Thermodynamics, strange quark matter, and strange stars

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Upon application of both the general ensemble theory and basic thermodynamical principles, we derive in detail the thermodynamics of strange matter with density-dependent particle masses, which resolves the problem of inconsistencies in the thermodynamical properties of the earlier approaches. We then recalculate the properties of strange quark matter with this new thermodynamical treatment and our recently determined quark mass scaling, and find that the density behavior of the sound velocity is opposite to the previous finding, but consistent with one of our recent publications. The structure equations for strange stars are integrated with the presently obtained equation of state. We find that the mass-radius relation is similar to previous results except the maximum mass is smaller in our case if strange quark matter is absolutely stable.

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I. INTRODUCTION

During the ten plus years which have elapsed since Witten's conjecture [1] that strange quark matter (SQM), rather than the normal nuclear matter, might be the true ground state of quantum chromodynamics (QCD), much theoretical and observational effort has been made on the investigation of its properties and potential astrophysical significance [2]. Because of the well-known difficulty of QCD in the nonperturbative domain, phenomenological models reflecting the characteristics of the strong interaction are widely used in the study of hadrons, and many of them have been successfully applied to investigating the stability and properties of SQM. One of the most famous models is the MIT bag model with which Jaffe *et al.* [3] find that SQM is absolutely stable around the normal nuclear density for a wide range of parameters. A vast number of further investigations [4–7] are performed with fruitful results. A recent important result is that young millisecond pulsars are more likely to be strange stars rather than neutron stars [8]. Another alternative model is the mass-density-dependent model with which Chakrabarty *et al*. obtained significantly different results [9,10]. However, Benvenuto and Lugones [11] pointed out that it is caused by the wrong thermodynamical treatment. They added an extra term to the expression of both pressure and energy, and got similar results to those in the bag model. A recent investigation indicates a link of SQM to the study of quark condensates [12] while a more recent work has carefully studied the relation between the charge and critical density of SQM [13].

Lately, we have demonstrated that the previous treatments have unreasonable vacuum limits [14]. In addition to this problem, there exists another serious problem, i.e., the zero pressure does not appear in the lowest-energy state. In fact, there are two important problems in the quark mass-density-dependent model. One is how to determine the quark mass scaling. The other is how to treat the thermodynamics with density-dependent particle masses self-consistently. We have mainly concentrated on the first problem in Ref. [14]. The present paper will concentrate more on the second problem. We find that the extra term provided in Ref. [11] should indeed be appended to the expression of pressure. However, it should not appear in that of energy according to both the general ensemble theory and basic thermodynamical principles. After our modification, the zero pressure point appears exactly at the lowest-energy state, and thus the thermodynamics with density-dependent particle masses becomes self-consistent, which leads to completely different density behavior of the sound velocity in SQM and different structure of strange stars.

We organize this paper as follows. In the subsequent section, we give detailed arguments on why the additional term in the pressure should not appear in the energy. The thermodynamical expressions needed later are all derived carefully in this section. Then in Sec. III, we apply the new thermodynamical formulas and our recently determined quark mass scaling to investigating the properties of SQM. We find that the density behavior of the sound velocity is opposite to the previous result [11], but consistent with one of our recent publications [14]. On application of the present equation of state, we integrate the equations of stellar structure for strange stars in Sec. IV, which indicates that the structure of strange stars is similar to previous results. However, the maximum quark star mass is smaller in our case if SQM is absolutely stable. Section V is a short summary.

II. THERMODYNAMICS OF SYSTEMS WITH DENSITY-DEPENDENT PARTICLE MASSES

Let us explore directly from the general ensemble theory what the expression of pressure and energy should look as if the particle masses are dependent on density. We express the density matrix as

$$\rho = \frac{1}{\Xi} e^{-\beta(E_{N_i,\alpha} - \Sigma_i \mu_i N_i)}, \tag{1}$$

where Ξ is the partition function, β is the reverse temperature, N_i are the particle numbers, and μ_i are the correspond-

