R ratios and moments of nuclear structure functions

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We study implications of a model which links nuclear and nucleon structure functions. For this model, computed Callen-Gross functions $\kappa^A(x,Q^2)=2xF_1^A(x,Q^2)/F_2^A(x,Q^2)$ are for finite Q^2 close to their asymptotic value 1. Using those κ, we compute R ratios for $Q^2\gtrsim 5$ GeV^2. We review approximate methods for the extraction of R from inclusive scattering and EMC data. We also calculate ratios of moments of F_k^A and find these to describe the data and in particular their Q^2 dependence. The above observables, as well as inclusive cross sections, are sensitive tests for the underlying relation between nucleonic and nuclear structure functions. In view of the overall agreement, we speculate that the above relation effectively circumvents a QCD calculation.

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In the following we discuss two topics related to nuclear structure functions (SF), namely ratios R^A of cross sections for longitudinal and transverse virtual photons, and ratios of moments of SF. We start with the cross section per nucleon for inclusive scattering of high-energy electrons from nuclei

$$\frac{d^2\sigma_{eA}(E;\theta,\nu)/A}{d\Omega\, d\nu}=\frac{2}{A}\sigma_M(E;\theta,\nu)\left[\frac{xM^2}{Q^2}F_2^A(x,Q^2)+\tan^2(\theta/2)F_1^A(x,Q^2)\right]. \qquad (1)$$

The inclusive and the Mott cross section σ_M for point-nucleons are measured as functions of beam energy E, scattering angle θ, and energy loss ν. The above nuclear SF $F_k^A(x,Q^2)$ describe the scattering of unpolarized electrons from randomly oriented targets. These depend on the square of the four-momentum $Q^2=\boldsymbol{q}^2-\nu^2$ and the Bjorken variable x, corresponding to the nucleon mass M with range $0\leqslant Q^2/2M\nu\leqslant A$.

The interest in F_k^A stems from the interplay between nucleonic and subnucleonic dynamics which one wishes to study. These are in principle obtained by the Rosenbluth extraction for a single-photon exchange cross section (1), which requires data for fixed x and Q^2 at different scattering angles θ. Since $\sin^2(\theta/2)=Q^2/[4E(E-Q^2/2Mx)]$, varying the scattering angle amounts to varying the beam energy E. Instead of the SF in Eq. (1), one extracts the above-mentioned ratio R^A [1]

$$R^A=d^2\sigma_L/d^2\sigma_T=\left(1+\frac{4M^2x^2}{Q^2}\right)\frac{1}{\kappa^A(x,Q^2)}-1, \qquad (2a)$$

$$\kappa^A(x,Q^2)=\frac{2xF_1^A(x,Q^2)}{F_2^A(x,Q^2)}. \qquad (2b)$$

We shall name $\kappa^A(x,Q^2)$ the nuclear Callen-Gross (CG) function.

There exists a rather extensive body of data from which R has been extracted, but the information does not cover wide x,Q^2 ranges and is not accurate, reflecting a similar uncertainty in F_k^A. Below we shall discuss computed results for R and standard approximations.

Equation (1) holds irrespective of the dynamics underlying the description of the nuclei. With nucleons as dominant degrees of freedom, it is appealing to relate SF of nuclei to those of nucleons, which are considered to be composite for the high Q^2 involved. We shall use below a proposed relation [2]

$$F_k^A(x,Q^2)=\int_x^A\frac{dz}{z^{2-k}}f^{PN}(z,Q^2)F_k^{\langle N\rangle}\left(\frac{x}{z},Q^2\right), \qquad (3)$$

where $F_k^{\langle N\rangle}$ are properly p,n-weighted SF's of *free* nucleons $F_k^p,F_k^n\approx F_k^D/2-F_k^p$. Those contain information on the substructure of the nucleon and we shall use data compiled for F_1^N [3], and parametrizations for F_2^N [4]. Dynamics enter through the SF of a nucleus with point-particles f^{PN}, probed at high Q^2.

