One-body density matrix and momentum distribution in s-p and s-d shell nuclei

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Analytical expressions of the one- and two-body terms in the cluster expansion of the one-body density matrix and momentum distribution of the s-p and s-d shell nuclei with $N=Z$ are derived. They depend on the harmonic oscillator parameter b and the parameter β which originates from the Jastrow correlation function. These parameters have been determined by least squares fit to the experimental charge form factors. The inclusion of short-range correlations increases the high momentum component of the momentum distribution $n(\mathbf{k})$ for all nuclei we have considered while there is an A dependence of $n(\mathbf{k})$ both at small values of k and the high momentum component. The A dependence of the high momentum component of $n(\mathbf{k})$ becomes quite small when the nuclei ^{24}Mg, ^{28}Si, and ^{32}S are treated as $1d$-$2s$ shell nuclei having the occupation probability of the $2s$ state as an extra free parameter in the fit to the form factors.

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I. INTRODUCTION

The momentum distribution (MD) is of interest in many research subjects of modern physics, including those referring to helium, electronic, nuclear, and quark systems [1–3]. In the last two decades, there has been significant effort for the determination of the MD in nuclear matter and finite nucleon systems [4–24]. MD is related to the cross sections of various kinds of nuclear reactions. The experimental evidence obtained from inclusive and exclusive electron scattering on nuclei established the existence of a high-momentum component for momenta $k>2$ fm^{-1} [25–28]. It has been shown that, in principle, mean field theories cannot describe correctly MD and density distribution simultaneously [13] and the main features of MD depend a little on the effective mean field considered [16]. The reason is that MD is sensitive to short-range and tensor nucleon-nucleon correlations which are not included in the mean field theories. Thus, theoretical approaches, which take into account short range correlations (SRC) due to the character of the nucleon-nucleon forces at small distances, are necessary to be developed.

In the various approaches, the MD of the closed shell nuclei ^{4}He, ^{16}O, and ^{40}Ca as well as of ^{208}Pb and nuclear matter is usually studied. There is no systematic study of the one body density matrix (OBDM) and MD which include both the case of closed and open shell nuclei. This would be helpful in the calculations of the overlap integrals and reactions in that region of nuclei if one wants to go beyond the mean field theories [29]. For that reason, in the present work, we attempt to find some general expressions for the OBDM $\rho(\mathbf{r},\mathbf{r}')$ and MD $n(\mathbf{k})$ which could be used both for closed and open shell nuclei. This work is a continuation of our previous study [30] on the form factors and densities of the s-p and s-d shell nuclei. The expression of $\rho(\mathbf{r},\mathbf{r}')$ was found, first, using the factor cluster expansion of Clark and co-workers [31–33] and Jastrow correlation function which introduces SRC for closed shell nuclei and then was extrapolated to the case of $N=Z$ open shell nuclei. $n(\mathbf{k})$ was found by Fourier transform of $\rho(\mathbf{r},\mathbf{r}')$. These expressions are functionals of the harmonic oscillator (HO) orbitals and depend on the HO parameter b and the correlation parameter β. The values of the parameters b and β, which we have used for the closed shell nuclei ^{4}He, ^{16}O, and ^{40}Ca, are the ones which have been determined in Ref. [30] by fit of the theoretical $F_{ch}(q)$, derived with the same cluster expansion, to the experimental one. For the open shell nuclei ^{12}C, ^{24}Mg, ^{28}Si, and ^{32}S we provide new values for these parameters, which have been found to give a better fit to the experimental form factors than in our previous analysis [30]. It is found that the high-momentum tail of the MD of all the nuclei we have considered appears for $k>2$ fm^{-1} and also there is an A dependence of the values of $n(k)$ for 2 fm$^{-1}<k<5$ fm^{-1}. This A dependence of MD was first investigated considering ^{24}Mg, ^{28}Si, and ^{32}S as $1d$ shell nuclei. Next we treated the above nuclei as $1d$-$2s$ shell nuclei having the occupation probability of the $2s$ state as an extra free parameter in the fit of the form factors. The A dependence is quite small in the second case.

The paper is organized as follows. In Sec. II the general expressions of the correlated OBDM and MD are derived using a Jastrow correlation function. In Sec. III the analytical expressions of the above quantities for the s-p and s-d shell nuclei, in the case of the HO orbitals, are given. Numerical results are reported and discussed in Sec. IV, while the summary of the present work is given in Sec. V.