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ing chemical potentials. The microscopic energy $E_{N_i,\alpha}$ is a function of the system volume V, the particle masses m_i, the particle numbers N_i, and the other quantum numbers α. The pressure of the system is

$$P=\frac{1}{\Xi}\sum_{\{N_i\},\alpha}\left(-\frac{\partial E_{N_i,\alpha}}{\partial V}\right)e^{-\beta(E_{N_i,\alpha}-\Sigma_i\mu_iN_i)}$$
$$=\frac{1}{\Xi}\sum_{\{N_i\},\alpha}\left[\frac{1}{\beta}\frac{\partial}{\partial V}e^{-\beta(E_{N_i,\alpha}-\Sigma_i\mu_iN_i)}\right]$$
$$=\frac{1}{\beta}\frac{\partial \ln\Xi}{\partial V}=-\frac{\partial(V\Omega)}{\partial V}, \quad (2)$$

where

$$\Omega\equiv-\frac{1}{V\beta}\ln\Xi \quad (3)$$

is the thermodynamical potential density which is generally a function of the temperature T, the chemical potentials μ_i, and the particle masses m_i. If the particle masses have nothing to do with the baryon number density $n_b=N/(3V)$ (N is the total particle number), we simply get

$$P=-\Omega. \quad (4)$$

If the masses depend on density or volume, one should have

$$P=-\Omega+n_b\frac{\partial\Omega}{\partial n_b}. \quad (5)$$

This is just the right thing that has been done in Ref. [11] where the derivation is

$$P=-\left.\frac{\partial(\Omega/n_b)}{\partial(1/n_b)}\right|_{T,\mu_i}=n_b\frac{\partial\Omega}{\partial n_b}-\Omega. \quad (6)$$

For canonical ensemble, the particle numbers N_i remain fixed. Thus, this derivation is obvious. However, it is not so obvious for grand canonical ensemble because the particle number is not necessarily constant when the temperature T and chemical potentials μ_i are unchanged. We will give a more convincing derivation a little later.

The additional term is of crucial importance for pressure balance. In Ref. [11], however, the extra term was incorrectly appended to the expression of energy. Now, let us calculate the statistic average for the energy:

$$\overline{E}=\frac{1}{\Xi}\sum_{\{N_i\},\alpha}E_{N_i,\alpha}e^{-\beta(E_{N_i,\alpha}-\Sigma_i\mu_iN_i)}$$
$$=\frac{1}{\Xi}\sum_{\{N_i\},\alpha}\left(-\frac{\partial}{\partial\beta}+\sum_i\mu_iN_i\right)e^{-\beta(E_{N_i,\alpha}-\Sigma_i\mu_iN_i)}$$
$$=-\frac{\partial}{\partial\beta}\ln\Xi+\sum_i\mu_i\overline{N}_i, \quad (7)$$

where

$$\overline{N}_i=\frac{1}{\Xi}\sum_{\{N_i\},\alpha}N_ie^{-\beta(E_{N_i,\alpha}-\Sigma_i\mu_iN_i)}$$
$$=\frac{1}{\Xi}\sum_{\{N_i\},\alpha}\left[\frac{1}{\beta}\frac{\partial}{\partial\mu_i}e^{-\beta(E_{N_i,\alpha}-\Sigma_i\mu_iN_i)}\right]_{V,T,\{m_k\}}$$
$$=\frac{1}{\beta}\frac{\partial}{\partial\mu_i}\ln\Xi\bigg|_{V,T,\{m_k\}}=-V\frac{\partial\Omega}{\partial\mu_i}\bigg|_{T,\{m_k\}} \quad (8)$$

is the average number for particle type i. Therefore, the energy density of the system is

$$E=\frac{\overline{E}}{V}=\frac{\partial(\beta\Omega)}{\partial\beta}+\sum_i\mu_in_i \quad (9)$$

$$=\Omega+\beta\frac{\partial\Omega}{\partial\beta}+\sum_i\mu_in_i \quad (10)$$

$$=\Omega+\sum_i\mu_in_i-T\frac{\partial\Omega}{\partial T}, \quad (11)$$

where n_i is the number density of particle type i:

$$n_i\equiv\frac{\overline{N}_i}{V}=-\frac{\partial\Omega}{\partial\mu_i}\bigg|_{T,\{m_k\}}. \quad (12)$$

It is clear from Eq. (11) that only when Eq. (4) holds can one get Eq. (8) in Ref. [11]. Therefore, we should not, as done in Ref. [11], use that expression to calculate the energy density. Instead, we will calculate E directly from Eq. (11) in this paper.