The above SF f^{PN} depends on Q^2 as is the rule for SF of any systems of fully interacting particles. This is not the case in the plane wave impulse approximation (PWIA), where one neglects the residual interaction between the knocked-out nucleon and the spectator nucleus (cf. for instance Refs. [6–8]).

Several heuristic nonperturbative arguments lead to Eq. (3). One is a cluster model for quarks in nuclei, with nucleons as clusters [2]. A second approach uses the light-cone approach of Akhulinitchev *et al.* [6] with momentum fractions replaced by Bjorken variables for finite Q^2 [2,9,10]. Trust in the approximate validity of Eq. (3) comes primarily from a comparison of its consequences and data, in particular the quite striking agreement between predicted inclusive cross sections [9,11,12] and data [13,14]. The same provides estimates of the kinematic limit $Q^2\gtrsim(1-1.5)$ GeV^2 and for $x\gtrsim 0.15$, below which pionic [15] and antiscreening effects, neglected in Eq. (3), grow in importance.

An additional restriction originates in the improper treatment of the c.m. motion of a nucleus in virtually all nuclear models. Incurred errors affect ground-state wave functions

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TABLE I. ''Exact'' R for low x and medium-high Q^2, the high Q^2 limit, and data for binned $\langle Q^2\rangle$ [18,22]. R^{exp} contains statistical and systematic errors. The first row for $x=0.1$ are extrapolations, down from $x\gtrsim 0.1$.

x	$Q^2(\mathrm{GeV}^2)$ / R	5	10	20	50
	$R^{\text{``exact''}}$	0.284	0.226	0.221	0.218
0.08	$R_L^{(2)}$	0.005	0.002	0.001	0.000
	$R^{exp}(\langle Q^2\rangle\approx 7)$	$0.27\pm 0.06\pm 0.02$			
	$R^{\text{``exact''}}$	0.216	0.203	0.185	0.176
0.12	$R_L^{(2)}$	0.010	0.005	0.003	0.001
	$R^{exp}(\langle Q^2\rangle\approx 12)$		$0.12\pm 0.05\pm 0.02$		
	$R^{\text{``exact''}}$	0.192	0.169	0.146	0.120
0.18	$R_L^{(2)}$	0.023	0.011	0.005	0.002
	$R^{exp}(\langle Q^2\rangle)\approx 23)$			$0.06\pm 0.06\pm 0.02$	
	$R^{\text{``exact''}}$	0.159	0.120	0.089	0.044
0.27	$R_L^{(2)}$	0.051	0.025	0.013	0.006
	$R^{exp}(\langle Q^2\rangle\approx 30)$			$0.04\pm 0.04\pm 0.01$	
	$R^{\text{``exact''}}$	0.144	0.119	0.064	0.009
0.36	$R_L^{(2)}$	0.091	0.025	0.013	0.006
	$R^{exp}(\langle Q^2\rangle\approx 50)$			$-0.04\pm 0.04\pm 0.01$	
0.5	$R^{\text{``exact''}}$	0.140	0.113	0.048	≈ 0
	$R_L^{(2)}$	0.178	0.089	0.044	0.018
0.7	$R^{\text{``exact''}}$	0.223	0.170	0.120	≈ 0
	$R_L^{(2)}$	0.348	0.170	0.085	0.035

and derived density matrices which enter a calculation of the above f^{PN} [11,12]. Those decrease with increasing A, and we therefore choose $A\geqslant 12$.

We have demonstrated before that the calculated f^{PN} are only weakly A-dependent [11], as are the weighted $F_k^{\langle N\rangle}$. The same holds therefore for nuclear SF F_k^A, even for large neutron excess $\delta N/A$ [12]. Experimental evidence can be found in Fig. 2a of Ref. [11] and Table I in Ref. [9].

At this point we mention that the proper expression (3) for F_1^A actually mixes F_1^N and F_2^N, and modifies the one for F_2^A [5,3]. Below we shall mention numerical consequences.

