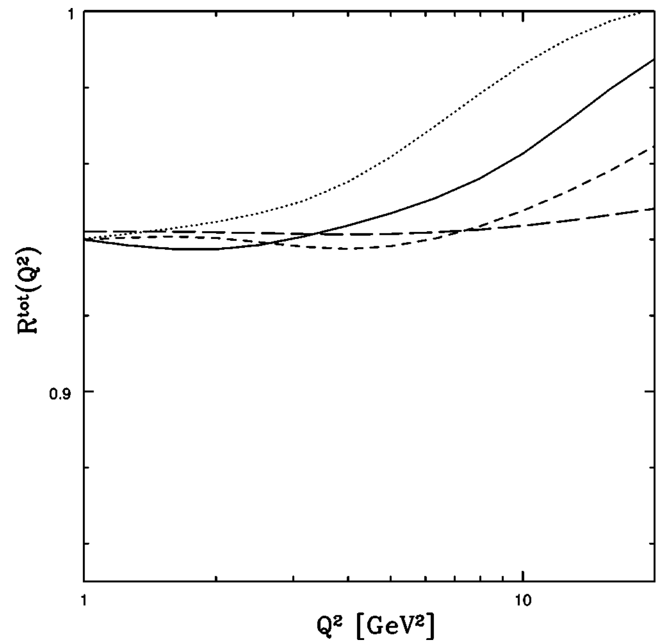

FIG. 3. Values of $R^{\mathrm{tot}}(Q^2)$ [Eq. (46)] at $x=1$ for the deuteron target. The different lines refer to the different cases described in the previous figure.

butions give a small correction at low Q^2 (small l) while they tend to cancel completely the elastic contribution at large Q^2 (large l).

To study quantitatively the behavior of the ratio which characterizes the strength of the FSI, viz.

$$R_N(Q^2)=1+\left(\frac{F_2^{N/d,(1)}(x,Q^2)}{F_2^{N/d,(0)}(x,Q^2)}\right)_{x=1} \tag{45}$$

one has to specify the value of the mass parameter M^2. Its meaning is that of the squared average excitation mass of the ejectile $M^2=(m^*)^2-m^2$, where m^* is the average mass of the ejectile. Previous calculations, based on the coupled-channel Glauber formalism have shown that m^* lies between the lowest N^* resonance mass 1.44 GeV and the average continuum mass 2.4 GeV, thus a reasonable value could be 1.8 GeV [10,17,18]. Predictions for $R_N(Q^2)$ for these values of m^* are shown in Fig. 2 at $x=1$. For the deuteron wave function we have used the parametrization of Ref. [20], corresponding to a realistic nucleon-nucleon interaction, and for the nucleon-nucleon amplitude we have used both the experimental data [21] and the results of the partial wave analysis [22]. As a comparison we also show the pure Glauber predictions, which correspond to a very large value of m^*. The results in the pure Glauber case are in agreement with those obtained in Ref. [23], where both the Reid soft core and the Bonn deuteron wave functions have been used and interference effects between deuteron S and D waves have been taken into account explicitly. In all cases the FSI are found to be small, as to be expected due to the large deuteron size. As for the behavior with Q^2, the values of both the threshold and the rate at which $R(Q^2)$ goes to one depend on the value chosen for m^*, the threshold growing and the rate diminishing with m^*. For the value $m^*=1.8$ GeV and at $x=1$, the FSI changes in a sizable way when Q^2 is of the order of 10 $(\mathrm{GeV}/c)^2$, which agrees with the conclusions inferred from the conventional approach to color transparency.

The FFT approach allows one also to calculate the total structure function at $x\sim 1$, provided that the initial interaction involves a proton as a whole. By looking at the imaginary part of the rescattering amplitude one can in fact work out the FSI for the total structure which vanishes faster, as $1/Q^4$, as compared to $R_N(Q^2)$. The behavior is illustrated in Fig. 3 where we show

$$R^{\mathrm{tot}}(Q^2)=1+\left(\frac{F_2^{N/d,(1)}(x,Q^2)+F_2^{N^*/d,(1)}(x,Q^2)}{F_2^{N/d,(0)}(x,Q^2)}\right)_{x=1} \tag{46}$$

and where $F_2^{N/d,(1)}+F_2^{N^*/d,(1)}$ is evaluated by taking twice the imaginary part of Eq. (38). Looking at the continuous curve, corresponding to an excitation mass $m^*=1.8$ GeV, one observes that the threshold at which the FSI starts to vanish is practically the same as for the proton production, while the effect of FSI is sizably smaller in this case.