For more evident arguments, let us see the following derivation starting from the basic derivative relation for an open system:

$$d(VE)=Td(VS)-PdV+\sum_i\mu_id\overline{N}_i, \quad (13)$$

where S is the entropy density of the system. Choosing T, V, and $\{\overline{N}_i\}$ as the independent macroscopic state variables, the combined statement of the first and second laws of thermodynamics, Eq. (13), can be expressed as

$$d(VA)=-VSdT-PdV+\sum_i\mu_id\overline{N}_i, \quad (14)$$

where $A\equiv E-TS$ is the Helmholtz free energy density. Then we have

$$P=-\left.\frac{d(VA)}{dV}\right|_{T,\{\bar{N}_i\}} \tag{15}$$

$$=-A-V\left.\frac{dA}{dV}\right|_{T,\{\bar{N}_i\}} \tag{16}$$

$$=-A+\sum_j n_j\left.\frac{dA(T,\{n_i\})}{dn_j}\right|_T. \tag{17}$$

This is a general expression for pressure. In obtaining the third equality, we have used the chain relation

$$-V\frac{d}{dV}f\left(\left\{n_i=\frac{\bar{N}_i}{V}\right\}\right)\bigg|_{\{\bar{N}_i\}}=\sum_j n_j\frac{d}{dn_j}f(\{n_i\}), \tag{18}$$

where f is an arbitrary function.

According to the basic relation between thermodynamics and statistics, we have

$$A=\Omega+\sum_i \mu_i n_i, \tag{19}$$

where Ω is the thermodynamical potential density. For a free Fermi system, it is

$$\Omega=-\sum_i \frac{g_iT}{2\pi^2}\int_0^\infty \ln[1+e^{-\beta(\sqrt{p^2+m_i^2}-\mu_i)}]p^2dp \tag{20}$$

$$\equiv\sum_i \Omega_i(T,\mu_i,m_i), \tag{21}$$

where g_i is the degeneracy factor which is 6 for quarks and 2 for electrons. In order to include the interaction between particles, we regard the particle masses as density-dependent, namely,

$$m_i=m_i\left(n_b\equiv\sum_j n_j\bigg/3\right). \tag{22}$$

Because we have chosen T, V, and $\{\bar{N}_i\}$ as independent state variables, the chemical potential μ_i should also be regarded as a function of T and $\{n_k\}$, namely,

$$\mu_i=\mu_i(T,\{n_k\}). \tag{23}$$

So, the total derivative of $\Omega(T,\{\mu_k\},\{m_k\})$ with respect to n_j should be taken as

$$\left.\frac{d\Omega}{dn_j}\right|_T=\sum_i \left.\frac{\partial\Omega}{\partial\mu_i}\right|_{T,\{m_k\}}\left.\frac{d\mu_i}{dn_j}\right|_T+\left.\frac{\partial\Omega}{\partial n_b}\right|_{T,\{\mu_k\}}\frac{\partial n_b}{\partial n_j} \tag{24}$$

$$=-\sum_i n_i\left.\frac{d\mu_i}{dn_j}\right|_T+\frac{1}{3}\left.\frac{\partial\Omega}{\partial n_b}\right|_{T,\{\mu_k\}}. \tag{25}$$

Here we have used Eq. (12) and the fact that $\partial n_b/\partial n_j=1/3$.

Substituting Eq. (19) into Eq. (17) gives

$$P=-A+\sum_j n_j\frac{d}{dn_j}\left[\Omega+\sum_i \mu_i n_i\right]_T \tag{26}$$

$$=-A+\sum_j n_j\left[\left.\frac{d\Omega}{dn_j}\right|_T+\sum_i\left(n_i\left.\frac{d\mu_i}{dn_j}\right|_T+\mu_i\frac{dn_i}{dn_j}\right)\right] \tag{27}$$

$$=-A+\sum_i \mu_i n_i+\sum_j \frac{n_j}{3}\left.\frac{\partial\Omega}{\partial n_b}\right|_{T,\{\mu_k\}} \tag{28}$$

$$=-\Omega+n_b\left.\frac{\partial\Omega}{\partial n_b}\right|_{T,\{\mu_k\}} \tag{29}$$

$$=\sum_i\left(-\Omega_i+n_b\frac{\partial m_i}{\partial n_b}\frac{\partial\Omega_i}{\partial m_i}\right). \tag{30}$$