We return to Eq. (3), which through Eq. (2a) implies

$$\kappa^A(x,Q^2)\approx\kappa^{\langle N\rangle}(x,Q^2)\approx\kappa^D(x,Q^2),$$

$$R^A(x,Q^2)\approx R(x,Q^2), \tag{4}$$

in agreement with data [1,16]. Using first the CG *relation* for nucleons

$$\epsilon_{CG}^N=\lim_{Q^2\to\infty}\kappa^N(x,Q^2)=1 \tag{5}$$

one finds from Eqs. (2b) and (4), its nuclear analog

$$\epsilon_{CG}^A\approx\lim_{Q^2\to\infty}\kappa^A(x,Q^2), \tag{6}$$

with an error, again estimated to be $\mathcal{O}(\delta N/A)$. Gluonic corrections $\mathcal{O}(\ln(Q^2))$, not given in Eqs. (5) and (6), appear to be small for x in the range $x\gtrsim 0.15$ under discussion.

Using Eq. (5), the nuclear CG *relation* (6) can be proven directly from Eq. (3) [17]. In contradistinction, the equality of nuclear and nucleonic CG *functions* (4) is compatible with Eq. (3), but does not necessarily follow from it.

First we mention a remarkable observation for the computed CG functions

$$|\kappa(x,Q^2)-1|\approx(0.11-0.12),$$

$$(0.2-0.3)\lesssim x\lesssim(0.7-0.75);\quad Q^2\geqslant 5\ \mathrm{GeV}^2. \tag{7}$$

In the indicated x-interval and over a wide Q^2-range, CG functions appear to be close to their asymptotic limit, the nuclear CG relation. It is also intriguing that without any apparent cause, a sign change occurs at a weakly Q^2-dependent $x_s\approx 0.5-0.6$. The above is in agreement with data from high energy $\nu,\bar{\nu}$ inclusive scattering (see Fig. 18 in [18]). The small $\kappa-1$ shall be shown to entail disproportionally large effects. For later use we remark on the estimated accuracy of the computed CG function (2b), which appears limited in various ranges.

(i) Disregarding other than valence quarks requires smoothing of F_k^N for $x\lesssim 0.15-0.20$, which entails the same for F_k^A. We thus prefer to use extrapolated values for nuclear CG functions, below $x\lesssim 0.15$.

(ii) Equation (3) shows that f^{PN} draws on an ever smaller support of dwindling intensity and accuracy, rendering $F_k^A(x,Q^2)$ unreliable beyond $x\geqslant 1.3-1.5$.

(iii) The parametrizations for F_2^p,F_2^D [3] hold for $Q^2\leqslant 20\ \mathrm{GeV}^2$, causing uncertainties in F_k^A for larger Q^2.

(iv) With SF for $x \geqslant 1.2$ falling orders of magnitudes from the maximum values, one expects inaccuracies if F_k^A and κ for growing x.

We mentioned above corrections to Eq. (3), consequences of which we now assess. Using the expressions given in the cited references [5,3], one first notes that the resulting changes vanish in the Bjorken limit, but have to be computed for moderate Q^2, relevant to the data.

One then infers from the above sources that the modification in the integrand for F_1^A is proportional to $\langle (p_x/M)^2 \rangle$, which is negligibly small. Finally the correction to F_2^A appears to amount to the insertion in the integrand in Eq. (3) of a factor

$$C(\rho,z) = (1 - \rho + \rho/z)^2, \tag{8}$$

with $\rho = (1 + Q^2/(2Mz)^2)^{-1}$. We considered $Q^2 = 1{,}2\,\mathrm{GeV}^2$ and the range $0.2 \lesssim x \lesssim 0.7$ in Eq. (7) and found that the effect on the CG function $\kappa(x,Q^2)$, Eq. (2b), is negligible for small x, grows slowly with x and reaches 13% for $x = 0.7$. In the indicated range the effect is hardly dependent on Q^2. As predicted, corrections decrease rapidly with increasing Q^2.

