PHYSICAL REVIEW C, VOLUME 62, 031901(R)

Choice of colliding beams to study deformation effects in relativistic heavy ion collisions

S. Das Gupta and C. Gale
Physics Department, McGill University, 3600 University St., Montréal, Québec, Canada H3A 2T8

(Received 6 March 2000; published 23 August 2000)

It has been suggested that collisions between deformed shapes will lead to interesting effects on various observables such as K production and elliptic flow. Simple formulas can be written down which show how to choose the colliding beams which will maximize the effects of deformation.

PACS number(s): 25.75.−q, 21.60.Ev, 24.10.Lx

In the nuclear periodic table there are large regions where nuclei are deformed with prolate intrinsic shapes. Recent calculations [1,2] suggest that if one can arrange to have tip-on-tip collisions (collision axis=symmetry axis) of two prolate shapes, the physical results are significantly different from when the collisions are side-on-side (the collision axis is perpendicular to the symmetry axis). If unpolarized beams are used, interesting events will be buried among many uninteresting ones. The occurrence of the interesting events can be enhanced by using polarized beams. We perform a quantitative estimate here.

For an unpolarized beam, the average density is spherical. One still will see effects of deformation because we shall assume that heavy ion collisions sample many-body correlations contained in $|\Psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_A)|^2$: the collision knocks out all particles. Having a particle at $\vec{r}_1$ influences the probability of seeing a particle at $\vec{r}_2$, etc. But not having control of the overall orientation is a problem: we would, for example, like to enhance the chance of tip-on-tip collision. It is then necessary to control even the one-body density.

We use the rotational model [3] to extract answers. The wave function of the ground state of an odd-A deformed nucleus can be written as

$$\Psi_{IMK}=\sqrt{\frac{2I+1}{16\pi^2}}\,[D^I_{MK}{}^*(\Omega)\Phi_K(x') + (-)^{I-K}D^I_{M-K}{}^*(\Omega)\Phi_{-K}(x')]. \tag{1}$$

The symmetrization will play no role in what follows so we will use

$$\Psi_{IMK}=\sqrt{\frac{2I+1}{8\pi^2}}D^I_{M,K}{}^*(\Omega)\Phi_K(x'). \tag{2}$$

We use the convention of Rose [4] for D functions. The quantity $\Phi_K(x')$ consists of two parts:

$$\Phi_K(x')=\Phi_0(x')\phi_k(x'). \tag{3}$$

The $\Phi_0(x')$ is the intrinsic deformed state of the even-even core and $\phi_k(x')$ is the Nilsson model type orbital: $\phi_k(x')=\Sigma_j c_{jk}|jk\rangle$. One may wish to include antisymmetrization between the core and the extra nucleon but for what we do later this will not matter. For many applications $\Phi_0(x')$ plays no role and is suppressed. As usual, $\Omega\equiv\alpha,\beta,\gamma$ are the Eulerian angles specifying the orientation of the deformed intrinsic state with respect to the lab. All of this is, of course, a very standard rotational model.

We need to choose I,M such that the one-body density in the lab is as deformed as possible. This is not so transparent. However, if the expectation value of the quadrupole moment is large the density should show large deformation since the quadrupole mode is the basic deviation from sphericity. To evaluate the expectation value of the quadrupole operator $r^2Y_{20}(x)$ in the lab we express the operator in the body-fixed system: $r^2Y_{20}(x)=\Sigma_m D^2_{0m}{}^*(\Omega)r^2Y_{2m}(x')$. One then obtains

$$\langle\Psi_{IMK}|r^2Y_{20}(x)|\Psi_{IMK}\rangle=(I2M0|IM)(I2K0|IK) \times\langle\Phi_K(x')|r^2Y_{20}(x')|\Phi_K(x')\rangle. \tag{4}$$

In this equation, the first two terms on the right-hand side are Clebsch-Gordan coefficients, the third one is the deformation of the intrinsic state. One might want to argue that the last orbital is just one of many orbitals and can be dropped: $\langle\Phi_K(x')|r^2Y_{20}(x')|\Phi_K(x')\rangle\approx\langle\Phi_0(x')|r^2Y_{20}(x')|\Phi_0(x')\rangle$. In that case the only role of the last odd particle is to align the nucleus. The product of the two Clebsch-Gordan coefficients is the reduction in perfect alignment brought about by quantum mechanics.

Since we are in the ground state, $K=I$, the key factor is

$$R=(I2M0|IM)(I2I0|II). \tag{5}$$

Clearly for $M=I$ one has approximate alignment in the direction of the symmetry axis. The value of $(I2I0|II)^2$ is $[I(2I-1)]/[(I+1)(2I+3)]$. This goes to 1 as $I\to\infty$. This is the limit at which the frequency of the tip-on-tip collision is 100%.

For an odd-A nucleus, assuming the direction of z is defined by the collision axis, we need to have $|I,1/2\rangle$ to have the ‘‘best’’ body-on-body collisions. The reduction factor is simply calculated by the above formula.

The arguments presented here for odd-A nuclei should hold for odd-odd nuclei as well. It is advantageous to choose a nucleus with a large ground state spin. Ground state spins of 9/2 in the deformed regions are available. Perhaps the most advantageous nucleus from this point of view is ^{176}Lu which has 7 as its ground state spin.

If a density in the intrinsic state $\rho_d(x')$ is assumed, the one-body density of the state Ψ_{IMK} can be numerically computed from $[(2I+1)/4\pi]\int|d^I_{M,K}(\beta)|^2\sin\beta d\beta d\gamma\rho_d(x',\beta,\gamma)$.

To conclude, we find that in order to study the role of deformation in high energy heavy ion collisions it will be judicious to choose nuclei with high spin in the ground state. By choosing $|II\rangle$ states where the z axis is the collision axis one can enhance tip-on-tip collisions. By choosing $|I1/2\rangle$ states one can enhance body-on-body collisions. Quantum mechanics will prevent a perfect alignment.

We are thankful to E. Shuryak for emphasizing to us the many-body correlation aspects of relativistic heavy ion collisions, and to J. Barrette, C. M. Ko, and C. Pruneau for useful exchanges. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada and in part by the Fonds FCAR of the Québec Government.

[1] E. Shuryak, Phys. Rev. C **61**, 034905 (2000).
[2] Bao-An Li, Phys. Rev. C **61**, 021903 (2000).
[3] A. Bohr and B. R. Mottelson, *Nuclear Structure, Vol. II* (Benjamin, Reading, MA, 1975).
[4] M. E. Rose, *Elementary Theory of Angular Momentum* (Wiley, New York, 1957).