II. CORRELATED ONE-BODY DENSITY MATRIX AND MOMENTUM DISTRIBUTION

A nucleus with A nucleons is described by the wave function $\Psi(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_A)$ which depends on $3A$ coordinates as well as on spins and isospins. The evaluation of the single particle characteristics of the system needs the one-body density matrix [34,35]

$$\rho(\mathbf{r},\mathbf{r}')=\int \Psi^*(\mathbf{r},\mathbf{r}_2,\ldots,\mathbf{r}_A)\times\Psi(\mathbf{r}',\mathbf{r}_2,\ldots,\mathbf{r}_A)d\mathbf{r}_2\cdots d\mathbf{r}_A, \tag{1}$$

where the integration is carried out over the radius vectors $\mathbf{r}_2,\ldots,\mathbf{r}_A$ and summation over spin and isospin variables is implied. $\rho(\mathbf{r},\mathbf{r}')$ can also be represented by the form

$$\rho(\mathbf{r},\mathbf{r}')=\frac{\langle\Psi|\mathbf{O}_{\mathbf{rr}'}(A)|\Psi'\rangle}{\langle\Psi|\Psi\rangle}=N\langle\Psi|\mathbf{O}_{\mathbf{rr}'}(A)|\Psi'\rangle=N\langle\mathbf{O}_{\mathbf{rr}'}(A)\rangle, \tag{2}$$

where $\Psi'=\Psi(\mathbf{r}_1',\mathbf{r}_2',\ldots,\mathbf{r}_A')$ and N is the normalization factor. The one-body ''density operator'' $\mathbf{O}_{\mathbf{rr}'}(A)$, has the form

$$\mathbf{O}_{\mathbf{rr}'}(A)=\sum_{i=1}^{A}\delta(\mathbf{r}_i-\mathbf{r})\delta(\mathbf{r}_i'-\mathbf{r}')\prod_{j\neq i}^{A}\delta(\mathbf{r}_j-\mathbf{r}_j'). \tag{3}$$

The diagonal elements of the OBDM give the density distribution $\rho(\mathbf{r},\mathbf{r})=\rho(\mathbf{r})$, while the MD is given by the Fourier transform of $\rho(\mathbf{r},\mathbf{r}')$,

$$n(\mathbf{k})=\frac{1}{(2\pi)^3}\int\exp[i\mathbf{k}(\mathbf{r}-\mathbf{r}')]\rho(\mathbf{r},\mathbf{r}')d\mathbf{r}\,d\mathbf{r}'. \tag{4}$$

If we denote the model operator, which introduces SRC, by $\mathcal{F}$, an eigenstate Φ of the model system corresponds to an eigenstate

$$\Psi=\mathcal{F}\Phi \tag{5}$$

of the true system.

Several restrictions can be made on the model operator $\mathcal{F}$ [36] and it is required that $\mathcal{F}$ be translationally invariant and symmetrical in its arguments $1\cdots i\cdots A$ and possesses the cluster property [33].

In order to evaluate the correlated one-body density matrix $\rho_{\rm cor}(\mathbf{r},\mathbf{r}')$, we consider, first, the generalized integral

$$I(\alpha)=\langle\Psi|\exp[\alpha I(0)\mathbf{O}_{\mathbf{rr}'}(A)]|\Psi'\rangle, \tag{6}$$

corresponding to the one-body ''density operator'' $\mathbf{O}_{\mathbf{rr}'}(A)$, from which we have

$$\langle\mathbf{O}_{\mathbf{rr}'}(A)\rangle=\left[\frac{\partial\ln I(\alpha)}{\partial\alpha}\right]_{\alpha=0}. \tag{7}$$

For the cluster analysis of Eq. (7), we consider the subproduct integrals $I_i(\alpha)$, $I_{ij}(\alpha)$, ..., for the subsystems of the A-nucleons system corresponding to the density operators $\mathbf{O}_{\mathbf{rr}'}(1),\mathbf{O}_{\mathbf{rr}'}(2),\ldots$. The factor cluster decomposition of these integrals, following the factor cluster expansion of Ristig, Ter Low, and Clark [31–33], gives

$$\langle\mathbf{O}_{\mathbf{rr}'}\rangle=\langle\mathbf{O}_{\mathbf{rr}'}\rangle_1+\langle\mathbf{O}_{\mathbf{rr}'}\rangle_2+\cdots+\langle\mathbf{O}_{\mathbf{rr}'}\rangle_A. \tag{8}$$

Three- and many-body terms will be neglected in the present analysis. Thus, in the two-body approximation, $\rho_{\rm cor}(\mathbf{r},\mathbf{r}')$, defined by Eq. (2), is written

$$\rho_{\rm cor}(\mathbf{r},\mathbf{r}')\approx N[\langle\mathbf{O}_{\mathbf{rr}'}\rangle_1+\langle\mathbf{O}_{\mathbf{rr}'}\rangle_{22}-\langle\mathbf{O}_{\mathbf{rr}'}\rangle_{21}], \tag{9}$$

where $\langle\mathbf{O}_{\mathbf{rr}'}\rangle_1=\rho_{\rm SD}(\mathbf{r},\mathbf{r}')$, the uncorrelated OBDM associated with the Slater determinant and

$$\langle\mathbf{O}_{\mathbf{rr}'}\rangle_{22}=\sum_{i<j}^{A}\langle ij|\mathcal{F}^{\dagger}(r_{12})\mathbf{O}_{\mathbf{rr}'}(2)\mathcal{F}(r_{12}')|i'j'\rangle_a. \tag{10}$$