We return to the R ratio (2a) and shall now discuss three approximations R_n for $R^A \approx R$, defined by a corresponding choice for the CG function κ_n. For each of these one has from Eq. (2a)

$$R(x,Q^2) = \beta_n(x,Q^2) R_n(x,Q^2) + (\beta_n(x,Q^2) - 1). \tag{9}$$

Deviations of $\beta_n(x,Q^2) = \kappa_n(x,Q^2)/\kappa(x,Q^2)$ from 1 manifestly determine the quality of the approximation.

(A) A high-Q^2 approximation, defined by $\kappa_L = 1$ (i.e., $\beta_L = \kappa^{-1}$), approximately valid for $1 \lesssim x \lesssim 0.6$:

$$R^{\text{``exact''}}(x,Q^2) = \beta_L(x,Q^2) R_L(x,Q^2) + (\beta_L(x,Q^2) - 1) \tag{10a}$$

$$\approx R_L(x,Q^2) + (\beta_L(x,Q^2) - 1), \tag{10b}$$

$$R_L^{(1)}(x,Q^2) = \frac{4M^2x^2}{Q^2} + (\beta_L(x,Q^2) - 1), \tag{10c}$$

$$R_L^{(2)}(x,Q^2) = \frac{4M^2x^2}{Q^2}. \tag{10d}$$

Equation (10a) is the same as Eqs. (2a). The corresponding R is dubbed ''exact,'' because it results from computed values of F_k^A, Eq. (3) [12], which implies some model. $R^{\text{``exact''}}$ should be distinguished from intrinsic approximations for R.

(B) The NE approximation for $x \approx 1$ rests on the decomposition of F_k^N in Eq. (3) into p,n-weighted nucleon-elastic (NE) and nucleon-inelastic (NI) parts. Retention of the NE part generates through Eq. (3) corresponding NE parts in the *nuclear* SF, thus with $\eta = Q^2/4M^2$ [see Eq. (6a) in Ref. [9]]

$$F_1^{N(NE)}(x,Q^2) = \frac{x}{2}[G_M^N(Q^2)]^2 \delta(x-1),$$

$$F_2^{N(NE)}(x,Q^2) = \frac{[G_E^N(Q^2)]^2 + \eta [G_M^N(Q^2)]^2}{1+\eta} \delta(x-1), \tag{11a}$$

$$F_1^{A(NE)}(x,Q^2) = \frac{1}{2} f^{PN}(x,Q^2)[G_M^N(Q^2)]^2,$$

$$F_2^{A(NE)}(x,Q^2) = x f^{PN}(x,Q^2) \frac{[G_E^N(Q^2)]^2 + \eta [G_M^N(Q^2)]^2}{1+\eta}. \tag{11b}$$

The corresponding CG function can be simplified by exploiting the approximate scaling of the static electromagnetic form factors in the NE part (11a), $1/[(\mu_M^p)^2 + (\mu_M^n)^2] = 0.0874$ [19] [see remark at the end of the third paragraph before Eq. (15)]

$$\kappa_{NE}^A = 2x F_1^{A(NE)}/F_2^{A(NE)} \approx (0.0874 + \eta)/(1+\eta). \tag{12}$$

Inserting Eq. (12) into Eq. (3) gives

$$R(x,Q^2) = \beta_{NE}(x,Q^2) R_{NE}(x,Q^2) + (\beta_{NE}(x,Q^2) - 1), \tag{13a}$$

$$R_{NE}^{(1)}(x,Q^2) = \frac{0.31}{Q^2} + \left(\frac{0.31}{Q^2} + 1\right)\left(\frac{x^2-1}{1+\eta}\right), \tag{13b}$$

$$R_{NE}^{(2)}(x,Q^2) \approx \frac{0.31}{Q^2}, \tag{13c}$$

with Q^2 expressed in GeV^2. Equation (13c) is the result of Bosted *et al.* [19], while Eq. (13b) provides x-dependent corrections.

