Two-loop contribution to high mass dilepton production by a quark-gluon plasma

J. I. Kapusta* and S. M. H. Wong†
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
(Received 20 March 2000; published 19 July 2000)

We calculate the order α_s finite temperature correction to dilepton production in quark-gluon plasma arising from the two-loop photon self-energy diagrams for high invariant mass $M \gg T$.

PACS number(s): 12.38.Mh, 11.10.Wx, 13.85.Qk, 25.75.Dw

Very early in the history of quark-gluon plasma studies it was realized that quark-antiquark annihilation into lepton pairs could provide information on the highest temperatures achieved [1–5]. An example of this would be chiral symmetry in the hot plasma [6]. Soon after, the thermal rates were used [7] in conjunction with the Bjorken model [8] for the dynamical evolution of the hot matter produced in heavy ion collisions, and by now this has become an industry. In this Brief Report we shall examine the effect of the order α_s correction to the thermal rate in the high invariant mass limit.

In the rest frame of the thermal system the rate of production of lepton pairs is given by the formula [9–13]

$$E_+E_-\frac{dR}{d^3p_+d^3p_-}=\frac{2}{(2\pi)^6}\frac{e^2}{M^4}(p_+^\mu p_-^\nu+p_+^\nu p_-^\mu -g^{\mu\nu}p_+\cdot p_-)\frac{\mathrm{Im}\,\Pi_{\mu\nu}(k)}{e^{E/T}-1}. \qquad (1)$$

Here $k=p_++p_-$ is the momentum of the virtual photon expressed as the sum of the positive and negative lepton momenta $E=k^0$ and $k^2=M^2$. The imaginary part of the retarded photon self-energy is labeled $\mathrm{Im}\,\Pi^{\mu\nu}$. In the limit of interest here, namely $M\gg T\gg|\mathbf{k}|$, the photon self-energy does not distinquish between longitudinal and transverse polarizations, and may be written as

$$\Pi^{\mu\nu}=\left(\frac{k^\mu k^\nu}{k^2}-g^{\mu\nu}\right)\Pi. \qquad (2)$$

When the lepton mass is small in comparison to M the rate then simplifies to

$$E_+E_-\frac{dR}{d^3p_+d^3p_-}=-\frac{2}{(2\pi)^6}\frac{e^2}{M^2}\frac{\mathrm{Im}\,\Pi}{e^{M/T}-1}. \qquad (3)$$

This expression is valid to lowest order in α and to all orders in the strong interactions.

The lowest order contribution to $\mathrm{Im}\,\Pi$ arises from the one loop self-energy with a light quark circulating in the loop. The vacuum piece is

$$\mathrm{Im}\,\Pi_{\text{one-loop}}=-\frac{e_q^2}{4\pi}M^2, \qquad (4)$$

where e_q is the electric charge of the quark. The finite temperature piece is suppressed by $e^{-M/2T}\ll 1$ and may be dropped. For three light flavors the resulting rate is

$$E_+E_-\frac{dR}{d^3p_+d^3p_-}=\frac{\alpha^2}{12\pi^5}e^{-M/T}. \qquad (5)$$

This is the same as one gets by using kinetic theory to calculate the thermal rate for the reaction $q+\bar{q}\to l^++l^-$, of course. An order α_s correction arises from the two-loop photon self-energy diagrams displayed in Fig. 1. Using the techniques developed in Ref. [14] to open up the loops of these diagrams corresponds to a number of processes. Some of these processes just modify the vacuum self-energy, such as an interference term between the tree vertex for $q\bar{q}\gamma$ and a one-loop modified vertex. Such processes simply modify the rate given above by multiplying by a factor $1+\mathrm{order}(\alpha_s)$ and are not given here. Other processes explicitly involve finite temperature many-body effects. Among them are reactions such as $q+g\to q+l^++l^-$ and $q+\bar{q}\to g+l^++l^-$. Also among them are some that involve interference between the tree level reaction, $q+\bar{q}\to l^++l^-$ together with a spectator quark or gluon, and a three-body initial state involving $q+\bar{q}+g$ or $q+q+\bar{q}$ or $q+\bar{q}+\bar{q}$. These are displayed explicitly in Fig. 2. All such finite temperature contributions were computed by us in the process of determining the shift in mass and width of the Z boson in quark-gluon plasma [15]. Those results may be taken over directly by setting the quark axial vector coupling constant g_A to zero and the quark vector coupling constant g_V to the electromagnetic one e_q

$$\mathrm{Im}\,\Pi_{\text{two-loop}}=-\frac{10}{9}\alpha_s e_q^2T^2\ln\left(\frac{M}{1.05\alpha_sT}\right). \qquad (6)$$

The proportionality to T^2 is natural given that the only other relevant scale is M. It cannot be stressed enough how impor-

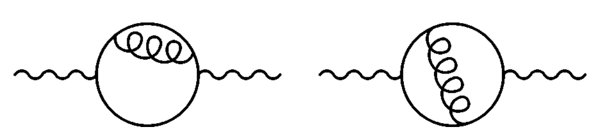

FIG. 1. Two-loop contributions to the photon self-energy due to QCD interactions.