The term $\langle\mathbf{O}_{\mathbf{rr}'}\rangle_{21}$ is as the term $\langle\mathbf{O}_{\mathbf{rr}'}\rangle_{22}$ without the operator $\mathcal{F}(r_{12})$. If the two-body operator $\mathcal{F}(r_{12})$ is taken to be the Jastrow correlation function [37], $f(r_{ij})=1-\exp[-\beta(\mathbf{r}_i-\mathbf{r}_j)^2]$, then

$$\mathcal{F}^{\dagger}(r_{12})\mathbf{O}_{\mathbf{rr}'}(2)\mathcal{F}(r_{12}')=\mathbf{O}_{\mathbf{rr}'}(2)[1-g_1(\mathbf{r},\mathbf{r}',\mathbf{r}_2)-g_2(\mathbf{r},\mathbf{r}',\mathbf{r}_2)+g_3(\mathbf{r},\mathbf{r}',\mathbf{r}_2)], \tag{11}$$

where

$$g_1(\mathbf{r},\mathbf{r}',\mathbf{r}_2)=\exp[-\beta(r^2+r_2^2)]\exp[2\beta\mathbf{r}\mathbf{r}_2],$$

$$g_2(\mathbf{r},\mathbf{r}',\mathbf{r}_2)=g_1(\mathbf{r}',\mathbf{r},\mathbf{r}_2),$$

$$g_3(\mathbf{r},\mathbf{r}',\mathbf{r}_2)=\exp[-\beta(r^2+r'^2)]\exp[-2\beta r_2^2]\times\exp[2\beta(\mathbf{r}+\mathbf{r}')\mathbf{r}_2], \tag{12}$$

and $\rho_{\rm cor}(\mathbf{r},\mathbf{r}')$ takes the form

$$\rho_{\rm cor}(\mathbf{r},\mathbf{r}')\approx N[\langle\mathbf{O}_{\mathbf{rr}'}\rangle_1-O_{22}(\mathbf{r},\mathbf{r}',g_1)-O_{22}(\mathbf{r},\mathbf{r}',g_2)+O_{22}(\mathbf{r},\mathbf{r}',g_3)], \tag{13}$$

where

$$O_{22}(\mathbf{r},\mathbf{r}',g_l)=\sum_{i<j}^{A}\langle ij|\mathbf{O}_{\mathbf{rr}'}(2)g_l(\mathbf{r},\mathbf{r}',\mathbf{r}_2)|i'j'\rangle_a=\int g_l(\mathbf{r},\mathbf{r}',\mathbf{r}_2)[\rho_{\rm SD}(\mathbf{r},\mathbf{r}')\rho_{\rm SD}(\mathbf{r}_2,\mathbf{r}_2)-\rho_{\rm SD}(\mathbf{r},\mathbf{r}_2)\rho_{\rm SD}(\mathbf{r}_2,\mathbf{r}')]d\mathbf{r}_2. \tag{14}$$

In the above expression of $\rho_{\rm cor}(\mathbf{r},\mathbf{r}')$, the one-body contribution to the OBDM is well known and is given by the equation

$$\langle\mathbf{O}_{\mathbf{rr}'}\rangle_1=\rho_{\rm SD}(\mathbf{r},\mathbf{r}')=\frac{1}{\pi}\sum_{nl}\eta_{nl}(2l+1)\phi_{nl}^{*}(r)\phi_{nl}(r')P_l(\cos\omega_{rr'}), \tag{15}$$

where η_{nl} are the occupation probabilities of the states nl (0 or 1 in the case of closed shell nuclei) and $\phi_{nl}(r)$ is the radial part of the SP wave function and $\omega_{rr'}$ the angle between the vectors $\mathbf{r}$ and $\mathbf{r}'$.

The term $O_{22}(\mathbf{r},\mathbf{r}',g_l)$, performing the spin-isospin summation and the angular integration, takes the general form

$$O_{22}(\mathbf{r},\mathbf{r}',g_l)=4\sum_{n_il_i,n_jl_j}\eta_{n_il_i}\eta_{n_jl_j}(2l_i+1)(2l_j+1)\times\Bigg[4A^{n_il_in_jl_j,0}_{n_il_in_jl_j}(\mathbf{r},\mathbf{r}',g_l)-\sum_{k=0}^{l_i+l_j}\langle l_i0l_j0|k0\rangle^2\times A^{n_jl_jn_il_i,k}_{n_il_in_jl_j}(\mathbf{r},\mathbf{r}',g_l)\Bigg],\quad l=1,2,3, \tag{16}$$