(C) An empirical estimate for moderate Q^2, which is assumed to be independent of x and A [19–21]

$$R_C(x,Q^2) \approx \frac{\delta}{Q^2}; \quad 0.2 \lesssim \delta \lesssim 0.5. \tag{14}$$

The estimates (10d) and (13c) for $x \approx 1$, and Eq. (14) predict $R \propto 1/Q^2$, but only (A) and (B) for $x \neq 1$ prescribe definite x dependence. Since by definition R depends on x, it is likely that extracted coefficients of $1/Q^2$ effectively hide actual x dependence.

Were it not for the listed inaccuracies in computed CG functions, the latter would through Eq. (2a) or Eq. (10a) provide a standard for all approximate R ratios. We now discuss those and start with the large Q^2 approximation. In view of the observation (7), the CG function $\kappa \approx 1$ holds also for moderate Q^2 and over a relatively wide x-range. For medium x^2/Q^2, which does not require large Q^2, $R \approx R^{(2)}$ may suffice. However, in the deep-inelastic region for small enough x^2/Q^2, even for a few % deviation of β_L from 1, the second part in Eq. (10c) exceeds $R_L^{(2)}$, and Eq. (10c) should therefore be used there.

In Table I we present results for relatively low x, $0.12 \lesssim x \lesssim 0.7$ and for $Q^2 \geqslant 5\,\mathrm{GeV}^2$. The first row gives $R^{\text{``exact''}}$, Eq. (10a), computed from Eq. (3), except the entry for x

TABLE II. R ratios (13b) and (13c) for $x \approx 1$, medium Q^2, and the x-independent R_c, Eq. (14). For $x=0.9,1.0$; $Q^2=5$ and $10\,\mathrm{GeV}^2$ we also entered $R^{\text{``exact''}}(x,Q^2)$. See text for discussion.

x	R	$Q^2(\mathrm{GeV}^2)$: 2	5	10
0.9	$R_{NE}^{(1)}$	<0	<0	<0
	$R_{NE}^{(2)}$	0.155	0.062	0.032
	$R^{\text{``exact''}}$		0.292	0.308
1.0	$R_{NE}^{(1)}$	0.155	0.062	0.032
	$R_{NE}^{(2)}$	0.155	0.062	0.032
	$R^{\text{``exact''}}$		0.329	0.404
1.05	$R_{NE}^{(1)}$	0.231	0.117	0.059
	$R_{NE}^{(2)}$	0.155	0.062	0.031
x	$R_C([0.4 \leqslant \delta \leqslant 0.6])$	0.2–0.3	0.08–0.12	0.04–0.06

$=0.12$ which, as explained above, has been extrapolated down from slightly larger x. The second row is the asymptotic limit $R_L^{(2)}$, Eq. (10d). We do not display $R_L^{(1)}$, since it virtually coincides with $R^{\text{``exact''}}$. One notices that for higher x, the asymptotic limit is either close to, or exceeds the exact answer. This reflects on κ, Eq. (2b) to be close to, or exceeding 1, in turn entailing a negative correction to $R_L^{(2)}$. This agrees with the observation (7). The last column contains a few scattered $\nu,\overline{\nu}$ data for the indicated x and binned $\langle Q\rangle^2$ [18,22]. Given the substantial statistical and systematic errors and the imprecisely given spreading due to binning, the agreement is reasonable.

Next we discuss the NE approximation, the validity of which depends foremost on the weight of $F_k^{N(NE)}(x,Q^2)$ in F_k^A. When using Eq. (3), that weight is determined by f^{PN}, for which there is only theoretical information. Computations show that only for $Q^2 \lesssim 2\,\mathrm{GeV}^2$, $F_k^{A(NE)}(x,Q^2)$ dominates for $x \lesssim (1.1-1.2)$. For growing Q^2 NI parts compete for ever growing x and ultimately overtake [12].