V. CONCLUSIONS

We have studied the FFT effects by introducing the dependence on the virtualities into the elementary amplitudes. Two options have been considered for generalizing the standard on-shell Glauber picture to take into account the virtuality of the ejected nucleon: the Feynman diagram and S-matrix factorization approaches. The latter choice seems to be more convincing, since it preserves both the overall unitarity and the AKG cutting rules. The single rescattering term is, however, the same in both approaches, so that its calculation seems to be reliable. Moreover the single rescattering term can be understood also in terms of the conventional multichannel picture of the FSI, showing in this way that the present approach is essentially equivalent to the conventional one at the single rescattering level. In addition to a better understanding, which is gained when a given mechanism of interaction can be described from different perspectives, an advantage of the actual approach lies in its far simpler implementation, as compared with the standard multichannel description of the FSI.

Numerical estimates are made for the deuteron target, where all FSI are described by the single rescattering term. Our result is that for the QE (e,e') reaction the FFT effects become clearly visible at rather high values of Q^2, namely for $Q^2 \simeq 10\ (\mathrm{GeV}/c)^2$, in accordance with the conclusions drawn within the CT approach.

ACKNOWLEDGMENTS

This work was partially supported by the Ministero dell'Università e della Ricerca Scientifica (MURST) through the funds COFIN99. M.A. Braun is deeply thankful to the Universities of Perugia and Trieste and to INFN, Sezioni di Perugia and Trieste for their hospitality and financial support. The authors gratefully acknowledge L. A. Kaptari for discussions and the European Centre for Theoretical Physics (ECT*) in Trento, where part of this work was completed.

APPENDIX: THE RESCATTERING AMPLITUDE OF FIG. 1

One standardly starts by the integrations over the zero components of the momenta. Since the poles coming from the propagators of the active nucleon all lie in the upper half-plane, one can integrate over k_{i0} or k'_{i0}, $i=2,\ldots,A$, just taking the residue at the pole of the corresponding propagator $P(k_i)$ or $P(k'_i)$. The two propagators of the active nucleon in the initial and final state together with the factors $i\Phi(k_i)i\Phi(k'_i)$ then combine into a product of two nuclear wave functions

$$[2m(2\pi)^3]^{A-1}\phi(k_i)\phi(k'_i).$$

Passing to the coordinate space wave functions $\psi(r_i)$ one then integrates over the transverse momenta. It is quite trivial, since all interactions, as well as the left propagators of the active nucleon do not depend on the small transverse momenta. So all the dependence is in the exponentials. One chooses one of the spectator momenta (say the A's) and k'_1 as dependent variables. Integration will then go over $A-1$ momenta $k_1,k_2,\ldots,k_{A-1}$ and n momenta $k'_2,k'_3,\ldots,k'_{n+1}$. The latter n integrations can be substituted by n integrations over the transferred momenta $q_2,q_3,\ldots,q_{n+1}$. One obtains

$$\begin{aligned} i\mathcal{A}=&\int\prod_{j=1}^{A-1}\frac{dk_{zj}}{2m(2\pi)}\prod_{j=2}^{n+1}\frac{dq_{zj}}{2m(2\pi)}P(k_1^{(1)})\\ &\times\prod_{j=2}^{n+1}P(k_1^{(j)})if_j i\gamma(k_1,q)i\gamma(k'_1,q)(2m)^{A-1}\prod dz_i\\ &\times\exp\Big(-i\sum k_{zj}z_j\Big)\prod dz'_i\exp\Big(i\sum k'_{zj}z'_j\Big)d^2b_1\\ &\times\prod_{j=n+2}^{A-1}d^2b_j\psi(b_1=b_2=b_3+\cdots b_{n+1},b_j;z_i)\\ &\times\psi(b_1=b_2=b_3+\cdots b_{n+1},b_j;z'_i). \end{aligned} \tag{A1}$$