where

$$A^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{r},\mathbf{r}',g_1)=\frac{1}{4\pi}\phi^*_{n_1l_1}(r)\phi_{n_3l_3}(r')\exp[-\beta r^2]\times P_{l_3}(\cos\omega_{rr'})\int_0^\infty\phi^*_{n_2l_2}(r_2)\phi_{n_4l_4}(r_2)\times\exp[-\beta r_2^2]i_k(2\beta rr_2)r_2^2dr_2, \tag{17}$$

and the matrix element $A^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{r},\mathbf{r}',g_2)$ can be found from Eq. (17) replacing $\mathbf{r}\leftrightarrow\mathbf{r}'$ and $n_1l_1\leftrightarrow n_3l_3$ while the matrix element corresponding to the factor g_3 is

$$A^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{r},\mathbf{r}',g_3)=\frac{1}{4\pi}\phi^*_{n_1l_1}(r)\phi_{n_3l_3}(r')\times\exp[-\beta(r^2+r'^2)]\Omega^k_{l_1l_3}(\omega_{rr'})\times\int_0^\infty\phi^*_{n_2l_2}(r_2)\phi_{n_4l_4}(r_2)\exp[-2\beta r_2^2]\times i_k(2\beta|\mathbf{r}+\mathbf{r}'|r_2)r_2^2dr_2. \tag{18}$$

In Eqs. (17) and (18) $i_k(z)$ is the modified spherical Bessel function, while the factor $\Omega^k_{l_1l_3}(\omega_{rr'})$ depends on the directions of $\mathbf{r}$ and $\mathbf{r}'$.

The expression of the term $O_{22}(\mathbf{r},\mathbf{r}',g_l)$ depends on the SP wave functions and so it is suitable to be used for analytical calculations with the HO orbitals and in principle for numerical calculations with more realistic SP orbitals. Expressions (15) and (16) were derived for the closed shell nuclei with $N=Z$, where η_{nl} is 0 or 1. For the open shell nuclei (with $N=Z$) we use the same expressions, where now $0\leqslant\eta_{nl}\leqslant 1$. In this way the mass dependence of the correlation parameter β and the OBDM or MD can be studied.

It should be noted that Eq. (16) is also valid for the cluster expansion of the density distribution and the form factor as it has been found in Ref. [30] and also in the cluster expansion of the MD. The only difference is the expressions of the matrix elements A.

The MD for the above mentioned nuclei can be found by taking the Fourier transform of $\rho(\mathbf{r},\mathbf{r}')$. It takes the form

$$n_{\rm cor}(\mathbf{k})\approx N[\langle\tilde{\mathbf{O}}_{\mathbf{k}}\rangle_1-2\tilde{O}_{22}(\mathbf{k},g_1)+\tilde{O}_{22}(\mathbf{k},g_3)], \tag{19}$$

where $\langle\tilde{\mathbf{O}}_{\mathbf{k}}\rangle_1=n_{\rm SD}(\mathbf{k})$ and the term $\tilde{O}_{22}(\mathbf{k},g_l)$, as in the case of OBDM, is given again by the right-hand side of Eq. (16) replacing the matrix elements $A^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{r},\mathbf{r}',g_l)$, defined by Eqs. (17) and (18), by the Fourier transform of them, that is by the matrix elements

$$\tilde{A}^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{k},g_l)=\frac{1}{(2\pi)^3}\int A^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{r},\mathbf{r}',g_l)\times\exp[i\mathbf{k}(\mathbf{r}-\mathbf{r}')]d\mathbf{r}d\mathbf{r}',\quad l=1,3. \tag{20}$$

As in the case of the OBDM, expression (19) is suitable for the study of the MD for the s-p and s-d shell nuclei and also for the study of the mass dependence of the kinetic energy of these nuclei. The mean value of the kinetic energy is given by the right-hand side of Eq. (19) replacing $\langle\tilde{\mathbf{O}}_{\mathbf{k}}\rangle_1$ by $\langle\mathbf{T}\rangle_1$ and $\tilde{O}_{22}(\mathbf{k},g_l)$ by $T_{22}(g_l)$, where

$$\langle\mathbf{T}\rangle_1=\frac{\hbar^2}{2m}\int k^2n_{\rm SD}(\mathbf{k})d\mathbf{k},$$

$$T_{22}(g_l)=\frac{\hbar^2}{2m}\int k^2\tilde{O}_{22}(\mathbf{k},g_l)d\mathbf{k},\quad l=1,3. \tag{21}$$

III. ANALYTICAL EXPRESSIONS

In the case of the HO wave functions, analytical expressions of the one-body terms $\langle\mathbf{O}_{\mathbf{rr}'}\rangle_1$ and $\langle\tilde{\mathbf{O}}_{\mathbf{k}}\rangle_1$ as well as of the matrix elements $A^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{r},\mathbf{r}',g_l)$ and $\tilde{A}^{n_3l_3n_4l_4,k}_{n_1l_1n_2l_2}(\mathbf{k},g_l)$ can be found [38]. From these expressions, the analytical expressions of the terms $O_{22}(\mathbf{r},\mathbf{r}',g_l)$ and $\tilde{O}_{22}(\mathbf{k},g_l)$, defined by Eq. (16), can also be found [38].