Disregarding NI contributions to R_{NE} for $x \neq 1$, corrections in the immediate neighborhood of the QEP can be estimated by choosing β_{NE} close to 1. One thus finds $R(1.05,5)/R^{NE}(1,5)=1.86$ which ratio rapidly increases with β_{NE}. One also checks from Eq. (13b) that for $1 \lesssim Q^2(\mathrm{GeV}^2) \lesssim 5$, $R_{NE}(x \lesssim 0.9,Q^2)$ reaches unphysical negative values. Only the disregarded NI part can restore R to positive values. For $1.5 \gtrsim Q^2(\mathrm{GeV}^2) \gtrsim 5$ and for instance $x=1.1$ on the elastic side of the QEP, $2 \gtrsim R_{NE}(1.1,Q^2)/R_{NE}(1,Q^2) \gtrsim 1.5$, which ratio again grows with x: NI terms may or may not off-set that growth. Table II compares the NE approximations $R_{NE}^{(1)},R_{NE}^{(2)}$ with R_C: the agreement is tolerable. Aware of the warnings after Eq. (7), we nevertheless compute and enter some ''exact'' values, which appear to exceed the NE values by far. CG functions $\kappa(1,Q^2)$ which fit R_{NE} would have to be 25–30 % larger than the computed ones, which we estimate to be outside the limits of our accuracy. In particular the negative $R_{NE}(0.9,Q^2)$ makes one believe that the NE estimates may not be precise [23,24].

Equation (13c) has been applied to extract R and F_2^A from inclusive scattering data for medium-Q^2 data for $x \approx 1$ [21,25]. Data by Bosted *et al.* for $0.75 \lesssim x \lesssim 1.15$ are quite erratic, but $R(\langle x\rangle,Q^2)$, averaged over x, shows a trend in agreement with Eq. (13c).

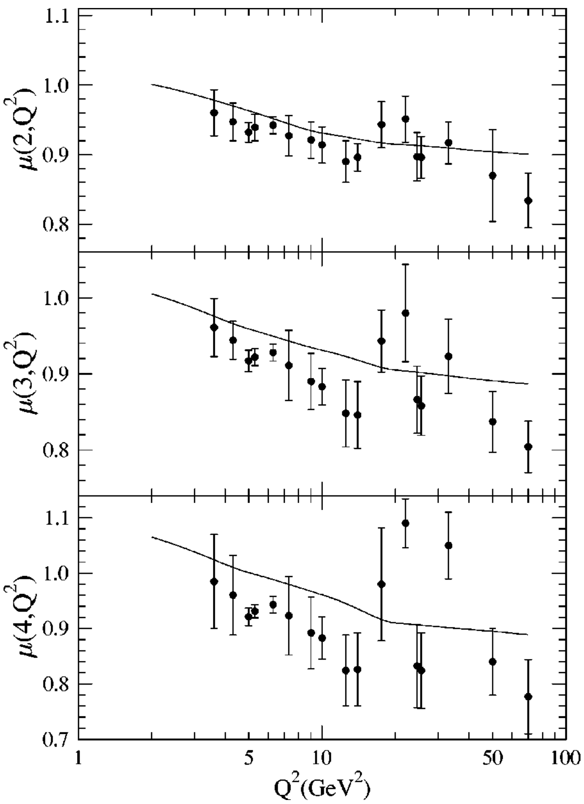

FIG. 1. Second, third, and fourth moments $\mu(m,Q^2)$. Data are taken from Ref. [28]. The drawn lines are numerical results from either Eq. (15c) or from Eq. (16c), using Eq. (15b).

In addition there are data for about the same Q^2 range, but more restricted x [25], which are in agreement with either Eq. (13c) or Eq. (14). There clearly are substantial corrections just off the QEP. In particular for the data of Bosted *et al.*, the above warns that the use of simple x-independent R ratios may lead to extracted F_2^A, which have inaccuracies, exceeding those estimated.