At zero temperature, the corresponding thermodynamical potential density can be obtained from Eq. (20) by carrying out the resulting integration in the limit of $T\to 0$:

$$\Omega=-\sum_i \frac{g_i}{48\pi^2}\bigg[\mu_i(\mu_i^2-m_i^2)^{1/2}(2\mu_i^2-5m_i^2)+3m_i^4\ln\frac{\mu_i+\sqrt{\mu_i^2-m_i^2}}{m_i}\bigg]. \tag{31}$$

We thus have, from Eqs. (12), (11), and (30),

$$n_i=\frac{g_i}{6\pi^2}(\mu_i^2-m_i^2)^{3/2}, \tag{32}$$

$$E=\sum_i m_in_iF(x_i), \tag{33}$$

$$P=\sum_i m_in_ix_i^2G(x_i)-\sum_i m_in_if(x_i), \tag{34}$$

where

$$x_i\equiv\frac{p_{f,i}}{m_i}\equiv\frac{\left(\frac{6\pi^2}{g_i}n_i\right)^{1/3}}{m_i}=\frac{\sqrt{\mu_i^2-m_i^2}}{m_i} \tag{35}$$

is the ratio of the Fermi momentum $p_{f,i}$ to the mass that related to particle type i. With the hyperbolic sine function $\sinh^{-1}(x)\equiv\ln(x+\sqrt{x^2+1})$, the functions $F(x_i)$, $G(x_i)$, and $f(x_i)$ are defined as

$$F(x_i)\equiv\tfrac{3}{8}[x_i\sqrt{x_i^2+1}(2x_i^2+1)-\sinh^{-1}(x_i)]/x_i^3, \tag{36}$$

$$G(x_i)\equiv\tfrac{1}{8}[x_i\sqrt{x_i^2+1}(2x_i^2-3)+3\sinh^{-1}(x_i)]/x_i^5, \tag{37}$$

$$f(x_i)\equiv-\frac{3}{2}\frac{n_b}{m_i}\frac{dm_i}{dn_b}[x_i\sqrt{x_i^2+1}-\sinh^{-1}(x_i)]/x_i^3. \tag{38}$$

One can see, from Eqs. (33) and (34), that an additional term appears in the pressure expression, but not in the energy expression. We can specially confirm this result further as such.

From Eq. (13), one has an alternative general expression for pressure

$$P=-\left.\frac{d(VE)}{dV}\right|_{S,\{\bar{N}_k\}} \tag{39}$$

$$=-E+\sum_j n_j \left.\frac{dE}{dn_j}\right|_S . \tag{40}$$

According to the Pauli principle and the relativistic energy-momentum relation $\varepsilon_i=\sqrt{p^2+m_i^2}$, the energy density of the system at zero temperature should be

$$E(\{n_i\},\{m_j(n_b)\})=\sum_i \frac{g_i}{2\pi^2}\int_0^{p_{f,i}} \varepsilon_i p^2 dp, \tag{41}$$

which, after the integration is carried out, is just the same as Eq. (33). Because the entropy is also zero (or constant) at zero temperature, we can substitute Eq. (41) into Eq. (40), and accordingly get

$$P=-E+\sum_j n_j\left(\frac{\partial E}{\partial n_j}+\sum_i \frac{\partial E}{\partial m_i}\frac{\partial m_i}{\partial n_b}\frac{\partial n_b}{\partial n_j}\right) \tag{42}$$

$$=-E+\sum_j n_j\frac{\partial E}{\partial n_j}+\sum_i\sum_j \frac{n_j}{3}\frac{\partial E}{\partial m_i}\frac{\partial m_i}{\partial n_b} \tag{43}$$

$$=-\Omega+\sum_i n_b \frac{\partial m_i}{\partial n_b}\frac{\partial E}{\partial m_i}, \tag{44}$$

which leads to Eq. (34) exactly.