The expressions of the one-body terms $\langle\mathbf{O}_{\mathbf{rr}'}\rangle_1$ and $\langle\tilde{\mathbf{O}}_{\mathbf{k}}\rangle_1$ have the forms

$$\langle\mathbf{O}_{\mathbf{rr}'}\rangle_1=\rho_{\rm SD}(\mathbf{r},\mathbf{r}')=\frac{2}{\pi^{3/2}b^3}\Big[2\eta_{1s}+3\eta_{2s}-2\eta_{2s}(r_b^2+r_b'^2)+4\eta_{1p}r_br_b'\cos\omega_{rr'}+\frac{4}{3}[\eta_{2s}+\eta_{1d}(3\cos^2\omega_{rr'}-1)\times r_b^2r_b'^2]\Big]\exp[-(r_b^2+r_b'^2)/2], \tag{22}$$

$$\langle\tilde{\mathbf{O}}_{\mathbf{k}}\rangle_1=n_{\rm SD}(\mathbf{k})=\frac{2b^3}{\pi^{3/2}}\exp[-k_b^2]\sum_{k=0}^{2}C_{2k}k_b^{2k}, \tag{23}$$

where $C_0=2\eta_{1s}+3\eta_{2s}$, $C_2=4(\eta_{1p}-\eta_{2s})$, and $C_4=\frac{4}{3}(2\eta_{1d}+\eta_{2s})$.

TABLE I. The values of the parameters b and β, of the mean kinetic energy per nucleon $\langle \mathbf{T} \rangle$, and of the rms charge radii $\langle r_{ch}^2 \rangle^{1/2}$, for various s-p and s-d shell nuclei, determined by a fit to the experimental $F_{ch}(q)$. Case 1 refers to the HO wave function without SRC and case 2 when SRC's are included. Case 2* is the same as case 2 but with the occupation probability of the state 2s taken to be a free parameter. The experimental rms charge radii are from Ref. [40].

Case	Nucleus	b [fm]	β [fm^{-2}]	$\langle \mathbf{T} \rangle$ [MeV]			$\langle r_{ch}^2 \rangle^{1/2}$ [fm]	
				HO	SRC	Total	Theor.	Expt.
1	^{4}He	1.4320		15.166		15.166	1.7651	1.676(8)
2	^{4}He	1.1732	2.3126	22.594	7.310	29.904	1.6234	
1	^{12}C	1.6251		17.010		17.010	2.4901	2.471(6)
2	^{12}C	1.5190	2.7468	19.469	6.111	25.580	2.4261	
1	^{16}O	1.7610		15.044		15.044	2.7377	2.730(25)
2	^{16}O	1.6507	2.4747	17.121	6.493	23.614	2.6802	
1	^{24}Mg	1.8495		16.162		16.162	3.1170	3.075(15)
2	^{24}Mg	1.8103	4.2275	16.870	4.239	21.109	3.0948	
2*	^{24}Mg	1.7473	2.4992	18.109	6.505	24.614	3.0638	
1	^{28}Si	1.8941		16.099		16.099	3.2570	3.086(18)
2	^{28}Si	1.8236	3.0020	17.369	5.564	22.933	3.2159	
2*	^{28}Si	1.7774	2.4440	18.283	6.922	25.205	3.1835	
1	^{32}S	2.0016		14.878		14.878	3.4830	3.248(11)
2	^{32}S	1.9368	3.0659	15.891	4.976	20.867	3.4425	
2*	^{32}S	1.8121	2.6398	18.154	6.761	24.915	3.2822	
1	^{36}Ar	1.8800		17.273		17.273	3.3270	3.327(15)
2	^{36}Ar	1.8007	2.2937	18.827	8.590	27.417	3.3343	
1	^{40}Ca	1.9453		16.437		16.437	3.4668	3.479(3)
2	^{40}Ca	1.8660	2.1127	17.863	8.754	26.617	3.5156	

The substitution of $A^{n_3 l_3 n_4 l_4, k}_{n_1 l_1 n_2 l_2}(\mathbf{r},\mathbf{r}',g_l)$ to the expression of $O_{22}(\mathbf{r},\mathbf{r}',g_l)$ which is given by Eq. (16) leads to the analytical expression of the two-body term of the OBDM, which is of the form

$$O_{22}(\mathbf{r}_b,\mathbf{r}'_b)=f_1(r_b,r'_b,\cos\omega_{rr'})\exp\left[-\frac{1+3y}{2(1+y)}r_b^2-\frac{1}{2}r_b'^2\right] +f_1(r'_b,r_b,\cos\omega_{rr'})\exp\left[-\frac{1+3y}{2(1+y)}r_b'^2 -\frac{1}{2}r_b^2\right]+f_3(r_b,r'_b,\cos\omega_{rr'})\exp\left[-\frac{1+2y}{2} \times(r_b^2+r_b'^2)\right]\exp\left[\frac{y^2}{1+2y}(\mathbf{r}_b+\mathbf{r}'_b)^2\right], \quad (24)$$

where $f_l(r_b,r'_b,\cos\omega_{rr'})$, $(l=1,3)$ are polynomials of r_b, r'_b $(r_b=r/b)$, and $\cos\omega_{rr'}$ which depend also on $y=\beta b^2$ and the occupation probabilities of the various states.