We now address a second topic regarding the moments of various SF

$$\mathcal{M}_k^A(m;Q^2)=\int_0^A dx x^m F_k^A(x,Q^2), \tag{15a}$$

$$\mathcal{M}_k^N(m;Q^2)=\int_0^1 dx x^m F_k^N(x,Q^2), \tag{15b}$$

$$\mu^A(m;Q^2)=\int_0^A dx x^m f^{PN}(x,Q^2). \tag{15c}$$

Moments $\mathcal{M}_k^N$ are related to higher twist corrections of SF of nucleons [26], and the same holds for their nuclear counterparts, had those been calculated in QCD. Our interest in those moments is the sensitivity of SF for large x and consequently the trust in the calculated F_k^A for that range. One readily derives from Eq. (11) [27]

$$F_k^A(0,Q^2)=\mu^A(-2+k;Q^2)F_k^N(0,Q^2), \quad (16a)$$

$$\mathcal{M}_k^A(m,Q^2)=\mu^A(m-1+k;Q^2)\mathcal{M}_k^N(m;Q^2), \quad (16b)$$

$$\mu^A(m+1;Q^2)=\frac{\mathcal{M}_1^A(m+1;Q^2)}{\mathcal{M}_1^N(m+1;Q^2)}=\frac{\mathcal{M}_2^A(m;Q^2)}{\mathcal{M}_2^N(m;Q^2)}, \quad (16c)$$

and in particular

$$\mu^A(0,Q^2)=\int_0^A dx f^{PN}(x,Q^2)=\int_0^A dx f^{as}(x)=1, \quad (17)$$

which expresses unitarity. All other relations (16) for finite Q^2 rest on the representation (3) and embody effects of the binding medium on moments of F_k^N through $\mu(n,Q^2)$. For instance, the deviation of $\mu^A(2,Q^2)$ from 1 measures the difference of the momentum fraction of a quark in a nucleus and in the nucleon at given Q^2.

We have computed the lowest moments and ratios μ from computed F_k^A, f^{PN} and parametrized F_k^N. With expected inaccuracies in F_k^A for $x \gtrsim 1.5$ one ought not to trust calculated higher moments. Yet we found consistent values for the different ratios in Eq. (16c) for $Q \leqslant 20\,\mathrm{GeV}^2$, and the moments of f^{PN}. Those for Fe are entered in Fig. 1 and agree reasonably well with the available data. We note in particular the rendition of the observed Q^2-dependence and the predicted slow vanishing for $Q^2 \to \infty$, as opposed to a results by Cothran *et al.* [28]. The authors used a generalized convolution like Eq. (3), with a Q^2-independent PWIA for f^{PN}, leading to the same for $\mu(m)$. Q^2 dependences, estimated for off-shell nucleons, produce far too small moment ratios with the wrong Q^2 behavior.

The above is reminiscent of previously considered, but not identical moments. We recall discrepancies between data and computed results for relatively low-q, longitudinal responses S_L, and the integral of the latter, the Coulomb sum rule [29,30]. All have occasionally been ascribed to the influence of the binding medium on the size of a nucleon, i.e., on the second moment of the *static* charge density. Apart from possible conventional accounts of those differences [31], one notes that Eq. (3) does not relate to static moments of charge distributions, but to dynamical SF.

The above and Refs. [9–12] conclude a program to determine observables which depend on nuclear SF, in turn computed from the basic relation (2a) between SF for composite nuclei, free nucleons, and of a nucleus composed of point nucleons. The various observables occasionally extend over wide ranges, and test to various measures the x,Q^2 dependence of F_k^A. It is gratifying to note frequently good agreement of data and predictions based on Eq. (3), which of course does not prove the underlying conjecture, but inspires trust in its approximate correctness.

The above clearly requires an explanation, because results have been obtained, circumventing QCD. It seems an attractive suggestion that in the tested x,Q^2 region, the relation (3) is the result of an effective theory, as has been argued originally [2] and somehow mimicking notions of QCD.

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