Now one integrates over the longitudinal momenta. Evidently the integrand does not depend on the k_z of the spectators. So these interactions are done trivially and convert the double integration over z into a single one for the spectators. Together with the integration over their transverse coordinates this turns the product of the wave functions into the nuclear ρ matrix for $n+1$ nucleons taking part in the interaction. One is left with the $2n+1$ integrations over the z components of k_i, $i=1,2,\ldots,n+1$, and $q_i i=2,3,\ldots,n+1$. All propagators of the active nucleon and also the amplitudes if_j depend only on v_i, $i=1,\ldots,n+1$. So it is convenient to pass from the $n+1$ variables $k_{1z},q_{2z},\ldots,q_{n+1,z}$ to variables v_i, $i=1,\ldots,n+1$. The rest integration variables are $k_{2z},\ldots,k_{n+1,z}$. The dependence on them is concentrated in the exponentials so that integration over them turns the double integrations over z into a single one for z_j with $j=2,3,\ldots,n+1$. The left integrations are over v_j which are done as explained in Sec. II.

The standard Glauber model corresponds to an approximation in which the dependence of the amplitudes f on their virtualities is neglected and thus the form-factor $F(v)=1$ and

$$J(z)=\theta(z). \tag{A2}$$

The integrations in Eq. (8) become

$$\begin{aligned} &\int_{z_1}^{z_{n+2}}dz_{n+1}\int_{z_1}^{z_{n+1}}dz_n\cdots\int_{z_1}^{z_3}dz_2\\ &\quad\to(1/n!)\int_{-\infty}^{+\infty}dz_{n+2}\int_{-\infty}^{z_{n+2}}\prod_{j=2}^{n+1}\int_{z_1}^{z_{n+2}}dz_j, \end{aligned}$$

where one uses the symmetry of the integrand in the variables $z_2,z_3,\ldots,z_{n+1}$. Each integration converts $\rho(b,z)$ it into the profile function between the two longitudinal points (12) and we get Eq. (11).

Various discontinuities of the amplitude (8) correspond graphically to various cuts of the diagram shown in Fig. 1.

Evidently there are two possibilities and correspondingly two possible types of discontinuities. The cut may pass through a fast nucleon line. The relevant discontinuity then corresponds to an intermediate state of a fast nucleon and the nucleus debris. Alternatively the cut may pass through an amplitude *f*. Then the intermediate state consists of an arbitrary ejectile state plus the nucleus debris. If we are interested only in the states with one fast nucleon plus the nucleus debris, we have to select only elastic intermediate states in the cut amplitude *f*.

Technically the discontinuity is obtained by making substitutions

$$\frac{1}{-v-i0}\to 2\pi i\,\delta(v) \tag{A3}$$

for the cut fast nucleon propagator or

$$f\to 2i\,\mathrm{Im}\,f=i\sigma \tag{A4}$$

for the cut amplitude. In the latter case only the elastic part of the contribution to the cross section should be taken if one is only interested in the states with one fast nucleon. Also all the parts of the amplitude to the right of the cut should be taken complex conjugate.

When the discontinuity passes through the fast nucleon line connecting the points z_k and z_{k+1} we should make the substitution

$$J(z_{k+1}-z_k)\to 1.$$

In all J's to the right of the cut we have to change the sign of the $i0$ in the denominator, so that for $j>k$

$$J(z_{j+1}-z_j)\to -J^*(z_j-z_{j+1}). \tag{A5}$$

Finally of n amplitudes $n-k+1$ have to be taken complex conjugate. As a result, the total discontinuity corresponding to cut fast nucleon lines is given by the integral (8) in which

$$I(z_1,\dots,z_{n+2})\equiv f^n\prod_{j=1}^{n+1} iJ(z_{j+1}-z_j)$$

is substituted by

$$I_1(z_1,\dots,z_{n+2})=\sum_{k=1}^{n+1}\prod_{j=1}^{k-1} ifJ(z_{j+1}-z_j)\times\prod_{k+1}^{n+1}(if)^*J^*(z_j-z_{j+1}). \tag{A6}$$