III. PROPERTIES OF STRANGE QUARK MATTER IN THE NEW THERMODYNAMICAL TREATMENT

Having derived in detail the thermodynamics with variable particle masses in the previous section, we now apply it to the investigation of SQM. As is usually done in the literature [3,9–14], we assume the SQM to be a Fermi gas mixture of u,d,s quarks and electrons with chemical equilibrium maintained by the weak interactions

$$d,s\leftrightarrow u+e+\bar{\nu}_e, \quad s+u\leftrightarrow u+d, \ldots .$$

Because of these reactions, the chemical potentials μ_i $(i=u,d,s,e)$ should satisfy

$$\mu_d=\mu_s\equiv\mu, \tag{45}$$

$$\mu_u+\mu_e=\mu. \tag{46}$$

For the bulk SQM in weak equilibrium, the previous investigations got a slightly positive charge [3]. Our recent study [13] demonstrates that negative charges could lower the critical density. However, too much negative charge can make it impossible to maintain flavor equilibrium. Thus the charge of SQM is not allowed to shift too far away from zero at both positive and negative directions. Therefore, one also has two additional equations for a given baryon number density n_b:

$$\tfrac{1}{3}(n_u+n_d+n_s)=n_b, \tag{47}$$

$$\tfrac{2}{3}n_u-\tfrac{1}{3}n_d-\tfrac{1}{3}n_s-n_e=0. \tag{48}$$

The first is the definition of the baryon number density, the second is from the charge neutrality requirement, and n_i $(i=u,d,s,e)$ is related to μ_i and m_i by Eq. (32).

Because the results from lattice calculations [15] show that quark matter does not become asymptotically free soon after the phase transition (instead, it approaches the free gas equation of state very slowly), one should consider the strong interaction between quarks in a proper way. We do this by including the interaction effect within the variable quark masses. Because of the characteristics of the strong interaction (quark confinement and asymptotic freedom), one can write down the simplest and most symmetric parametrization for the quark masses m_q $(q=u,d,s)$ [14]:

$$m_q=m_{q0}+\frac{D}{n_b^z}, \tag{49}$$

where m_{q0} is the corresponding quark current mass, z is a fixed exponential. Previously, z was regarded as 1. Our recent study [14] indicates that it is more reasonable to take $z=1/3$. The parameter D is usually determined by stability arguments, i.e., at zero pressure $(P=0)$, the energy per baryon E/n_b is greater than 930 MeV for two flavor quark matter in order not to contradict standard nuclear physics, but less than 930 MeV for three flavor symmetric quark matter so that SQM can have the possibility of absolute stability. Obviously, the rang of D determined by this method depends on different thermodynamical treatments. Within the thermodynamics derived in the preceding section, D is in the range $(155–171\ \text{MeV})^2$ when taking $z=1/3$.

Because the light quark current masses are very small, their value uncertainties are not important. So we take the fixed central values $m_{u0}=5$ MeV and $m_{d0}=10$ MeV in our calculation. The electron mass is very small (0.511 MeV). As for s quarks, we take 80 and 90 MeV, corresponding, respectively, to $D^{1/2}=156$ and 160 MeV.

For a given n_b, we solve for μ_i $(i=u,d,s,e)$ from Eqs. (45)–(48), and calculate the energy density and pressure of SQM, respectively, from Eqs. (33) and (34) with the quark masses replaced by Eq. (49). First, we draw the configuration of the SQM for the parameter set $m_{s0}=80$ MeV and $D^{1/2}=156$ MeV in Fig. 1. At high densities, all of the u, d, and s quarks tend to become a triplicate. When the density becomes lower, d fraction increases while s fraction decreases, and becomes zero at a definite density which is called critical density in Ref. [13] because SQM cannot maintain chemical equilibrium below that density. The u fraction is nearly unchanged. It in fact increases very slowly. To keep charge

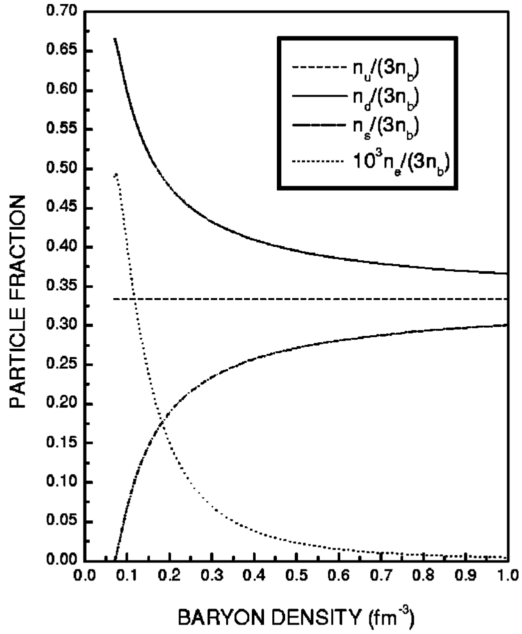

FIG. 1. The configuration of SQM varies with density. At high densities, all of the u, d, and s quarks tend to become a triplicate. When the density becomes lower, the d fraction increases while the s fraction decreases. The u fraction is nearly unchanged. It only increases very slowly.

neutrality, the electron fraction also increases. However, because of the electron's very small mass, the electron fraction is so little that we multiply it by 1000 to draw it in the figure.