The substitution of $\tilde{A}^{n_3 l_3 n_4 l_4, k}_{n_1 l_1 n_2 l_2}(\mathbf{k},g_l)$ to the expression of $\tilde{O}_{22}(\mathbf{k},g_l)$ leads to the analytical expression of the two-body term of the MD, which is of the form

$$\tilde{O}_{22}(k)=\tilde{f}_1(k_b^2)\exp\left[-\frac{1+2y}{1+3y}k_b^2\right]+\tilde{f}_3(k_b^2)\exp\left[-\frac{1}{1+2y}k_b^2\right], \quad (25)$$

where $\tilde{f}_l(k_b^2)$, $(l=1,3)$ are polynomials of k_b^2 $(k_b=bk)$ which depend also on $y=\beta b^2$ and the occupation probabilities of the various states. Similar expressions have been found for the mean value of the kinetic energy.

IV. RESULTS AND DISCUSSION

The calculations of the MD for the various s-p and s-d shell nuclei, with $N=Z$, have been carried out on the basis of Eq. (19) and the analytical expressions of the one- and two-body terms which were given in Sec. III. Two cases have been examined, named case 1 and case 2 corresponding to

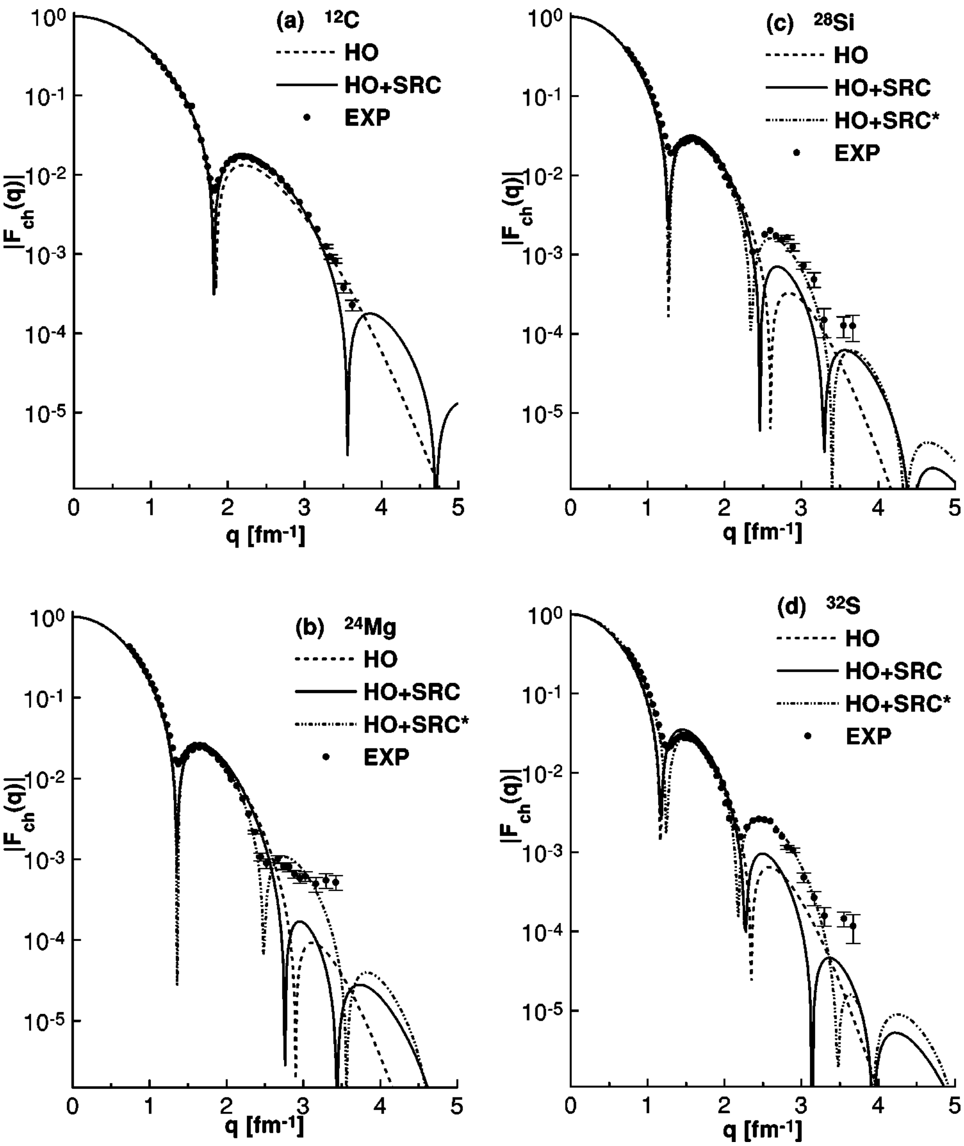

FIG. 1. The charge form factor of the nuclei: ^{12}C (a), ^{24}Mg (b), ^{28}Si (c), and ^{32}S (d) for various cases. Case HO+SRC* corresponds to the case when the occupation probability η_{2s} is treated as a free parameter. The experimental points of ^{12}C are from Ref. [41] and for the other nuclei from Ref. [42].

the analytical calculations with HO orbitals without and with SRC, respectively.

The parameters b and β of the model in case 1 and for ^{4}He, ^{16}O, ^{36}Ar, and ^{40}Ca in case 2 were the ones which have been determined in our previous work [30] by fit of the theoretical $F_{ch}(q)$, derived with the same cluster expansion, to the experimental one. These values of the parameters are given in Table I. The values of the correlation parameter β of the open shell nuclei which have been reported in Ref. [30] were quite large. That is the correlations for these nuclei were quite small. The MD of the open shell nuclei, which we found with these values of the parameters, had a high momentum tail at values of k larger than expected. As that seems to us quite unreasonable we tried to redetermine more carefully the parameters of the model by fit of the theoretical $F_{ch}(q)$ to the experimental one in order to obtain a better fit.