Now let the cut pass through the kth rescattering amplitude. Then the latter should be changed according to Eq. (A3). For $j>k$ again one should make the substitution (A4) and take all the amplitudes conjugate. Thus the second type of discontinuities will be given by the integral (8) with $I(z_1,\dots,z_{n+2})$ substituted by

$$I_2(z_1,\dots,z_{n+2})=i\frac{\sigma^{el}}{ff^*}\sum_{k=2}^{n+1}\prod_{j=1}^{k-1} ifJ(z_{j+1}-z_j)\times\prod_{k}^{n+1}(if)^*J^*(z_j-z_{j+1}). \tag{A7}$$

It is instructive to see how the found discontinuities transform in the case when the amplitudes do not depend on their virtualities and $F(v)=1$. For the discontinuity (A5) we then get the z integrations

$$\int_{z_1}^{\infty}dz_k\int_{z_1}^{z_k}dz_{k-1}\cdots\int_{z_1}^{z_3}dz_2\int_{z_{n+2}}^{\infty}dz_{k+1}\times\int_{z_{n+2}}^{z_{k+1}}dz_{k+2}\cdots\int_{z_{n+2}}^{z_n}dz_{n+1},$$

which using the symmetry of the integrand can be transformed into

$$\frac{1}{(k-1)!(n-k-1)!}\int_{z_1}^{\infty}\prod_{j=2}^{k}dz_j\int_{z_{n+2}}^{\infty}\prod_{j=k+1}^{n+1}dz_j. \tag{A8}$$

Doing the integrations we obtain a product

$$\frac{1}{(k-1)!(n-k+1)!}[ifT(b,z_1)]^{k-1}[ifT(b,z_{n+2})^*]^{n-k+1}.$$

Summation over k and n gives

$$\mathrm{Disc}_1\mathcal{A}(b,z,z')=iA[1+ifT(b,z)-if^*T(b,z')]^{A-1}, \tag{A9}$$

where $T(b,z)=T(b,\infty,z)$.

For the discontinuity (A6) the z integrations are

$$\int_{z_>}^{\infty}dz_k\int_{z_1}^{z_k}dz_{k-1}\cdots\int_{z_1}^{z_3}dz_2\int_{z_{n+2}}^{z_k}dz_{k+1}\times\int_{z_{n+2}}^{z_{k+1}}dz_{k+2}\cdots\int_{z_{n+2}}^{z_n}dz_{n+1},$$

where $z_>=\max(z_1,z_{n+2})$ Fixing $z_k\equiv\zeta$ and again using the symmetry of the integrand in the two groups of the left variables we arrange the integrations as in Eq. (A7) and doing them get

$$\frac{1}{(k-2)!(n-k+1)!}\int_{z_>}^{\infty}d\zeta\rho(b,\zeta)[ifT(b,\zeta,z_1)]^{k-2}\times[ifT(b,\zeta,z_{n+2})^*]^{n-k+1}.$$

After summation over k and n the part due to cut amplitudes is found to be

$$\mathrm{Disc}_2\mathcal{A}(b,z,z')=iA(A-1)\sigma^{el}\times\int_{z_>}^{\infty}d\zeta\rho(b,\zeta)[1+ifT(b,\zeta,z)-if^*T(b,\zeta,z')]^{A-2} \quad (A10)$$

with $z_>=\max(z,z')$. Using

$$\frac{\partial}{\partial\zeta}[1+ifT(b,\zeta,z)-if^*T(b,\zeta,z')]^{A-1}=-(A-1)\sigma^{\mathrm{tot}}\rho(b,\zeta)[1+ifT(b,\zeta,z)-if^*T(b,\zeta,z')]^{A-2} \quad (A11)$$

one can do the integration over ζ in Eq. (A11):

$$\mathrm{Disc}_2\mathcal{A}(b,z,z')=-iA\frac{\sigma^{\mathrm{el}}}{\sigma^{\mathrm{tot}}}\{[1+ifT(b,z)-if^*T(b,z')]^{A-1}-[1+ifT(b,z_>,z)-if^*T(b,z_>,z')]^{A-1}\}. \quad (A12)$$