In Fig. 2, we show the density dependence of the energy per baryon, $VE/N = E/n_b$, vs baryon number density n_b for the parameter set I: $m_{s0} = 80$ MeV, $D^{1/2} = 156$ MeV, and II: $m_{s0} = 90$ MeV, $D^{1/2} = 160$ MeV. For the first parameter set, SQM is absolutely stable while for the second set it is nearly metastable. The points marked with a circle are the zero pressure points where the system pressure becomes zero. It can be clearly seen that the zero pressure points are exactly located at the lowest-energy state. In fact, this is a basic requirement of thermodynamics because one can obtain from Eq. (39)

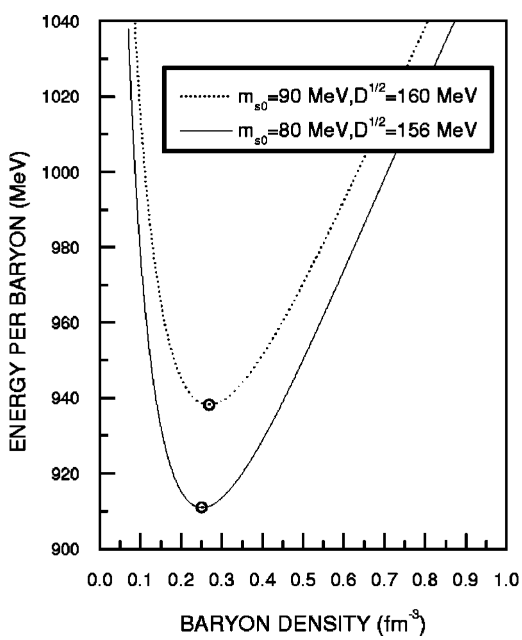

FIG. 2. Energy per baryon E/n_b vs baryon number density n_b for different parameter groups. The zero pressure points marked with a circle are located at the lowest-energy state, which is not the case for most of the previous thermodynamical treatments of SQM in the quark mass-density-dependent model.

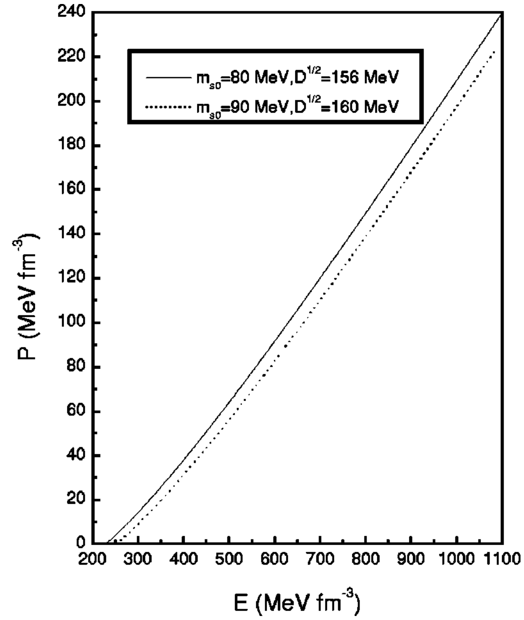

FIG. 3. Equation of state of strange quark matter (pressure P vs energy density E). It approaches the free gas equation of state at high densities. However, it is a little sunken at lower densities, contrary to the previous calculation.

$$P = -\frac{d(VE)}{dn_b}\frac{dn_b}{dV}\bigg|_{\{\bar{N}_k\}} = n_b^2\frac{d(E/n_b)}{dn_b}. \qquad (50)$$

However, this is not the case for most of the previous thermodynamical treatments of SQM in the mass-density-dependent model [11,9,10,12], which is another serious flaw in addition to the unreasonable vacuum limits mentioned before.

In Fig. 3, we give the relation between the pressure P and energy density E, i.e., the equation of state. It approaches the free gas equation of state at high densities. However, its shape is a little sunken at lower densities, contrary to the previous result [11] which is protuberant. This will lead to completely different lower density behavior of the sound velocity in SQM.