The new values of b and β for case 2 and for ^{12}C, ^{24}Mg, ^{28}Si, and ^{32}S are shown in Table I. The theoretical $F_{ch}(q)$ for these nuclei, which are shown in Fig. 1, are closer to the experimental data than they were in Ref. [30]. From the values of χ^2, which have been found in cases 1 and 2 and also from Fig. 1 it can been seen that the inclusion of SRC's improves the fit of the form factor of the above mentioned nuclei. Also, all the diffraction minima, even the third one which seems to exist in the experimental data of ^{24}Mg, ^{28}Si, and ^{32}S are reproduced in the correct place.

Although the values of the parameters b and β, for the open shell nuclei, are different from those reported in Ref. [30], their behavior, still, indicates that there should be a shell effect in the case of closed shell nuclei. This behavior has an effect on the MD of nuclei as it is seen from Fig. 2, where the MD, of the various s-p and s-d shell nuclei calculated with the values of b and β of Table I for case 2, have been plotted. It is seen that the inclusion of SRC's increases considerably the high momentum component of $n(k)$, for all nuclei we have considered. Also, while the general structure of the high momentum component of the MD for $A=4$, 12, 16, 24, 28, 32, 36, 40, is almost the same, in agreement with other studies [2,4,12,39], there is an A dependence of $n(k)$ both at small values of k and in the region 2 fm$^{-1}<k<5$ fm^{-1}. The A dependence of the high momentum component of $n(k)$ is larger in the open shell nuclei than in the

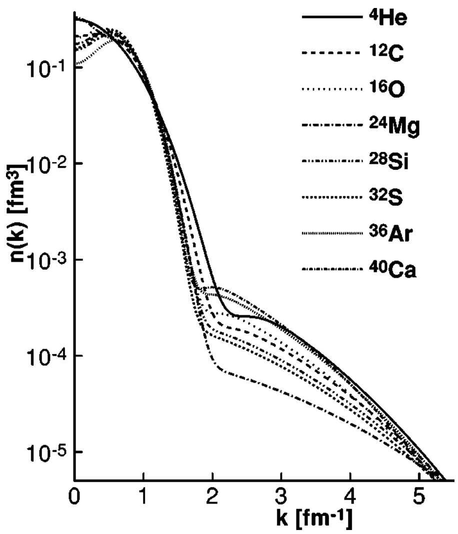

FIG. 2. The correlated momentum distribution for various s-p and s-d shell nuclei calculated with the parameters b and β of case 2 when the nuclei ^{24}Mg, ^{28}Si, ^{32}S, and ^{36}Ar were treated as $1d$ shell nuclei. The normalization is $\int n(\mathbf{k})d\mathbf{k}=1$.