*Electronic address: kapusta@physics.spa.umn.edu
†Electronic address: swong@nucth1.hep.umn.edu

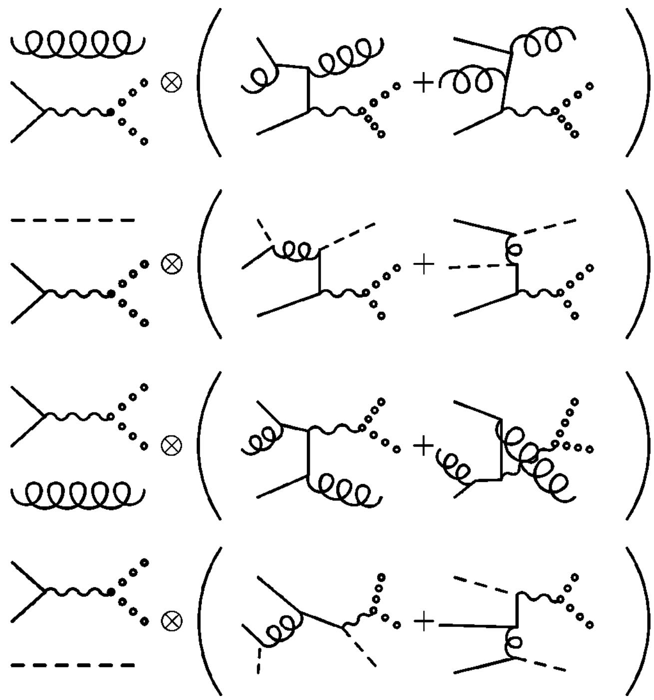

FIG. 2. Opening up the two-loop diagrams of Fig. 1 leads to various reactions, including the interference contributions shown here. The produced dilepton pair has been added and shown with large dotted lines. The quark line that originates from a loop within a single amplitude has been drawn with dashed lines.

tant it is to include the interference diagrams involving a spectator quark or gluon from the plasma. Without them the thermal rate would be infrared power divergent.

The relative contribution of the two-loop finite temperature contribution is just the ratio of the imaginary parts of Π

$$\frac{\mathrm{Im}\,\Pi_{\text{two-loop}}}{\mathrm{Im}\,\Pi_{\text{one-loop}}}=\frac{40\pi}{9}\alpha_s\frac{T^2}{M^2}\ln\left(\frac{M}{1.05\alpha_s T}\right). \qquad (7)$$

Since the correction arises from thermal quarks and gluons, the strong coupling should be evaluated at the thermal scale. The one-loop beta function gives rise to the running coupling (with three light flavors)

$$\alpha_s(T)=\frac{6\pi}{27\ln(T/50\ \ \mathrm{MeV})}. \qquad (8)$$

The argument of the logarithm is chosen such that α_s takes the values 0.5 and 0.25 at temperatures of 200 and 1000 MeV, respectively. The former temperature is comparable to or slightly above the minimum temperature to form quark-gluon plasma. The highest temperature expected at the RHIC is about 500 MeV, while the highest expected at the LHC is about 1 GeV. The two-loop contribution is subordinate to the one-loop result when M exceeds T by 2 or 3 times, so generally this means that M must exceed 2 or 3 GeV for higher order terms to be negligible. Remember that the result obtained here assumes that $M>T$ so that it cannot be extrapolated to small M.

As long as the pair momentum $\mathbf{k}$ is small compared to M the above calculation should apply. Extension to larger values of the momentum may easily be done by using the techniques developed in Ref. [14]. That may be of practical importance for a reliable description of the background to J/ψ production, absorption, and screening in the plasma.

This work was supported by the U.S. Department of Energy under Grant No. DE-FG02-87ER40328.

[1] E.L. Feinberg, Nuovo Cimento A **34**, 391 (1976).
[2] E.V. Shuryak, Phys. Lett. **78B**, 150 (1978).
[3] G. Domokos and J.I. Goldman, Phys. Rev. D **23**, 203 (1981); G. Domokos, *ibid.* **28**, 123 (1983).
[4] K. Kajantie and H.I. Miettenen, Z. Phys. C **9**, 341 (1981); **14**, 357 (1982).
[5] S. Chin, Phys. Lett. **119B**, 51 (1982).
[6] J. Kapusta, Phys. Lett. **136B**, 201 (1984).
[7] K. Kajantie, J. Kapusta, L. McLerran, and A. Mekjian, Phys. Rev. D **34**, 2746 (1986).
[8] J.D. Bjorken, Phys. Rev. D **27**, 140 (1983).
[9] L.D. McLerran and T. Toimela, Phys. Rev. D **31**, 545 (1985).
[10] H.A. Weldon, Phys. Rev. D **42**, 2384 (1990).
[11] C. Gale and J.I. Kapusta, Nucl. Phys. **B357**, 65 (1991).
[12] E. Braaten, R.D. Pisarski, and T.C. Yuan, Phys. Rev. Lett. **64**, 2242 (1990).
[13] S.M.H. Wong, Z. Phys. C **53**, 465 (1992).
[14] S.M.H. Wong and J.I. Kapusta, Report No. NUC-MINN-00/08-T (in preparation).
[15] J.I. Kapusta and S.M.H. Wong, Phys. Rev. D **62**, 037301 (2000).