The sum of Eqs. (A9) and (A12) gives the total discontinuity

$$\mathrm{Disc}\,\mathcal{A}(b,z,z')=iA\left(1-\frac{\sigma^{\mathrm{el}}}{\sigma^{\mathrm{tot}}}\right)[1+ifT(b,z)-if^*T(b,z')]^{A-1}+A\frac{\sigma^{\mathrm{el}}}{\sigma^{\mathrm{tot}}}[1+ifT(b,z_>,z)-if^*T(b,z_>,z')]^{A-1}. \quad (A13)$$

One immediately notes that this discontinuity (and the corresponding proton production probability) is different from the one obtained by squaring the nonforward production amplitudes for the process (e,e') calculated in the Glauber approach and summing over all final states. The latter is easily obtained as [17]

$$\mathrm{Disc}\,\mathcal{A}(b,z,z')=iA[1+ifT(b,z)-if^*T(b,z')+\sigma^{el}T(b,z_>)]^{A-1} \quad (A14)$$

which, for a source not extended in z so that $z=z'$ transforms into

$$\mathrm{Disc}\,\mathcal{A}(b,z,z)=iA[1-\sigma^{in}T(b,z)]^{A-1} \quad (A15)$$

with a clear probabilistic interpretation. The reason for this difference has long been known: the Glauber-Gribov picture of consecutive rescatterings corresponding to Fig. 1 is effectively valid for the amplitude itself but not for its discontinuities, due to the wrong space-time picture inherent in it.

If one is interested in the distribution of the produced fast nucleons in the momentum space then the discontinuities taken in Sec. II have to be further specified. It is quite simple to do it if the cut passes through one of the rescattering amplitudes. Then the inclusive cross section of interest is obtained by substituting the cut amplitude by the relevant inclusive cross section for the collision of the active nucleon (momentum k_1+q) with a nucleon at rest.

The contribution to the inclusive cross-section coming from a cut (ith) propagator is a bit more complicated. Now one has to substitute the propagator by

$$(2\pi)^4\delta(v_i)\delta^3(k^{(i)}-l)=2\pi\delta(v_i)\int d^3R\exp i(\mathbf{k}^{(\mathbf{i})}-\mathbf{l})\mathbf{R}.$$

The additional exponential function will somewhat change our derivation.

In the transverse part of the exponent apart from $-il_TR_T$, we have additional terms $ik_{1T}R_T$, which will shift the argument of the corresponding δ function by $-R_T$, and a term $-iq_{jT}R_T$, $j\leqslant i$ which will also shift the arguments in the corresponding δ functions by R_T. As a result the transverse coordinates in the nuclear wave functions in Eq. (44) become $b_j=b_j'=b_1$ for $j=2,\ldots,i$; $b_j=b_j'=b_1'=b_1-R_T$ for $j>i$. Shifting the b_1 integration by R_T we make them $b_j=b_j'=b_1+R_T$ for $j=2,\ldots,i$; $b_j=b_j'=b_1'=b_1$ for $j>i$.

As to the longitudinal part, the additional exponent in terms of v_i has the form

$$-iZ[(xm/Q^2)v_i+\Delta+q_z-l_z].$$

However, $v_i=0$ so that after integration over Z we obtain a factor $2\pi\delta(l_z-q_z-\Delta)$. Thus the observed fast nucleon carries the longitudinal momentum of the initial photon shifted by Δ. No other effect is introduced by the longitudinal exponent.

So in the end the inclusive cross section to produce a fast nucleon with the momentum $\mathbf{l}$, corresponding to the cut kth line will be given by the expression

$$I_1^{(i)}(\mathbf{l})=i\gamma^2(Q^2)(2\pi)\delta(l_z-q_z-\Delta)\frac{x^2m}{2}\frac{A!}{(A-n-1)!}\times\int d^2bd^2b'\exp[il_\perp(b-b')]\prod_{j=1}^{n+2}dz_j\times\exp[i\Delta(z_{n+2}-z_1)]\rho(bz_1|b'z_{n+2})\times\prod_{j=2}^{k}if\rho(bz_j)J(z_j-z_{j-1})\times\prod_{k+1}^{n+1}(if)^*\rho(b'z_j)J^*(z_j-z_{j+1}). \quad (A16)$$

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