The velocity of sound is plotted in the lower part of Fig. 4. The upper part is calculated by the same method in Ref. [11] with parameter set B there. Simultaneously given with a full horizontal line is the ultrarelativistic case $(1/\sqrt{3})$ for purpose of comparison. Obviously, they become nearly identical at high densities while the lower density behavior is opposite. The sound velocity in the previous treatment is higher than the ultrarelativistic case and will eventually ex-

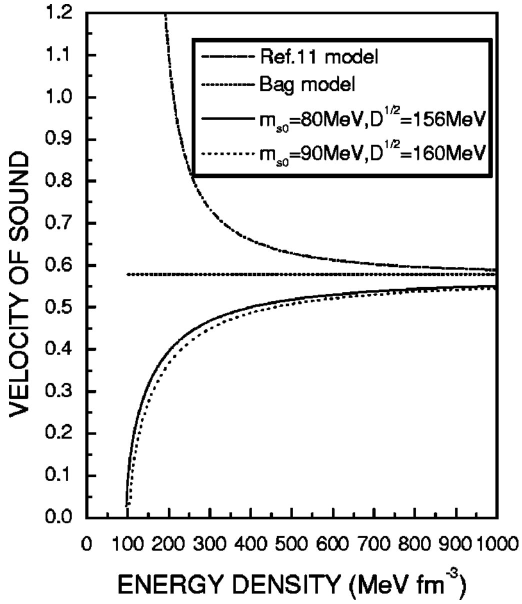

FIG. 4. Velocity of sound in strange quark matter. The solid horizontal line is the ultrarelativistic case. The lower half part is the results of our calculation while the upper part is calculated by the same method as in Ref. [11] for parameter set B there. Their lower density behavior is obviously opposite.

ceed the speed of light at lower densities, which is unreasonable from the point of view of the theory of relativity.

IV. STRUCTURE OF STRANGE STARS

It has long been proposed that some of the currently called neutron stars might be composed of strange quark matter and thus be in fact strange stars. A recent investigation shows that the newly discovered millisecond x-ray pulsar SAX J1808.4-3658 is a likely strange star candidate [16]. Previous authors have investigated the properties of strange stars by applying their obtained equation of state with interesting results [10,11]. We have now modified the thermodynamical treatment and updated the quark mass scaling. Therefore, it is meaningful to study the structure of strange stars in the new context from the astrophysical point of view.

As generally done, we assume the strange star to be a spherically symmetric object. Its stability is governed by the general relativistic equation of hydrostatic equilibrium known as the Tolman-Oppenheimer-Volkov equation [17]

$$\frac{dP}{dr} = -\frac{GmE}{r^2}\frac{(1+P/E)(1+4\pi r^3 P/m)}{1-2Gm/r}, \qquad (51)$$

with the subsidiary condition

$$dm/dr = 4\pi r^2 E, \qquad (52)$$

where $G = 6.707\times 10^{-45}$ MeV^{-2} is the gravitational constant, r is the distance from the core of the star, $E=E(r)$ is the energy density or mass density, $P=P(r)$ is the pressure, and $m=m(r)$ is the mass within the radius r.

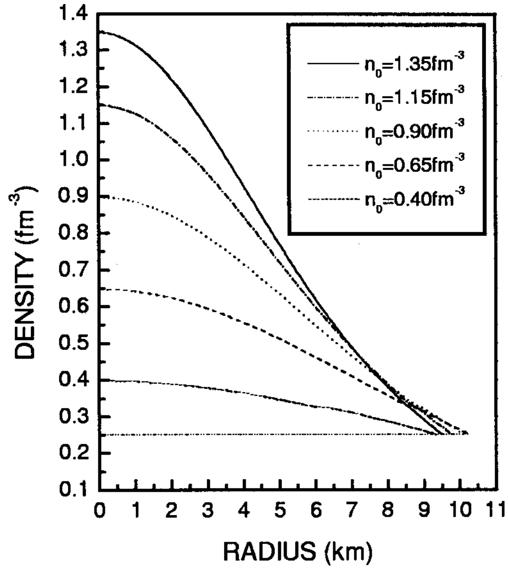

FIG. 5. Density profiles for the parameter set $m_{s0}=80$ MeV and $D=(156\ \text{MeV})^2$. The upmost line is for the largest acceptable central density $n_{0\max}$. The lowest horizontal line corresponds to the surface density of strange stars. The cross points of each line and the lowest horizontal line correspond to the radius R of the star.