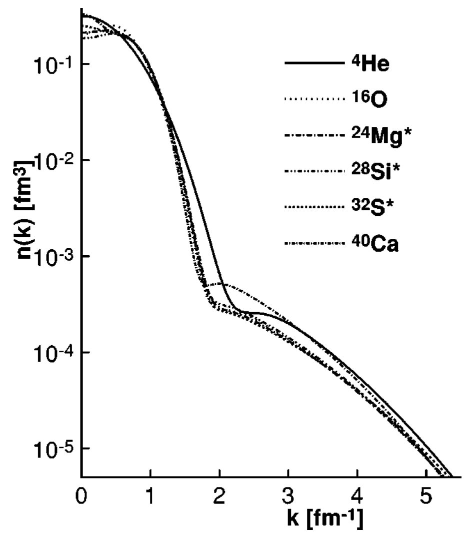

FIG. 3. The correlated momentum distribution for the closed shell nuclei ^{4}He, ^{16}O, ^{40}Ca calculated as in Fig. 2 and for the open shell nuclei ^{24}Mg, ^{28}Si, ^{32}S calculated with the parameters b, β, and η_{2s} of case 2* when they were treated as $1d$-$2s$ shell nuclei. The normalization is as in Fig. 2.

closed shell nuclei. It is seen that the high momentum component is almost the same for the closed shell nuclei ^{4}He, ^{16}O, and ^{40}Ca as expected from other studies [2,4,39].

In the previous analysis, the nuclei ^{24}Mg, ^{28}Si, and ^{32}S were treated as $1d$ shell nuclei, that is, the occupation probability of the $2s$ state was taken to be zero. The formalism of the present work has the advantage that the occupation probabilities of the various states can be treated as free parameters in the fitting procedure of $F_{ch}(q)$. Thus, the analysis can be made with more free parameters. For that reason we considered case 2* in which the occupation probability η_{2s} of the nuclei ^{24}Mg, ^{28}Si, and ^{32}S was taken to be a free parameter together with the parameters b and β. We found that the χ^2 values become better, compared to those of case 2 and the A dependence of the parameter β is not so large as it was before. The new values of b and β are shown in Table I and the theoretical $F_{ch}(q)$ in Fig. 1. The values of the occupation probability η_{2s} of the abovementioned three nuclei are 0.19982, 0.17988, and 0.50921, respectively, while the corresponding values of η_{1d}, which can be found from the values of η_{2s} through the relation $\eta_{1d}=[(Z-8)-2\eta_{2s}]/10$, are 0.36004, 0.56402, and 0.69816, respectively. The MD of these three nuclei together with the closed shell nuclei ^{4}He, ^{16}O, and ^{40}Ca found in case 2 are shown in Fig. 3. It is seen that the A dependence of the high momentum component is now not so large as it was in case 2. As $F_{ch}(q)$ calculated in case 2* is closer to the experimental data than in case 2, we might say that this result is in the correct direction, that is the high momentum component of the MD of nuclei is almost the same. We would like to mention that experimental data for $n(k)$ are not directly measured but are obtained by means of y-scaling analysis [28] and only for ^{4}He and ^{12}C in s-p and s-d shell region. We expect that the above conclusion could be corroborated if new experimental data are obtained in the future for MD for several nuclei and we carry out a simultaneous fit both to MD and to form factors.

Finally, in Table I we give the one and the two-body terms of the mean kinetic energy $\langle \mathbf{T} \rangle$ of the various s-p and s-d shell nuclei calculated on the basis of Eq. (21), as well as the rms charge radii $\langle r_{ch}^2 \rangle^{1/2}$ which are compared with the experimental values. It is seen that the introduction of SRC's (in case 2) increases the mean kinetic energy relative to case $1((\langle \mathbf{T}_{\text{case 2}} \rangle - \langle \mathbf{T}_{\text{case 1}} \rangle)/\langle \mathbf{T}_{\text{case 2}} \rangle)$ about 50% in ^{4}He and 23% in ^{24}Mg. This relative increase follows the fluctuation of the parameter β. Also the values of the kinetic energy in percents, $100\langle \mathbf{T}_{\text{SRC}} \rangle / \langle \mathbf{T}_{\text{total}} \rangle$, as well as the ratio $\langle \mathbf{T}_{\text{total}} \rangle / \langle \mathbf{T}_{\text{HO}} \rangle$ follow the fluctuation of the parameter β. In closed shell nuclei there is an increase of the above values by increasing of the mass number.

V. SUMMARY

In the present work, general expressions for the correlated OBDM and MD have been found using the factor cluster expansion of Clark and co-workers. These expressions can be used for analytical calculations, with HO orbitals and in principle for numerical calculations with more realistic orbitals.

The analytical expressions of the OBDM, MD, and mean kinetic energy for the s-p and s-d shell nuclei, which have been found, are functions of the HO parameter b, the correlation parameter β, and the occupation probabilities of the various states. These expressions are suitable for the systematic study of the above quantities for the $N=Z$, s-p and s-d shell nuclei and also for the study of the dependence of these quantities on the various parameters.

It is found that, while the general structure of the MD at high momenta is almost the same for all the nuclei we have considered, in agreement with other studies, there is an A

dependence on $n(k)$ both at small values of k and the high momentum component. The A dependence of the high momentum component becomes quite small if the occupation probability of the $2s$ state for the nuclei ^{24}Mg, ^{28}Si, and ^{32}S is treated as a free parameter in the fitting procedure of the charge form factor.

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