For an initial baryon number density n_0 (accordingly P_0 and E_0), we can numerically solve Eqs. (51) and (52) with the aid of the equation of state, and obtain the corresponding $P=P(r,n_0)$ and $m=m(r,n_0)$, and consequently $n=n(r,n_0)$, the baryon number density at the radius r for the central density n_0. The radius R of the strange star is determined by the condition

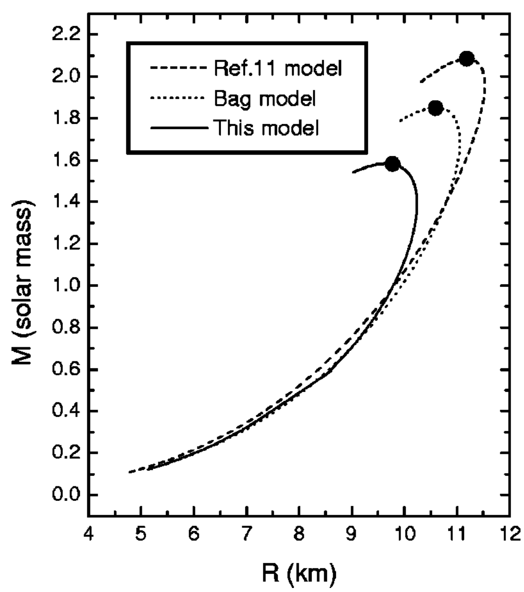

FIG. 6. The mass-radius relation for strange stars. The vertical axis is the star mass in unit of the solar mass while the horizontal axis is the star radius in units of kilometers. The solid line is obtained by the method in this paper. The dotted line is from the bag model. The dashed line is calculated with the same method as in Ref. [11] for the parameter set B there. The points marked with a full dot represent the maximum acceptable masses.

$$P(R,n_0)=0, \tag{53}$$

namely,

$$R=R(n_0). \tag{54}$$

Accordingly, the mass of the strange star is

$$M=m[R(n_0),n_0]\equiv M(n_0). \tag{55}$$

To make strange stars stable, we must require $dM/dn_0 >0$. For the above obtained equation of state, M first increases with n_0 up to a definite value $M_{\rm max}$ corresponding to the highest acceptable central density $n_{0\rm max}$. After that, M decreases with n_0, and the star becomes unstable.

For parameter set I, i.e., $m_{s0}=80$ MeV and $D =(156\ \text{MeV})^2$, we give the density profiles $n(r,n_0)$ in Fig. 5 as an example. The upmost line is for the largest acceptable central density $n_{0\rm max}$ ($\approx$1.35 fm^{-3}). The lowest horizontal line corresponds to the surface density n_s ($\approx$0.25 fm^{-3}) of strange stars which is independent of the central density, but a function of the equation of state. Each line will intersect with it. The cross points correspond to the radius R of the star. The maximum radius of the star appears in $n_0 \approx 0.65$ fm^{-3}.

In Fig. 6, we show the mass-radius relation of strange stars with a solid line. The point marked with a full dot represents the largest acceptable mass $M_{\rm max}$ ($\approx$1.58 times the solar mass). For comparison, we have also plotted the result from the bag model calculation with the bag constant $B^{1/4}=144$ MeV, and that in Ref. [11] with parameter set B there. We can see that the shapes of the three lines are similar to each other. However, the maximum quark star mass in our case is smaller than in previous calculations. Naturally, this observation depends on the parameters employed. If we choose a bigger m_{s0} and larger D, the case might be different. However, SQM would be less stable in that case.

V. SUMMARY

We have self-consistently derived the thermodynamics with density-dependent particle masses, which overcomes inconsistencies in the thermodynamical properties of the earlier approaches. We prove that an additional term should be appended to the expression of pressure, but it does not appear in that of energy. When applying the new thermodynamics and our recently determined quark mass scaling to the investigation of SQM, we find that the density behavior of the sound velocity is opposite to the previous calculation [11], but consistent with our recent publication [14]. With the presently obtained equation of state, we have numerically solved the structure equations for strange stars, and found a similar mass-radius relation to previous results, although the maximum quark star mass is a little smaller in our case.

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